Today's outline - September 25, 2024

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- Solutions to HW #2

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- Fresnel lenses & zone plates

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Reading Assignment: Chapter 4.3–4.4

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Reading Assignment: Chapter 4.3–4.4

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Today's outline - September 25, 2024

- Solutions to HW #2
- Fresnel lenses & zone plates
- Scattering review
- Kinematical scattering
- Liquid scattering

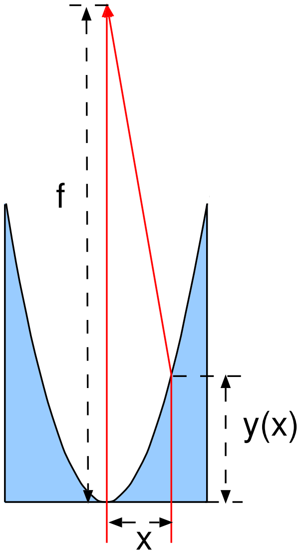
Reading Assignment: Chapter 4.3–4.4

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Homework Assignment #04:
Chapter 4: 2,4,6,7,10
due Monday, October 14, 2024

Elliptical lens surface review

When we lift the restriction on aperture, the ideal surface for a focusing lens becomes elliptical rather than parabolic



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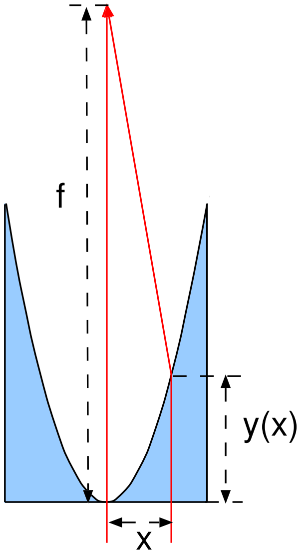
The two rays shown must be in phase when they reach the focal point and so we can write



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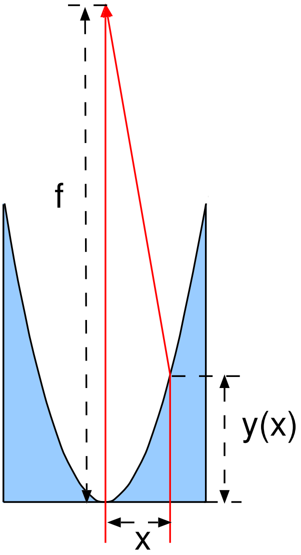
$$x^2 = 2f\delta y - (2\delta - \delta^2)y^2$$



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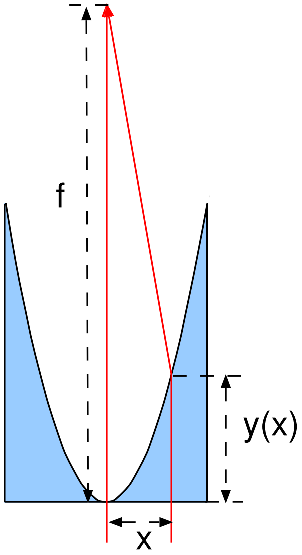
This is justthe equation of an ellipse



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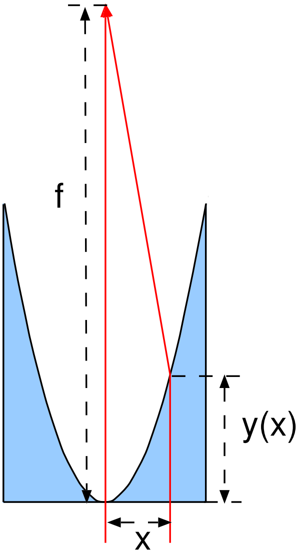
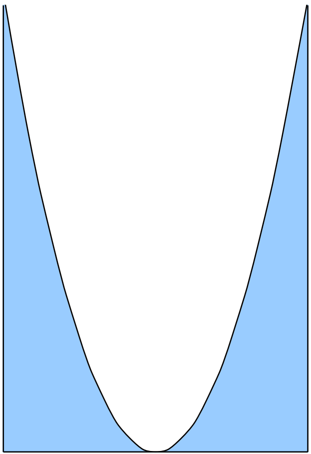
$$0 = x^2 + \frac{a^2}{b^2}y^2 - 2\frac{a^2}{b}y$$

$$a = f\sqrt{\frac{\delta}{2-\delta}}, \qquad b = \frac{f}{2-\delta}$$

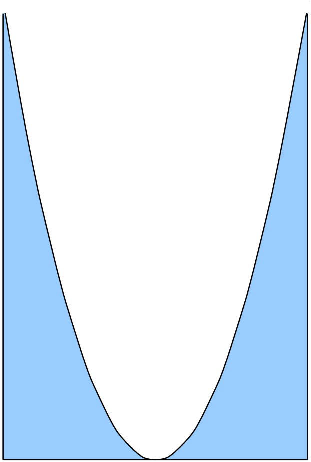


How to make a Fresnel lens



The ideal refracting lens has an elliptical shape but this is impractical to make. Assuming the parabolic approximation:

How to make a Fresnel lens



The ideal refracting lens has an elliptical shape but this is impractical to make. Assuming the parabolic approximation:

$$h(x) = \Lambda \left(\frac{x}{\sqrt{2\lambda_o f}} \right)^2$$

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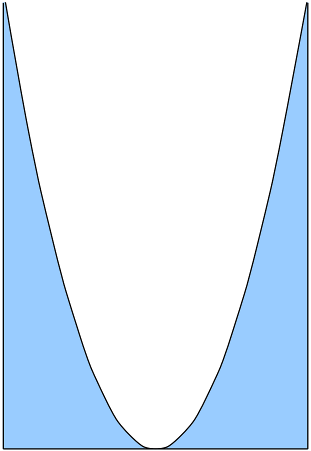

