Today's outline - September 04, 2024

Today's outline - September 04, 2024

- Absorption calculations

Today's outline - September 04, 2024

- Absorption calculations
- Undulator spectrum

Today's outline - September 04, 2024

- Absorption calculations
- Undulator spectrum
- Undulator coherence

Today's outline - September 04, 2024

- Absorption calculations
- Undulator spectrum
- Undulator coherence
- APS-U, ERLs and FELs

Today's outline - September 04, 2024

- Absorption calculations
- Undulator spectrum
- Undulator coherence
- APS-U, ERLs and FELs

Reading Assignment: Chapter 3.1–3.3

Today's outline - September 04, 2024

- Absorption calculations
- Undulator spectrum
- Undulator coherence
- APS-U, ERLs and FELs

Reading Assignment: Chapter 3.1–3.3

Homework Assignment #02:
Problems on Canvas
due Monday, September 16, 2024

Today's outline - September 04, 2024

- Absorption calculations
- Undulator spectrum
- Undulator coherence
- APS-U, ERLs and FELs

Reading Assignment: Chapter 3.1–3.3

Homework Assignment #02:
Problems on Canvas
due Monday, September 16, 2024

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Monday, September 30, 2024

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

$$\mu = \rho_a \sigma_a$$

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

rewriting in terms of the mass density, ρ_m, the atomic mass, M_a, and Avogadro's number, N_A, the absorption coefficient becomes

$$\mu = \rho_a \sigma_a$$

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

rewriting in terms of the mass density, ρ_m, the atomic mass, M_a, and Avogadro's number, N_A, the absorption coefficient becomes

$$\mu = \rho_a \sigma_a = \left(\frac{\rho_m N_A}{M_a} \right) \sigma_a$$

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

rewriting in terms of the mass density, ρ_m, the atomic mass, M_a, and Avogadro's number, N_A, the absorption coefficient becomes

$$\mu = \rho_a \sigma_a = \left(\frac{\rho_m N_A}{M_a} \right) \sigma_a$$

if the absorber is made up of a number of different atoms, this calculation can be generalized

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

rewriting in terms of the mass density, ρ_m, the atomic mass, M_a, and Avogadro's number, N_A, the absorption coefficient becomes

$$\mu = \rho_a \sigma_a = \left(\frac{\rho_m N_A}{M_a} \right) \sigma_a$$

if the absorber is made up of a number of different atoms, this calculation can be generalized

$$\mu = \sum_j \rho_j \sigma_{aj}$$

Absorption calculations

For an elemental material, the absorption coefficient, μ, is simply the product of the atomic density, ρ_a, times the atomic absorption cross-section, σ_a.

rewriting in terms of the mass density, ρ_m, the atomic mass, M_a, and Avogadro's number, N_A, the absorption coefficient becomes

$$\mu = \rho_a \sigma_a = \left(\frac{\rho_m N_A}{M_a} \right) \sigma_a$$

if the absorber is made up of a number of different atoms, this calculation can be generalized

$$\mu = \sum_j \rho_j \sigma_{aj}$$

where ρ_j and σ_{aj} are the atomic density and atomic absorption cross-section of each component

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

Absorption coefficient of a compound

μ[cm^{-1}] is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, μ_m[cm^2/g]

$$\mu_m = \mu/\rho$$

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

Absorption coefficient of a compound

μ[cm^{-1}] is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, μ_m[cm^2/g]

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

$$M_c = \sum_j x_j M_j$$

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

$$M_c = \sum_j x_j M_j$$

$$\mu_m = (N_A/M_c) \sum_j x_j \sigma_{aj}$$

Absorption coefficient of a compound

μ[cm^{-1}] is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, μ_m[cm^2/g]

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

$$M_c = \sum_j x_j M_j$$

$$\mu_m = (N_A/M_c) \sum_j x_j \sigma_{aj}$$

The "thickness" of a mass m of the compound, distributed over an area A is then:

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

$$M_c = \sum_j x_j M_j$$

$$\mu_m = (N_A/M_c) \sum_j x_j \sigma_{aj}$$

The "thickness" of a mass m of the compound, distributed over an area A is then:

$$z = \frac{m}{\rho_c A}$$

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

$$M_c = \sum_j x_j M_j$$

$$\mu_m = (N_A/M_c) \sum_j x_j \sigma_{aj}$$

The "thickness" of a mass m of the compound, distributed over an area A is then:

$$z = \frac{m}{\rho_c A}$$

This leads to an absorption per unit mass of μ_m/A and Beer's law becomes

Absorption coefficient of a compound

$\mu[\text{cm}^{-1}]$ is the linear absorption coefficient. It is useful in practice to define the mass absorption coefficient, $\mu_m[\text{cm}^2/\text{g}]$

$$\mu_m = \mu/\rho$$

Beer's Law now becomes

$$I = I_0 e^{-\mu z} = I_0 e^{-\mu_m \rho z}$$

Suppose we want to compute the absorption coefficient per unit mass of a compound if we distribute it over an area A

If the compound is made up of x_j atoms with atomic mass M_j and has a molecular mass M_c and density ρ_c, we can write:

$$M_c = \sum_j x_j M_j$$

$$\mu_m = (N_A/M_c) \sum_j x_j \sigma_{aj}$$

The "thickness" of a mass m of the compound, distributed over an area A is then:

$$z = \frac{m}{\rho_c A}$$

This leads to an absorption per unit mass of μ_m/A and Beer's law becomes

$$I = I_0 e^{-(\mu_m/A)m}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj} / M_j$ so we have

$$I = I_0 e^{-(\mu_m / A) m}, \ \mu_m = \frac{N_A}{M_c} \sum_j x_j \sigma_{aj}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \; \mu_m = \frac{N_A}{M_c} \sum_j x_j \sigma_{aj} = \frac{N_A}{M_c} \sum_j \frac{M_j}{N_A} x_j \sigma_j$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \ \mu_m = \frac{N_A}{M_c} \sum_j x_j \sigma_{aj} = \frac{N_A}{M_c} \sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c} \sum_j M_j x_j \sigma_j$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \ \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

$$\rho = 5.24\,\mathrm{g/cm^3}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj} / M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \; \mu_m = \frac{N_A}{M_c} \sum_j x_j \sigma_{aj} = \frac{N_A}{M_c} \sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c} \sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

$$\rho = 5.24 \, \mathrm{g/cm^3}$$

$$M_{Fe} = 55.895 \, \mathrm{g/mol}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

$$\rho = 5.24\,\mathrm{g/cm^3}$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

$$\rho = 5.24\,\text{g}/\text{cm}^3$$
$$M_{Fe} = 55.895\,\text{g}/\text{mol}$$
$$M_O = 16.000\,\text{g}/\text{mol}$$
$$M_c = 159.69\,\text{g}/\text{mol}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

$$\rho = 5.24\,\mathrm{g/cm^3}$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$
$$M_c = 159.69\,\mathrm{g/mol}$$

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj} / M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c} \sum_j x_j \sigma_{aj} = \frac{N_A}{M_c} \sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c} \sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

$$\rho = 5.24\,\mathrm{g/cm^3}$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$
$$M_c = 159.69\,\mathrm{g/mol}$$
$$\sigma_{Fe} = 138.860\,\mathrm{cm^2/g}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

$$\rho = 5.24\,\mathrm{g/cm^3}$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$
$$M_c = 159.69\,\mathrm{g/mol}$$
$$\sigma_{Fe} = 138.860\,\mathrm{cm^2/g}$$
$$\sigma_O = 46.666\,\mathrm{cm^2/g}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

assuming a 5 mm diameter pellet

$$\rho = 5.24\,\mathrm{g/cm^3}$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$
$$M_c = 159.69\,\mathrm{g/mol}$$
$$\sigma_{Fe} = 138.860\,\mathrm{cm^2/g}$$
$$\sigma_O = 46.666\,\mathrm{cm^2/g}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

assuming a 5 mm diameter pellet

$$\rho = 5.24\,\text{g/cm}^3$$
$$M_{Fe} = 55.895\,\text{g/mol}$$
$$M_O = 16.000\,\text{g/mol}$$
$$M_c = 159.69\,\text{g/mol}$$
$$\sigma_{Fe} = 138.860\,\text{cm}^2/\text{g}$$
$$\sigma_O = 46.666\,\text{cm}^2/\text{g}$$
$$A = \pi(0.25\,\text{cm})^2 = 0.1963\,\text{cm}^2$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A\sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \ \mu_m = \frac{N_A}{M_c}\sum_j x_j\sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A}x_j\sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

assuming a 5 mm diameter pellet

$$\mu_m = \frac{1}{159.69}\left[2\cdot 55.895\cdot 138.860 + 3\cdot 16.000\cdot 46.666\right]$$

