Today's outline - September 16, 2024

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab
- Kiessig fringes

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab
- Kiessig fringes
- Multilayers in the kinematical regime

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab
- Kiessig fringes
- Multilayers in the kinematical regime
- Parratt's exact recursive calculation

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab
- Kiessig fringes
- Multilayers in the kinematical regime
- Parratt's exact recursive calculation

Reading Assignment: Chapter 3.7–3.8

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab
- Kiessig fringes
- Multilayers in the kinematical regime
- Parratt's exact recursive calculation

Reading Assignment: Chapter 3.7–3.8

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Today's outline - September 16, 2024

- Limiting cases of Fresnel equations
- Reflection from a thin slab
- Kiessig fringes
- Multilayers in the kinematical regime
- Parratt's exact recursive calculation

Reading Assignment: Chapter 3.7–3.8

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Homework Assignment #04:
Chapter 4: 2,4,6,7,10
due Monday, October 14, 2024

Fresnel equation review

The scattering vector (or momentum transfer) is given by

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda} \sin \alpha$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda} \sin\alpha = 2k \sin\alpha$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha$$

and for small angles

Fresnel equation review

The scattering vector (or momentum transfer) is given by

and for small angles

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda} \sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

similarly for the critical angle we define

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

similarly for the critical angle we define

$$Q_c = 2k\sin\alpha_c$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

$$q = \frac{Q}{Q_c}$$

this leads to reduced scattering vectors

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

similarly for the critical angle we define

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha,$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

similarly for the critical angle we define

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

using the reduced scattering vectors, the three defining optical equations become

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

using the reduced scattering vectors, the three defining optical equations become

Snell's Law

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

using the reduced scattering vectors, the three defining optical equations become

Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu, \quad b_\mu = \frac{2k}{Q_c^2}\mu$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

using the reduced scattering vectors, the three defining optical equations become

Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu, \quad b_\mu = \frac{2k}{Q_c^2}\mu$$

Fresnel equations

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

using the reduced scattering vectors, the three defining optical equations become

Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu, \quad b_\mu = \frac{2k}{Q_c^2}\mu$$

Fresnel equations

$$r = \frac{q - q'}{q + q'},$$

Fresnel equation review

The scattering vector (or momentum transfer) is given by

$$Q = \frac{4\pi}{\lambda}\sin\alpha = 2k\sin\alpha \approx 2k\alpha$$

and for small angles

$$Q_c = 2k\sin\alpha_c \approx 2k\alpha_c$$

similarly for the critical angle we define

this leads to reduced scattering vectors

$$q = \frac{Q}{Q_c} \approx \frac{2k}{Q_c}\alpha, \quad q' \approx \frac{2k}{Q_c}\alpha'$$

using the reduced scattering vectors, the three defining optical equations become

Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu, \quad b_\mu = \frac{2k}{Q_c^2}\mu$$

Fresnel equations

$$r = \frac{q-q'}{q+q'}, \qquad t = \frac{2q}{q+q'}$$

Limiting cases - $q \gg 1$

Starting with Snell's Law $\quad q^2 = q'^2 + 1 - 2ib_\mu$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$,

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^{\,2} + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^{\,2} = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^2 = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^2 = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2\left(1 + 2i\,\frac{Im(q')}{q}\right)$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^{\,2} + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^{\,2} = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^{\,2} = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^2 = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^2 = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^{\,2} + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^{\,2} = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^{\,2} = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^2 = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^2 = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

$$r = \frac{(q - q')(q + q')}{(q + q')(q + q')}$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

$$q' = q + i\, Im(q')$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q'^2 = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\, Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

$$r = \frac{(q - q')(q + q')}{(q + q')(q + q')} = \frac{q^2 - q'^2}{(q + q')^2}$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\, Im(q')$$

$$q'^2 = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\, Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

$$r = \frac{(q - q')(q + q')}{(q + q')(q + q')} = \frac{q^2 - q'^2}{(q + q')^2} \approx \frac{1}{(2q)^2}$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^{\,2} + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^{\,2} = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^{\,2} = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

$$r = \frac{(q-q')(q+q')}{(q+q')(q+q')} = \frac{q^2 - q'^{\,2}}{(q+q')^2} \approx \frac{1}{(2q)^2}\,, \qquad t = \frac{2q}{q+q'} \approx 1$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^{\,2} + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^{\,2} = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\,Im(q')$$

$$q'^{\,2} = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\,Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

$$r = \frac{(q-q')(q+q')}{(q+q')(q+q')} = \frac{q^2 - q'^{\,2}}{(q+q')^2} \approx \frac{1}{(2q)^2}\,, \quad t = \frac{2q}{q+q'} \approx 1\,, \quad \Lambda \approx \frac{\alpha}{\mu}$$

Limiting cases - $q \gg 1$

Starting with Snell's Law

$$q^2 = q'^2 + 1 - 2ib_\mu$$

rearrange and simplify for $q \gg 1$ and real

$$q'^2 = q^2 - 1 + 2ib_\mu \approx q^2 + 2ib_\mu$$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

$$q' = q + i\, Im(q')$$

$$q'^2 = q^2\left(1 + i\,\frac{Im(q')}{q}\right)^2 \approx q^2 + 2iq\, Im(q')$$

comparing to the equation above

$$Im(q')q \approx b_\mu \;\rightarrow\; Im(q') \approx \frac{b_\mu}{q}$$

The reflection and transmission coefficients are thus

$$r = \frac{(q-q')(q+q')}{(q+q')(q+q')} = \frac{q^2 - q'^2}{(q+q')^2} \approx \frac{1}{(2q)^2}, \quad t = \frac{2q}{q+q'} \approx 1, \quad \Lambda \approx \frac{\alpha}{\mu}$$

the reflected wave is in phase with the incident wave, almost total transmission

Limiting cases - $q \ll 1$

Starting with Snell's Law again

$$q^2 = q'^2 + 1 - 2ib_\mu$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

$$q^2 = q'^2 + 1 - 2ib_\mu$$

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

$$q^2 = q'^2 + 1 - 2ib_\mu$$

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

thus the reflection and transmission coefficients become

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')}$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'}$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

$$q^2 = q'^2 + 1 - 2ib_\mu$$

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

thus the reflection and transmission coefficients become

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'} = -1$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'} = -1$$

$$t = \frac{2q}{q + q'}$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'} = -1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q'}$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'} = -1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q'} = -2iq$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'} = -1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q'} = -2iq$$

$$\Lambda \approx \frac{1}{Q_c}$$

Limiting cases - $q \ll 1$

Starting with Snell's Law again

when $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

thus the reflection and transmission coefficients become

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu \approx -1$$

$$q' \approx i$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{-q'}{+q'} = -1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q'} = -2iq$$

$$\Lambda \approx \frac{1}{Q_c}$$

The reflected wave is out of phase with the incident wave, there is only small transmission in the form of an evanescent wave, and the penetration depth is very short.

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$
$$q'^2 = q^2 - 1 + 2ib_\mu$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1)$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1 + i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

adding and subtracting b_μ,

q' is complex with real and imaginary parts of equal magnitude.

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1 + i)^2$$

$$q' \approx \sqrt{b_\mu}(1 + i)$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1 + i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

$$q' \approx \sqrt{b_\mu}(1 + i)$$

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

$$q' \approx \sqrt{b_\mu}(1+i)$$

$$r = \frac{(q - q')}{(q + q')}$$

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

adding and subtracting b_μ,

q' is complex with real and imaginary parts of equal magnitude.