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when $h(x) = 100\Lambda \sim 1000\mu$m

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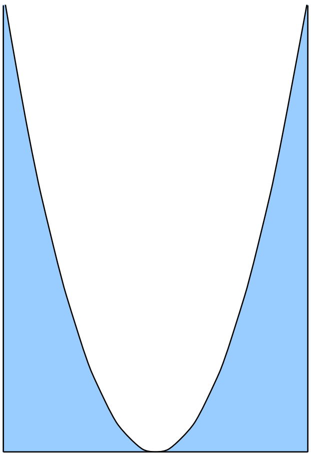

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$$x = 10\sqrt{2\lambda_o f} \sim 100\mu\text{m}$$

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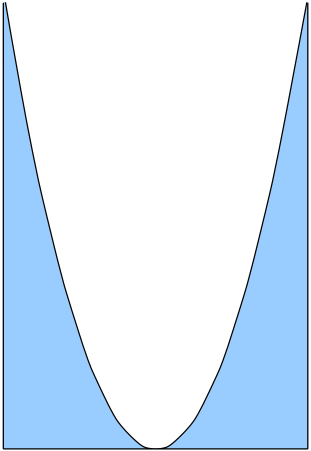

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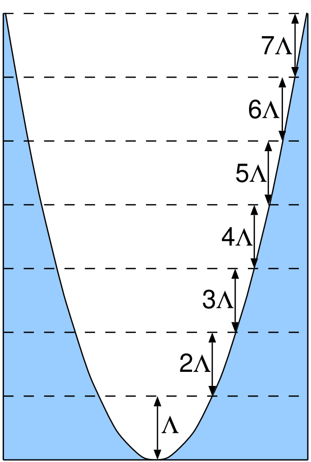
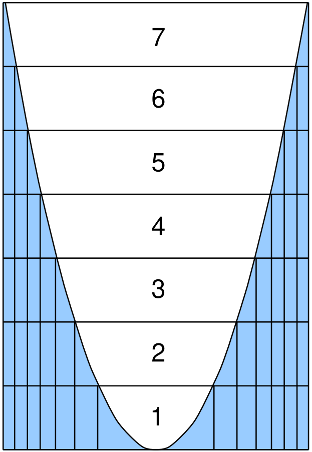
aspect ratio too large for a stable structure and absorption would be too large!

How to make a Fresnel lens



Mark off the longitudinal zones (of thickness Λ) where the waves inside and outside the material are in phase.

How to make a Fresnel lens



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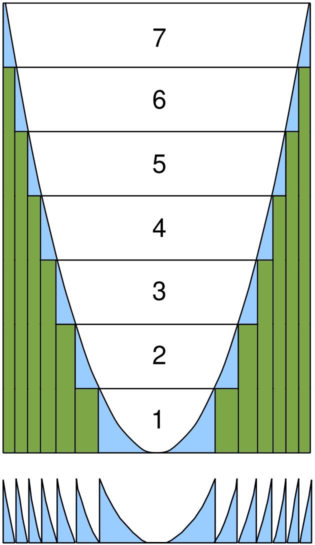
Each block of thickness Λ serves no purpose for refraction but only attenuates the wave.

How to make a Fresnel lens



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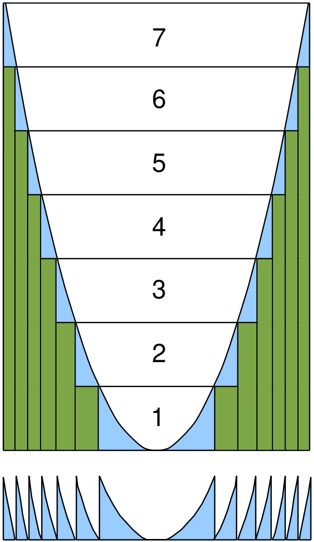
Each block of thickness Λ serves no purpose for refraction but only attenuates the wave.

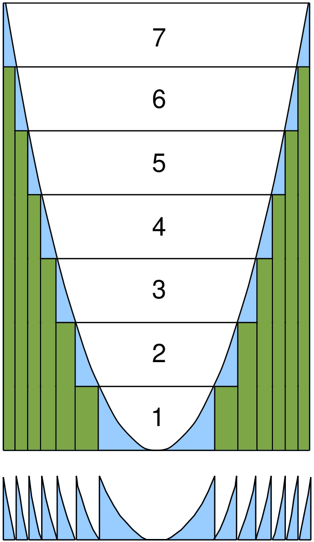
This material can be removed and the remaining material collapsed to produce a Fresnel lens which has the same optical properties as the parabolic lens as long as $f \gg N\Lambda$ where N is the number of zones.

Fresnel lens dimensions



The outermost zones become very small and thus limit the overall aperture of the zone plate. The dimensions of outermost zone, N can be calculated by first defining a scaled height and lateral dimension

Fresnel lens dimensions



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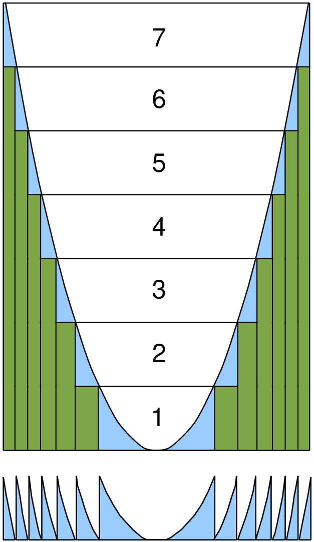
$$\nu = \frac{h(x)}{\Lambda}$$