$$\rho = 5.24\,\mathrm{g/cm^3}$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$
$$M_c = 159.69\,\mathrm{g/mol}$$
$$\sigma_{Fe} = 138.860\,\mathrm{cm^2/g}$$
$$\sigma_O = 46.666\,\mathrm{cm^2/g}$$
$$A = \pi(0.25\,\mathrm{cm})^2 = 0.1963\,\mathrm{cm^2}$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \;\; \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

assuming a 5 mm diameter pellet

$$\mu_m = \frac{1}{159.69}\left[2 \cdot 55.895 \cdot 138.860 + 3 \cdot 16.000 \cdot 46.666\right]$$
$$= 111.23\,\mathrm{cm}^2/\mathrm{g}$$

$$\rho = 5.24\,\mathrm{g/cm}^3$$
$$M_{Fe} = 55.895\,\mathrm{g/mol}$$
$$M_O = 16.000\,\mathrm{g/mol}$$
$$M_c = 159.69\,\mathrm{g/mol}$$
$$\sigma_{Fe} = 138.860\,\mathrm{cm}^2/\mathrm{g}$$
$$\sigma_O = 46.666\,\mathrm{cm}^2/\mathrm{g}$$
$$A = \pi(0.25\,\mathrm{cm})^2 = 0.1963\,\mathrm{cm}^2$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \quad \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

assuming a 5 mm diameter pellet

$$\mu_m = \frac{1}{159.69}\left[2 \cdot 55.895 \cdot 138.860 + 3 \cdot 16.000 \cdot 46.666\right]$$

$$= 111.23\,\mathrm{cm}^2/\mathrm{g} \quad \longrightarrow \quad \mu_m/A = 566.7\,\mathrm{g}^{-1}$$

$$\rho = 5.24\,\mathrm{g/cm}^3$$

$$M_{Fe} = 55.895\,\mathrm{g/mol}$$

$$M_O = 16.000\,\mathrm{g/mol}$$

$$M_c = 159.69\,\mathrm{g/mol}$$

$$\sigma_{Fe} = 138.860\,\mathrm{cm}^2/\mathrm{g}$$

$$\sigma_O = 46.666\,\mathrm{cm}^2/\mathrm{g}$$

$$A = \pi(0.25\,\mathrm{cm})^2 = 0.1963\,\mathrm{cm}^2$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \;\; \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

$$\rho = 5.24\,\text{g/cm}^3$$
$$M_{Fe} = 55.895\,\text{g/mol}$$
$$M_O = 16.000\,\text{g/mol}$$
$$M_c = 159.69\,\text{g/mol}$$

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

$$\sigma_{Fe} = 138.860\,\text{cm}^2/\text{g}$$
$$\sigma_O = 46.666\,\text{cm}^2/\text{g}$$

assuming a 5 mm diameter pellet

$$A = \pi(0.25\,\text{cm})^2 = 0.1963\,\text{cm}^2$$

$$\mu_m = \frac{1}{159.69}[2 \cdot 55.895 \cdot 138.860 + 3 \cdot 16.000 \cdot 46.666]$$
$$= 111.23\,\text{cm}^2/\text{g} \quad \longrightarrow \quad \mu_\text{m}/\text{A} = 566.7\,\text{g}^{-1}$$
$$\mu = \mu_m \rho = 582.9\,\text{cm}^{-1},$$

Absorption of Fe_2O_3 at 5 keV

The most commonly tabulated cross-sections are not the atomic cross-sections but the mass cross sections, $\sigma_j = N_A \sigma_{aj}/M_j$ so we have

$$I = I_0 e^{-(\mu_m/A)m}, \ \mu_m = \frac{N_A}{M_c}\sum_j x_j \sigma_{aj} = \frac{N_A}{M_c}\sum_j \frac{M_j}{N_A} x_j \sigma_j = \frac{1}{M_c}\sum_j M_j x_j \sigma_j$$

the molecular mass and density of Fe_2O_3 are

begin by finding tabulated values of the cross-section for the elements Fe and O at 5 keV

assuming a 5 mm diameter pellet

$$\mu_m = \frac{1}{159.69}\left[2 \cdot 55.895 \cdot 138.860 + 3 \cdot 16.000 \cdot 46.666\right]$$

$$= 111.23\,\mathrm{cm^2/g} \quad \longrightarrow \quad \mu_m/A = 566.7\,\mathrm{g^{-1}}$$

$$\mu = \mu_m \rho = 582.9\,\mathrm{cm^{-1}}, \quad \longrightarrow \quad 1/\mu = 17.2\,\mu\mathrm{m}$$

$$\rho = 5.24\,\mathrm{g/cm^3}$$

$$M_{Fe} = 55.895\,\mathrm{g/mol}$$

$$M_O = 16.000\,\mathrm{g/mol}$$

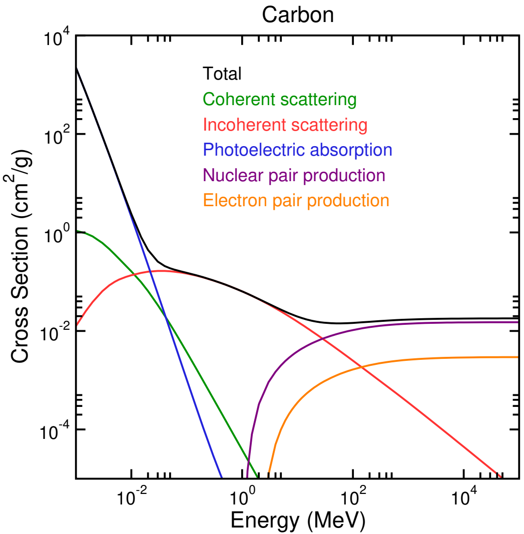
$$M_c = 159.69\,\mathrm{g/mol}$$

$$\sigma_{Fe} = 138.860\,\mathrm{cm^2/g}$$

$$\sigma_O = 46.666\,\mathrm{cm^2/g}$$

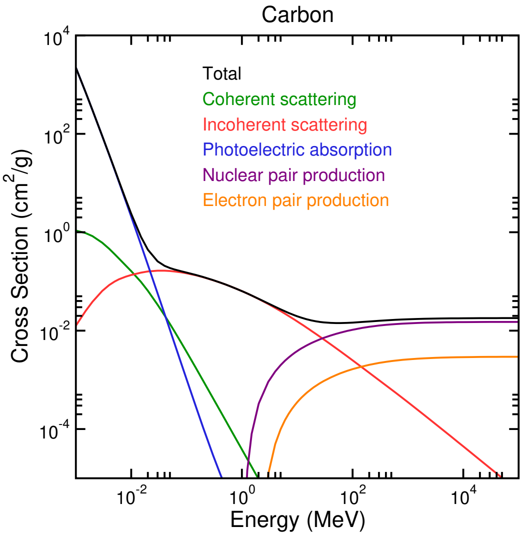
$$A = \pi(0.25\,\mathrm{cm})^2 = 0.1963\,\mathrm{cm^2}$$

Comparison of cross sections



Comparison of cross sections

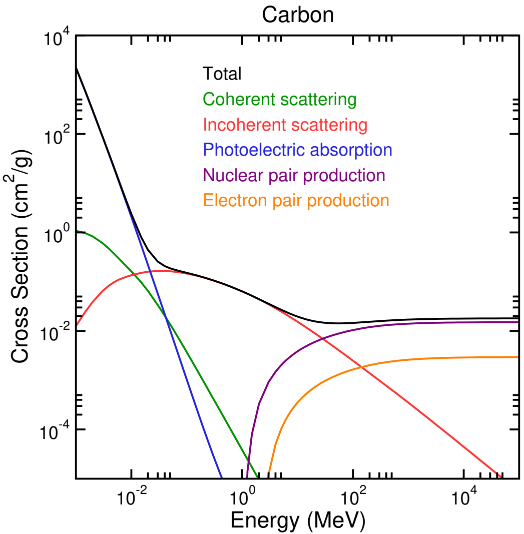
Photoelectric absorption dominates at low energies



