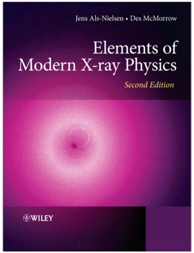
PHYS 570 - Introduction to Synchrotron Radiation

Term:	Spring 2026
Meetings:	Tuesday & Thursday 11:25-12:40
Location:	032 Rettaliata Engineering
Instructor:	Carlo Segre
Office:	166d/172 Pritzker Science
Phone:	312.567.3498
email:	segre@illinoistech.edu
Office Hours:	Tuesday 15:30-16:30 & Thursday 14:30-15:30 or by appointment.
Book:	*Elements of Modern X-Ray Physics, 2nd ed.*, J. Als-Nielsen and D. McMorrow (Wiley, 2011)
Web Site:	http://csrri.iit.edu/~segre/phys570/26S



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- Write a General User Proposal in the format used by the Advanced Photon Source

Course syllabus

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- Final project - writing a General User Proposal
  - Start thinking about a suitable project right away
  - Synchrotron technique must differ from journal article used in final presentation
  - Make proposal and get approval before starting

Optional activities

- Visits to Advanced Photon Source
  - All students who plan to attend will need to request badges from APS
  - Go to the APS User Portal and start the new user checklist: https://www.aps.anl.gov/Users-Information/Getting-Started/User-Checklist
  - Use MRCAT (Sector 10) as location of experiment
  - Use Carlo Segre as local contact
  - State that your beamtime will be in the second week of March
  - Schedule to be determined

Optional activities (cont.)

- Hands on data analysis training
  - GSAS for Rietveld refinement of powder diffraction data
    https://subversion.xray.aps.anl.gov/trac/pyGSAS
  - Demeter: XAS processing and analysis
    https://bruceravel.github.io/demeter/
  - Larch: Data analysis tools for x-ray spectroscopy
    https://xraypy.github.io/xraylarch/

Course grading

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Weekly or bi-weekly

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Grading scale

A	–	80%	to	100%
B	–	65%	to	80%
C	–	50%	to	65%
E	–	0%	to	50%

Topics to be covered (at a minimum)

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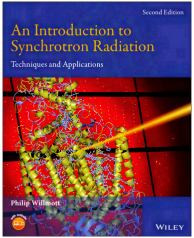

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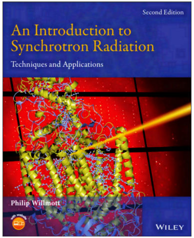

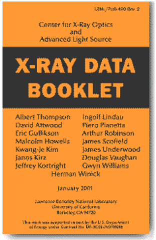

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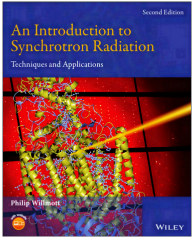

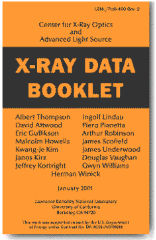

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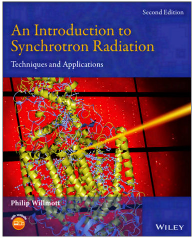

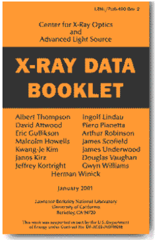

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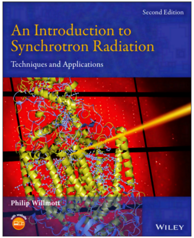

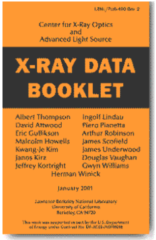

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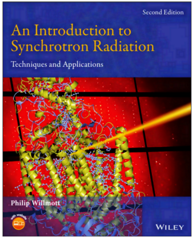

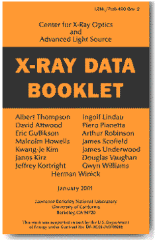

- X-ray Oriented Programs: https://www.aps.anl.gov/Science/Scientific-Software/XOP

Today's outline - January 13, 2026

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Reading Assignment: Chapter 1.1–1.6; 2.1–2.2

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The broad range of techniques make synchrotron x-ray sources to nearly any science or engineering field

The classical x-ray

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$$\begin{aligned}
\lambda &= hc/\mathcal{E} \\
&= (4.1357 \times 10^{-15}\,\text{eV} \cdot \text{s})(2.9979 \times 10^{8}\,\text{m/s})/\mathcal{E} \\
&= (4.1357 \times 10^{-18}\,\text{keV} \cdot \text{s})(2.9979 \times 10^{18}\,\text{Å/s})/\mathcal{E} \\
&= 12.398\,\text{Å} \cdot \text{keV}/\mathcal{E} \quad \text{to give units of Å}
\end{aligned}$$

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We will only discuss the first three.