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

$$q^2 = q'^{\,2} + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^{\,2} \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

$$q' \approx \sqrt{b_\mu}(1+i)$$

$$r = \frac{(q - q')}{(q + q')} \approx \frac{q}{q} \approx 1$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

$$q' \approx \sqrt{b_\mu}(1+i)$$

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

$$r = \frac{(q - q')}{(q + q')} \approx \frac{q}{q} \approx 1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q}$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

$$q' \approx \sqrt{b_\mu}(1+i)$$

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

$$r = \frac{(q - q')}{(q + q')} \approx \frac{q}{q} \approx 1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q} = 2$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

q' is complex with real and imaginary parts of equal magnitude.

$$q' \approx \sqrt{b_\mu}(1+i)$$

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

$$r = \frac{(q-q')}{(q+q')} \approx \frac{q}{q} \approx 1$$

$$t = \frac{2q}{q+q'} \approx \frac{2q}{q} = 2$$

$$\Lambda \approx \frac{1}{Q_c\, Im(q')} \approx \frac{1}{Q_c\sqrt{b_\mu}}$$

Limiting cases - $q \sim 1$

Using Snell's Law, with $q \sim 1$,

$$q^2 = {q'}^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1) = b_\mu(1+i)^2$$

$$q' \approx \sqrt{b_\mu}(1+i)$$

q' is complex with real and imaginary parts of equal magnitude.

$$r = \frac{(q - q')}{(q + q')} \approx \frac{q}{q} \approx 1$$

$$t = \frac{2q}{q + q'} \approx \frac{2q}{q} = 2$$

since $\sqrt{b_\mu} \ll 1$, the reflection and transmission coefficients become

$$\Lambda \approx \frac{1}{Q_c\, Im(q')} \approx \frac{1}{Q_c \sqrt{b_\mu}}$$

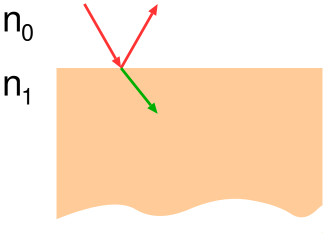
The reflected wave is in phase with the incident, there is significant (larger amplitude than the reflection) transmission with a large penetration depth.

Review of interface effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium.

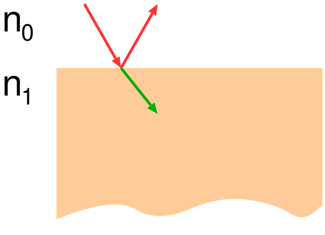
Review of interface effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium.



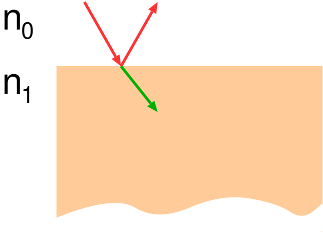
Review of interface effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.



Review of interface effects

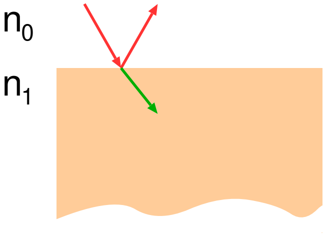
We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.



$$r = \frac{Q - Q'}{Q + Q'}$$

Review of interface effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.

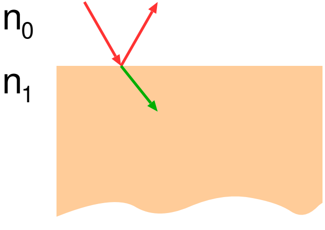


$$r = \frac{Q - Q'}{Q + Q'}$$

$$t = \frac{2Q}{Q + Q'}$$

Review of interface effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.



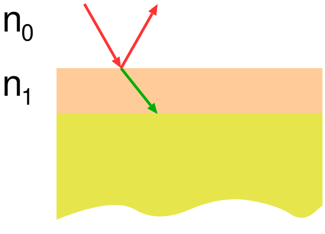
$$r = \frac{Q - Q'}{Q + Q'}$$

$$t = \frac{2Q}{Q + Q'}$$

We have assumed that the transmitted wave eventually attenuates to zero in all cases due to absorption.

Review of interface effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.



$$r = \frac{Q - Q'}{Q + Q'}$$

$$t = \frac{2Q}{Q + Q'}$$

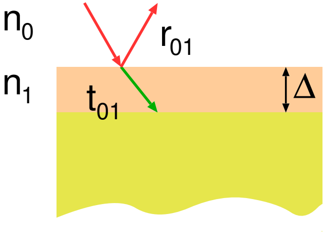
We have assumed that the transmitted wave eventually attenuates to zero in all cases due to absorption. We now consider what happens if there is a second interface encountered by the transmitted wave before it dies away. That is, a thin slab of material on top of an infinite substrate

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:

Reflection and transmission coefficients

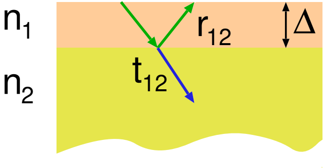
For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:



r_{01} – reflection in n_0 off n_1
t_{01} – transmission from n_0 into n_1

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:

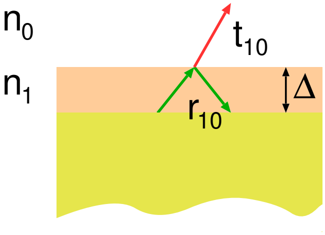


r_{01} – reflection in n_0 off n_1
t_{01} – transmission from n_0 into n_1

r_{12} – reflection in n_1 off n_2
t_{12} – transmission from n_1 into n_2

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:



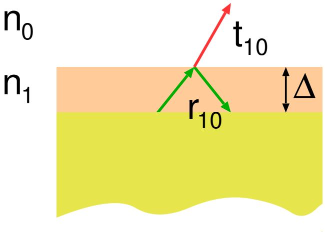
r_{01} – reflection in n_0 off n_1
t_{01} – transmission from n_0 into n_1

r_{12} – reflection in n_1 off n_2
t_{12} – transmission from n_1 into n_2

r_{10} – reflection in n_1 off n_0
t_{10} – transmission from n_1 into n_0

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:



r_{01} – reflection in n_0 off n_1
t_{01} – transmission from n_0 into n_1

r_{12} – reflection in n_1 off n_2
t_{12} – transmission from n_1 into n_2

r_{10} – reflection in n_1 off n_0
t_{10} – transmission from n_1 into n_0

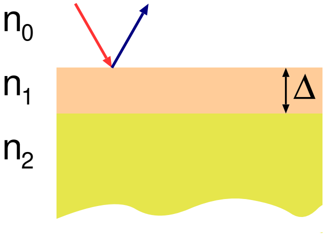
Build the composite reflection coefficient from all possible events

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed

Overall reflection from a slab

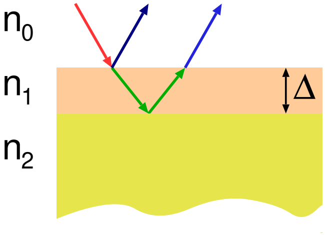
The composite reflection coefficient for each ray emerging from the top surface is computed



r_{01}

Overall reflection from a slab

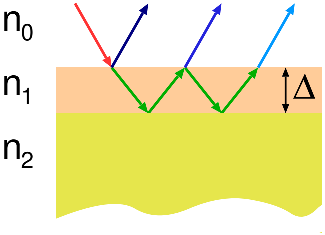
The composite reflection coefficient for each ray emerging from the top surface is computed



$$r_{01}$$
$$+$$
$$t_{01}r_{12}t_{10}$$

Overall reflection from a slab

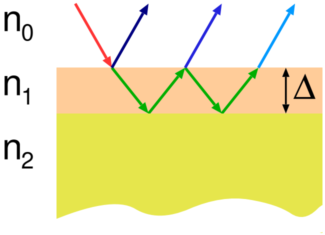
The composite reflection coefficient for each ray emerging from the top surface is computed



$$r_{01}$$
$$+$$
$$t_{01} r_{12} t_{10}$$
$$+$$
$$t_{01} r_{12} r_{10} r_{12} t_{10}$$