Comparison of cross sections

Photoelectric absorption dominates at low energies

Thomson scattering (coherent) drops rapidly with energy

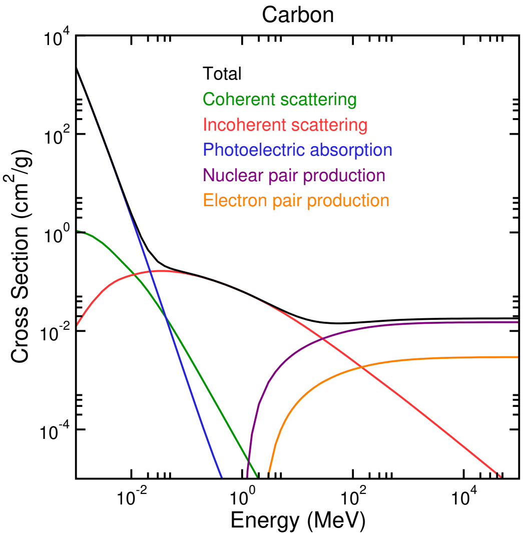


Comparison of cross sections

Photoelectric absorption dominates at low energies

Thomson scattering (coherent) drops rapidly with energy

Compton scattering (incoherent) dominates at medium energies



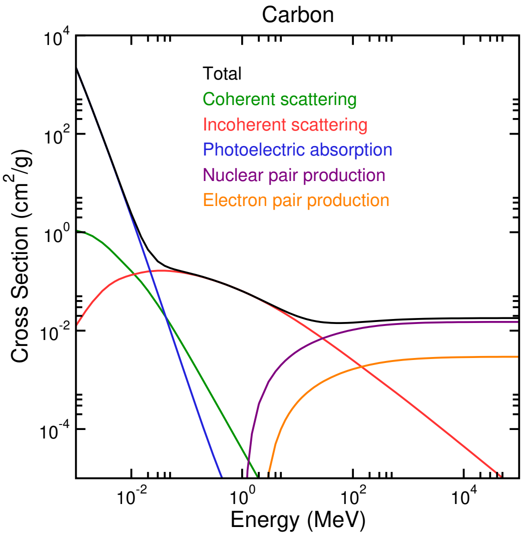
Comparison of cross sections

Photoelectric absorption dominates at low energies

Thomson scattering (coherent) drops rapidly with energy

Compton scattering (incoherent) dominates at medium energies

Pair production dominates at high energies



Comparison of cross sections

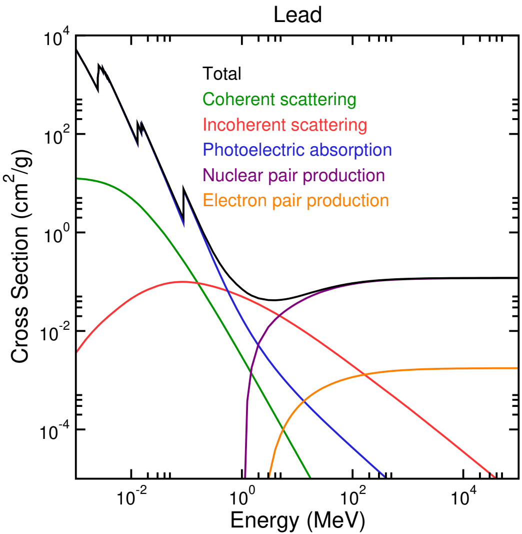
Photoelectric absorption dominates at low energies

Thomson scattering (coherent) drops rapidly with energy

Compton scattering (incoherent) dominates at medium energies

Pair production dominates at high energies

Each portion of the cross-section is element-dependent



Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Indeed, x-rays do interact with magnetic materials (and electrons which have magnetic moment and spin) but the strength of the interaction is comparatively weak.

Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Indeed, x-rays do interact with magnetic materials (and electrons which have magnetic moment and spin) but the strength of the interaction is comparatively weak.

$$\frac{A_{magnetic}}{A_{charge}} = \frac{\hbar\omega}{mc^2}$$

Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Indeed, x-rays do interact with magnetic materials (and electrons which have magnetic moment and spin) but the strength of the interaction is comparatively weak.

$$\frac{A_{magnetic}}{A_{charge}} = \frac{\hbar\omega}{mc^2}$$

For an x-ray of energy 5.11 keV, interacting with an electron with mass 0.511 MeV.

Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Indeed, x-rays do interact with magnetic materials (and electrons which have magnetic moment and spin) but the strength of the interaction is comparatively weak.

$$\frac{A_{magnetic}}{A_{charge}} = \frac{\hbar\omega}{mc^2} = \frac{5.11 \times 10^3 \text{ eV}}{0.511 \times 10^6 \text{ eV}}$$

For an x-ray of energy 5.11 keV, interacting with an electron with mass 0.511 MeV.

Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Indeed, x-rays do interact with magnetic materials (and electrons which have magnetic moment and spin) but the strength of the interaction is comparatively weak.

$$\frac{A_{magnetic}}{A_{charge}} = \frac{\hbar\omega}{mc^2} = \frac{5.11 \times 10^3 \text{ eV}}{0.511 \times 10^6 \text{ eV}} = 0.01$$

For an x-ray of energy 5.11 keV, interacting with an electron with mass 0.511 MeV.

Magnetic interactions

We have focused on the interaction of x-rays and charged particles. However, electromagnetic radiation also consists of a traveling magnetic field. In principle, this means it should interact with magnetic materials as well.

Indeed, x-rays do interact with magnetic materials (and electrons which have magnetic moment and spin) but the strength of the interaction is comparatively weak.

$$\frac{A_{magnetic}}{A_{charge}} = \frac{\hbar\omega}{mc^2} = \frac{5.11 \times 10^3 \text{ eV}}{0.511 \times 10^6 \text{ eV}} = 0.01$$

For an x-ray of energy 5.11 keV, interacting with an electron with mass 0.511 MeV. Only with the advent of synchrotron radiation sources has magnetic x-ray scattering become a practical experimental technique.

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0$$

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0$$
$$= 0.934\lambda_u[\text{cm}]\text{B}_0[\text{T}]$$

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0 = 0.934\lambda_u[\mathrm{cm}]\mathrm{B}_0[\mathrm{T}]$$

the electron path length through the undulator, $S\lambda_u$

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0 = 0.934\lambda_u[\mathrm{cm}]\mathrm{B}_0[\mathrm{T}]$$

the electron path length through the undulator, $S\lambda_u$

$$S\lambda_u \approx \lambda_u\left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right)$$

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0$$
$$= 0.934\lambda_u[\mathrm{cm}]\mathrm{B}_0[\mathrm{T}]$$

the electron path length through the undulator, $S\lambda_u$

$$S\lambda_u \approx \lambda_u\left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right)$$

the fundamental wavelength of the undulator, λ_1

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0 = 0.934\lambda_u[\text{cm}]\text{B}_0[\text{T}]$$

the electron path length through the undulator, $S\lambda_u$

$$S\lambda_u \approx \lambda_u \left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right)$$

the fundamental wavelength of the undulator, λ_1

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0 = 0.934\lambda_u[\text{cm}]\text{B}_0[\text{T}]$$

the electron path length through the undulator, $S\lambda_u$

$$S\lambda_u \approx \lambda_u \left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right)$$

the fundamental wavelength of the undulator, λ_1 and the origin of the odd and even harmonics

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Undulator review

For an undulator of period λ_u we have derived the following undulator parameters and their relationships:

the K parameter, a dimensionless quantity which represents the "strength" of the undulator

$$K = \frac{e}{2\pi mc}\lambda_u B_0 = 0.934\lambda_u[\text{cm}]\text{B}_0[\text{T}]$$

the electron path length through the undulator, $S\lambda_u$

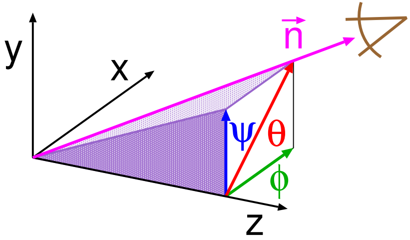
$$S\lambda_u \approx \lambda_u\left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right)$$

the fundamental wavelength of the undulator, λ_1 and the origin of the odd and even harmonics

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

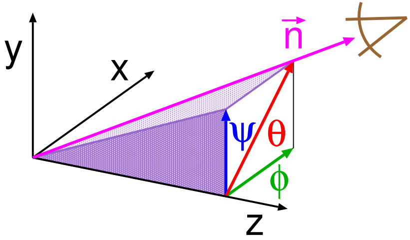
Now let us look at additional properties

