Today's outline - September 23, 2024

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- Mirrors

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Reading Assignment: Chapter 4.1–4.2

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Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

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- Mirrors
- Ideal refractive surface
- Fresnel lenses and zone plates
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Reading Assignment: Chapter 4.1–4.2

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Homework Assignment #04:
Chapter 4: 2,4,6,7,10
due Monday, October 14, 2024

Tangential focusing mirror

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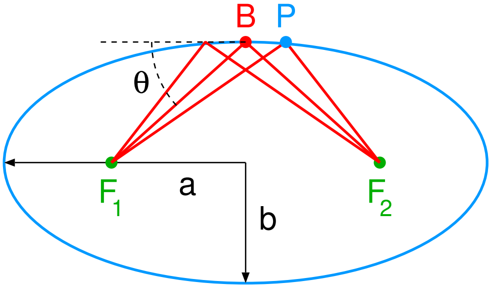

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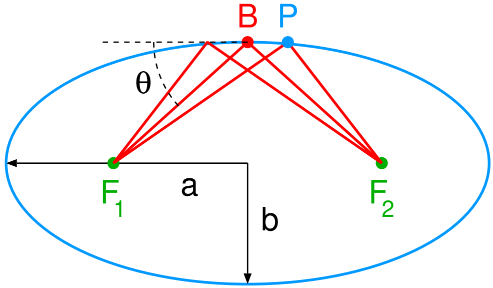
$$F_1P + F_2P = 2a$$



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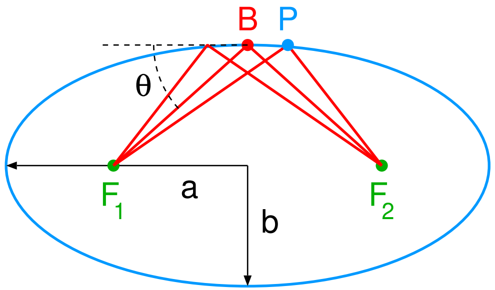

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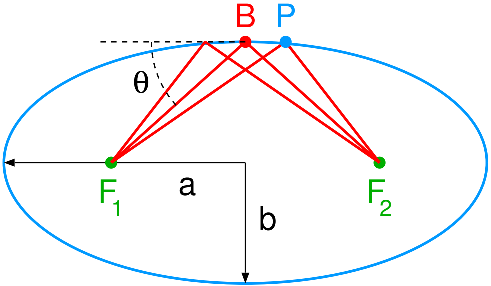

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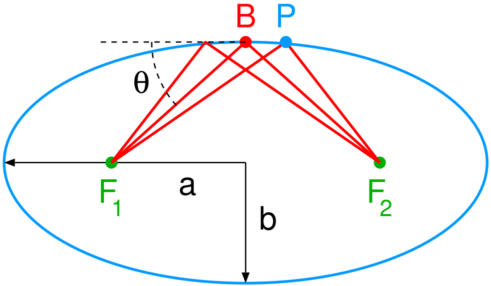

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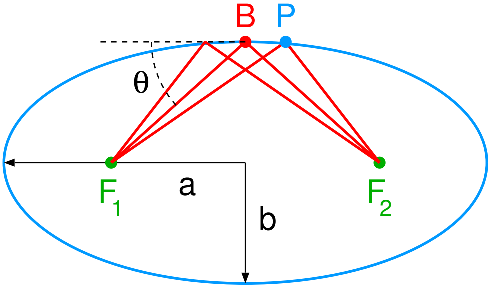

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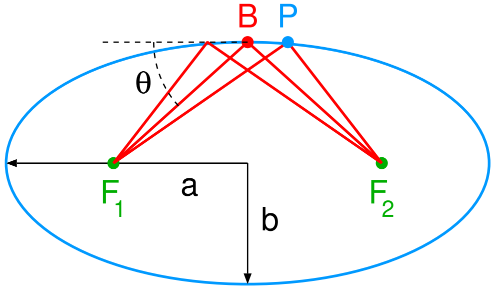

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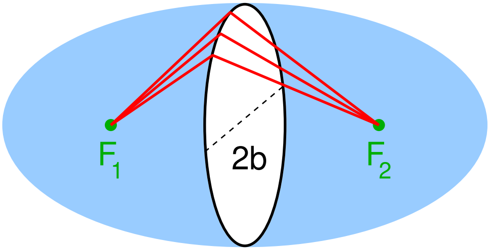
Saggital focusing mirror

Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of saggital focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.



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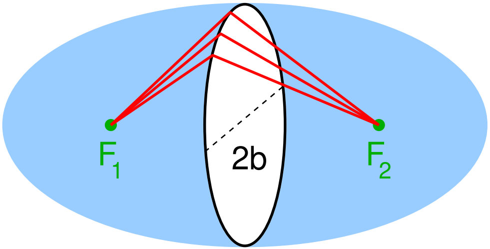

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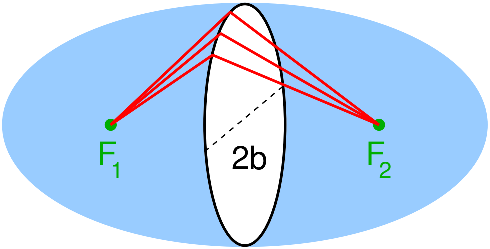

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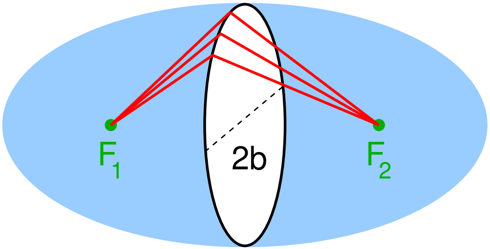
$$\rho_{tangential} = a = \frac{2f}{\sin\theta}$$



Types of focusing mirrors

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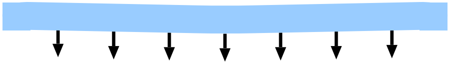

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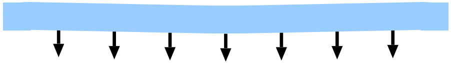

A cost effective way to focus in both directions is a toroidal mirror which has a fixed bend in the transverse direction but which can be bent longitudinally to change the vertical focus.