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed



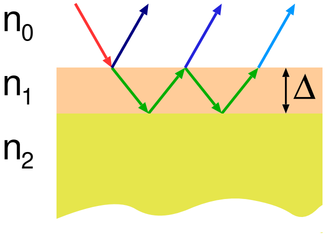
$$r_{01}$$
$$+$$
$$t_{01}r_{12}t_{10}$$
$$+$$
$$t_{01}r_{12}r_{10}r_{12}t_{10}$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta}$$

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed



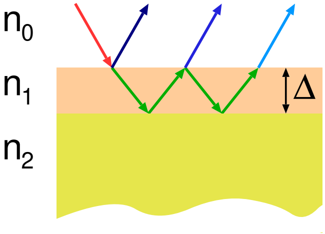
$$r_{01}$$
$$+$$
$$t_{01}r_{12}t_{10}$$
$$+$$
$$t_{01}r_{12}r_{10}r_{12}t_{10}$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta} = e^{iQ_1\Delta}$$

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed



$$r_{01} + t_{01} r_{12} t_{10} \cdot p^2 + t_{01} r_{12} r_{10} r_{12} t_{10}$$

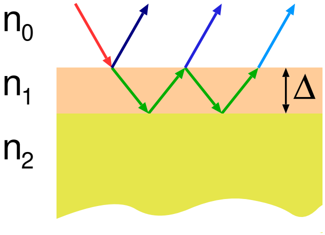
Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta} = e^{iQ_1\Delta}$$

which multiplies the reflection coefficient

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed



$$r_{01} + t_{01} r_{12} t_{10} \cdot p^2 + t_{01} r_{12} r_{10} r_{12} t_{10} \cdot p^4$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta} = e^{iQ_1\Delta}$$

which multiplies the reflection coefficient with each pass through the slab

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

factoring out the second term from all the rest

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \sum_{m=0}^{\infty} \left(r_{10} r_{12} p^2 \right)^m$$

factoring out the second term from all the rest

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01}r_{12}t_{10}p^2 + t_{01}r_{10}r_{12}^2t_{10}p^4 + t_{01}r_{10}^2r_{12}^3t_{10}p^6 + \cdots$$

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2\sum_{m=0}^{\infty}\left(r_{10}r_{12}p^2\right)^m$$

factoring out the second term from all the rest

summing the geometric series as previously

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \sum_{m=0}^{\infty} \left(r_{10} r_{12} p^2 \right)^m$$

factoring out the second term from all the rest

$$= r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

summing the geometric series as previously

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01}r_{12}t_{10}p^2 + t_{01}r_{10}r_{12}^2t_{10}p^4 + t_{01}r_{10}^2r_{12}^3t_{10}p^6 + \cdots$$

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2\sum_{m=0}^{\infty}\left(r_{10}r_{12}p^2\right)^m$$

factoring out the second term from all the rest

$$= r_{01} + t_{01}t_{10}r_{12}p^2\frac{1}{1 - r_{10}r_{12}p^2}$$

summing the geometric series as previously

The individual reflection and transmission coefficients can be determined using the Fresnel equations. Recall

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \sum_{m=0}^{\infty} \left(r_{10} r_{12} p^2 \right)^m$$

factoring out the second term from all the rest

$$= r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

summing the geometric series as previously

The individual reflection and transmission coefficients can be determined using the Fresnel equations. Recall

$$r = \frac{Q - Q'}{Q + Q'}$$

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \sum_{m=0}^{\infty} \left(r_{10} r_{12} p^2 \right)^m$$

factoring out the second term from all the rest

$$= r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

summing the geometric series as previously

The individual reflection and transmission coefficients can be determined using the Fresnel equations. Recall

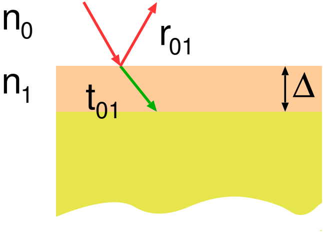
$$r = \frac{Q - Q'}{Q + Q'}, \qquad t = \frac{2Q}{Q + Q'}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface

Fresnel equation identity

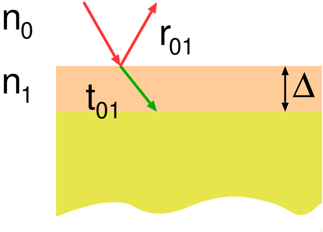
Applying the Fresnel equations to the top interface



$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface

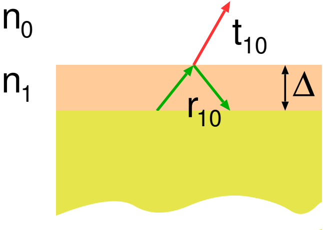


$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1}$$

$$t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface

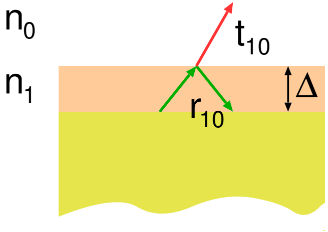


$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface

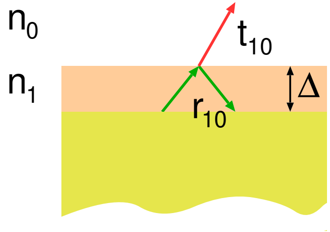


$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01} \qquad t_{10} = \frac{2Q_1}{Q_1 + Q_0}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface



$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

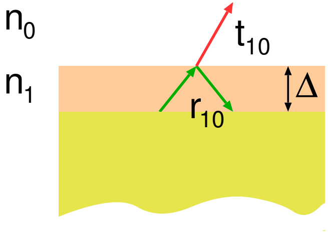
$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01} \qquad t_{10} = \frac{2Q_1}{Q_1 + Q_0}$$

we can, therefore, construct the following identity

$$r_{01}^2 + t_{01}t_{10}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface



$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

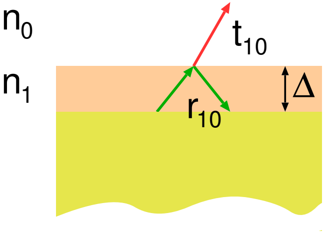
$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01} \qquad t_{10} = \frac{2Q_1}{Q_1 + Q_0}$$

we can, therefore, construct the following identity

$$r_{01}^2 + t_{01}t_{10} = \frac{(Q_0 - Q_1)^2}{(Q_0 + Q_1)^2} + \frac{2Q_0}{Q_0 + Q_1}\frac{2Q_1}{Q_1 + Q_0}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface



$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

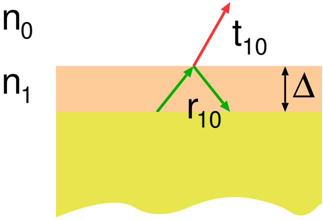
$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01} \qquad t_{10} = \frac{2Q_1}{Q_1 + Q_0}$$

we can, therefore, construct the following identity

$$r_{01}^2 + t_{01}t_{10} = \frac{(Q_0 - Q_1)^2}{(Q_0 + Q_1)^2} + \frac{2Q_0}{Q_0 + Q_1}\frac{2Q_1}{Q_1 + Q_0} = \frac{Q_0^2 - 2Q_0Q_1 + Q_1^2 + 4Q_0Q_1}{(Q_0 + Q_1)^2}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface



$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01} \qquad t_{10} = \frac{2Q_1}{Q_1 + Q_0}$$