Higher harmonics



Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

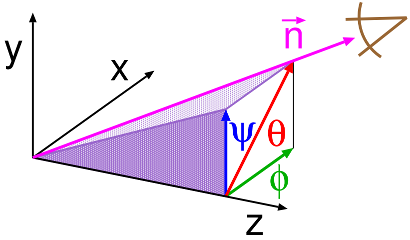
Higher harmonics



Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t')$$

Higher harmonics

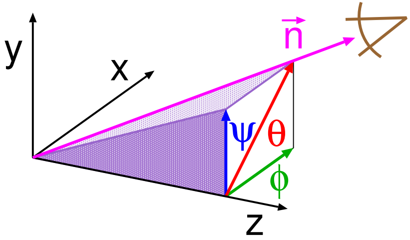


Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t')$$

Higher harmonics



$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t')$$

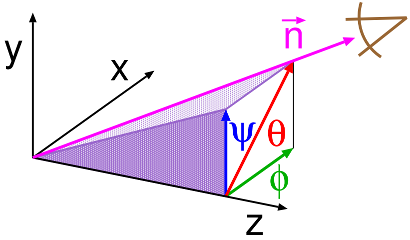
Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

$$\vec{n} = \left\{ \phi, \psi, \sqrt{1 - \theta^2} \right\}$$

$$\vec{\beta} = \beta \left\{ \alpha, 0, \sqrt{1 - \alpha^2} \right\}$$

Higher harmonics



$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t')$$

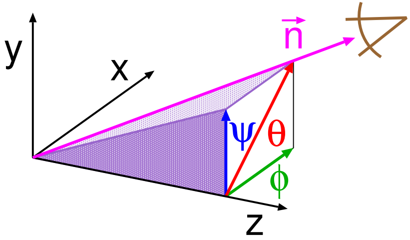
Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

$$\vec{n} \approx \left\{ \phi, \psi, (1 - \theta^2/2) \right\}$$

$$\vec{\beta} \approx \beta \left\{ \alpha, 0, (1 - \alpha^2/2) \right\}$$

Higher harmonics



Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

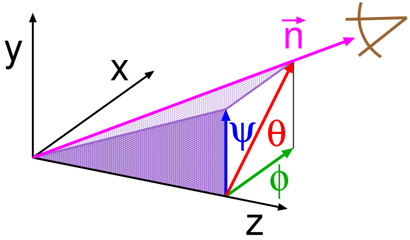
This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

$$\vec{n} \approx \left\{ \phi, \psi, (1 - \theta^2/2) \right\}$$

$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t') \approx 1 - \beta \left[\alpha\phi + \left(1 - \frac{\theta^2}{2} - \frac{\alpha^2}{2} \right) \right]$$

$$\vec{\beta} \approx \beta \left\{ \alpha, 0, (1 - \alpha^2/2) \right\}$$

Higher harmonics



Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

$$\vec{n} \approx \left\{ \phi, \psi, (1 - \theta^2/2) \right\}$$

$$\vec{\beta} \approx \beta \left\{ \alpha, 0, (1 - \alpha^2/2) \right\}$$

$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t') \approx 1 - \beta \left[\alpha\phi + \left(1 - \frac{\theta^2}{2} - \frac{\alpha^2}{2} \right) \right]$$

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2} \right) \left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2} \right)$$

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx 1 - 1 - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2}$$

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \cancel{1} - \cancel{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2}$$

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \cancel{1} - \cancel{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi$$

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \not{1} - \not{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi$$

This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \cancel{1} - \cancel{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi$$

This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

$$\omega_1 t = \omega_u t'$$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \not{1} - \not{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi$$

This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2} \sin\left(2\omega_u t'\right)$$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression , but they are not proportional because of the second

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \cancel{1} - \cancel{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi$$

This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2} \sin(2\omega_u t') - \frac{2K\gamma}{1 + (\gamma\theta)^2 + K^2/2} \phi \sin(\omega_u t')$$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression , but they are not proportional because of the second and third terms.

Higher harmonics

$$\frac{dt}{dt'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right)$$

$$\approx \cancel{1} - \cancel{1} - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi$$

This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2} \sin(2\omega_u t') - \frac{2K\gamma}{1 + (\gamma\theta)^2 + K^2/2} \phi \sin(\omega_u t')$$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression , but they are not proportional because of the second and third terms.

The motion of the electron, $\sin \omega_u t'$, is always sinusoidal, but because of the additional terms, the motion as seen by the observer, $\sin \omega_1 t$, is not.

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \cancel{\frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')}$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6}\sin(2\omega_u t')$$

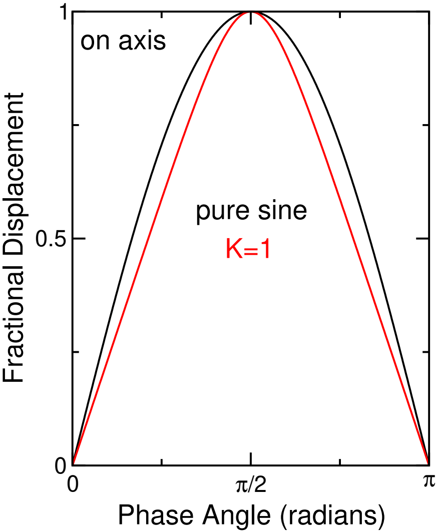
On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \cancel{\frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')}$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6}\sin(2\omega_u t')$$

Plotting $sin\omega_u t'$ and $sin\omega_1 t$ shows the deviation from sinusoidal.



On-axis undulator characteristics

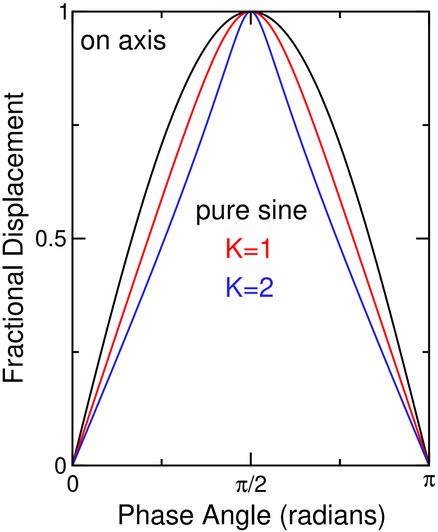
$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

Suppose we have $K=1$ and $\theta=0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6}\sin(2\omega_u t')$$

Plotting $sin\omega_u t'$ and $sin\omega_1 t$ shows the deviation from sinusoidal.

Similarly, for $K = 2$



On-axis undulator characteristics

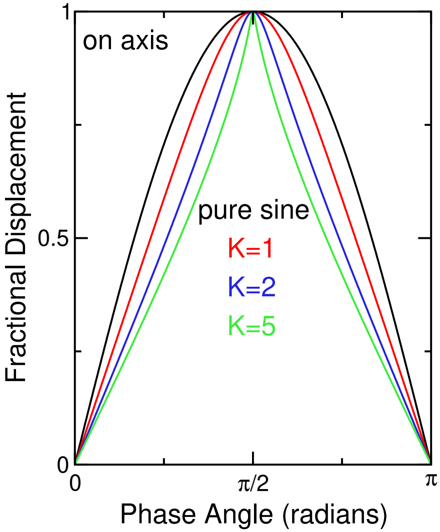
$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6}\sin(2\omega_u t')$$

Plotting $sin\omega_u t'$ and $sin\omega_1 t$ shows the deviation from sinusoidal.

Similarly, for $K = 2$ and $K = 5$, the deviation becomes more pronounced.



On-axis undulator characteristics

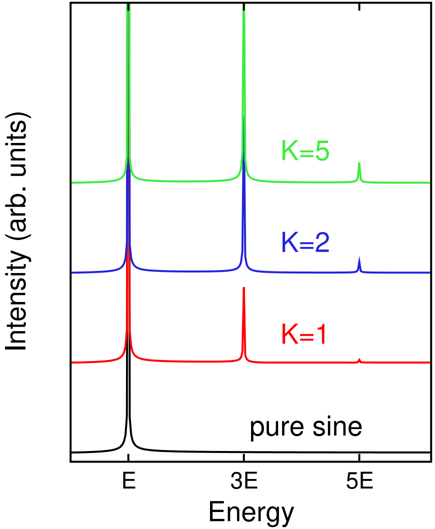
$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6}\sin(2\omega_u t')$$

Plotting $sin\omega_u t'$ and $sin\omega_1 t$ shows the deviation from sinusoidal.

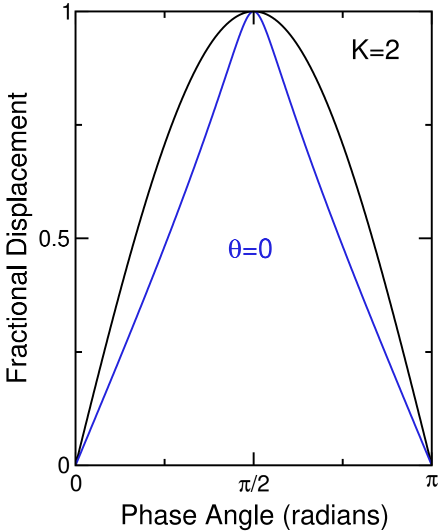
Similarly, for $K = 2$ and $K = 5$, the deviation becomes more pronounced. This shows how higher harmonics must be present in the radiation as seen by the observer.



Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

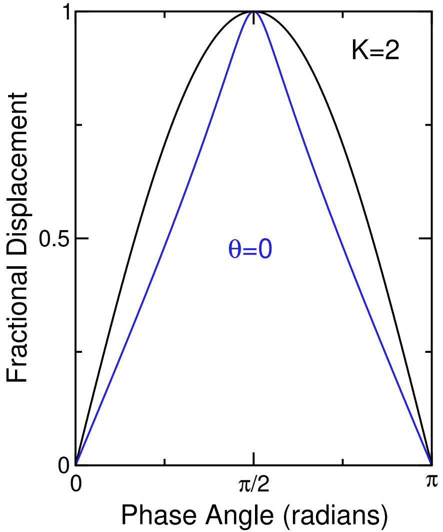


Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin\left(2\omega_u t'\right) - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin\left(\omega_u t'\right)$$

When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

$$\omega_1 t = \omega_u t' + \frac{1}{4}\sin\left(2\omega_u t'\right) + \sin\omega_u t'$$



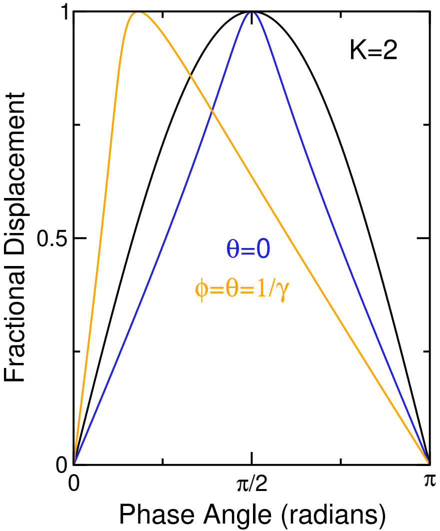
Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

$$\omega_1 t = \omega_u t' + \frac{1}{4}\sin(2\omega_u t') + \sin\omega_u t'$$

The last term introduces an antisymmetric term which skews the function



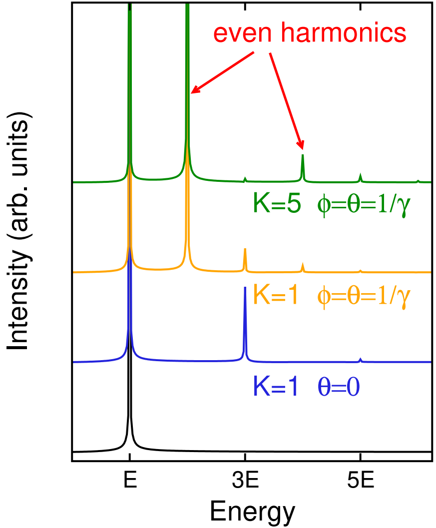
Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin\left(2\omega_u t'\right) - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin\left(\omega_u t'\right)$$

When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

$$\omega_1 t = \omega_u t' + \frac{1}{4}\sin\left(2\omega_u t'\right) + \sin\omega_u t'$$

The last term introduces an antisymmetric term which skews the function and leads to the presence of forbidden harmonics (2^{nd}, 4^{th}, etc) in the radiation from the undulator compared to the on-axis radiation.

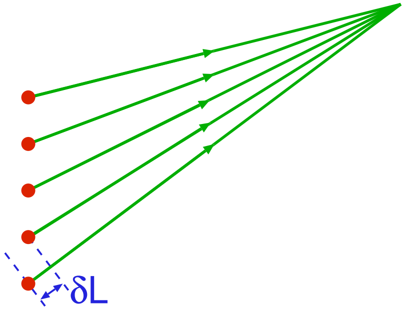


Diffraction grating

An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain.

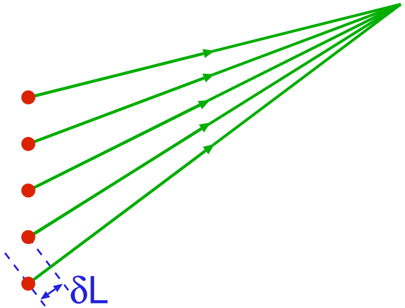
Diffraction grating

An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.



Diffraction grating

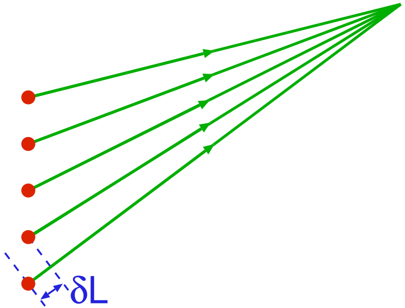
An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.



The radiation from each slit has to travel a slightly different distance to get to the detector.

Diffraction grating

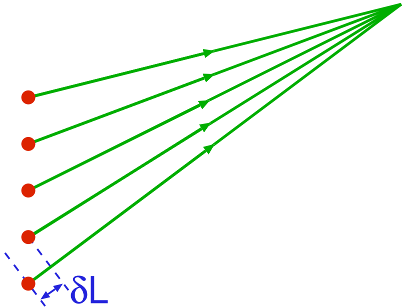
An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.



The radiation from each slit has to travel a slightly different distance to get to the detector. For consecutive slits this path length difference, $L_{m+1} - L_m = \delta L$,

Diffraction grating

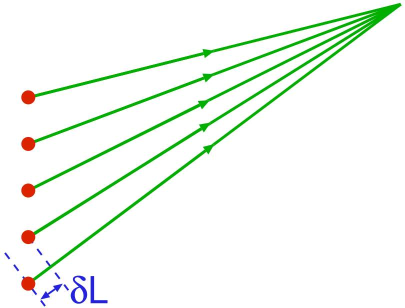
An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.



The radiation from each slit has to travel a slightly different distance to get to the detector. For consecutive slits this path length difference, $L_{m+1} - L_m = \delta L$, gives rise to a phase shift, $2\pi\epsilon = 2\pi\delta L/\lambda$.

Diffraction grating

An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.

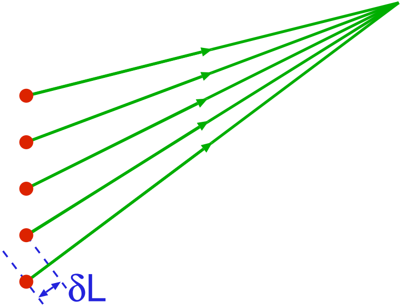


The radiation from each slit has to travel a slightly different distance to get to the detector. For consecutive slits this path length difference, $L_{m+1} - L_m = \delta L$, gives rise to a phase shift, $2\pi\epsilon = 2\pi\delta L/\lambda$. So at the detector, we have a sum of waves:

$$\sum_{m=0}^{N-1} e^{i(\vec{k}\cdot\vec{r}+2\pi m\epsilon)}$$

Diffraction grating

An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.

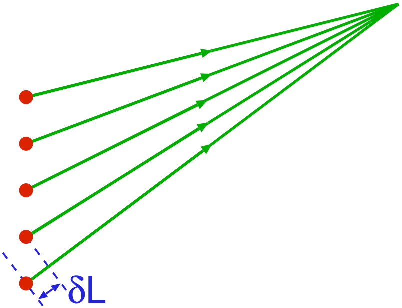


The radiation from each slit has to travel a slightly different distance to get to the detector. For consecutive slits this path length difference, $L_{m+1} - L_m = \delta L$, gives rise to a phase shift, $2\pi\epsilon = 2\pi\delta L/\lambda$. So at the detector, we have a sum of waves:

$$\sum_{m=0}^{N-1} e^{i(\vec{k}\cdot\vec{r}+2\pi m\epsilon)} = e^{i\vec{k}\cdot\vec{r}} \sum_{m=0}^{N-1} e^{i2\pi m\epsilon}$$

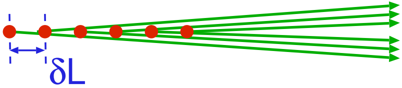
Diffraction grating

An N period undulator is basically like a diffraction grating, only in the time domain rather than the space domain. A diffraction grating consists of N coherent sources whose emission is detected at a single point.



The radiation from each slit has to travel a slightly different distance to get to the detector. For consecutive slits this path length difference, $L_{m+1} - L_m = \delta L$, gives rise to a phase shift, $2\pi\epsilon = 2\pi\delta L/\lambda$. So at the detector, we have a sum of waves:

$$\sum_{m=0}^{N-1} e^{i(\vec{k}\cdot\vec{r}+2\pi m\epsilon)} = e^{i\vec{k}\cdot\vec{r}} \sum_{m=0}^{N-1} e^{i2\pi m\epsilon}$$



the phase shift from each undulator pole depends on the wavelength λ_u

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$ and

$$S_N = 1 + kS_{N-1}$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$ and

$$S_N = 1 + kS_{N-1} = 1 + k(S_N - k^{N-1})$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$ and

$$S_N = 1 + kS_{N-1} = 1 + k(S_N - k^{N-1}) = 1 + kS_N - k^N$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$ and

$$S_N = 1 + kS_{N-1} = 1 + k(S_N - k^{N-1}) = 1 + kS_N - k^N$$

Solving for S_N, we have

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$ and

$$S_N = 1 + kS_{N-1} = 1 + k(S_N - k^{N-1}) = 1 + kS_N - k^N$$

Solving for S_N, we have

$$S_N - kS_N = 1 - k^N$$

Geometric series

The sum is simply a geometric series, S_N with $k = e^{i2\pi\epsilon}$

$$S_N = \sum_{m=0}^{N-1} k^m = 1 + k + k^2 + \cdots + k^{N-2} + k^{N-1}$$

We can develop a recursion relation by writing the expression for S_{N-1}

$$S_{N-1} = \sum_{m=0}^{N-2} k^m = 1 + k + k^2 + \cdots + k^{N-2}$$

so we can write that $S_{N-1} = S_N - k^{N-1}$ and

$$S_N = 1 + kS_{N-1} = 1 + k(S_N - k^{N-1}) = 1 + kS_N - k^N$$

Solving for S_N, we have

$$S_N - kS_N = 1 - k^N \quad \longrightarrow \quad S_N = \frac{1 - k^N}{1 - k}$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}}$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}}\right)\frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}}\right)\frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

$$S_N = \left(\frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)}\right) e^{i\pi(N-1)\epsilon}$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}} \right) \frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

$$S_N = \left(\frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)} \right) e^{i\pi(N-1)\epsilon}$$