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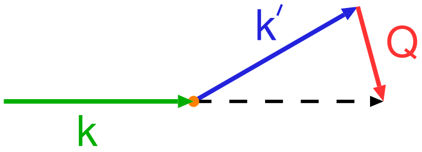

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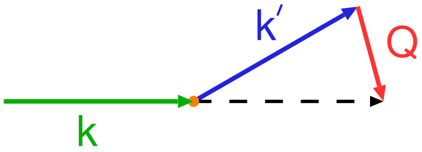

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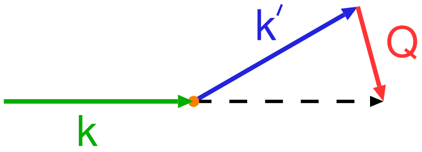

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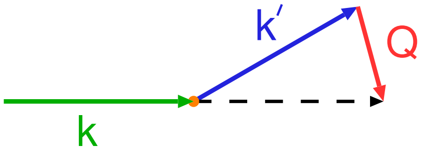

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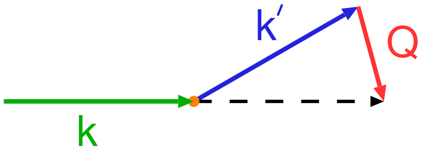

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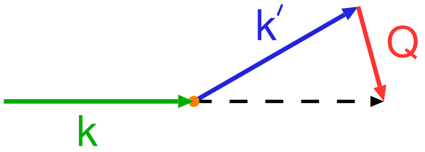

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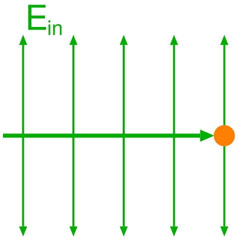
Start with the scattering from a single electron, then build up to more complexity

Thomson scattering

Assumptions:



Thomson scattering

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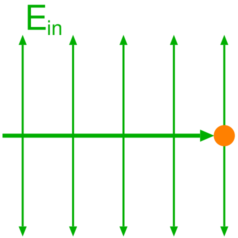

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Assumptions:

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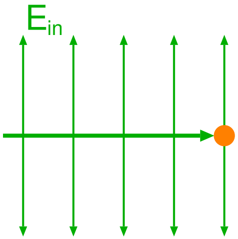

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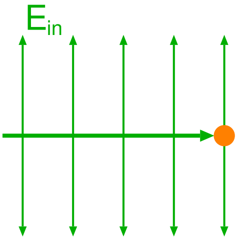

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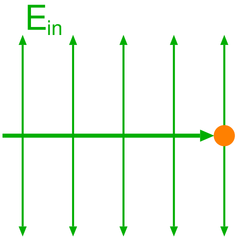

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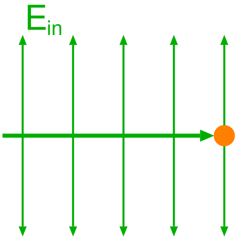

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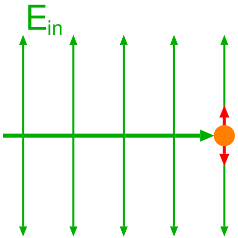

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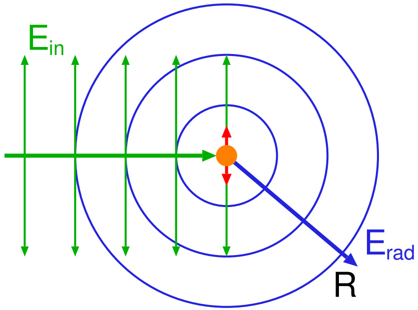

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The observer at R "sees" a scattered electric field $E_{rad}(R, t)$ at a later time $t = t' + R/c$