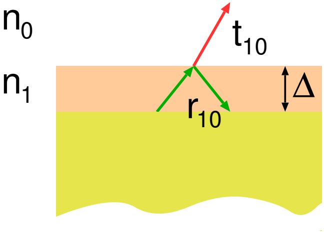
we can, therefore, construct the following identity

$$r_{01}^2 + t_{01}t_{10} = \frac{(Q_0 - Q_1)^2}{(Q_0 + Q_1)^2} + \frac{2Q_0}{Q_0 + Q_1}\frac{2Q_1}{Q_1 + Q_0} = \frac{Q_0^2 - 2Q_0Q_1 + Q_1^2 + 4Q_0Q_1}{(Q_0 + Q_1)^2}$$

$$= \frac{Q_0^2 + 2Q_0Q_1 + Q_1^2}{(Q_0 + Q_1)^2}$$

Fresnel equation identity

Applying the Fresnel equations to the top interface



$$r_{01} = \frac{Q_0 - Q_1}{Q_0 + Q_1} \qquad t_{01} = \frac{2Q_0}{Q_0 + Q_1}$$

$$r_{10} = \frac{Q_1 - Q_0}{Q_1 + Q_0} = -r_{01} \qquad t_{10} = \frac{2Q_1}{Q_1 + Q_0}$$

we can, therefore, construct the following identity

$$r_{01}^2 + t_{01}t_{10} = \frac{(Q_0 - Q_1)^2}{(Q_0 + Q_1)^2} + \frac{2Q_0}{Q_0 + Q_1}\frac{2Q_1}{Q_1 + Q_0} = \frac{Q_0^2 - 2Q_0Q_1 + Q_1^2 + 4Q_0Q_1}{(Q_0 + Q_1)^2}$$

$$= \frac{Q_0^2 + 2Q_0Q_1 + Q_1^2}{(Q_0 + Q_1)^2} = \frac{(Q_0 + Q_1)^2}{(Q_0 + Q_1)^2} = 1$$

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

Using the identity

$$t_{01}t_{10} = 1 - r_{01}^2$$

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

Using the identity

$$t_{01} t_{10} = 1 - r_{01}^2$$

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

Using the identity

$$t_{01}t_{10} = 1 - r_{01}^2$$

Expanding over a common denominator and recalling that $r_{10} = -r_{01}$.

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= \frac{r_{01} + r_{01}^2 r_{12}p^2 + \left(1 - r_{01}^2\right) r_{12}p^2}{1 - r_{10}r_{12}p^2}$$

Using the identity

$$t_{01}t_{10} = 1 - r_{01}^2$$

Expanding over a common denominator and recalling that $r_{10} = -r_{01}$.

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= \frac{r_{01} + r_{01}^2 r_{12}p^2 + \left(1 - r_{01}^2\right) r_{12}p^2}{1 - r_{10}r_{12}p^2}$$

$$r_{slab} = \frac{r_{01} + r_{12}p^2}{1 + r_{01}r_{12}p^2}$$

Using the identity

$$t_{01}t_{10} = 1 - r_{01}^2$$

Expanding over a common denominator and recalling that $r_{10} = -r_{01}$.

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

Using the identity

$$t_{01} t_{10} = 1 - r_{01}^2$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

Expanding over a common denominator and recalling that $r_{10} = -r_{01}$.

$$= \frac{r_{01} + r_{01}^2 r_{12} p^2 + \left(1 - r_{01}^2\right) r_{12} p^2}{1 - r_{10} r_{12} p^2}$$

In the case of $n_0 = n_2$ there is the further simplification of $r_{12} = -r_{01}$.

$$r_{slab} = \frac{r_{01} + r_{12} p^2}{1 + r_{01} r_{12} p^2}$$

Reflection coefficient of a slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01}t_{10}r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12}p^2 \frac{1}{1 - r_{10}r_{12}p^2}$$

$$= \frac{r_{01} + r_{01}^2 r_{12}p^2 + \left(1 - r_{01}^2\right) r_{12}p^2}{1 - r_{10}r_{12}p^2}$$

$$r_{slab} = \frac{r_{01} + r_{12}p^2}{1 + r_{01}r_{12}p^2} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Using the identity

$$t_{01}t_{10} = 1 - r_{01}^2$$

Expanding over a common denominator and recalling that $r_{10} = -r_{01}$.

In the case of $n_0 = n_2$ there is the further simplification of $r_{12} = -r_{01}$.

Kiessig fringes

$$p^2 = e^{iQ_1\Delta}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Kiessig fringes

$$p^2 = e^{iQ_1\Delta}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

If we plot the reflectivity

$$R_{slab} = \left|r_{slab}\right|^2$$

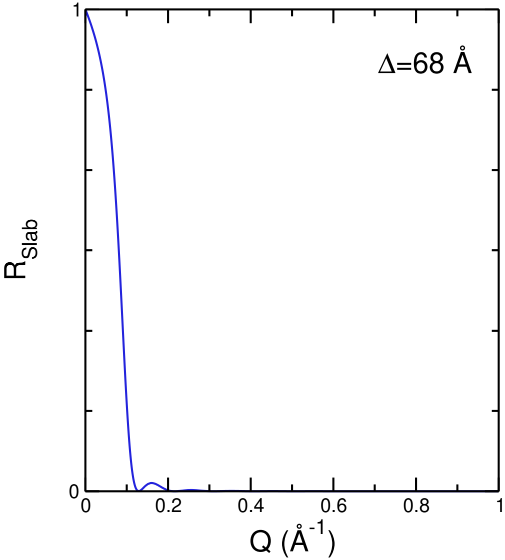
Kiessig fringes

$$p^2 = e^{iQ_1\Delta}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

If we plot the reflectivity

$$R_{slab} = \left|r_{slab}\right|^2$$



Kiessig fringes

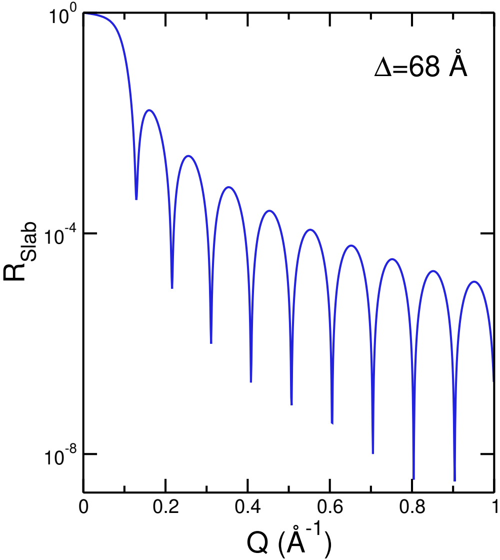
$$p^2 = e^{iQ_1\Delta}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

If we plot the reflectivity

$$R_{slab} = \left|r_{slab}\right|^2$$

These are the so=-called Kiessig fringes which arise from interference between reflections at the top and bottom of the slab.



Kiessig fringes

$$p^2 = e^{iQ_1\Delta}$$

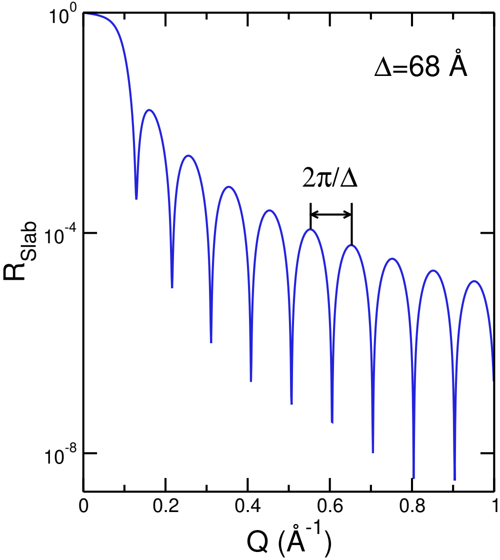
$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

If we plot the reflectivity

$$R_{slab} = \left|r_{slab}\right|^2$$

These are the so=-called Kiessig fringes which arise from interference between reflections at the top and bottom of the slab. They have an oscillation frequency

$$2\pi/\Delta = 0.092\text{Å}^{-1}$$



Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab.