Dual focusing options

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- Zone plates — in combination with an initial focusing element, gives smallest focal spot, but hard to vary energy

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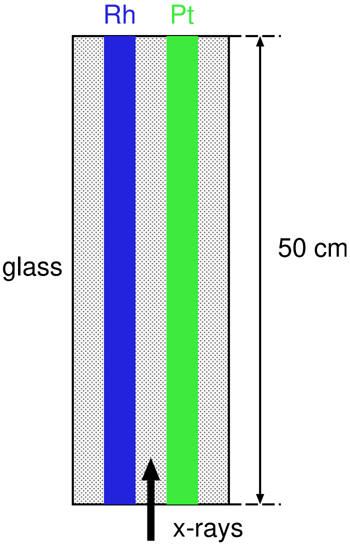
- Zone plates — in combination with an initial focusing element, gives smallest focal spot, but hard to vary energy
- Refractive lenses — good final focus, focus moves with energy, significant attenuation and hard to change focal length

The MRCAT mirror



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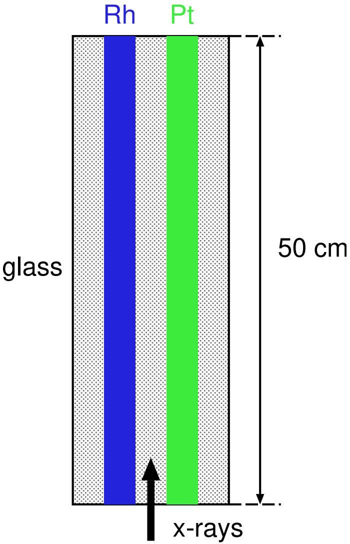
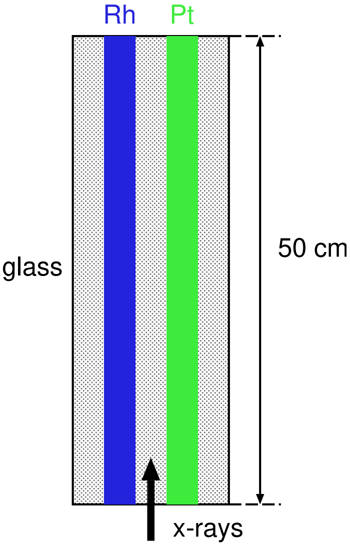


Ultra low expansion glass polished to a few Å roughness

The MRCAT mirror



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One platinum stripe and one rhodium stripe deposited along the length of the mirror on top of a chromium buffer layer

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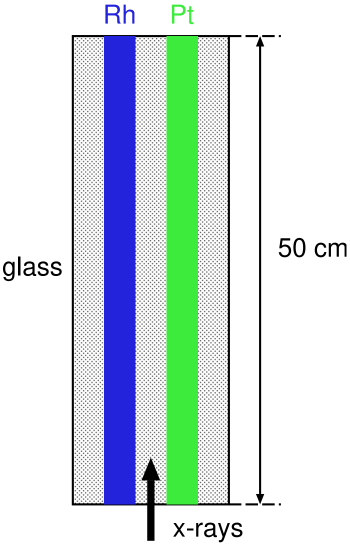

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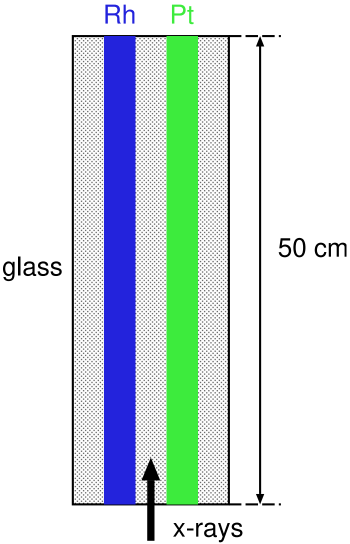

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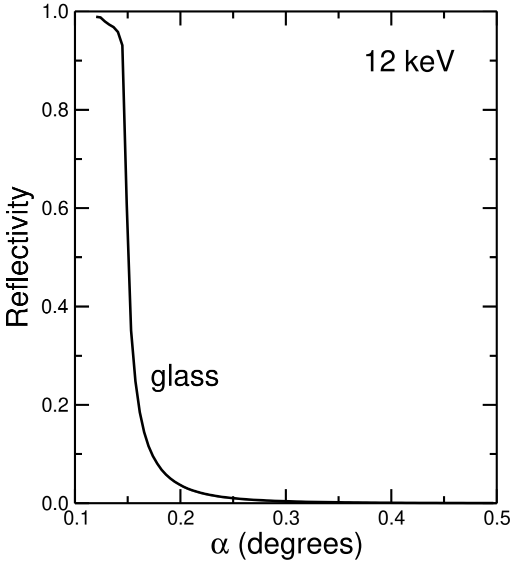
A bending mechanism to permit vertical focusing of the beam to $\sim 60\ \mu\text{m}$

Mirror performance

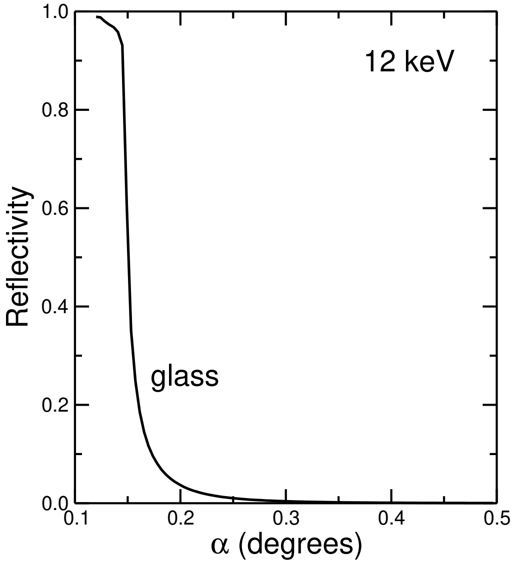
When illuminated with 12 keV x-rays on the glass "stripe", the reflectivity is measured as:



Mirror performance

When illuminated with 12 keV x-rays on the glass "stripe", the reflectivity is measured as:

With the Rh stripe, the thin slab reflection is evident and the critical angle is significantly higher.



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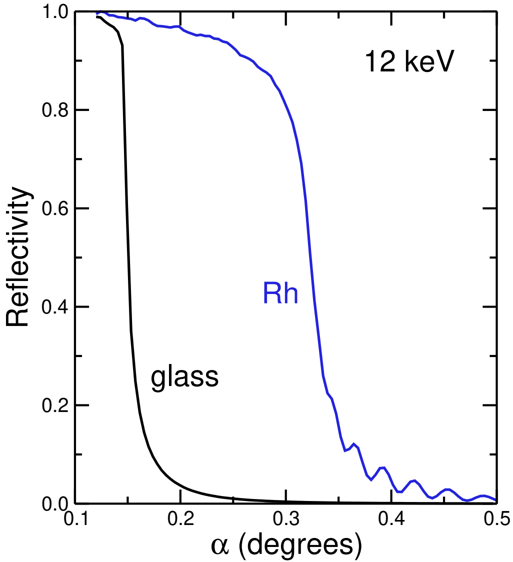

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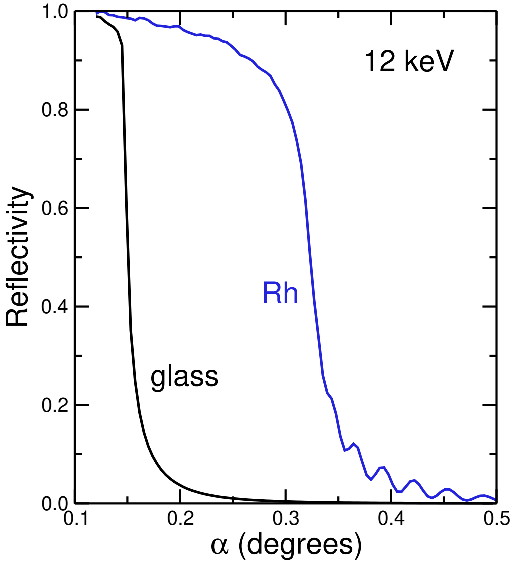
The Pt stripe gives a higher critical angle still but a lower reflectivity and it looks like an infinite slab.