Fresnel lens dimensions



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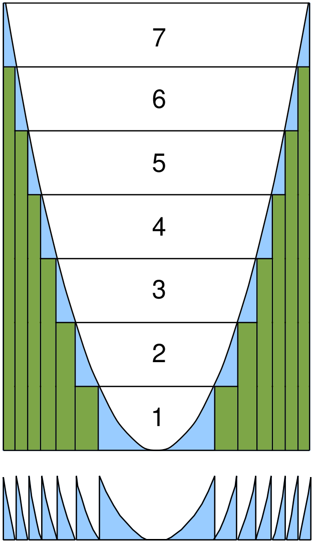
$$\nu = \frac{h(x)}{\Lambda} \qquad \xi = \frac{x}{\sqrt{2\lambda_o f}}$$

Fresnel lens dimensions



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Since $\nu = \xi^2$, the position of the N^{th} zone is $\xi_N = \sqrt{N}$ and the scaled width of the N^{th} (outermost) zone is

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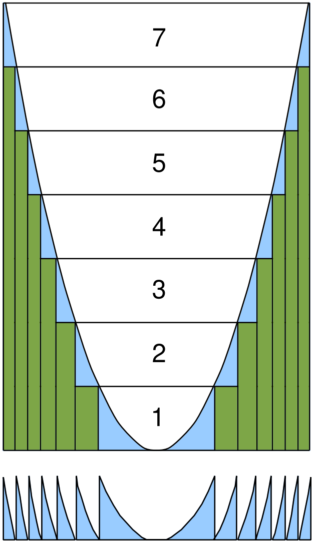

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$$\Delta\xi_N = \xi_N - \xi_{N-1} = \sqrt{N} - \sqrt{N-1}$$

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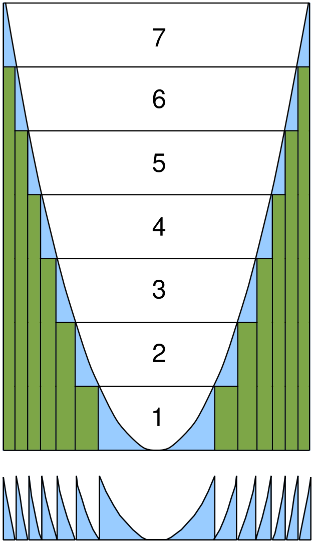

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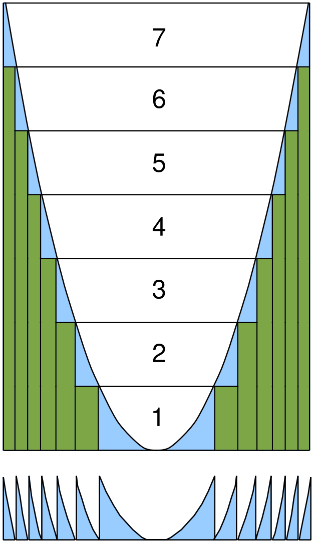
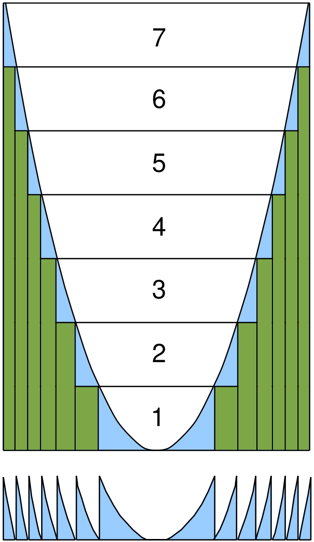


$$\Delta\xi_N = \xi_N - \xi_{N-1} = \sqrt{N} - \sqrt{N-1}$$
$$= \sqrt{N}\left(1 - \sqrt{1 - \frac{1}{N}}\right)$$

Fresnel lens dimensions



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$$\approx \sqrt{N}\left(1 - \left[1 - \frac{1}{2N}\right]\right)$$

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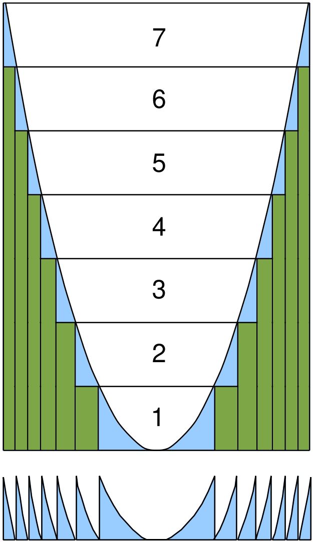

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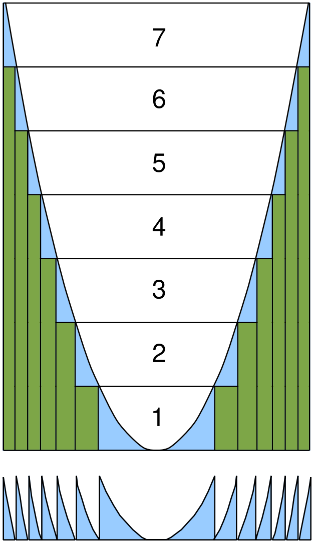

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$$\approx \sqrt{N}\left(1 - \left[1 - \frac{1}{2N}\right]\right)$$

$$\Delta\xi_N \approx \frac{1}{2\sqrt{N}}$$

The diameter of the entire lens is thus

$$2\xi_N = 2\sqrt{N} = \frac{1}{\Delta\xi_N}$$

Fresnel lens example

In terms of the unscaled variables

$$\Delta x_N = \Delta \xi_N \sqrt{2\lambda_o f}$$

Fresnel lens example

In terms of the unscaled variables

$$\Delta x_N = \Delta\xi_N\sqrt{2\lambda_o f} = \sqrt{\frac{\lambda_o f}{2N}}$$

Fresnel lens example

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Fresnel lens example

In terms of the unscaled variables

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If we take

$$\lambda_o = 1\text{Å} = 1 \times 10^{-10}\text{m}$$
$$f = 50\text{cm} = 0.5\text{m}$$
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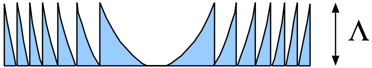
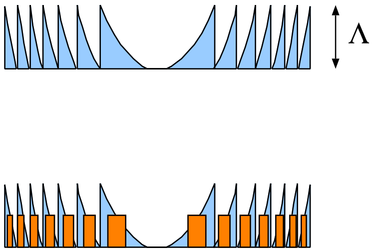
Making a Fresnel zone plate



The specific shape required for a zone plate is difficult to fabricate, consequently, it is convenient to approximate the nearly triangular zones with a rectangular profile.