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab.

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle

$$q \gg 1$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c \qquad r_{01} = \frac{q_0 - q_1}{q_0 + q_1}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c \qquad r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2}$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right) \approx \left(\frac{Q_c}{2Q_0}\right)^2\left(1 - e^{iQ\Delta}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right) \approx \left(\frac{Q_c}{2Q_0}\right)^2 \left(1 - e^{iQ\Delta}\right)$$

$$= -\frac{16\pi\rho r_0}{4Q^2} e^{iQ\Delta/2}\left(e^{iQ\Delta/2} - e^{-iQ\Delta/2}\right)$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right) \approx \left(\frac{Q_c}{2Q_0}\right)^2 \left(1 - e^{iQ\Delta}\right)$$

$$= -\frac{16\pi\rho r_0}{4Q^2} e^{iQ\Delta/2}\left(e^{iQ\Delta/2} - e^{-iQ\Delta/2}\right) = -i\left(\frac{4\pi\rho r_0 \Delta}{Q}\right)\frac{\sin(Q\Delta/2)}{Q\Delta/2} e^{iQ\Delta/2}$$

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1 \qquad |r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right) \approx \left(\frac{Q_c}{2Q_0}\right)^2\left(1 - e^{iQ\Delta}\right)$$

$$= -\frac{16\pi\rho r_0}{4Q^2} e^{iQ\Delta/2}\left(e^{iQ\Delta/2} - e^{-iQ\Delta/2}\right) = -i\left(\frac{4\pi\rho r_0\Delta}{Q}\right)\frac{\sin(Q\Delta/2)}{Q\Delta/2} e^{iQ\Delta/2}$$

Since $Q\Delta \ll 1$ for a thin slab

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right) \approx \left(\frac{Q_c}{2Q_0}\right)^2\left(1 - e^{iQ\Delta}\right)$$

$$= -\frac{16\pi\rho r_0}{4Q^2} e^{iQ\Delta/2}\left(e^{iQ\Delta/2} - e^{-iQ\Delta/2}\right) = -i\left(\frac{4\pi\rho r_0\Delta}{Q}\right)\cancel{\frac{\sin(Q\Delta/2)}{Q\Delta/2}}\,\cancel{e^{iQ\Delta/2}}$$

Since $Q\Delta \ll 1$ for a thin slab

Kinematical reflection from a thin slab

Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle refraction effects can be ignored and we are in the "kinematical" regime.

$$q \gg 1$$

$$|r_{01}| \ll 1 \qquad \alpha > \alpha_c$$

$$r_{01} = \frac{q_0 - q_1}{q_0 + q_1}\frac{q_0 + q_1}{q_0 + q_1} = \frac{q_0^2 - q_1^2}{(q_0 + q_1)^2} \approx \frac{1}{(2q_0)^2} = \left(\frac{Q_c}{2Q_0}\right)^2$$

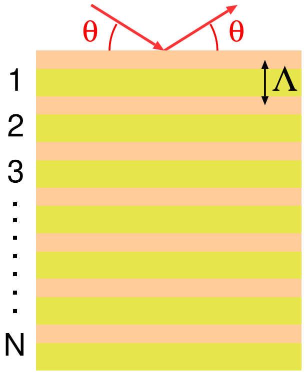
$$r_{slab} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2} \approx r_{01}\left(1 - p^2\right) \approx r_{01}\left(1 - e^{iQ\Delta}\right) \approx \left(\frac{Q_c}{2Q_0}\right)^2\left(1 - e^{iQ\Delta}\right)$$

$$= -\frac{16\pi\rho r_0}{4Q^2} e^{iQ\Delta/2}\left(e^{iQ\Delta/2} - e^{-iQ\Delta/2}\right) = -i\left(\frac{4\pi\rho r_0\Delta}{Q}\right)\cancel{\frac{\sin(Q\Delta/2)}{Q\Delta/2}}\cancel{e^{iQ\Delta/2}}$$

$$\approx -i\frac{\lambda\rho r_0\Delta}{\sin\alpha} = r_{thin\,slab}$$

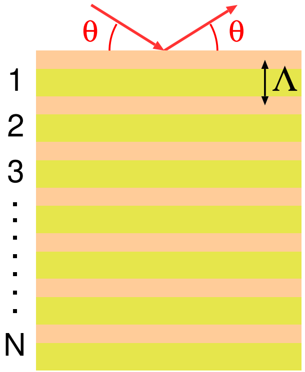
Since $Q\Delta \ll 1$ for a thin slab

Multilayers in the kinematical regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

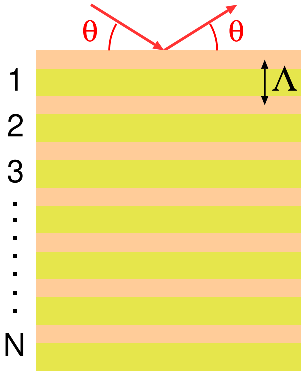
Multilayers in the kinematical regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast $(\rho_A > \rho_B)$.

r_1 is the reflectivity of a single bilayer

Multilayers in the kinematical regime

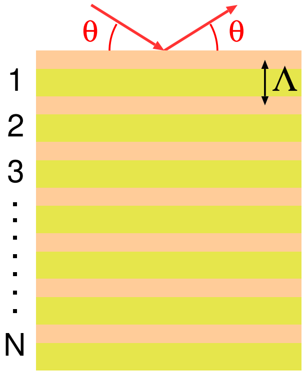


N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

r_1 is the reflectivity of a single bilayer

β is the average absorption per bilayer

Multilayers in the kinematical regime



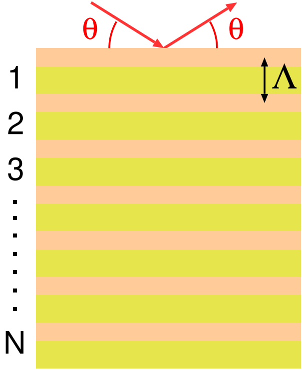
N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

r_1 is the reflectivity of a single bilayer

β is the average absorption per bilayer

$\zeta = Q\Lambda/2\pi$ is a dimensionless parameter related to the phase shift of a single bilayer

Multilayers in the kinematical regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast $(\rho_A > \rho_B)$.

r_1 is the reflectivity of a single bilayer

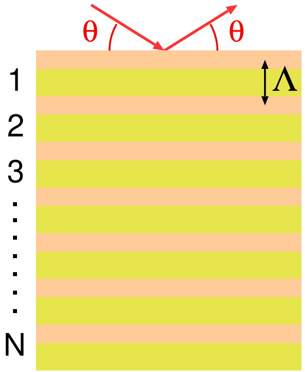
β is the average absorption per bilayer

$\zeta = Q\Lambda/2\pi$ is a dimensionless parameter related to the phase shift of a single bilayer

Form a stack of N bilayers

$$r_N(\zeta) = \sum_{\nu=0}^{N-1} r_1(\zeta) e^{i2\pi\zeta\nu} e^{-\beta\nu}$$

Multilayers in the kinematical regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

r_1 is the reflectivity of a single bilayer

β is the average absorption per bilayer

$\zeta = Q\Lambda/2\pi$ is a dimensionless parameter related to the phase shift of a single bilayer