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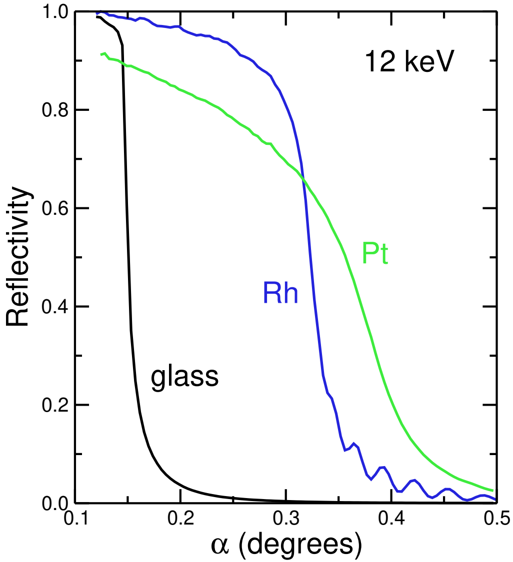

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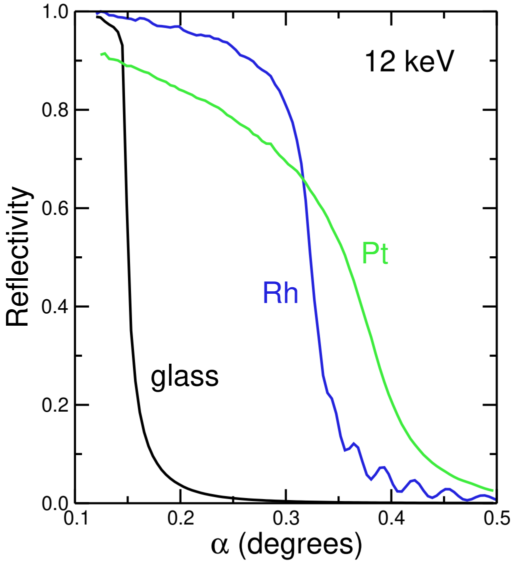
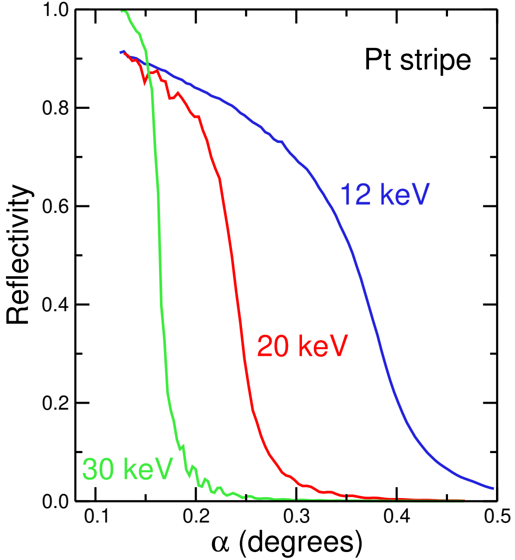
Why?



Mirror performance (cont.)



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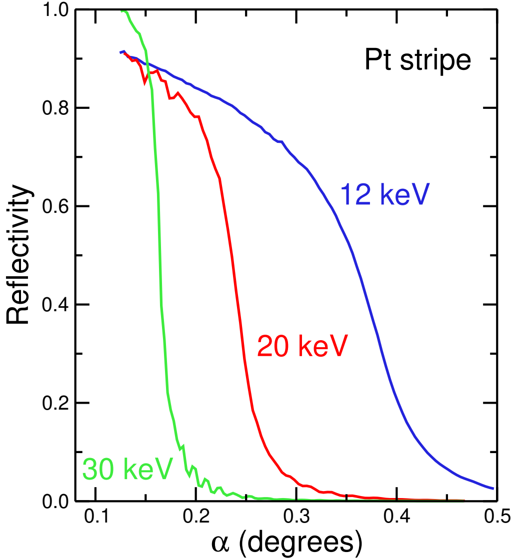

As we move up in energy the critical angle for the Pt stripe drops.

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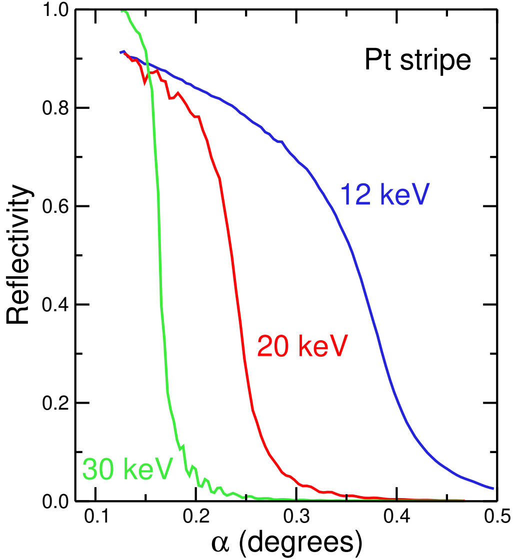

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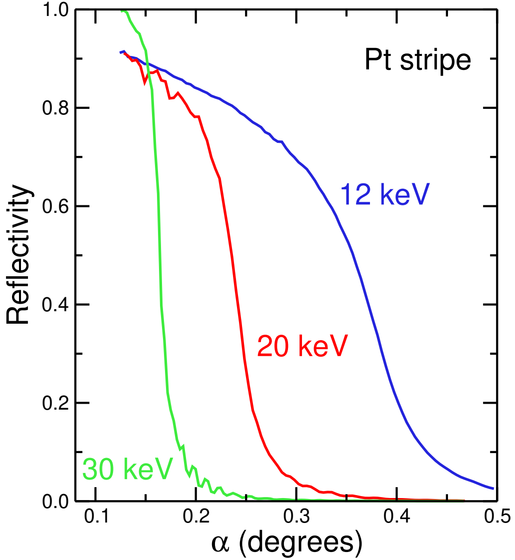
The reflectivity at low angles improves as we are well away from the Pt absorption edges at 11,565 eV, 13,273 eV, and 13,880 eV.

Mirror performance (cont.)



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As energy rises, the Pt layer begins to show the reflectivity of a thin slab.

