

• Photoelectric absorption



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- EXAFS theory





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- Reversibility in tin-based anode materials

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Reading Assignment: Chapter 8.1–8.3

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Homework Assignment #06: Chapter 6: 1,6,7,8,9 due Friday, November 15, 2024

Homework Assignment #07: Chapter 7: 2,3,9,10,11 due Monday, November 25, 2024





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bra	$\psi^*(x)$	$\langle\psi $	complex conjugate is implicit



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 $\begin{array}{ccc} & \text{integral} & \text{bra-ket} \\ \\ \text{bra} & \psi^*(x) & \langle \psi | & \text{complex conjugate is implicit} \\ \\ \text{ket} & \psi(x) & |\psi \rangle \\ \\ \text{normalization} & \int \psi^*(x)\psi(x)dx = 1 & \langle \psi | \psi \rangle = 1 \end{array}$



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integral bra-ket $\psi^*(x)$ bra $\langle \psi |$ complex conjugate is implicit ket $\psi(\mathbf{x})$ $|\psi\rangle$ $\int \psi^*(x)\psi(x)dx = 1 \qquad \langle \psi \mid \psi \rangle = 1$ normalization expectation value $\int \psi^* Q \psi dx$ $\langle \psi | Q \psi \rangle$ operator is applied to the right Carlo Segre (Illinois Tech) PHYS 570 - Fall 2024 November 06, 2024 2/16



From first-order perturbation theory, the absorption cross section is given by



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$$\sigma_{a} = \frac{2\pi}{\hbar c} \frac{V^{2}}{4\pi^{3}} \int |M_{if}|^{2} \delta(\mathcal{E}_{f} - \mathcal{E}_{i}) q^{2} \sin\theta dq d\theta d\varphi$$



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where the matrix element M_{if} between the initial, $\langle i |$, and final, $|f \rangle$, states is given by



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The interaction Hamiltonian is expressed in terms of the electromagnetic vector potential

$$\mathcal{H}_I = rac{eec{p}\cdotec{A}}{m} + rac{e^2A^2}{2m}$$

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$$\mathcal{H}_I = \frac{e\vec{p}\cdot\vec{A}}{m} + \frac{e^2A^2}{2m}$$

$$\vec{A} = \hat{\varepsilon} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[a_k e^{i\vec{k}\cdot\vec{r}} + a_k^{\dagger} e^{-i\vec{k}\cdot\vec{r}} \right]$$

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The interaction Hamiltonian is expressed in terms of the electromagnetic vector potential

The first term gives absorption

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$$\mathcal{H}_I = \frac{e\vec{p}\cdot\vec{A}}{m} + \frac{e^2A^2}{2m}$$

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where the matrix element M_{if} between the initial, $\langle i |$, and final, $|f \rangle$, states is given by

The interaction Hamiltonian is expressed in terms of the electromagnetic vector potential

The first term gives absorption while the second produces Thomson scattering so we take only the first into consideration now.

$$M_{if} = \langle i | \mathcal{H}_I | f \rangle$$

$$\mathcal{H}_{I} = \frac{e\vec{p}\cdot\vec{A}}{m} + \frac{e^{2}A^{2}}{2m}$$

$$\vec{A} = \hat{\varepsilon} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[a_k e^{i\vec{k}\cdot\vec{r}} + a_k^{\dagger} e^{-i\vec{k}\cdot\vec{r}} \right]$$



In order to evaluate the M_{if} matrix element we define the initial and final states



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the initial state has a photon and a K electron (no free electron)

similarly, the final state has no photon and an ejected free electron (ignoring the core hole and charged ion)

$$|i
angle = |1
angle_{\gamma}|0
angle_{e}$$



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 $\langle f|={}_e\langle 1|_\gamma\langle 0|$

Thus

$$M_{if} = rac{e}{m} \sqrt{rac{\hbar}{2\epsilon_0 V \omega}} \left[{}_e \langle 1 |_\gamma \langle 0 | (ec{p} \cdot \hat{arepsilon}) a e^{iec{k} \cdot ec{r}} + (ec{p} \cdot \hat{arepsilon}) a^{\dagger} e^{-iec{k} \cdot ec{r}} | 1
angle_\gamma | 0
angle_e
ight]$$

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In order to evaluate the M_{if} matrix element we define the initial and final states

the initial state has a photon and a K electron (no free electron)

similarly, the final state has no photon and an ejected free electron (ignoring the core hole and charged ion)

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Thus

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angle_\gamma | 0
angle_e
ight]$$

The calculation is simplified if the interaction Hamiltonian is applied to the left since the final state has only a free electron and no photon