Making a Fresnel zone plate



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In practice, since the outermost zones are very small, zone plates are generally fabricated as alternating zones (rings for 2D) of materials with a large Z-contrast, such as Au/Si or W/C.

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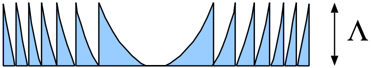

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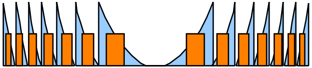

In practice, since the outermost zones are very small, zone plates are generally fabricated as alternating zones (rings for 2D) of materials with a large Z-contrast, such as Au/Si or W/C.

This kind of zone plate is not as efficient as a true Fresnel lens would be in the x-ray regime. Nevertheless, efficiencies up to 35% have been achieved.

Zone plate fabrication

Making high aspect ratio zone plates is challenging but a new process has been developed to make plates with an aspect ratio as high as 25.

M. Wojick et al., "X-ray zone plates with 25 aspect ratio using a 2-μm-thick ultra-nanocrystalline diamond mold," *Microsyst. Technol.* **20**, 2045-2050 (2014).

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Start with Ultra nano crystalline diamond (UNCD) films on SiN.

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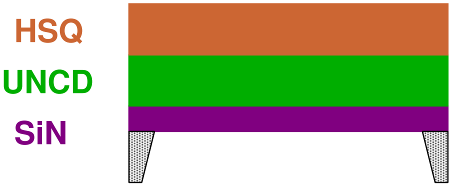

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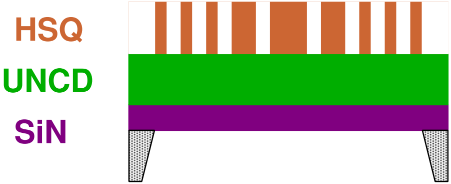
Start with Ultra nano crystalline diamond (UNCD) films on SiN. Coat with hydrogen silsesquioxane (HSQ). Pattern and develop the HSQ layer.



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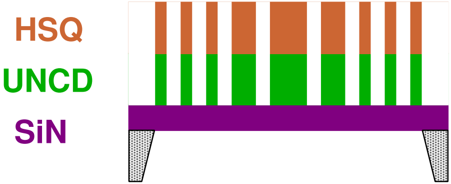

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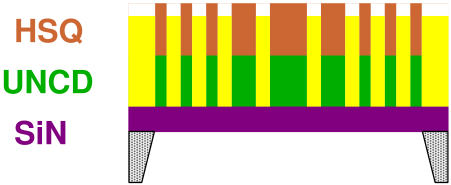

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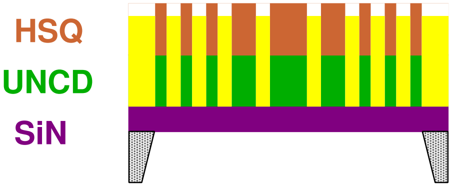
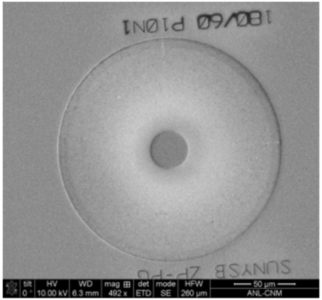
The whole 150nm diameter zone plate





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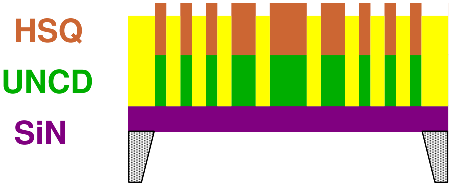
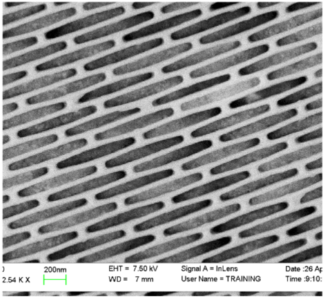


The whole 150nm diameter zone plate

Detail view of outer zones



M. Wojick et al., "X-ray zone plates with 25 aspect ratio using a 2-μm-thick ultra-nanocrystalline diamond mold," *Microsyst. Technol.* **20**, 2045-2050 (2014).

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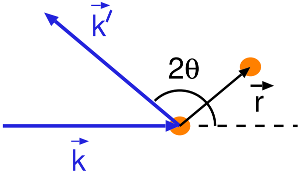

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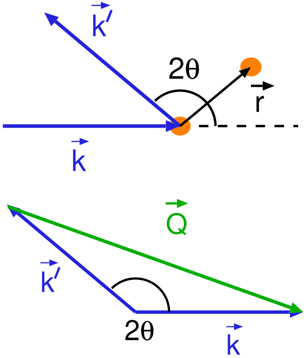

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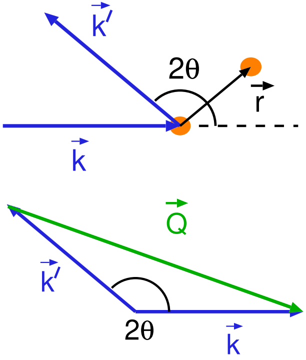

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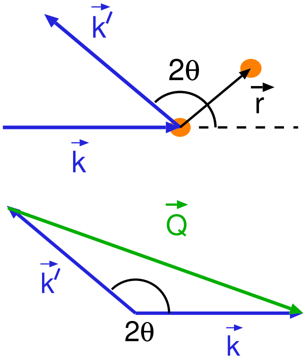