Refractive optics

Just as with visible light, it is possible to make refractive optics for x-rays

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visible light:

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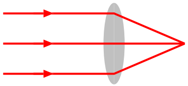
$$f \sim 0.1\text{m}$$



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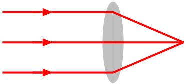

x-rays:

$$n \approx 1 - \delta, \; \delta \sim 10^{-5}$$

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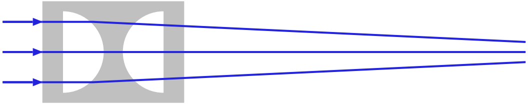

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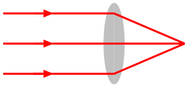

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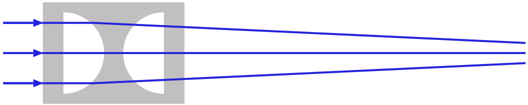

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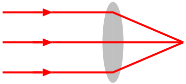

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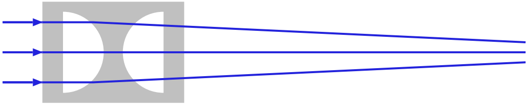
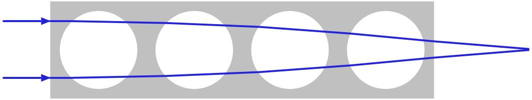
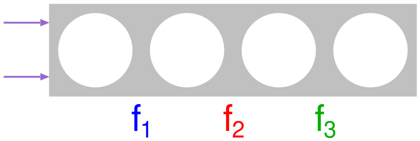


x-ray lenses are complementary to those for visible light getting manageable focal distances requires making compound lenses

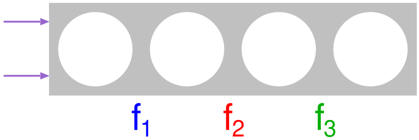


Focal length of a compound lens



Start with a 3-element compound lens, calculate effective focal length

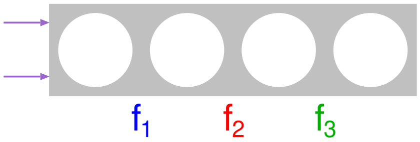
Focal length of a compound lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

Focal length of a compound lens



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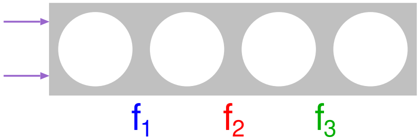

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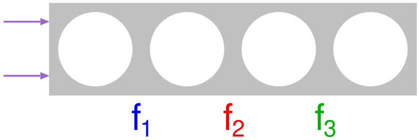
Focal length of a compound lens



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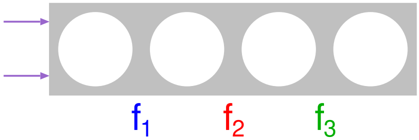
$f_1 = f,\ o_1 = \infty$

Focal length of a compound lens



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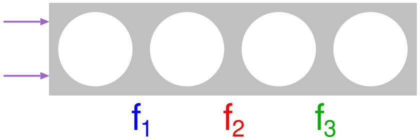
$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{o_1} \quad \longrightarrow \quad \frac{1}{i_1} = \frac{1}{f} \quad \longrightarrow \quad i_1 = f$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f, \ o_1 = \infty$

Focal length of a compound lens



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for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

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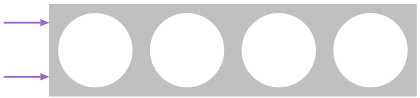

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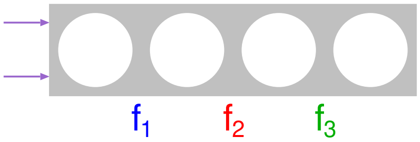

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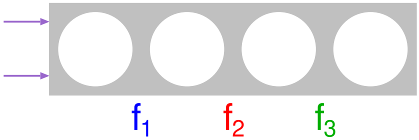

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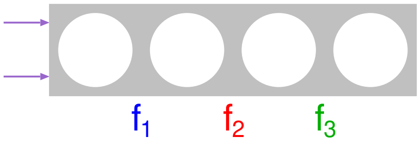
similarly for the third lens, $o_3 = -i_2$

Focal length of a compound lens



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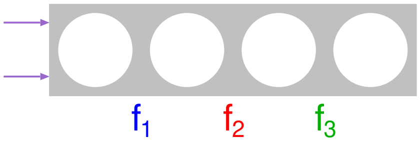

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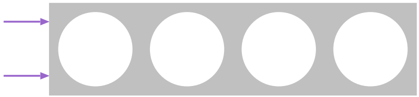
similarly for the third lens, $o_3 = -i_2$

$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{o_2} \longrightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{2}{f} \longrightarrow i_2 = \frac{f}{3}$$

Focal length of a compound lens



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Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1=f,\ o_1=\infty$

for the second lens, the image i_1 is a virtual object, $o_2=-i_1$

similarly for the third lens, $o_3=-i_2$

so for N lenses $f_{eff}=f/N$

Rephasing distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.

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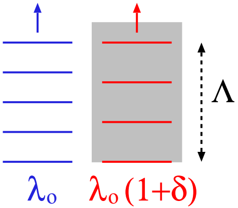

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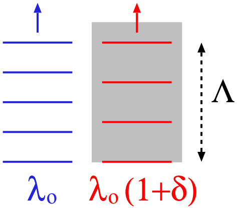

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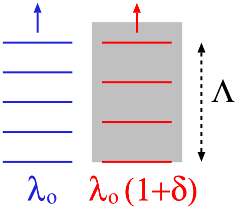

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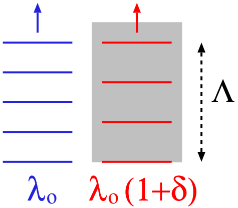

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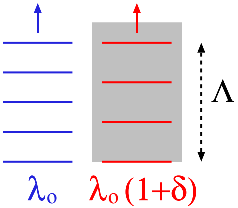

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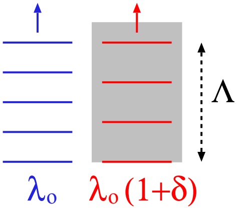

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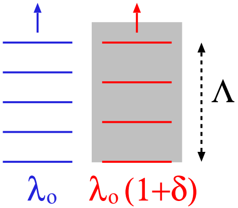

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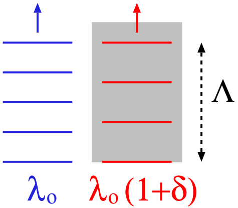

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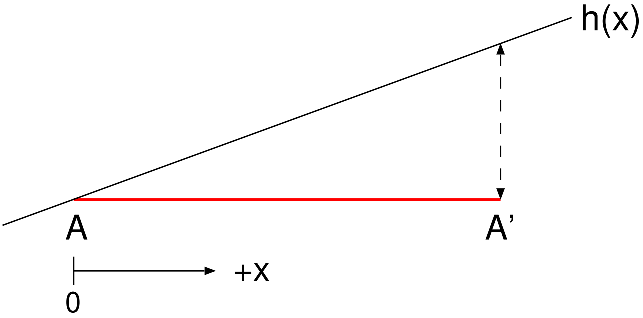
Ideal interface profile - "thin" lens



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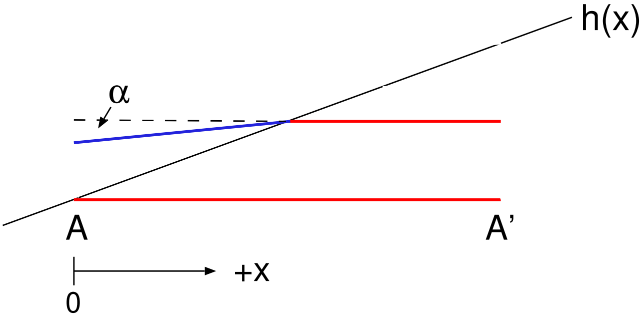

The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α.