Form a stack of N bilayers

$$r_N(\zeta) = \sum_{\nu=0}^{N-1} r_1(\zeta) e^{i2\pi\zeta\nu} e^{-\beta\nu} = r_1(\zeta) \frac{1 - e^{i2\pi\zeta N} e^{-\beta N}}{1 - e^{i2\pi\zeta} e^{-\beta}}$$

Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components

Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components

$$\rho \quad \longrightarrow \quad \rho_{AB} = \rho_A - \rho_B$$

Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components and assuming that material A is a fraction Γ of the bilayer thickness

$$\rho \quad \longrightarrow \quad \rho_{AB} = \rho_A - \rho_B$$

$$r_1(\zeta) = -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta} \int_{-\Gamma\Lambda/2}^{+\Gamma\Lambda/2} e^{i2\pi\zeta z/\Lambda} dz$$

Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components and assuming that material A is a fraction Γ of the bilayer thickness

$$\rho \quad \longrightarrow \quad \rho_{AB} = \rho_A - \rho_B$$

$$r_1(\zeta) = -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta} \int_{-\Gamma\Lambda/2}^{+\Gamma\Lambda/2} e^{i2\pi\zeta z/\Lambda} dz$$

$$= -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta} \frac{\Lambda}{i2\pi\zeta} \left[e^{i\pi\zeta\Gamma} - e^{-i\pi\zeta\Gamma} \right]$$

Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components and assuming that material A is a fraction Γ of the bilayer thickness

$$\rho \quad \longrightarrow \quad \rho_{AB} = \rho_A - \rho_B$$

$$r_1(\zeta) = -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta}\int_{-\Gamma\Lambda/2}^{+\Gamma\Lambda/2} e^{i2\pi\zeta z/\Lambda} dz$$

$$= -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta}\frac{\Lambda}{i2\pi\zeta}\left[e^{i\pi\zeta\Gamma} - e^{-i\pi\zeta\Gamma}\right]$$

$$e^{ix} - e^{-ix} = 2i\sin x$$

$$Q = 4\pi\sin\theta/\lambda = 2\pi\zeta/\Lambda$$

Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components and assuming that material A is a fraction Γ of the bilayer thickness

$$\rho \quad \longrightarrow \quad \rho_{AB} = \rho_A - \rho_B$$

$$r_1(\zeta) = -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta}\int_{-\Gamma\Lambda/2}^{+\Gamma\Lambda/2} e^{i2\pi\zeta z/\Lambda} dz$$

$$e^{ix} - e^{-ix} = 2i\sin x$$

$$= -i\frac{\lambda r_0 \rho_{AB}}{\sin\theta}\frac{\Lambda}{i2\pi\zeta}\left[e^{i\pi\zeta\Gamma} - e^{-i\pi\zeta\Gamma}\right]$$

$$Q = 4\pi\sin\theta/\lambda = 2\pi\zeta/\Lambda$$

$$r_1(\zeta) = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}$$

Absorption coefficient of a bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

Absorption coefficient of a bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

The incident x-ray has a path length $\Lambda/\sin\theta$ in a bilayer, a fraction Γ through n_A and a fraction $(1-\Gamma)$ through n_B.

Absorption coefficient of a bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

The incident x-ray has a path length $\Lambda/\sin\theta$ in a bilayer, a fraction Γ through n_A and a fraction $(1-\Gamma)$ through n_B. The amplitude absorption coefficient, β is

Absorption coefficient of a bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin\left(\pi\Gamma\zeta\right)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

The incident x-ray has a path length $\Lambda/\sin\theta$ in a bilayer, a fraction Γ through n_A and a fraction $(1-\Gamma)$ through n_B. The amplitude absorption coefficient, β is

$$\beta = 2\left[\frac{\mu_A}{2}\frac{\Gamma\Lambda}{\sin\theta}+\frac{\mu_B}{2}\frac{(1-\Gamma)\Lambda}{\sin\theta}\right]$$

Absorption coefficient of a bilayer

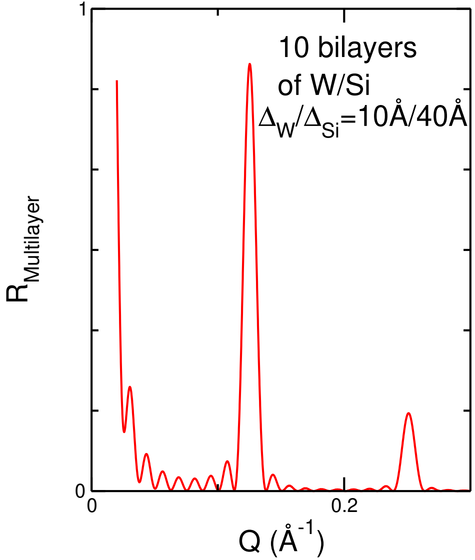
The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

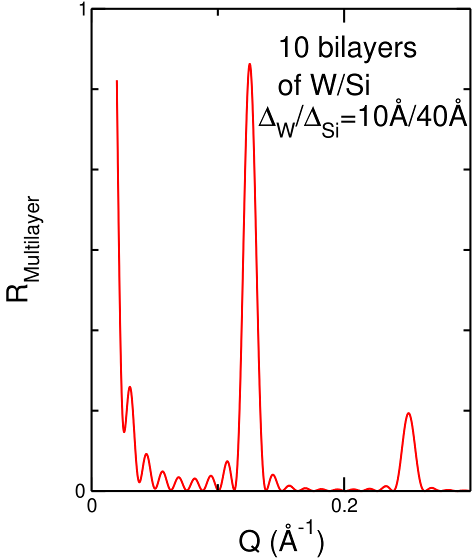
The incident x-ray has a path length $\Lambda/\sin\theta$ in a bilayer, a fraction Γ through n_A and a fraction $(1-\Gamma)$ through n_B. The amplitude absorption coefficient, β is

$$\beta = 2\left[\frac{\mu_A}{2}\frac{\Gamma\Lambda}{\sin\theta} + \frac{\mu_B}{2}\frac{(1-\Gamma)\Lambda}{\sin\theta}\right] = \frac{\Lambda}{\sin\theta}\left[\mu_A\Gamma + \mu_B(1-\Gamma)\right]$$