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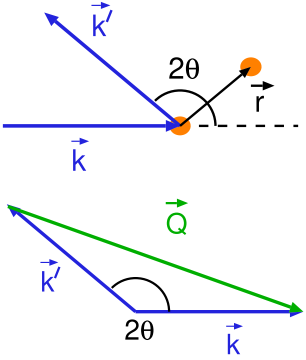

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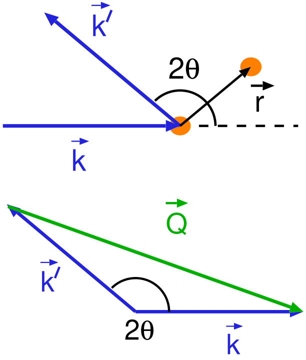

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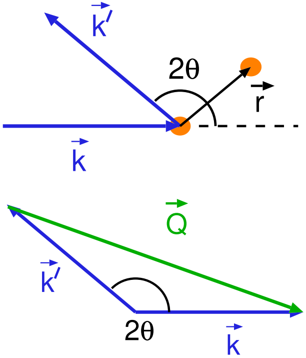

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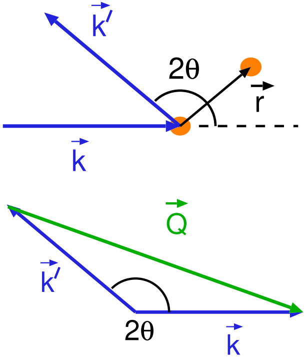

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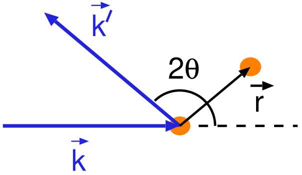

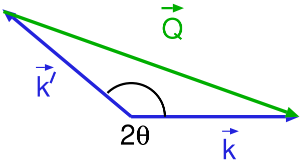

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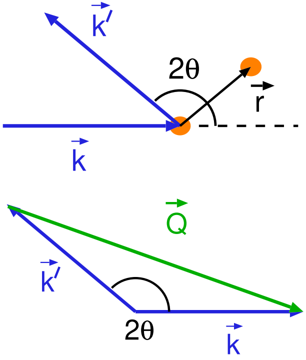

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We will now look at the consequences of this orientation and generalize to more than two electrons

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The expression

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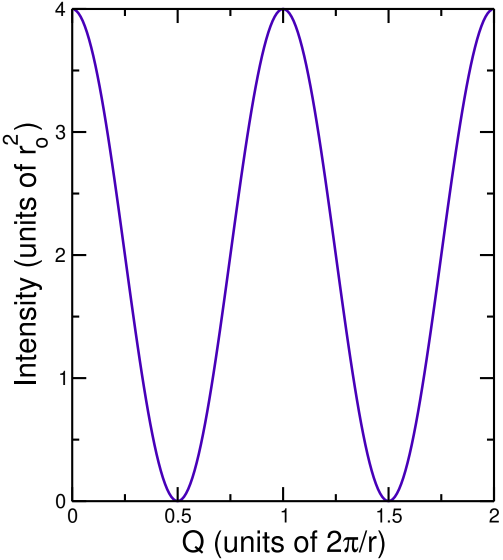

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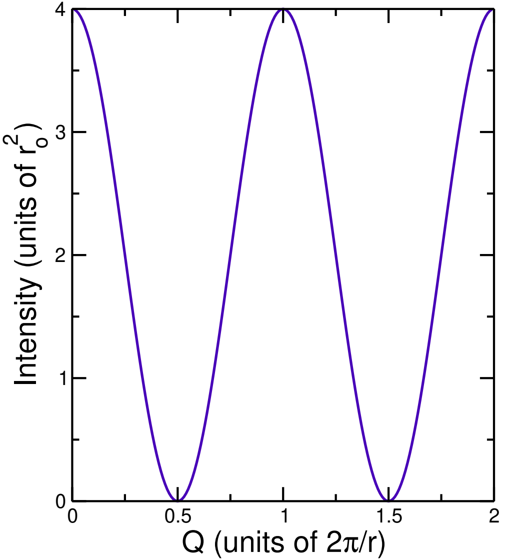
Fixed orientation is not the usual case, particularly for solution and small-angle scattering.



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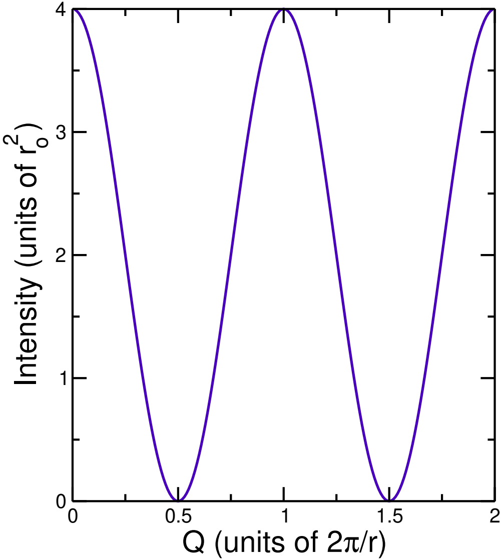

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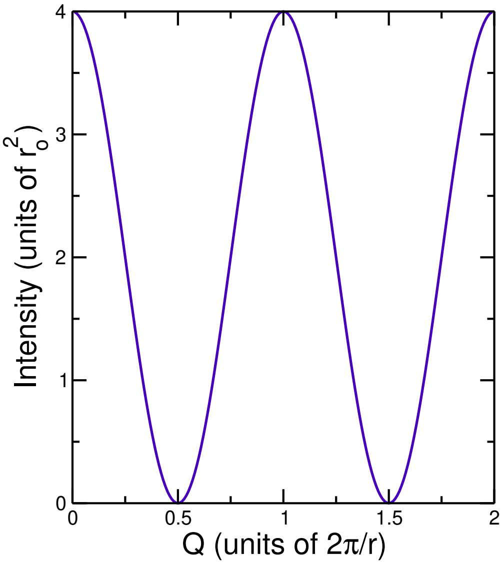