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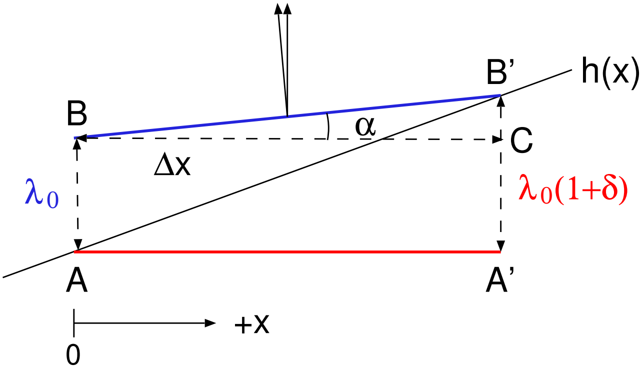

The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α.

Follow the path of two points on the wavefront, A and A' as they propagate to B and B'.

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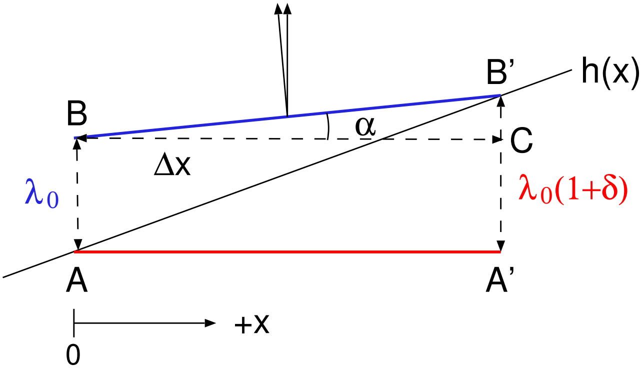

from the $AA'B'$ triangle

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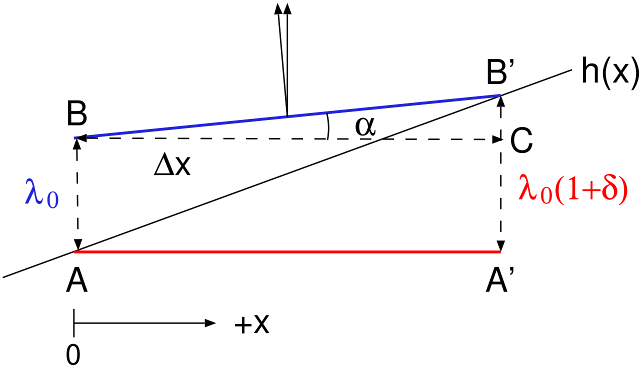

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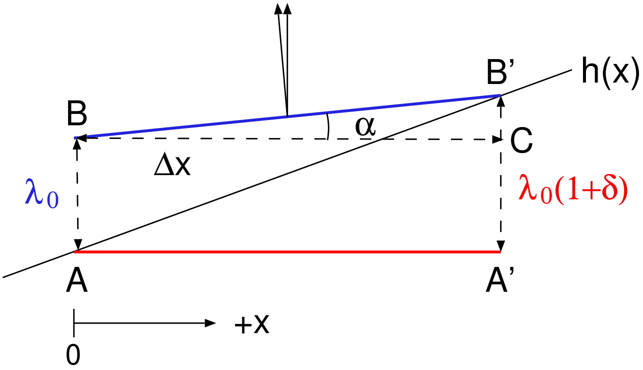

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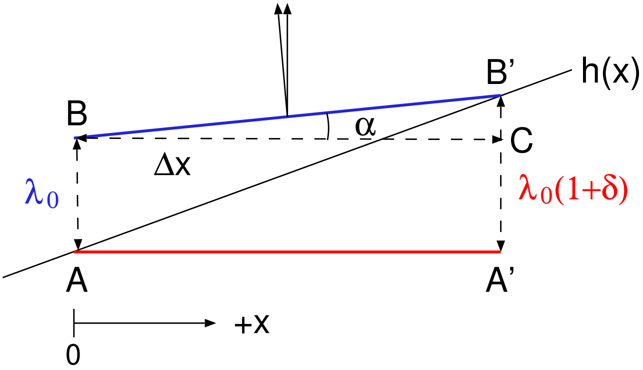

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and from the BCB' triangle

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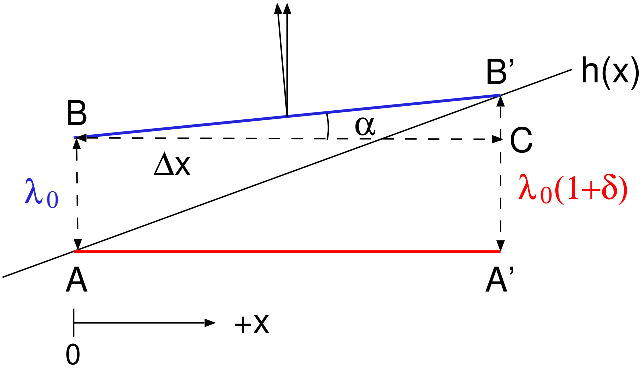

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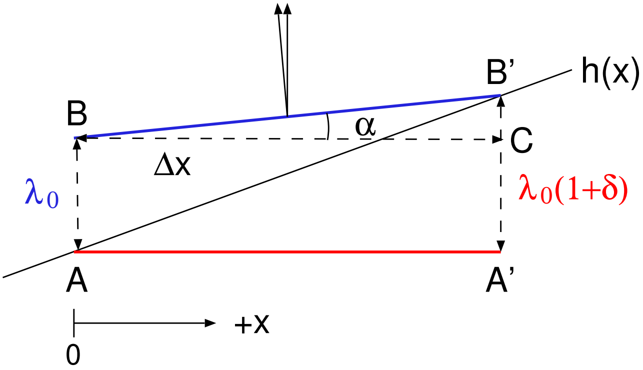

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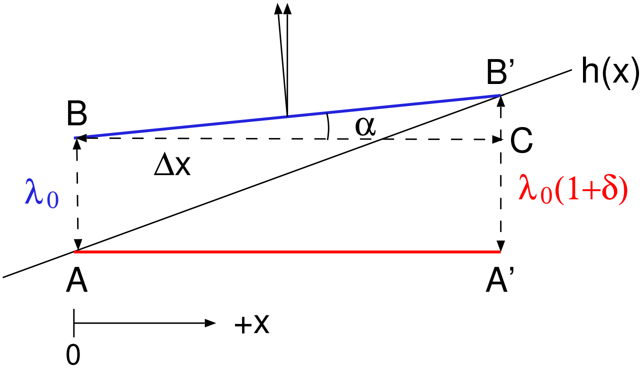

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using $\Lambda = \lambda_0/\delta$

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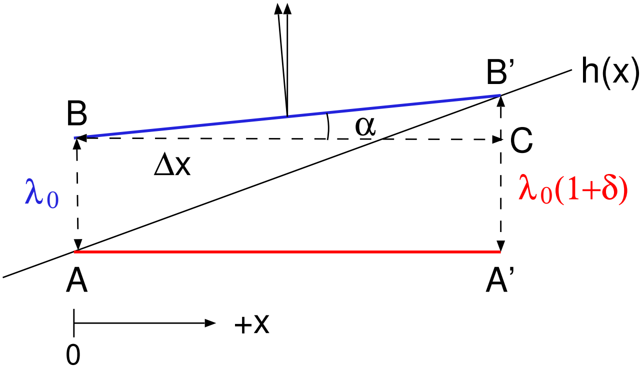