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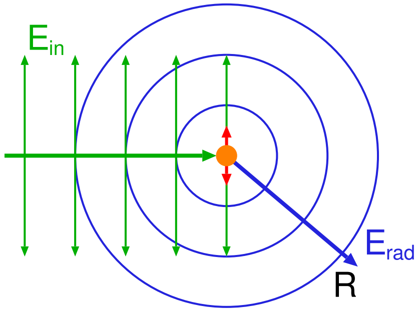


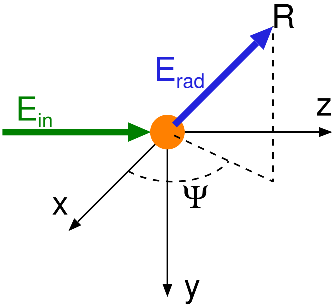
The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

The observer at R "sees" a scattered electric field $E_{rad}(R, t)$ at a later time $t = t' + R/c$

Using this, calculate the elastic scattering cross-section

Thomson scattering



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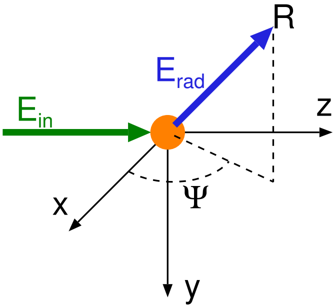

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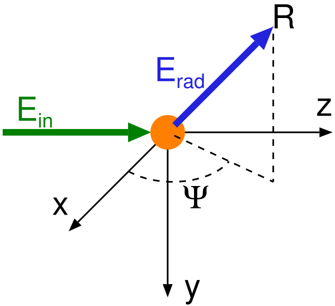

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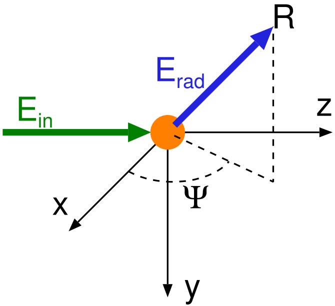

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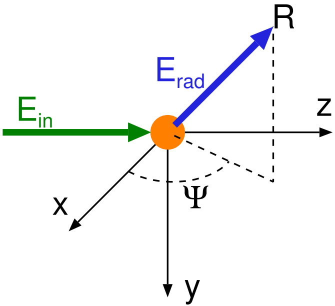

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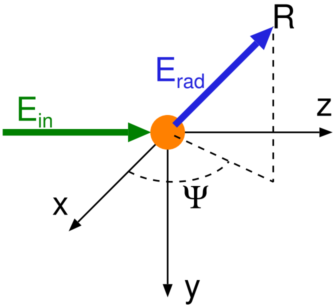

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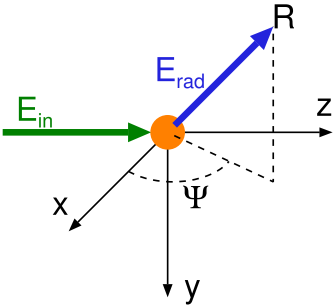

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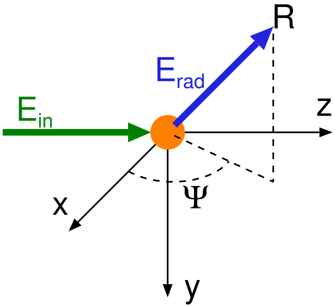

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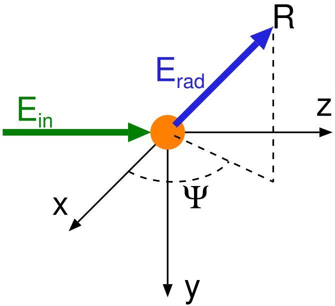

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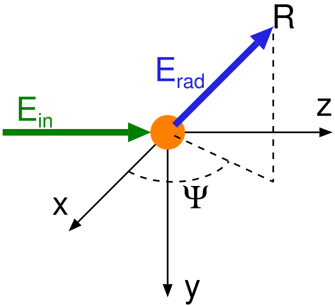

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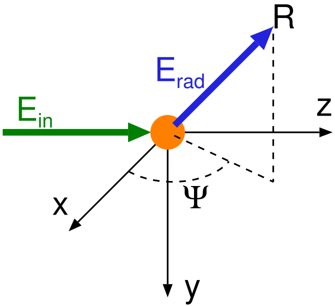