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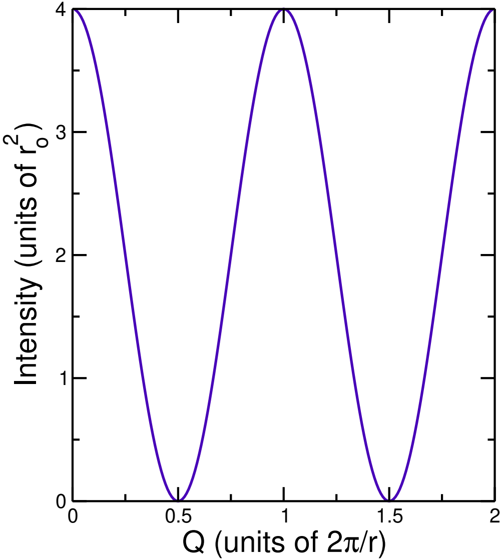

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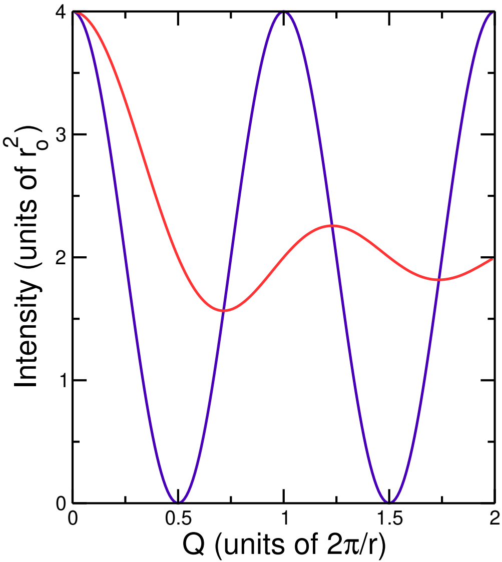

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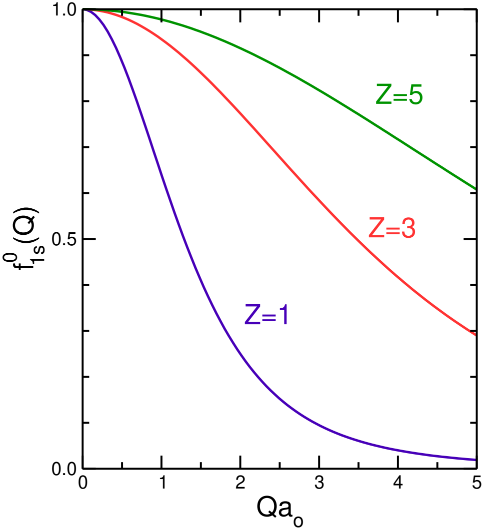

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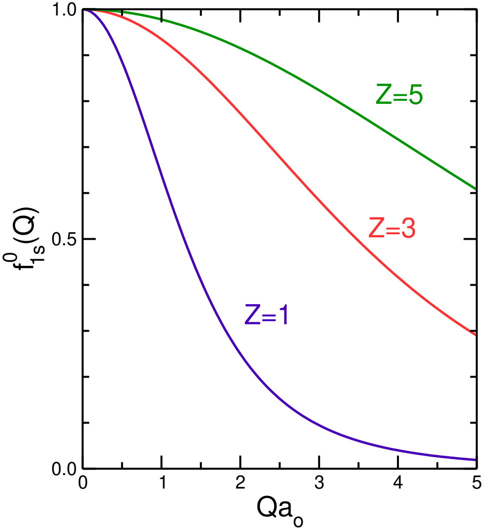
In principle, one can compute the full atomic form factors, however, it is more useful to tabulate the experimentally measured form factors



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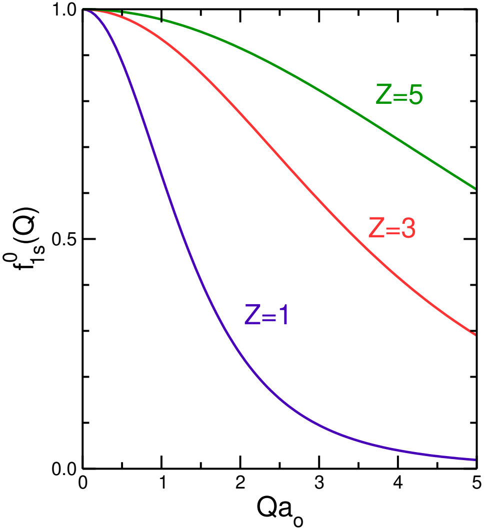

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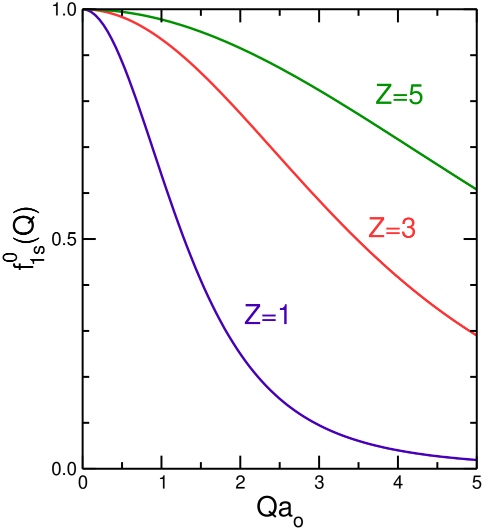

