

• Photoelectric absorption

- Photoelectric absorption
- EXAFS theory

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- Reversibility in tin-based anode materials

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Reading Assignment: Chapter 8.1–8.3

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Homework Assignment #06: Chapter 6: 1,6,7,8,9 due Friday, November 15, 2024 Homework Assignment #07: Chapter 7: 2,3,9,10,11 due Monday, November 25, 2024

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Calculation of σ_{a}

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$$
\vec{A} = \hat{\varepsilon} \sqrt{\frac{\hbar}{2 \epsilon_0 V \omega}} \left[a_k e^{i \vec{k} \cdot \vec{r}} + a_k^{\dagger} e^{-i \vec{k} \cdot \vec{r}} \right]
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The first term gives absorption

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Calculation of σ_3

From first-order perturbation theory, the absorption cross section is given by

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$$

where the matrix element M_{if} between the initial, $\langle i|$, and final, $|f\rangle$, states is given by

The interaction Hamiltonian is expressed in terms of the electromagnetic vector potential

The first term gives absorption while the second produces Thomson scattering so we take only the first into consideration now.

$$
M_{if}=\langle i|\mathcal{H}_I|f\rangle
$$

$$
\mathcal{H}_I = \frac{e\vec{p} \cdot \vec{A}}{m} + \frac{e^2 A^2}{2m}
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$$
\vec{A} = \hat{\varepsilon} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[a_k e^{i\vec{k}\cdot\vec{r}} + a_k^{\dagger} e^{-i\vec{k}\cdot\vec{r}} \right]
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similarly, the final state has no photon and an ejected free electron (ignoring the core hole and charged ion)

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|i\rangle=|1\rangle_{\gamma}|0\rangle_{e}
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Thus

$$
M_{if} = \frac{e}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[e \langle 1 |_{\gamma} \langle 0 | (\vec{p} \cdot \hat{\varepsilon}) a e^{i \vec{k} \cdot \vec{r}} + (\vec{p} \cdot \hat{\varepsilon}) a^{\dagger} e^{-i \vec{k} \cdot \vec{r}} | 1 \rangle_{\gamma} | 0 \rangle_{e} \right]
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$$

The calculation is simplified if the interaction Hamiltonian is applied to the left since the final state has only a free electron and no photon

The free electron state is an eigenfunction of the electron momentum operator

 $_{e}$ (1| $\vec{p} = (\hbar \vec{q})_{e}$ (1|

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The annihilation operator applied to the left creates a photon

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e^{\langle 1|\vec{\rho} = (\hbar \vec{q})_e \langle 1|}
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\gamma \langle n|a = (\sqrt{n+1})_\gamma \langle n+1|
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$$
{e}\langle 1|{\gamma}\langle 0|(\vec{\rho}\cdot\hat{\varepsilon})a=\hbar(\vec{q}\cdot\hat{\varepsilon})_{e}\langle 1|_{\gamma}\langle 1|,
$$

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$$
{e}\langle 1|{\gamma}\langle 0|(\vec{p}\cdot\hat{\varepsilon})a=\hbar(\vec{q}\cdot\hat{\varepsilon})_{e}\langle 1|_{\gamma}\langle 1|, \qquad \ \ \, e\langle 1|_{\gamma}\langle 0|(\vec{p}\cdot\hat{\varepsilon})a^{\dagger}=0
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Free electron approximation

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The free electron state is an eigenfunction of the electron momentum operator

The annihilation operator applied to the left creates a photon while the creation operator annihilates a photon when applied to the left.