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The free electron state is an eigenfunction of the electron momentum operator



 $_{e}\langle 1|ec{p}=(\hbarec{q})_{e}\langle 1|$

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The free electron state is an eigenfunction of the electron momentum operator

The annihilation operator applied to the left creates a photon



$$_{e}\langle 1|ec{p}=(\hbarec{q})_{e}\langle 1|$$
 $_{\gamma}\langle n|a=(\sqrt{n+1})_{\gamma}\langle n+1|$

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$${}_{\gamma}\langle n|a^{\dagger}=(\sqrt{n})_{\gamma}\langle n-1|$$

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The free electron state is an eigenfunction of the electron momentum operator

$${}_{e}\langle 1|_{\gamma}\langle 0|(ec{p}\cdot \hat{arepsilon}) a=\hbar(ec{q}\cdot \hat{arepsilon})_{e}\langle 1|_{\gamma}\langle 1|,$$



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$${}_{e}\langle 1|_{\gamma}\langle 0|(ec{p}\cdot \hat{arepsilon})a=\hbar(ec{q}\cdot \hat{arepsilon})_{e}\langle 1|_{\gamma}\langle 1|, \qquad {}_{e}\langle 1|_{\gamma}\langle 0|(ec{p}\cdot \hat{arepsilon})a^{\dagger}=0$$
Free electron approximation



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$$_{e}\langle 1|ec{
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The free electron state is an eigenfunction of the electron momentum operator

The annihilation operator applied to the left creates a photon while the creation operator annihilates a photon when applied to the left.

$${}_{e}\langle 1|_{\gamma}\langle 0|(ec{p}\cdot\hat{arepsilon})a=\hbar(ec{q}\cdot\hat{arepsilon})_{e}\langle 1|_{\gamma}\langle 1|, \qquad {}_{e}\langle 1|_{\gamma}\langle 0|(ec{p}\cdot\hat{arepsilon})a^{\dagger}=0$$

$$M_{if}=rac{e}{m}\sqrt{rac{\hbar}{2\epsilon_{0}V\omega}}\left[\hbar(ec{q}\cdot\hat{arepsilon})_{e}\langle1|_{\gamma}\langle1|e^{iec{k}\cdotec{r}}|1
angle_{\gamma}|0
angle_{e}+0
ight]$$

Free electron approximation



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$$M_{if} = \frac{e}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[\hbar (\vec{q} \cdot \hat{\varepsilon})_e \langle 1 |_{\gamma} \langle 1 | e^{i\vec{k} \cdot \vec{r}} | 1 \rangle_{\gamma} | 0 \rangle_e + 0 \right]$$
$$= \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon})_e \langle 1 | e^{i\vec{k} \cdot \vec{r}} | 0 \rangle_e$$

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Free electron approximation



$$_{e}\langle 1|ec{p}=(\hbarec{q})_{e}\langle 1|$$

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$$\begin{split} M_{if} &= \frac{e}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[\hbar (\vec{q} \cdot \hat{\varepsilon})_e \langle 1|_\gamma \langle 1| e^{i\vec{k} \cdot \vec{r}} |1 \rangle_\gamma |0 \rangle_e + 0 \right] \\ &= \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon})_e \langle 1| e^{i\vec{k} \cdot \vec{r}} |0 \rangle_e = \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon}) \int \psi_f^* e^{i\vec{k} \cdot \vec{r}} \psi_i d\vec{r} \end{split}$$

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$$M_{if} = \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V\omega}} (\vec{q} \cdot \hat{\varepsilon}) \int \psi_f^* e^{i\vec{k}\cdot\vec{r}} \psi_i d\vec{r}$$



$$M_{if} = \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V\omega}} (\vec{q} \cdot \hat{\varepsilon}) \int \psi_f^* e^{i\vec{k}\cdot\vec{r}} \psi_i d\vec{r} = \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V\omega}} (\vec{q} \cdot \hat{\varepsilon}) \phi(\vec{Q})$$



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The initial electron wavefunction is simply that of a 1s atomic state



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The initial electron wavefunction is simply that of a 1s atomic state while the final state is approximated as a plane wave



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$$= \sqrt{\frac{1}{V}} \int \psi_{1s}(\vec{r}) e^{i(\vec{k}-\vec{q})\cdot\vec{r}} d\vec{r}$$