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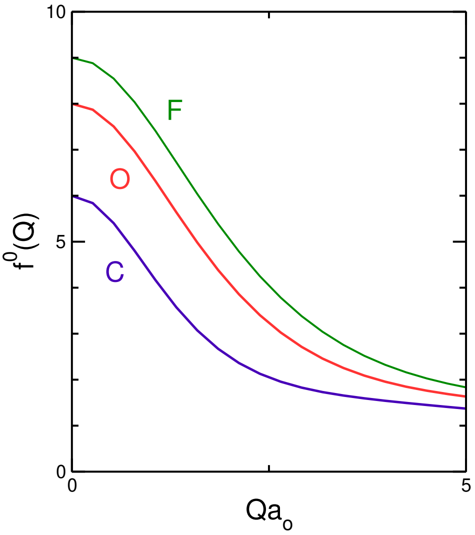

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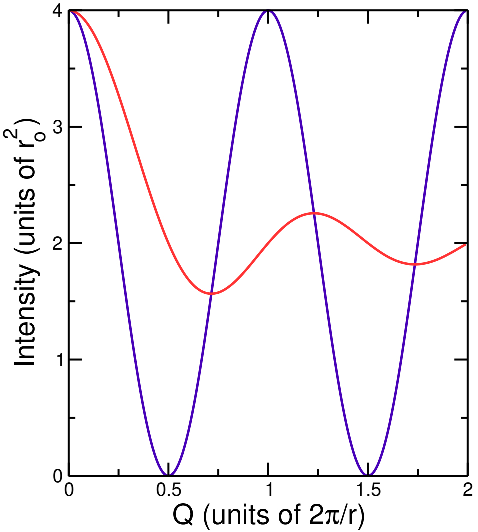

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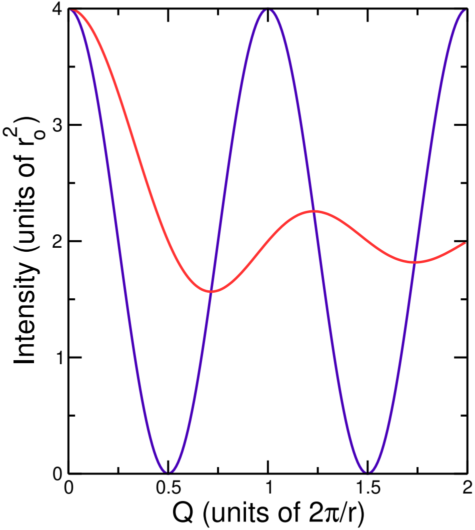

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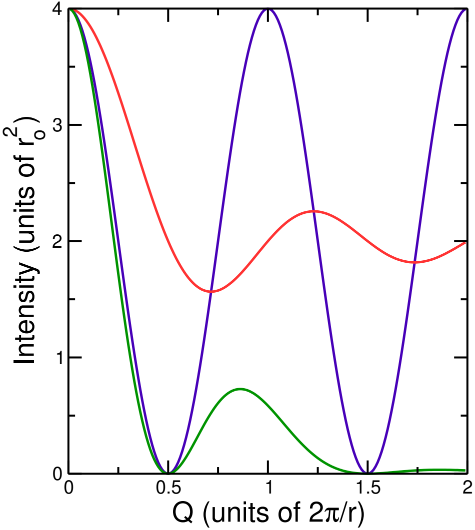

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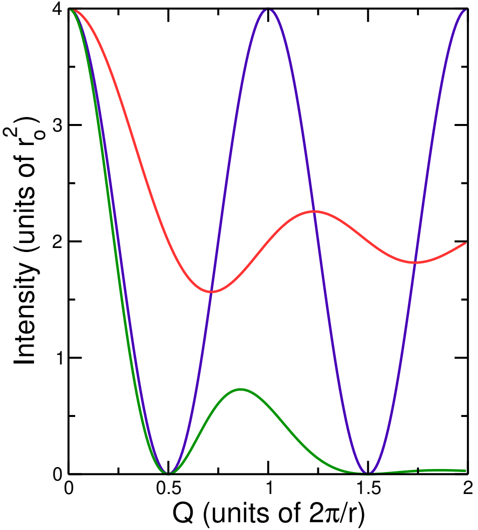

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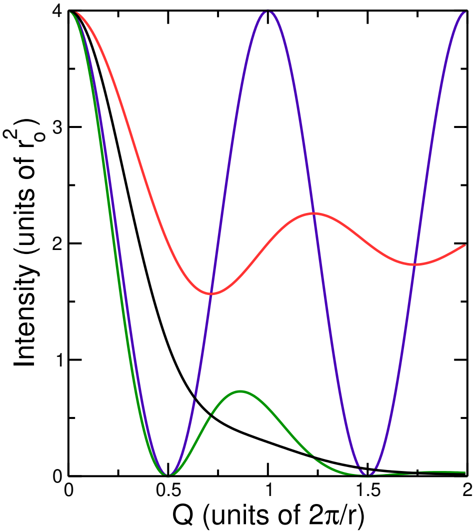

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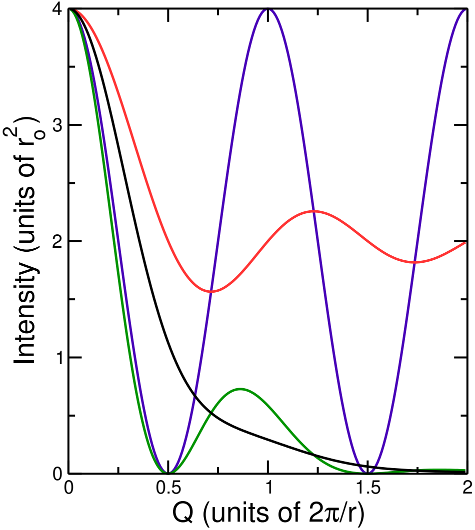