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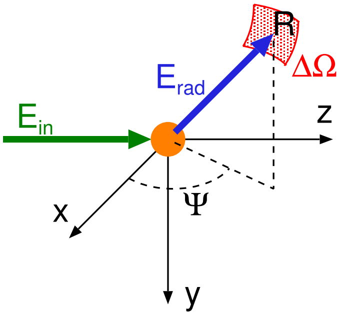
r_0 is called the Thomson scattering length or the “classical” radius of the electron

Scattering cross-section



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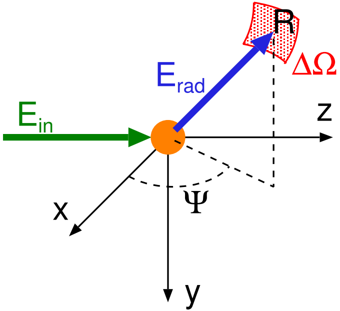

detector of solid angle $\Delta\Omega$ located a distance R from electron

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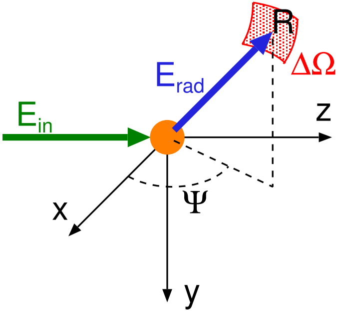

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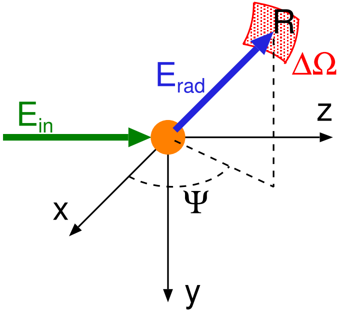

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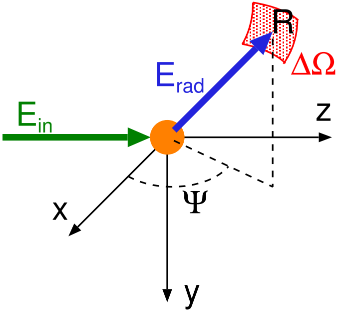

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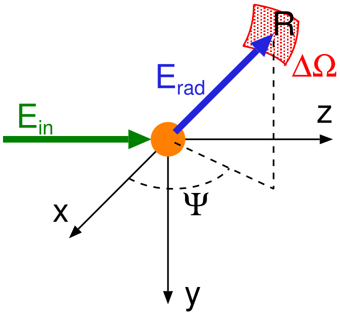

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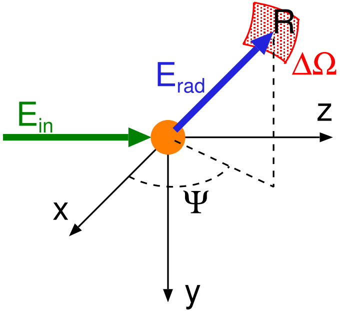

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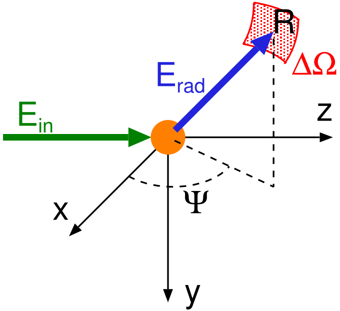

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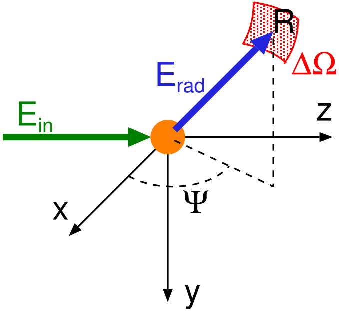

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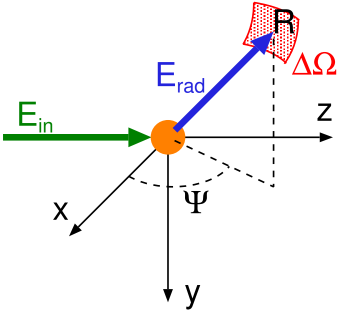

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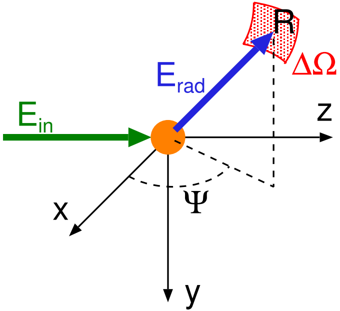