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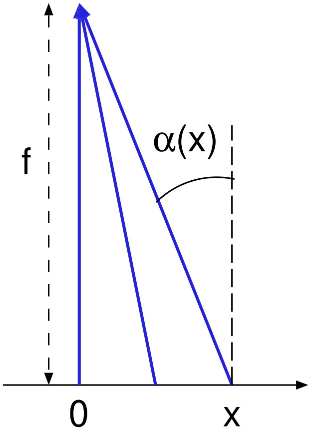
and from the BCB' triangle

using $\Lambda = \lambda_0/\delta$

$$\alpha(x) \approx \frac{\lambda_0 \delta}{\Delta x} = h'(x)\delta = h'(x)\frac{\lambda_0}{\Lambda}$$

Ideal interface profile - "thin" lens

If the desired focal length of this lens is f, the wave must be redirected at an angle which depends on the distance from the optical axis



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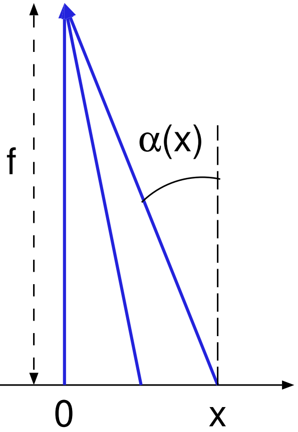
$$\alpha(x) = \frac{x}{f}$$



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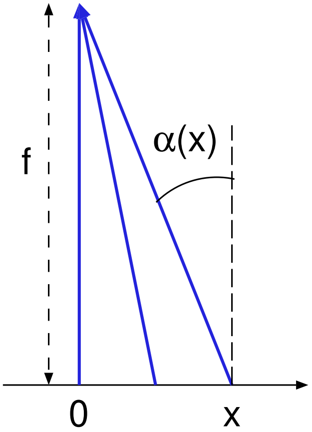
combining, we have

$$\frac{\lambda_0 h'(x)}{\Lambda} = \frac{x}{f}$$



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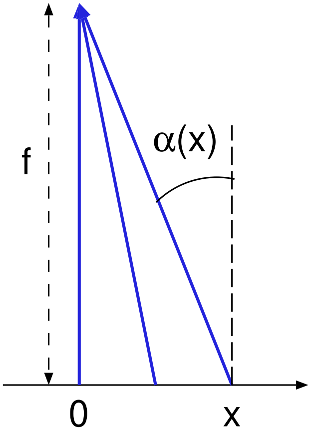

Ideal interface profile - "thin" lens

If the desired focal length of this lens is f, the wave must be redirected at an angle which depends on the distance from the optical axis

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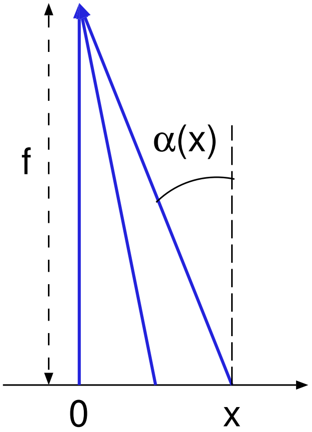

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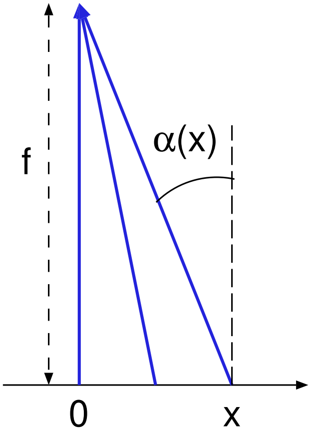

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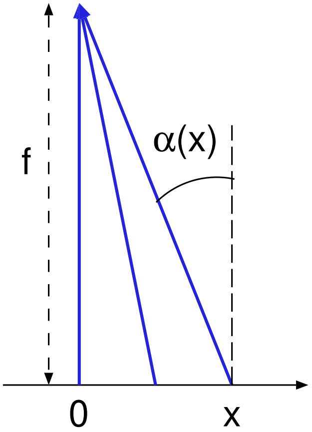

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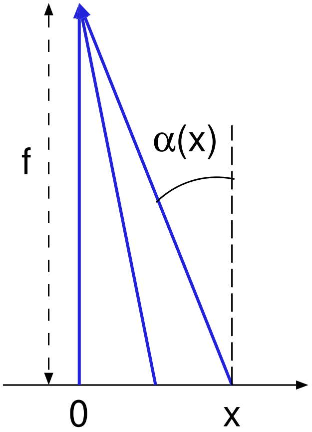

a parabola is the ideal surface shape for focusing by refraction for a “thin” lens with limited aperture

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

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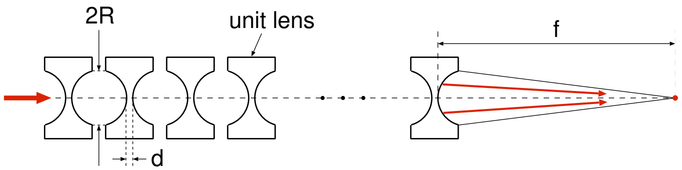
for $2N$ circular lenses we have

$$f_{2N} \approx \frac{R}{2N\delta}$$

Focussing by a beryllium lens



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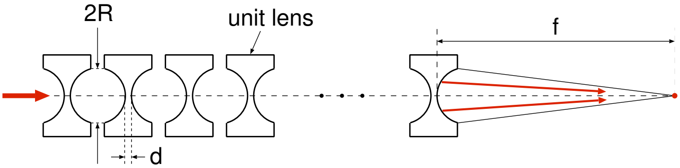

For 50 holes of radius $R = 1\text{mm}$ in beryllium (Be) at $E = 10\text{keV}$, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

$$f_N = \frac{R}{2N\delta}$$

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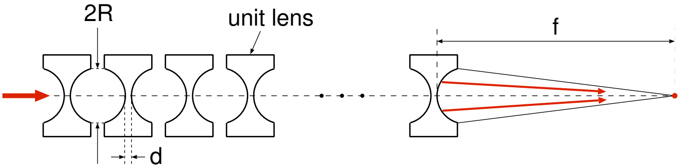

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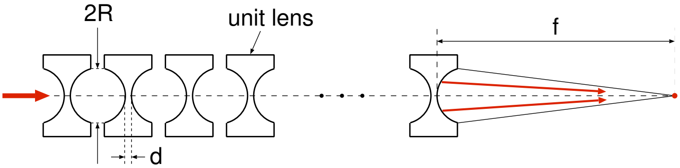

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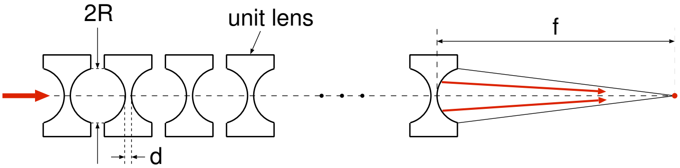

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depending on the wall thickness of the lenslets, the transmission can be up to 74%

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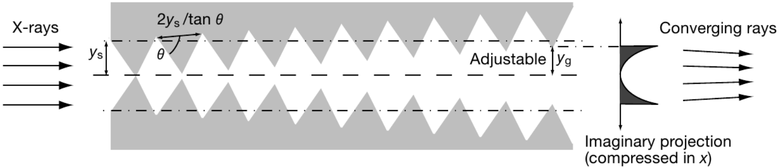
Alligator-type lenses

Perhaps one of the most original x-ray lenses has been made by using old vinyl records in an “alligator” configuration.