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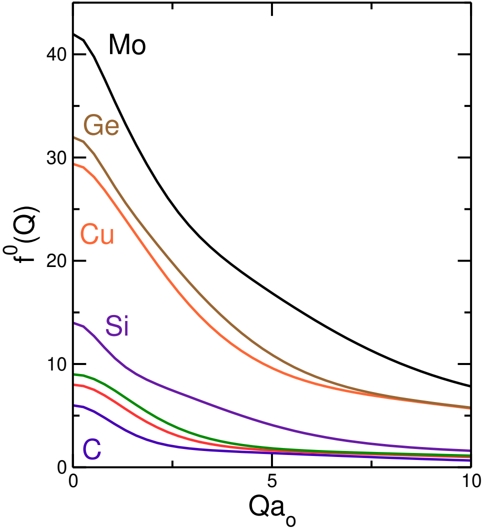

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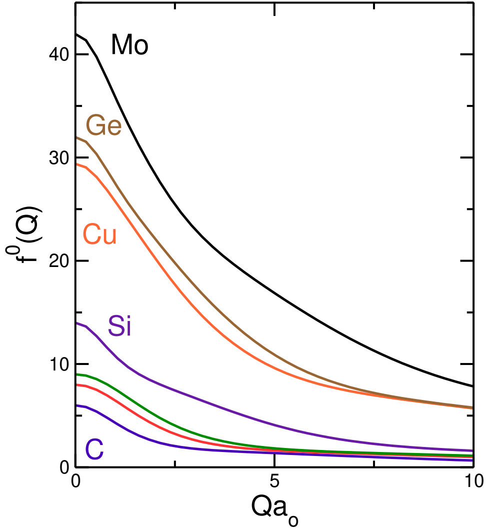

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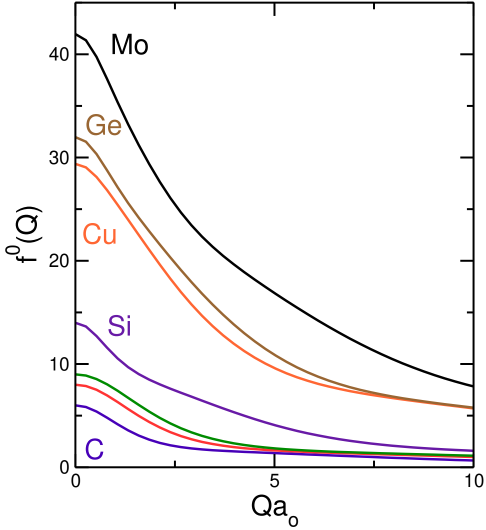

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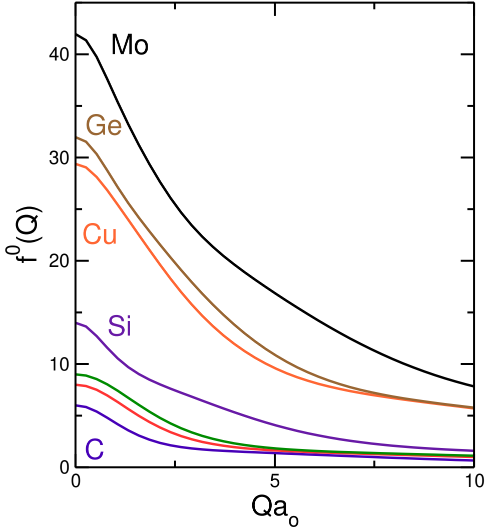

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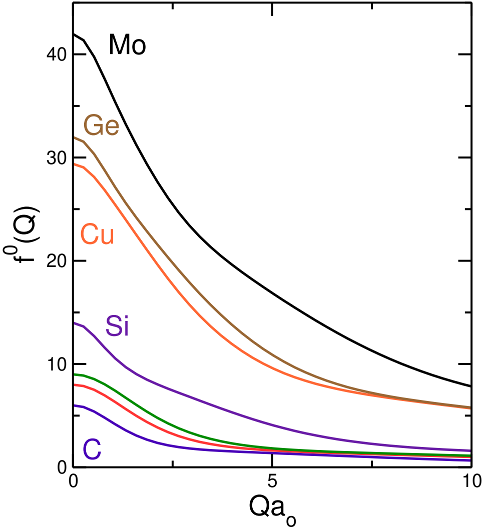

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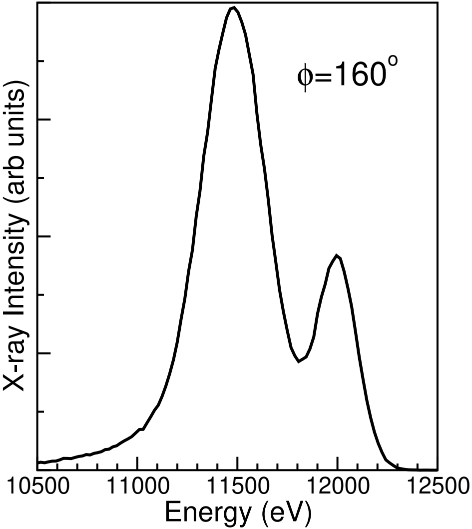

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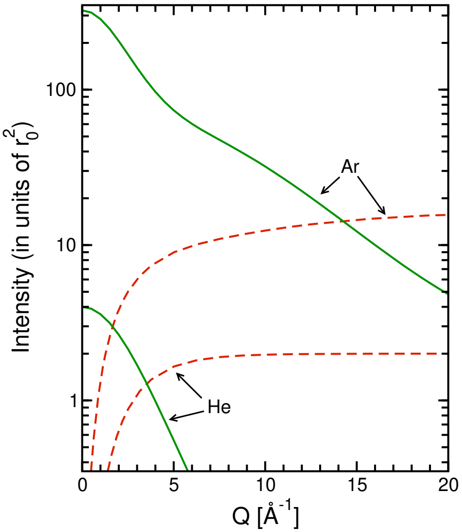

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