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$$\begin{split} \psi_i &= \psi_{1s}(\vec{r}) \qquad \psi_f = \sqrt{\frac{1}{V}} e^{i\vec{q}\cdot\vec{r}} \\ \phi(\vec{Q}) &= \sqrt{\frac{1}{V}} \int e^{-i\vec{q}\cdot\vec{r}} e^{i\vec{k}\cdot\vec{r}} \psi_{1s}(\vec{r}) d\vec{r} \\ &= \sqrt{\frac{1}{V}} \int \psi_{1s}(\vec{r}) e^{i(\vec{k}-\vec{q})\cdot\vec{r}} d\vec{r} \\ &= \sqrt{\frac{1}{V}} \int \psi_{1s}(\vec{r}) e^{i\vec{Q}\cdot\vec{r}} d\vec{r} \end{split}$$

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The initial electron wavefunction is simply that of a 1s atomic state while the final state is approximated as a plane wave

The integral thus becomes

which is the Fourier transform of the initial state 1s electron wave function

V

the overall matrix element squared for a particular photoelectron final direction (φ , θ) is



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$$|M_{if}|^2 = \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} (q^2 \sin^2 \theta \cos^2 \varphi) \phi^2(\vec{Q})$$



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and the final cross-section per K electron can now be computed as



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and the final cross-section per K electron can now be computed as

$$\sigma_{a} = \frac{2\pi}{\hbar c} \frac{V^2}{4\pi^3} \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} I_3$$

V

the overall matrix element squared for a particular photoelectron final direction ($arphi,\, heta)$ is

$$|M_{if}|^2 = \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} (q^2 \sin^2 \theta \cos^2 \varphi) \phi^2(\vec{Q})$$

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V

the overall matrix element squared for a particular photoelectron final direction ($arphi,\, heta)$ is

$$|M_{if}|^2 = \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} (q^2 \sin^2 \theta \cos^2 \varphi) \phi^2(\vec{Q})$$

and the final cross-section per K electron can now be computed as

$$\sigma_{a} = \frac{2\pi}{\hbar c} \frac{V^{2}}{4\pi^{3}} \left(\frac{e\hbar}{m}\right)^{2} \frac{\hbar}{2\epsilon_{0}V^{2}\omega} I_{3} = \left(\frac{e\hbar}{m}\right)^{2} \frac{1}{4\pi^{2}\epsilon_{0}c\omega} I_{3}$$

where the integral I_3 is given by

V

the overall matrix element squared for a particular photoelectron final direction ($arphi,\, heta)$ is

$$|M_{if}|^2 = \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} (q^2 \sin^2 \theta \cos^2 \varphi) \phi^2(\vec{Q})$$

and the final cross-section per K electron can now be computed as

$$\sigma_{a} = \frac{2\pi}{\hbar c} \frac{V^{2}}{4\pi^{3}} \left(\frac{e\hbar}{m}\right)^{2} \frac{\hbar}{2\epsilon_{0}V^{2}\omega} I_{3} = \left(\frac{e\hbar}{m}\right)^{2} \frac{1}{4\pi^{2}\epsilon_{0}c\omega} I_{3}$$

where the integral I_3 is given by

$$I_3 = \int \phi^2(\vec{Q}) q^2 \sin^2 \theta \cos^2 \varphi \delta(\mathcal{E}_f - \mathcal{E}_i) q^2 \sin \theta dq d\theta d\phi$$

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Calculated cross sections





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Calculated cross sections





Calculated cross sections





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X-ray Absorption Fine-Structure (XAFS) is the modulation of the x-ray absorption coefficient at energies near and above an x-ray absorption edge. XAFS is also referred to as X-ray Absorption Spectroscopy (XAS) and is broken into 2 regimes:



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• is sensitive to local atomic coordination



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- is sensitive to chemical / oxidation state
- applies to any element
- works at low concentrations
- has minimal sample requirements

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The EXAFS experiment





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The EXAFS experiment



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normalize by fitting pre-edge and post-edge trends





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remove "smooth" μ_0 background

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Fourier transform to get real space EXAFS





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Modern codes, such as FEFF9, are able to accurately compute XANES features.

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X-ray absorption needs an available state for the photoelectron to go into

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An atom absorbs an x-ray of energy E, destroying a core electron with energy E_0 and creating a photoelectron with energy $(E - E_0)$. The core hole is eventually filled, and a fluorescence x-ray or Auger electron is ejected from the atom.

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The amplitude and phase of the back-scattered photoelectron at the absorbing atom will vary with energy, causing the oscillations in $\mu(E)$






$$\mu(E) = \mu_0(E) + \Delta \mu(E)$$





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 $|f\rangle$ is the final state which has a photoelectron, a hole in the core, and no photon. This is altered by the neighboring atom: the photoelectron scatters.