Therefore, for the diffraction grating we can calculate the intensity at the detector as

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}}\right) \frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

$$S_N = \left(\frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)}\right) e^{i\pi(N-1)\epsilon}$$

Therefore, for the diffraction grating we can calculate the intensity at the detector as

$$I = \left| e^{i\vec{k}\cdot\vec{r}} \sum_{m=0}^{N-1} e^{i2\pi m\epsilon} \right|^2$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}} \right) \frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

$$S_N = \left(\frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)} \right) e^{i\pi(N-1)\epsilon}$$

Therefore, for the diffraction grating we can calculate the intensity at the detector as

$$I = \left| e^{i\vec{k}\cdot\vec{r}} \sum_{m=0}^{N-1} e^{i2\pi m\epsilon} \right|^2 = \left| e^{i\vec{k}\cdot\vec{r}} S_N \right|^2$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}} \right) \frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

$$S_N = \left(\frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)} \right) e^{i\pi(N-1)\epsilon}$$

Therefore, for the diffraction grating we can calculate the intensity at the detector as

$$I = \left| e^{i\vec{k}\cdot\vec{r}} \sum_{m=0}^{N-1} e^{i2\pi m\epsilon} \right|^2 = \left| e^{i\vec{k}\cdot\vec{r}} S_N \right|^2 = \left| e^{i\vec{k}\cdot\vec{r}} \frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)} e^{i\pi(N-1)\epsilon} \right|^2$$

Intensity from a diffraction grating

Restoring the expression for $k = e^{i2\pi\epsilon}$, we have:

$$\sum_{m=0}^{N-1} e^{i2\pi m\epsilon} = S_N = \frac{1 - e^{i2\pi N\epsilon}}{1 - e^{i2\pi\epsilon}} = \left(\frac{e^{-i\pi N\epsilon} - e^{i\pi N\epsilon}}{e^{-i\pi\epsilon} - e^{i\pi\epsilon}}\right) \frac{e^{i\pi N\epsilon}}{e^{i\pi\epsilon}}$$

$$S_N = \left(\frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)}\right) e^{i\pi(N-1)\epsilon}$$

Therefore, for the diffraction grating we can calculate the intensity at the detector as

$$I = \left| e^{i\vec{k}\cdot\vec{r}} \sum_{m=0}^{N-1} e^{i2\pi m\epsilon} \right|^2 = \left| e^{i\vec{k}\cdot\vec{r}} S_N \right|^2 = \left| e^{i\vec{k}\cdot\vec{r}} \frac{\sin(\pi N\epsilon)}{\sin(\pi\epsilon)} e^{i\pi(N-1)\epsilon} \right|^2$$

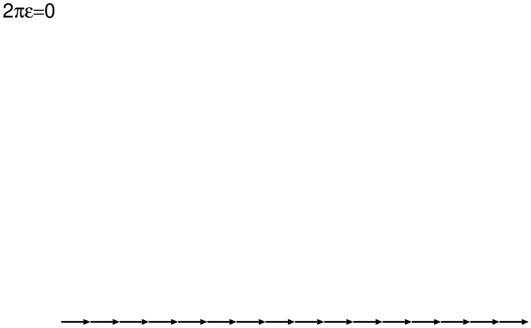
$$I = \frac{\sin^2(\pi N\epsilon)}{\sin^2(\pi\epsilon)}$$

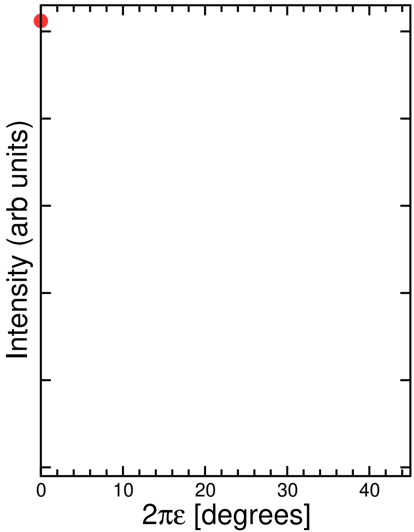
Beam coherence

The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

Beam coherence

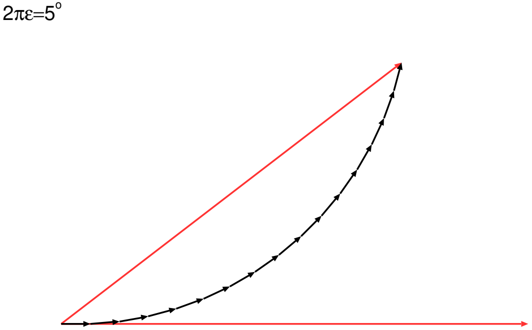
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

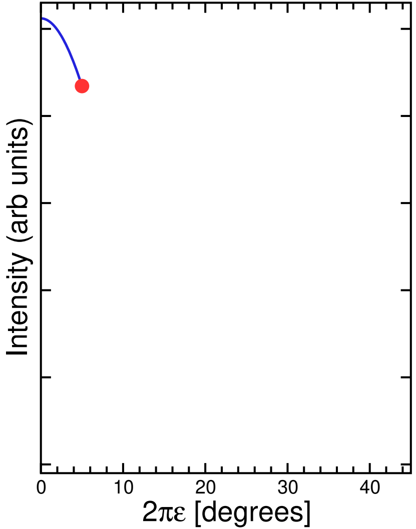




Beam coherence

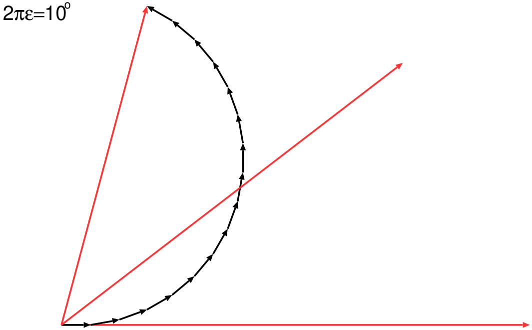
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

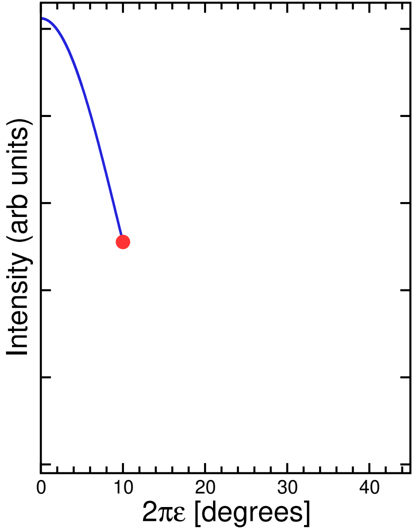




Beam coherence

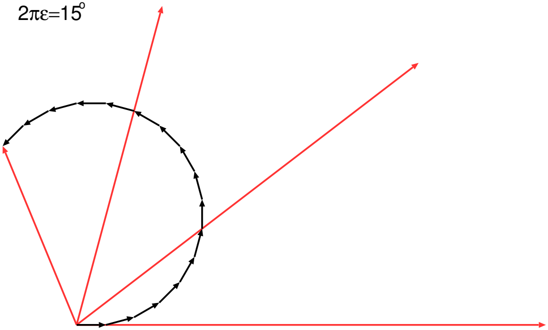
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

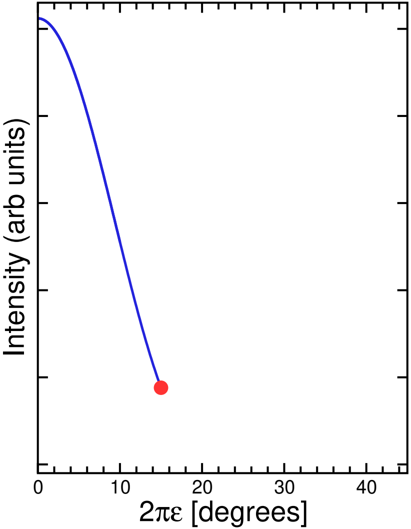




Beam coherence

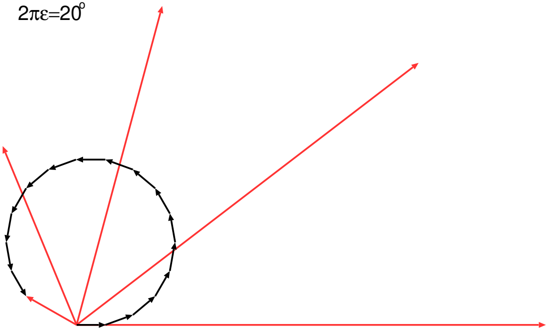
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