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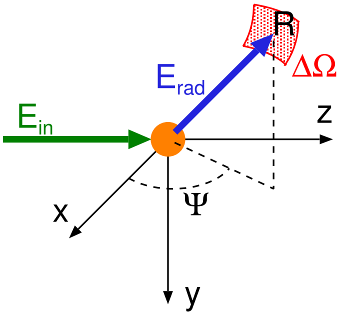

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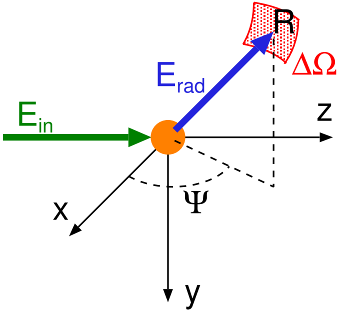

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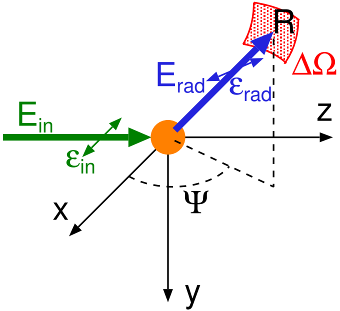

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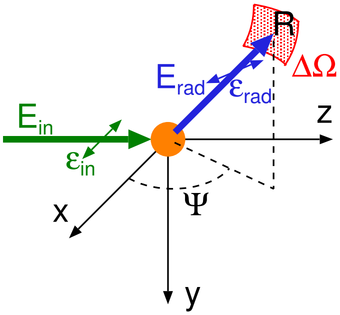

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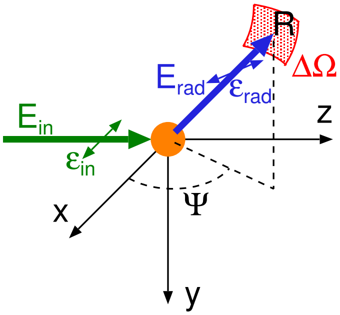

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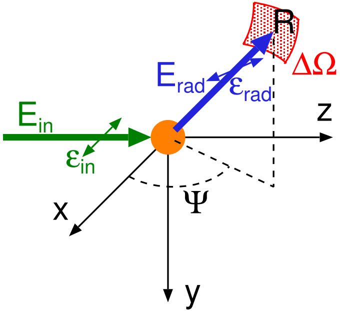

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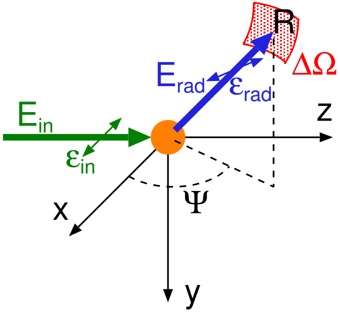

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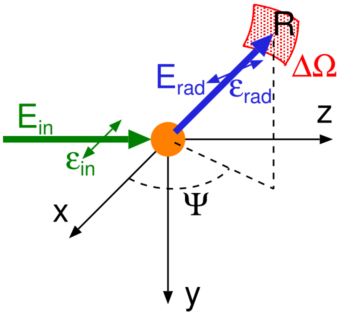

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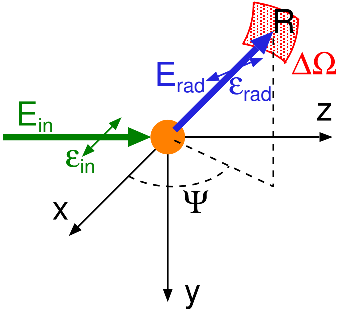

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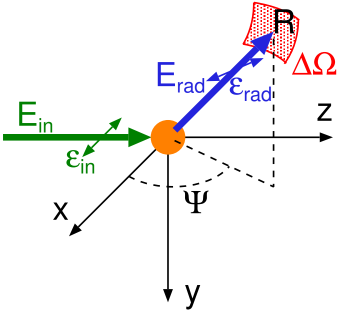