Reflectivity calculation

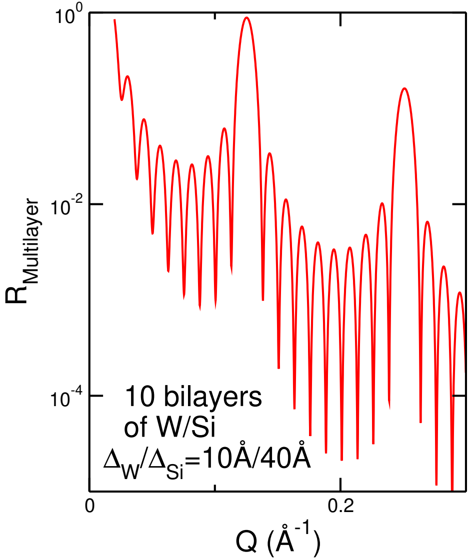


Reflectivity calculation



When $\zeta = Q\Lambda/2\pi$ is an integer, we have peaks

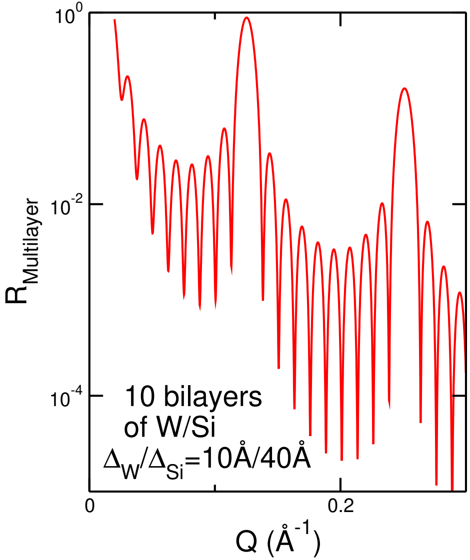
Reflectivity calculation



When $\zeta = Q\Lambda/2\pi$ is an integer, we have peaks

As N becomes larger, these peaks would become more prominent

Reflectivity calculation

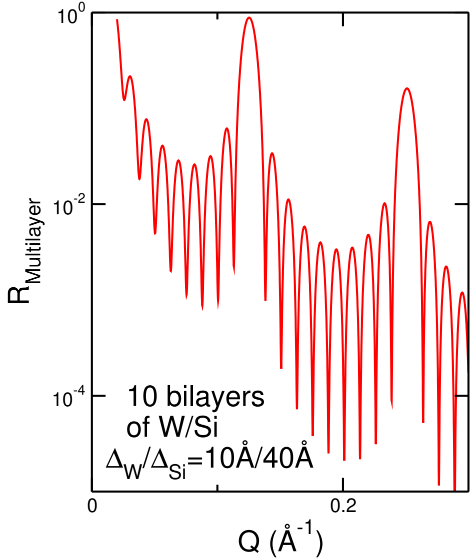


When $\zeta = Q\Lambda/2\pi$ is an integer, we have peaks

As N becomes larger, these peaks would become more prominent

This is effectively a diffraction grating for x-rays

Reflectivity calculation



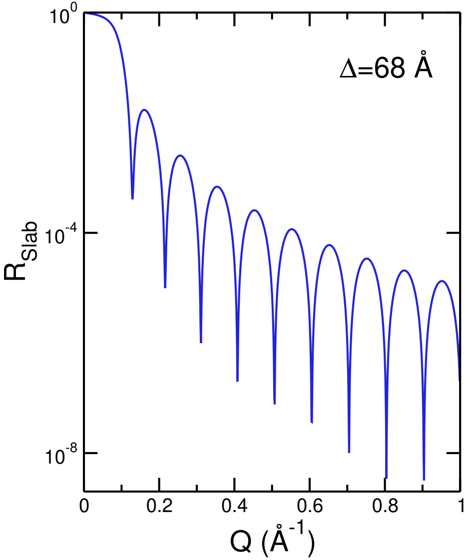
When $\zeta = Q\Lambda/2\pi$ is an integer, we have peaks

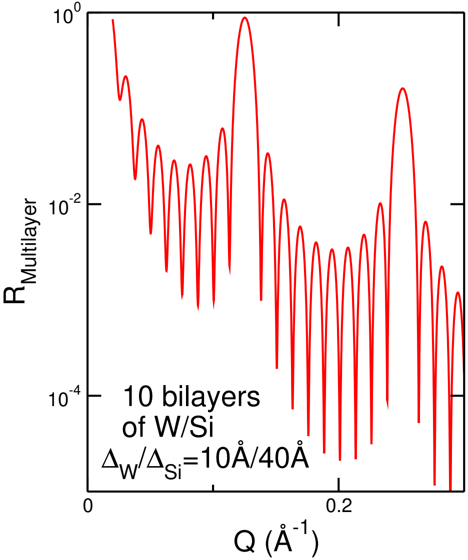
As N becomes larger, these peaks would become more prominent

This is effectively a diffraction grating for x-rays

Multilayers are used commonly on laboratory sources as well as at synchrotrons as mirrors

Slab - multilayer comparison





Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

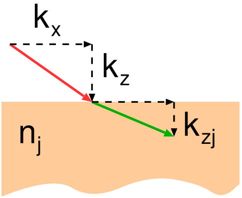
take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$



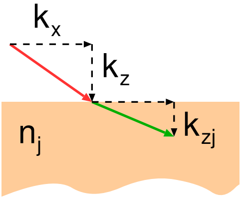
Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$

$$k_{zj}^2 = (n_j k)^2 - k_x^2$$



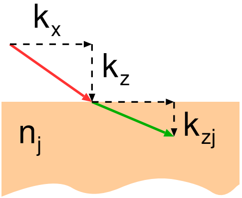
Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$

$$k_{zj}^2 = (n_j k)^2 - k_x^2 = (1 - \delta_j + i\beta_j)^2 k^2 - k_x^2$$



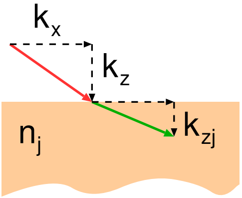
Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$

$$\begin{aligned} k_{zj}^2 &= (n_j k)^2 - k_x^2 = (1 - \delta_j + i\beta_j)^2 k^2 - k_x^2 \\ &\approx k_z^2 - 2\delta_j k^2 + 2i\beta_j k^2 \end{aligned}$$

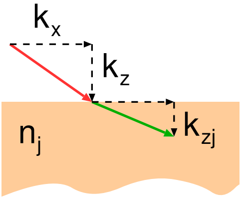


Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$



$$
\begin{aligned}
k_{zj}^2 &= (n_j k)^2 - k_x^2 = (1 - \delta_j + i\beta_j)^2 k^2 - k_x^2 \\
&\approx k_z^2 - 2\delta_j k^2 + 2i\beta_j k^2
\end{aligned}
$$

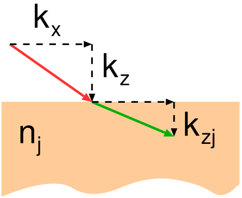
and the wavevector transfer in the j^{th} layer

Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$



$$
\begin{aligned}
k_{zj}^2 &= (n_j k)^2 - k_x^2 = (1 - \delta_j + i\beta_j)^2 k^2 - k_x^2 \\
&\approx k_z^2 - 2\delta_j k^2 + 2i\beta_j k^2
\end{aligned}
$$

$$Q_j = 2k_j \sin\alpha_j = 2k_{zj}$$

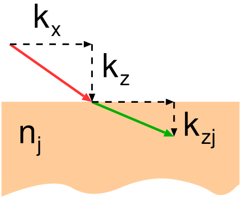
and the wavevector transfer in the jth layer

Parratt's recursive method

Treat the multilayer as a stratified medium on top of an infinitely thick substrate.

take Δ_j as the thickness of each layer and $n_j = 1 - \delta_j + i\beta_j$ as the index of refraction of each layer

because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$



$$
\begin{aligned}
k_{zj}^2 &= (n_j k)^2 - k_x^2 = (1 - \delta_j + i\beta_j)^2 k^2 - k_x^2 \\
&\approx k_z^2 - 2\delta_j k^2 + 2i\beta_j k^2
\end{aligned}
$$

$$
Q_j = 2k_j \sin\alpha_j = 2k_{zj} = \sqrt{Q^2 - 8k^2\delta_j + 8ik^2\beta_j}
$$

and the wavevector transfer in the jth layer

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

$$r'_{j,j+1} = \frac{Q_j - Q_{j+1}}{Q_j + Q_{j+1}}$$

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

$$r'_{j,j+1} = \frac{Q_j - Q_{j+1}}{Q_j + Q_{j+1}}$$

Now start calculating the reflectivity from the bottom of the N^{th} layer, closest to the substrate, where multiple reflections are not present

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

$$r'_{j,j+1} = \frac{Q_j - Q_{j+1}}{Q_j + Q_{j+1}}$$

Now start calculating the reflectivity from the bottom of the N^{th} layer, closest to the substrate, where multiple reflections are not present

$$r'_{N,\infty} = \frac{Q_N - Q_\infty}{Q_N + Q_\infty}$$

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

$$r'_{j,j+1} = \frac{Q_j - Q_{j+1}}{Q_j + Q_{j+1}}$$

Now start calculating the reflectivity from the bottom of the N^{th} layer, closest to the substrate, where multiple reflections are not present

$$r'_{N,\infty} = \frac{Q_N - Q_\infty}{Q_N + Q_\infty}$$

The reflectivity from the top of the N^{th} layer, including multiple reflections is now calculated (note no prime!)