$$
{\mathsf{e}}\langle 1|{\gamma}\langle 0|(\vec{\rho}\cdot\widehat{\varepsilon})a=\hbar(\vec{q}\cdot\widehat{\varepsilon})_{\mathsf{e}}\langle 1|_{\gamma}\langle 1|, \qquad \, {_{\mathsf{e}}}\langle 1|_{\gamma}\langle 0|(\vec{\rho}\cdot\widehat{\varepsilon})a^{\dagger}=0
$$

$$
M_{\text{if}} = \frac{e}{m}\sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[\hbar (\vec{q}\cdot\hat{\varepsilon})_e \langle 1|_\gamma \langle 1|e^{i\vec{k}\cdot\vec{r}}|1\rangle_\gamma |0\rangle_e + 0\right]
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$$
\begin{aligned} M_{\text{if}} &= \frac{e}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[\hbar (\vec{q} \cdot \hat{\varepsilon})_e \langle 1|_{\gamma} \langle 1| e^{i \vec{k} \cdot \vec{r}} | 1 \rangle_{\gamma} | 0 \rangle_e + 0 \right] \\ &= \frac{e \hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon})_e \langle 1| e^{i \vec{k} \cdot \vec{r}} | 0 \rangle_e \end{aligned}
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Free electron approximation

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M_{if} = \frac{e}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} \left[\hbar (\vec{q} \cdot \hat{\varepsilon}) e \langle 1|_{\gamma} \langle 1| e^{i\vec{k}\cdot \vec{r}} | 1 \rangle_{\gamma} | 0 \rangle_e + 0 \right]
$$

= $\frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon}) e \langle 1| e^{i\vec{k}\cdot \vec{r}} | 0 \rangle_e = \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon}) \int \psi_f^* e^{i\vec{k}\cdot \vec{r}} \psi_i d\vec{r}$

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M_{if} = \frac{e\hbar}{m} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega}} (\vec{q} \cdot \hat{\varepsilon}) \int \psi_f^* e^{i\vec{k} \cdot \vec{r}} \psi_i d\vec{r}
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The initial electron wavefunction is simply that of a 1s atomic state

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The initial electron wavefunction is simply that of a 1s atomic state while the final state is approximated as a plane wave

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The initial electron wavefunction is simply that of a 1s atomic state while the final state is approximated as a plane wave

The integral thus becomes

which is the Fourier transform of the initial state 1s electron wave function

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the overall matrix element squared for a particular photoelectron final direction (φ , θ) is

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|M_{if}|^2 = \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} (q^2 \sin^2 \theta \cos^2 \varphi) \phi^2(\vec{Q})
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and the final cross-section per K electron can now be computed as

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$$
\sigma_a = \frac{2\pi}{\hbar c} \frac{V^2}{4\pi^3} \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} I_3
$$

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|M_{if}|^2 = \left(\frac{e\hbar}{m}\right)^2 \frac{\hbar}{2\epsilon_0 V^2 \omega} (q^2 \sin^2 \theta \cos^2 \varphi) \phi^2(\vec{Q})
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$$

Calculated cross sections

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- works at low concentrations
- has minimal sample requirements

The EXAFS experiment

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The EXAFS experiment

I_o = incident intensity

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Any excess energy from the x-ray is given to an ejected photoelectron, which expands as a spherical wave, reaches the neighboring electron clouds, and scatters back to the core hole, creating interference patterns called XANES and EXAFS.

normalize by fitting pre-edge and post-edge trends

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Fourier transform to get real space EXAFS

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XANES can be used as a fingerprint of phases and XANES analysis can be as simple as making linear combinations of "known" spectra to get composition.

Modern codes, such as FEFF9, are able to accurately compute XANES features.

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An atom absorbs an x-ray of energy E, destroying a core electron with energy E_0 and creating a photoelectron with energy $(E - E_0)$. The core hole is eventually filled, and a fluorescence x-ray or Auger electron is ejected from the atom.

With another atom nearby, the ejected photoelectron can scatter from a neighboring atom and return back to the absorbing atom

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The amplitude and phase of the back-scattered photoelectron at the absorbing atom will vary with energy, causing the oscillations in $\mu(E)$

$$
\mu(E)=\mu_0(E)+\Delta\mu(E)
$$

$$
\mu(E) = \mu_0(E) + \Delta \mu(E) = \mu_0(E)[1 + \chi(E)]
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 $|f\rangle$ is the final state which has a photoelectron, a hole in the core, and no photon. This is altered by the neighboring atom: the photoelectron scatters.