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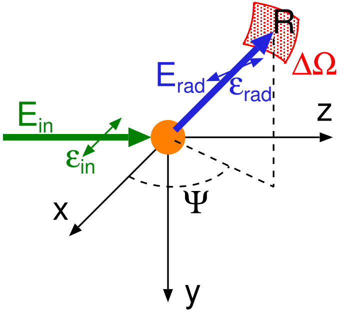

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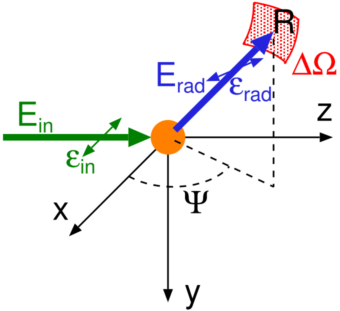

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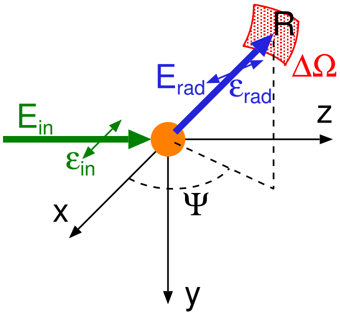

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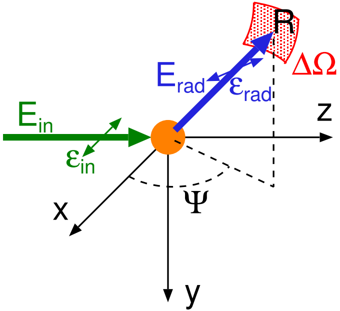

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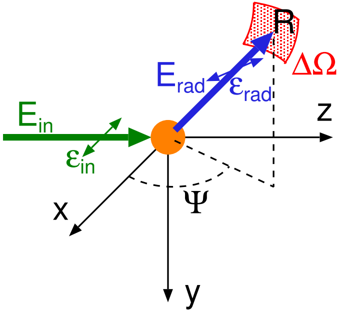

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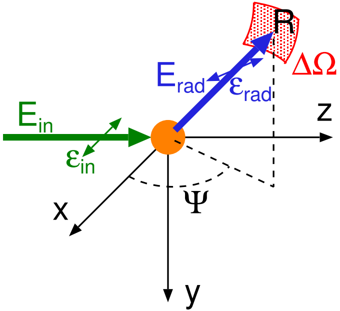

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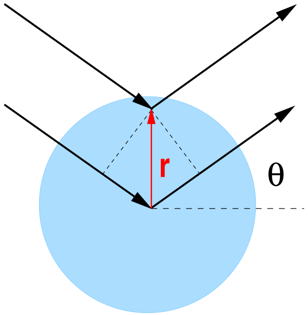
Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex



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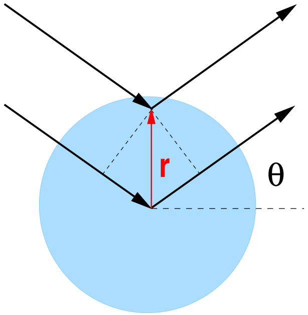

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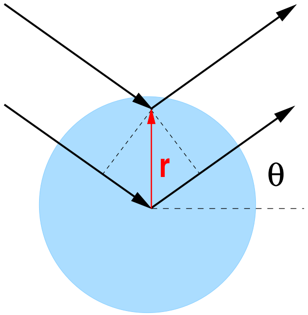

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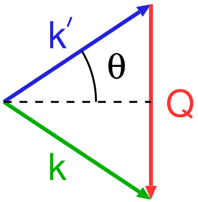

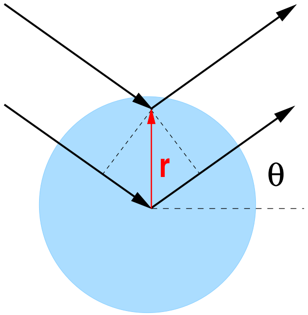

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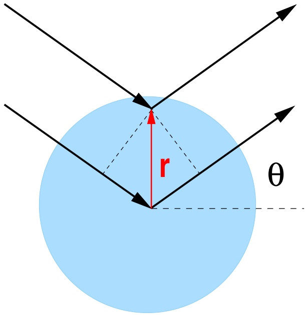