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

$$r'_{j,j+1} = \frac{Q_j - Q_{j+1}}{Q_j + Q_{j+1}}$$

Now start calculating the reflectivity from the bottom of the N^{th} layer, closest to the substrate, where multiple reflections are not present

$$r'_{N,\infty} = \frac{Q_N - Q_\infty}{Q_N + Q_\infty}$$

The reflectivity from the top of the N^{th} layer, including multiple reflections is now calculated (note no prime!)

$$r_{N-1,N} = \frac{r'_{N-1,N} + r'_{N,\infty} p_N^2}{1 + r'_{N-1,N} r'_{N,\infty} p_N^2}$$

Parratt reflectivity calculation

The reflectivity from the interface between layer j and $j+1$, not including multiple reflections is

$$r'_{j,j+1} = \frac{Q_j - Q_{j+1}}{Q_j + Q_{j+1}}$$

Now start calculating the reflectivity from the bottom of the N^{th} layer, closest to the substrate, where multiple reflections are not present

$$r'_{N,\infty} = \frac{Q_N - Q_\infty}{Q_N + Q_\infty}$$

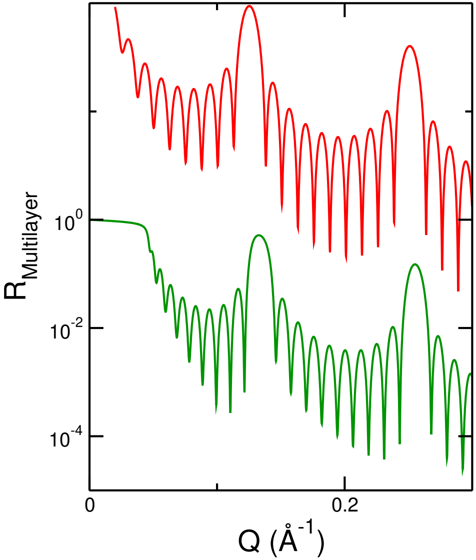
The reflectivity from the top of the N^{th} layer, including multiple reflections is now calculated (note no prime!)

$$r_{N-1,N} = \frac{r'_{N-1,N} + r'_{N,\infty} p_N^2}{1 + r'_{N-1,N} r'_{N,\infty} p_N^2}$$

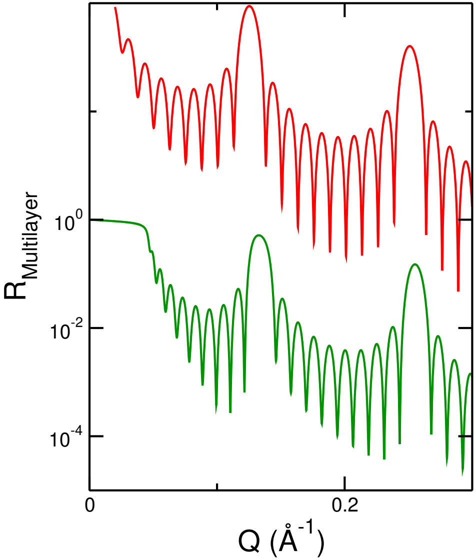
The recursive relation can be seen from the calculation of reflectivity of the next layer up

$$r_{N-2,N-1} = \frac{r'_{N-2,N-1} + r_{N-1,N} p_{N-1}^2}{1 + r'_{N-2,N-1} r_{N-1,N} p_{N-1}^2}$$

Kinematical - Parratt comparison

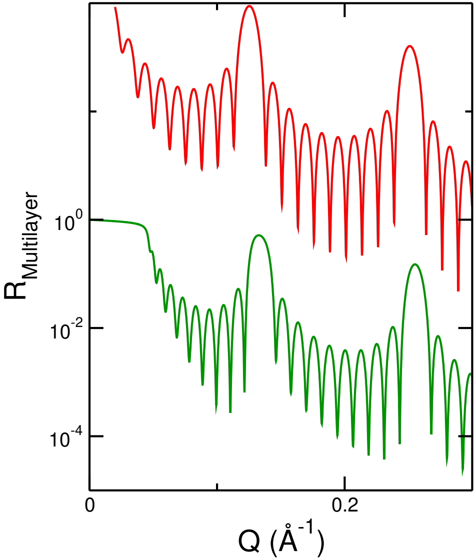


Kinematical - Parratt comparison



Kinematical approximation gives a reasonably good approximation to the correct calculation, with a few exceptions.

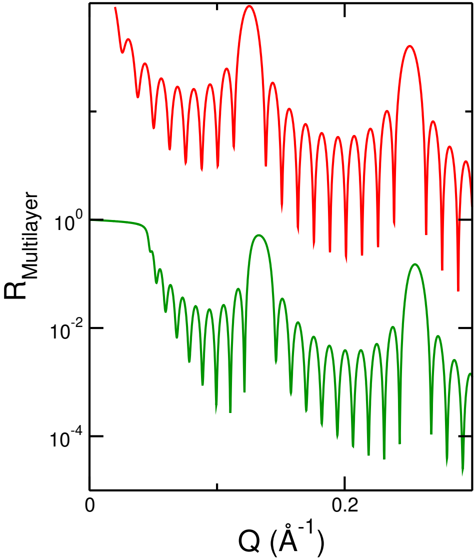
Kinematical - Parratt comparison



Kinematical approximation gives a reasonably good approximation to the correct calculation, with a few exceptions.

Parratt calculation gives $R_{Par} = 1$ as $Q \to 0$ while kinematical diverges ($R_{Kin} \to \infty$).

Kinematical - Parratt comparison

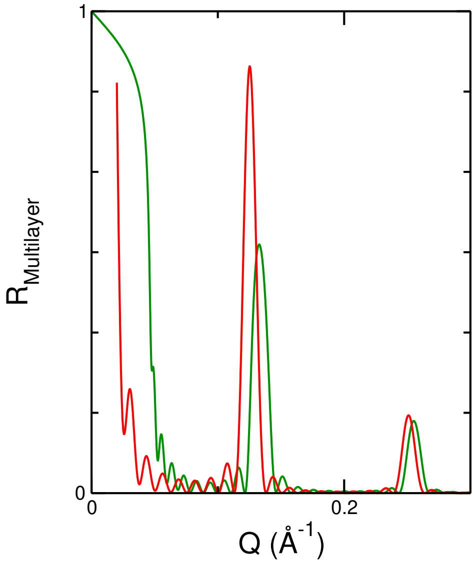


Kinematical approximation gives a reasonably good approximation to the correct calculation, with a few exceptions.

Parratt calculation gives $R_{Par} = 1$ as $Q \to 0$ while kinematical diverges ($R_{Kin} \to \infty$).

Parratt peaks shifted to slightly higher values of Q

Kinematical - Parratt comparison



Kinematical approximation gives a reasonably good approximation to the correct calculation, with a few exceptions.

Parratt calculation gives $R_{Par} = 1$ as $Q \to 0$ while kinematical diverges ($R_{Kin} \to \infty$).

Parratt peaks shifted to slightly higher values of Q

Peaks in kinematical calculation are somewhat higher reflectivity than true value.