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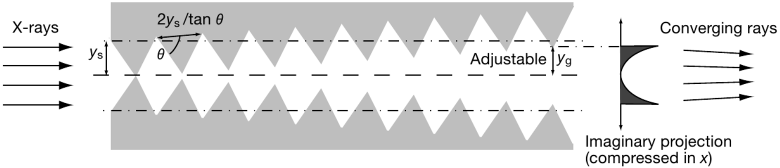

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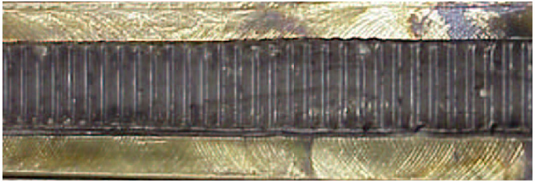

This design has also been used to make lenses out of lithium metal.

E.M. Dufresne et al., "Lithium metal for x-ray refractive optics", *Appl. Phys. Lett.* **79**, 4085 (2001).

Extruded Al lens

The compound refractive lenses (CRL) are useful for fixed focus but are difficult to use if a variable focal distance and a long focal length is required.

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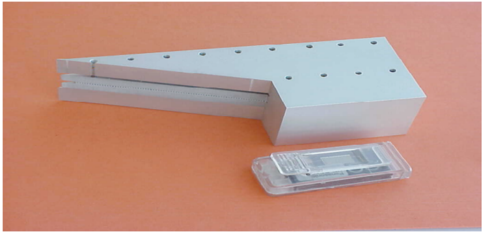

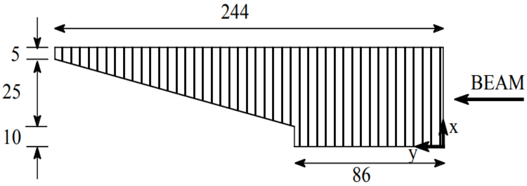

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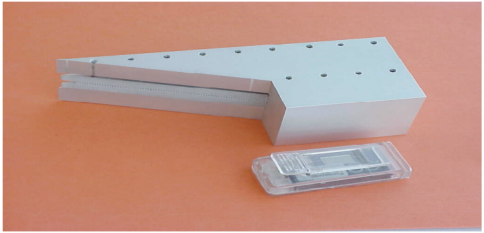

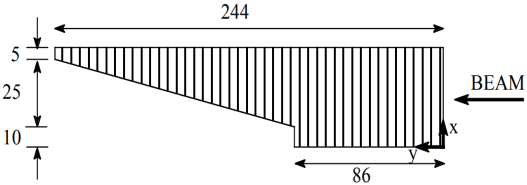

Extruded aluminum lens with parabolic figure

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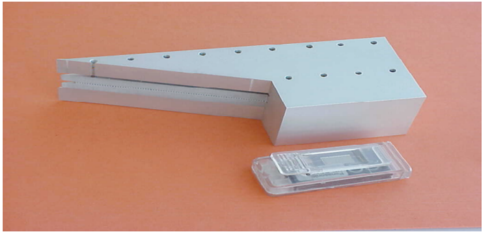

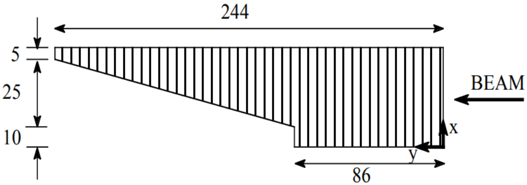

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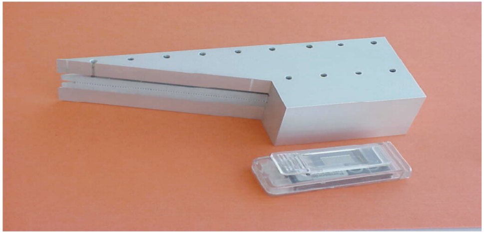

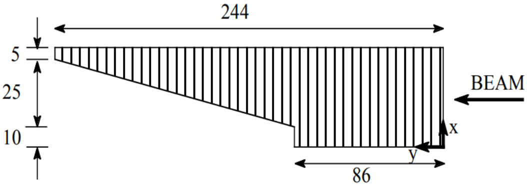

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Horizontal translation allows change in focal length but it is quantized, not continuous

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Variable focal length CRL

A continuously variable focal length is very important for two specific reasons: tracking sample position, and keeping the focal length constant as energy is changed.

B. Adams and C. Rose-Petruck, "X-ray focusing scheme with continuously variable lens," *J. Synchrotron Radiation* **22**, 16-22 (2015).

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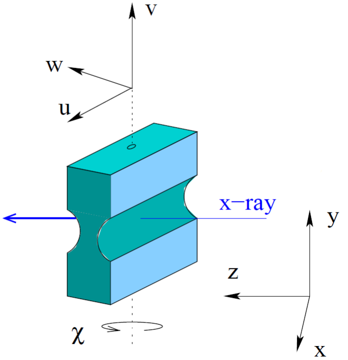

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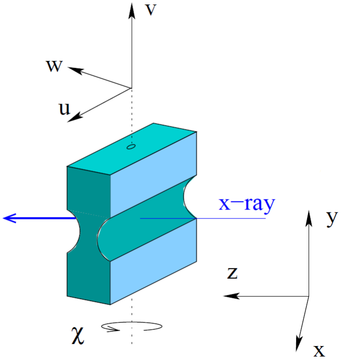
Start with a 2 hole CRL. Rotate by an angle χ about vertical axis



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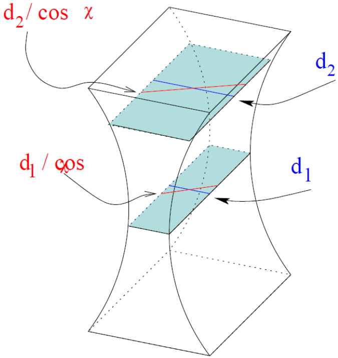

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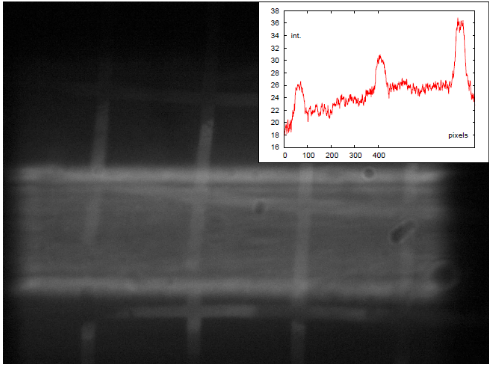
at $E = 5.5$keV and $\chi = 0^\circ$, height is over 120μm