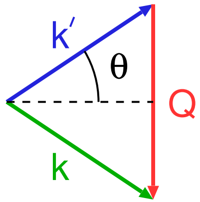

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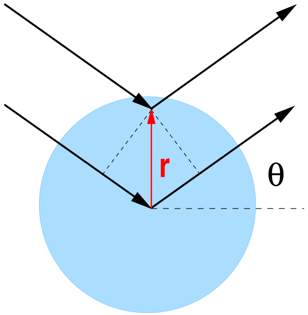

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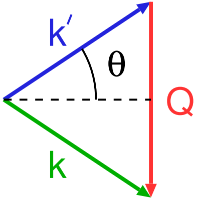

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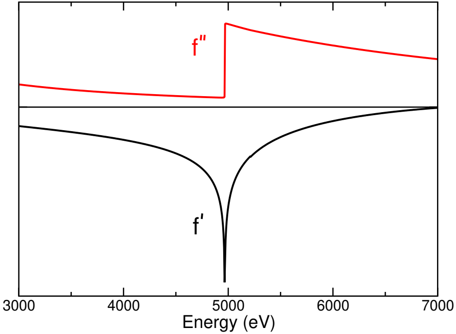

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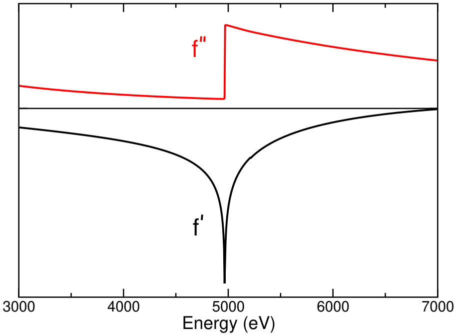

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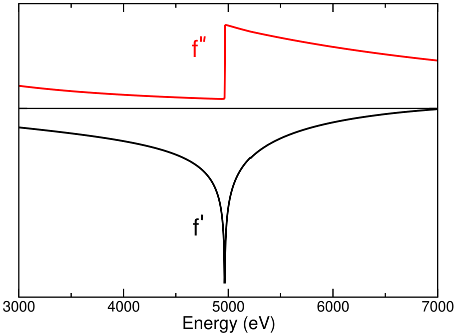

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$$f(\mathbf{Q}, \hbar\omega) = f^0(\mathbf{Q}) + f'(\hbar\omega) + if''(\hbar\omega)$$

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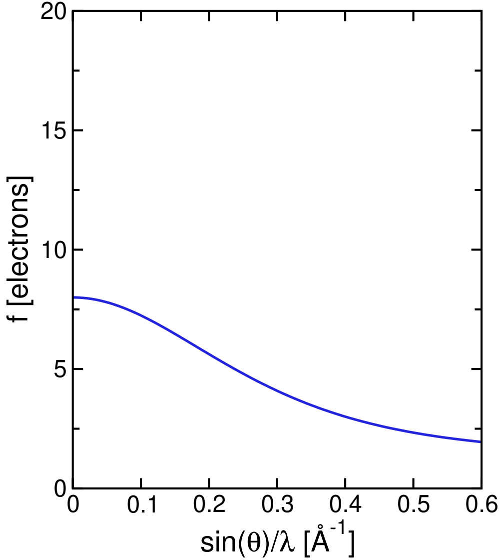

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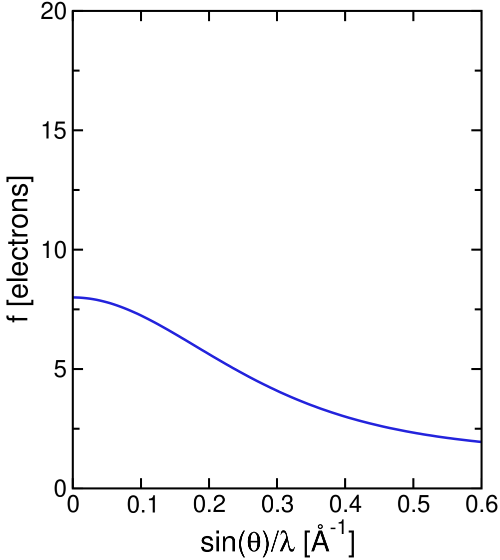