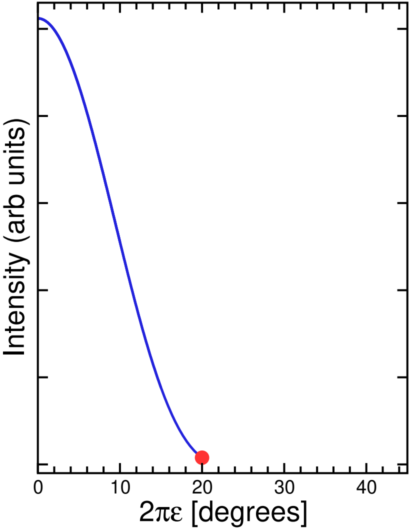




Beam coherence

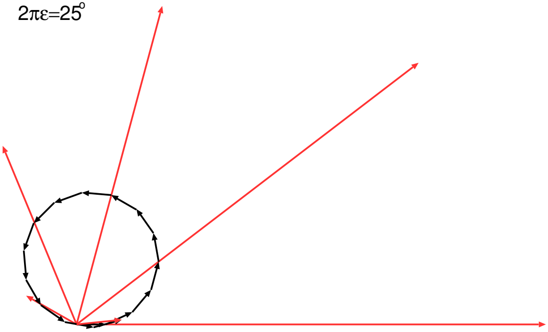
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

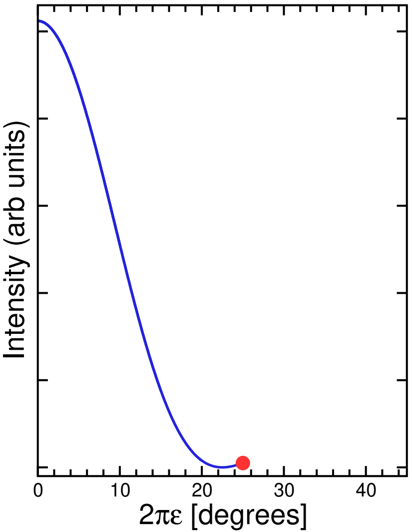




Beam coherence

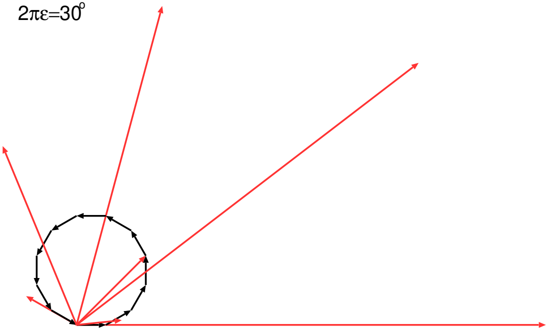
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

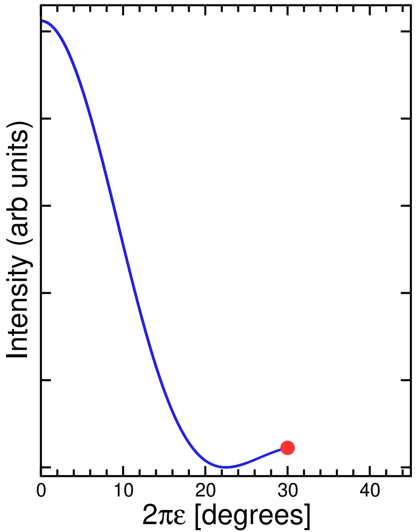




Beam coherence

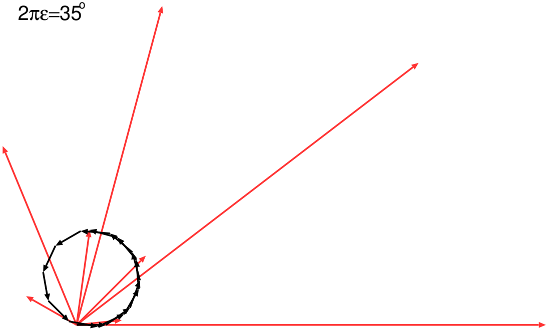
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

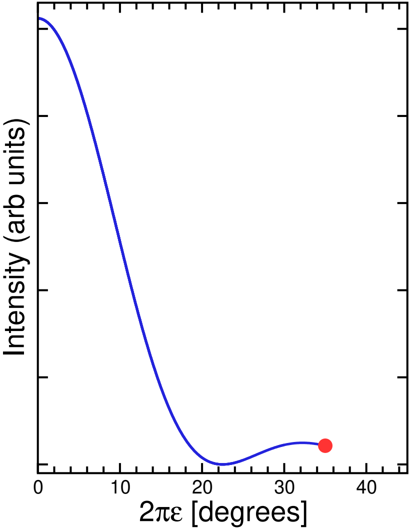




Beam coherence

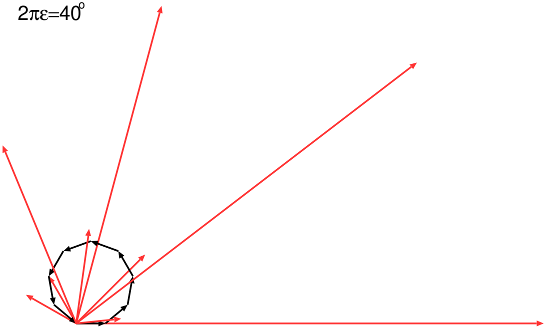
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

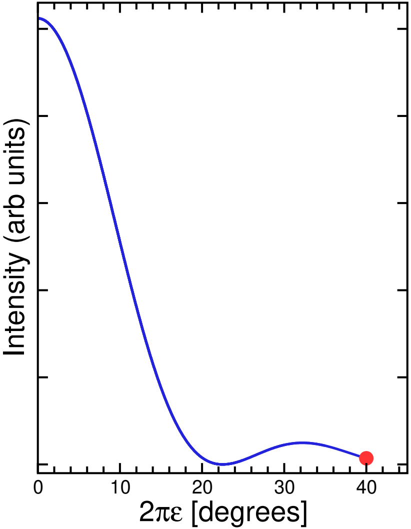




Beam coherence

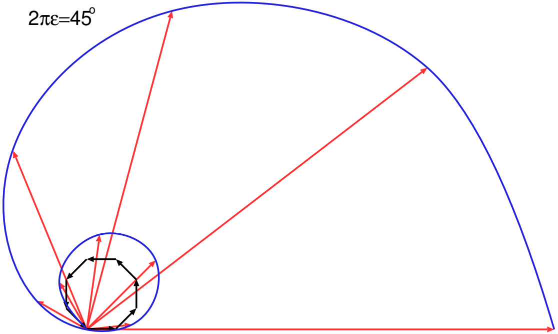
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

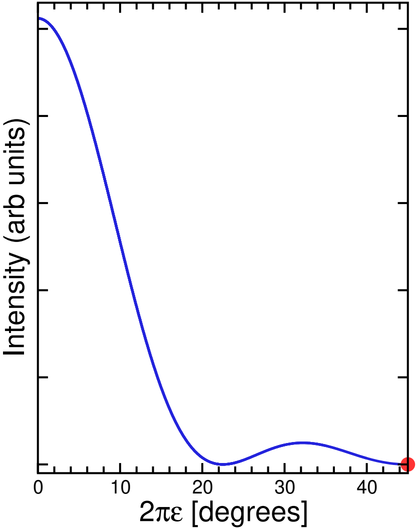




Beam coherence

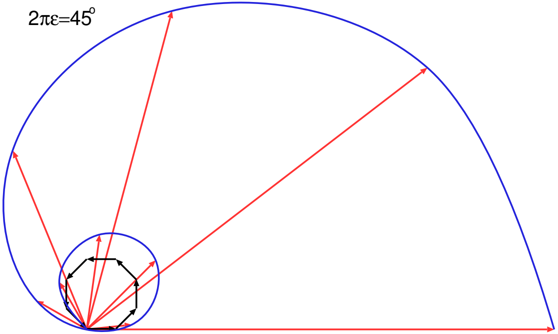
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.