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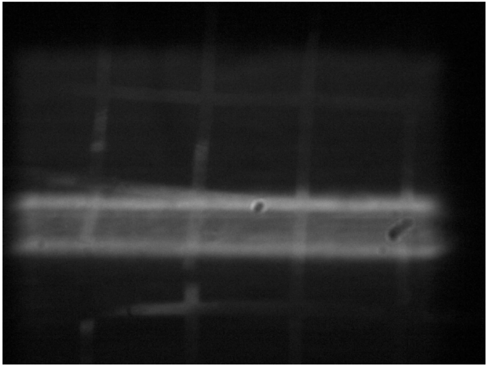

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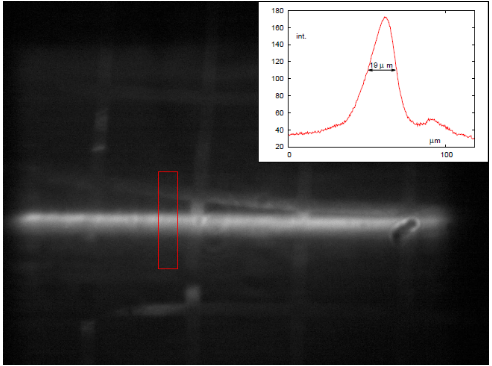
Optimal focus is 20μm at $\chi = 40^\circ$



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Polycapillary optics

A polycapillary is a focusing optic made up of an array of thousands of thin-walled hollow tubes which are $> 65\%$ empty space

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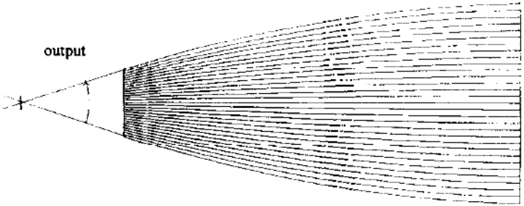

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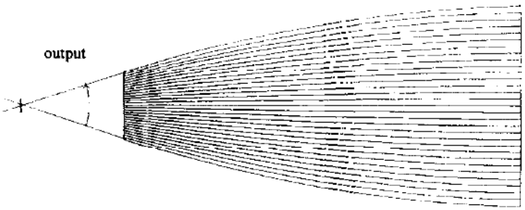

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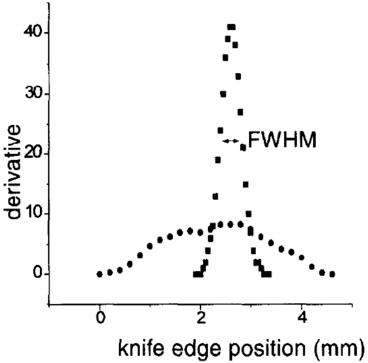

Improving polycapillary optic performance

One drawback of a glass capillary is that the transmission at high energies is reduced because of critical angle restrictions

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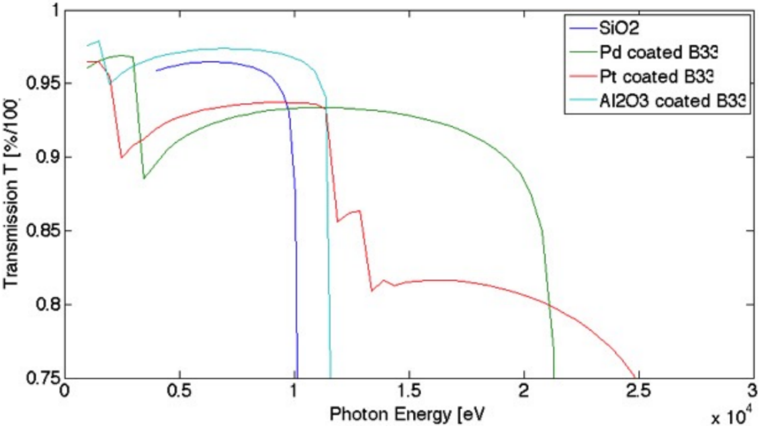

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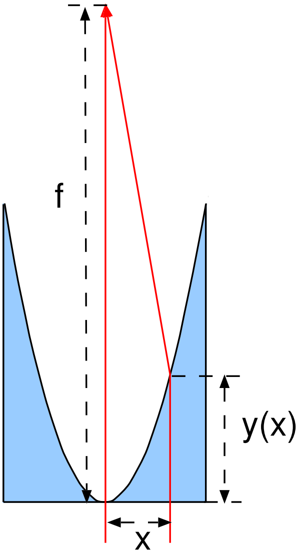

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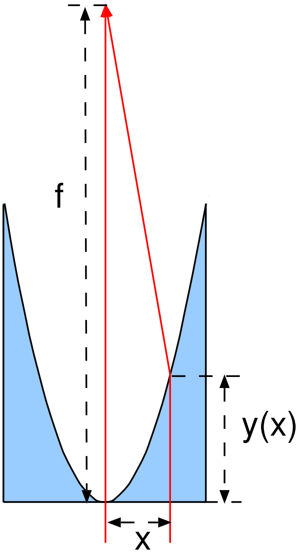

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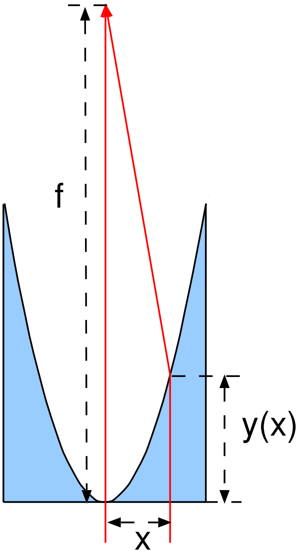

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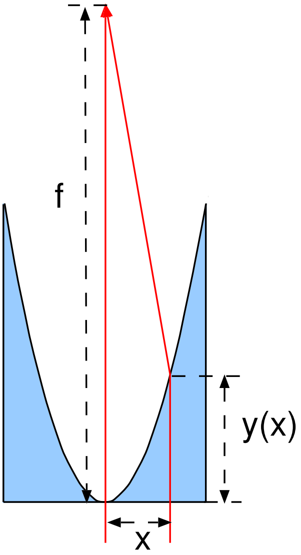

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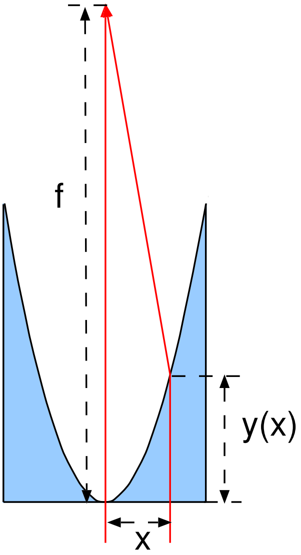
$$2f\delta y - (2\delta - \delta^2)y^2 = x^2$$



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Ellipse

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The ideal surface for a thick lens is an ellipse