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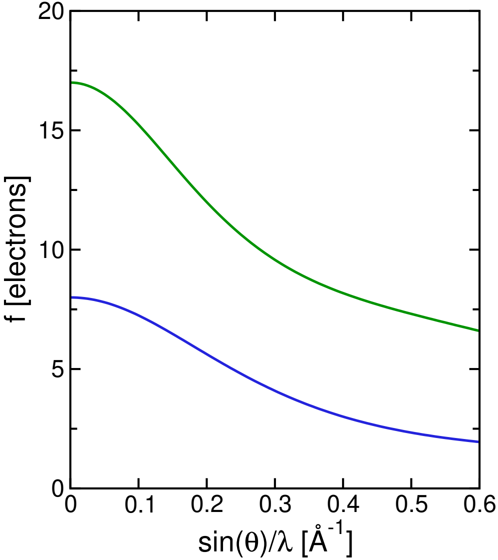

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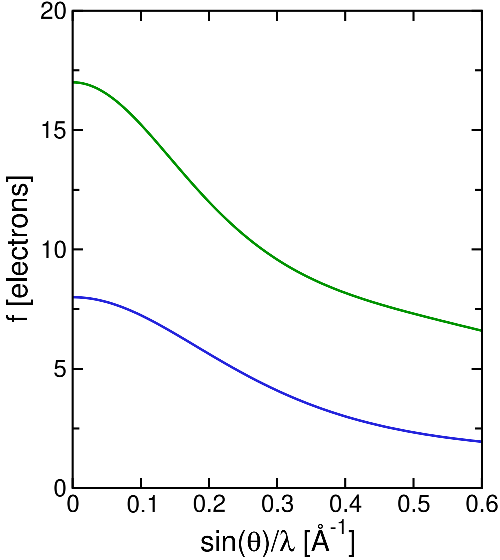

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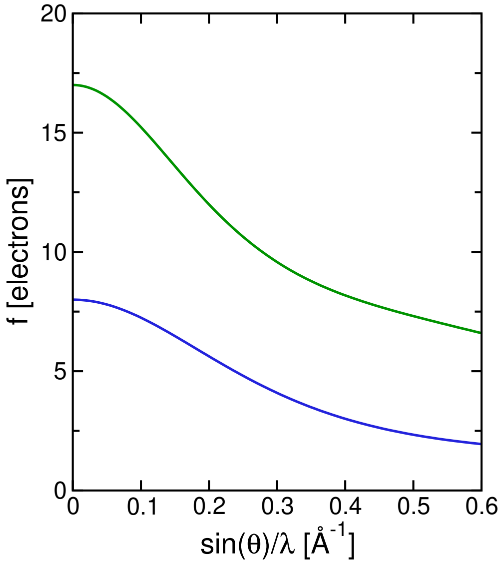

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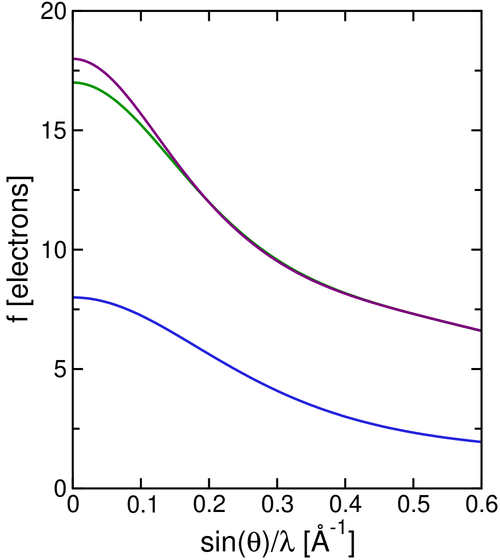

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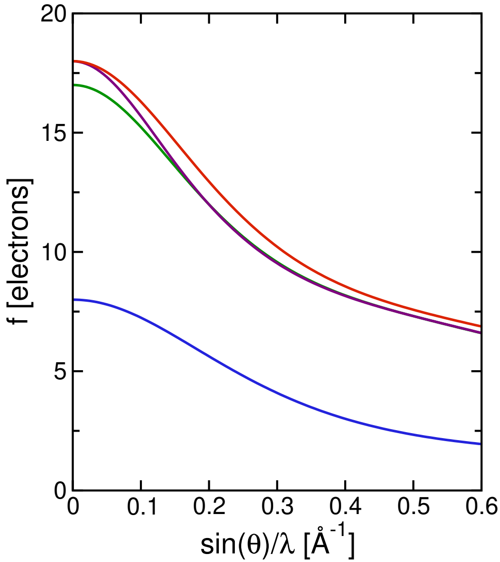

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