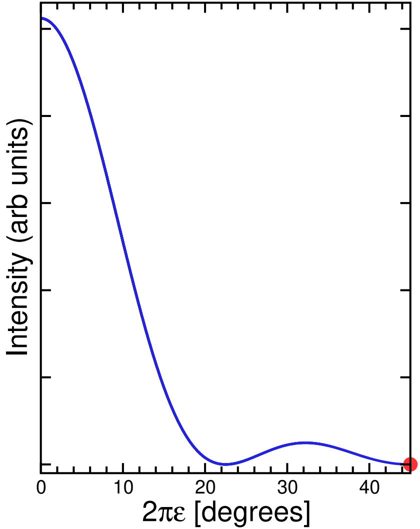




Beam coherence

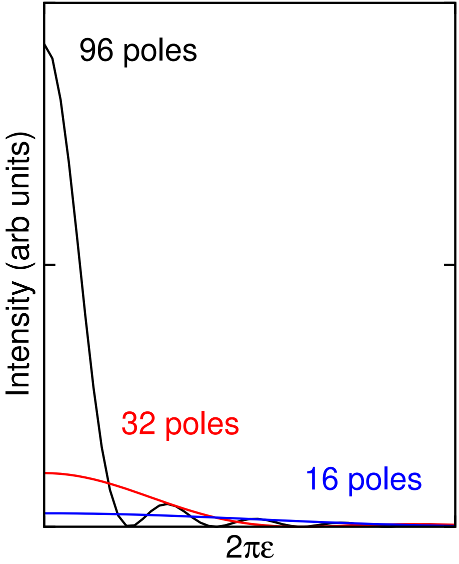
The coherence of an undulator depends on the amount each pole's emission is out of phase with the others, ϵ.





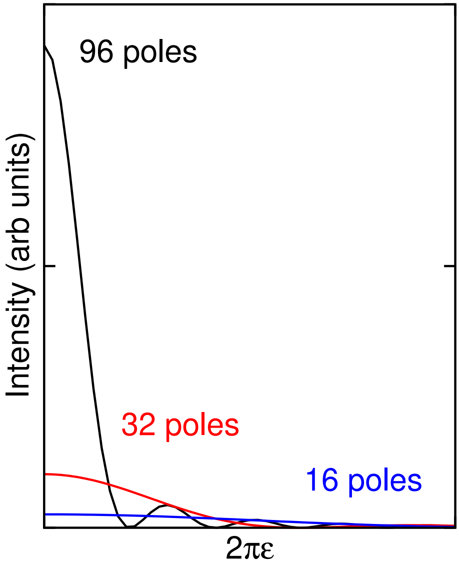
With the height and width of the peak dependent on the number of poles.

Undulator monochromaticity



The more poles in the undulator, the more monochromatic the beam since a slight change in $\epsilon = \delta L/\lambda$ implies a slightly different wavelength λ

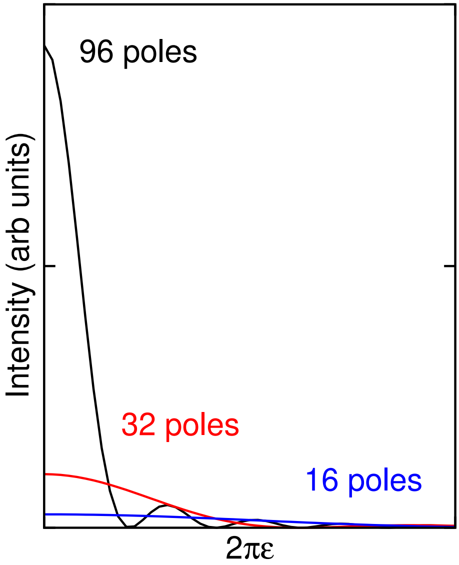
Undulator monochromaticity



The more poles in the undulator, the more monochromatic the beam since a slight change in $\epsilon = \delta L/\lambda$ implies a slightly different wavelength λ

Another way to look at this is that the longer the undulator, the longer the pulse train in time and the narrower the frequency distribution in its Fourier Transform

Undulator monochromaticity

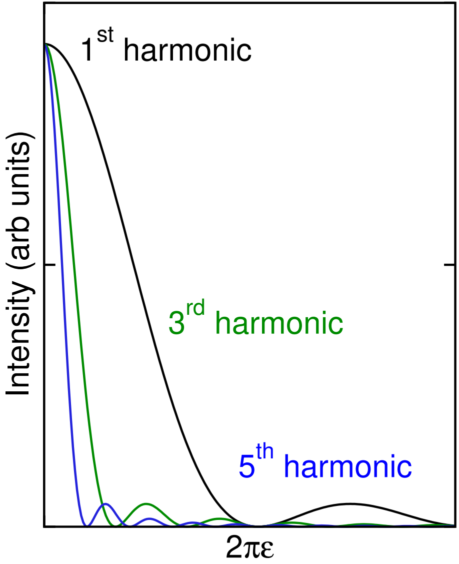


The more poles in the undulator, the more monochromatic the beam since a slight change in $\epsilon = \delta L/\lambda$ implies a slightly different wavelength λ

Another way to look at this is that the longer the undulator, the longer the pulse train in time and the narrower the frequency distribution in its Fourier Transform

The APS has a 72 pole undulator of 3.3 cm period

Undulator monochromaticity



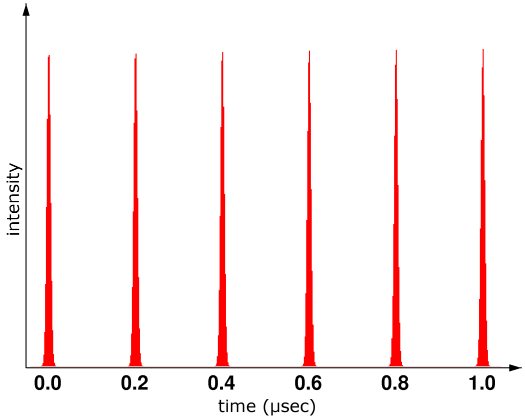
The more poles in the undulator, the more monochromatic the beam since a slight change in $\epsilon = \delta L/\lambda$ implies a slightly different wavelength λ

Another way to look at this is that the longer the undulator, the longer the pulse train in time and the narrower the frequency distribution in its Fourier Transform

The APS has a 72 pole undulator of 3.3 cm period

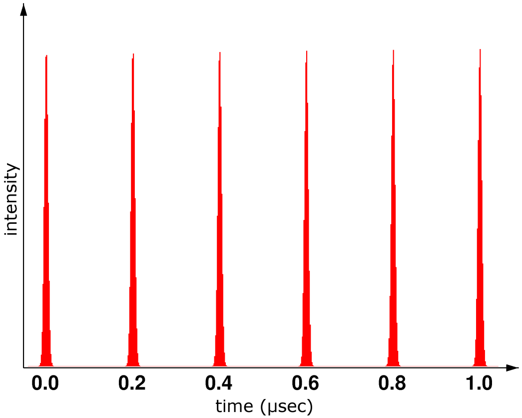
Higher order harmonics have narrower energy bandwidth but lower peak intensity

Synchrotron time structure



There are two important time scales for a storage ring such as the APS: pulse length and interpulse spacing

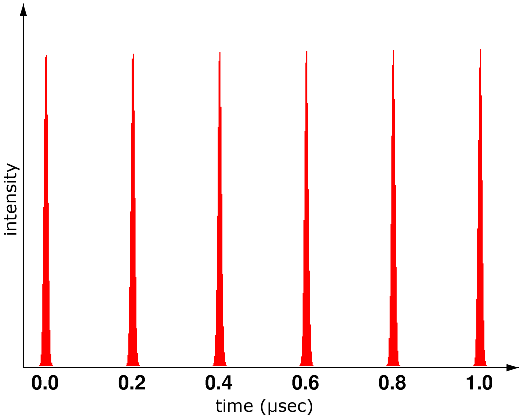
Synchrotron time structure



There are two important time scales for a storage ring such as the APS: pulse length and interpulse spacing

The APS pulse length in 24-bunch mode is 90 ps while the pulses come every 154 ns

Synchrotron time structure

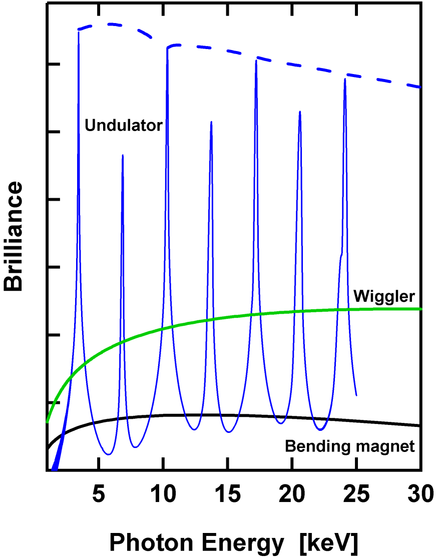


There are two important time scales for a storage ring such as the APS: pulse length and interpulse spacing

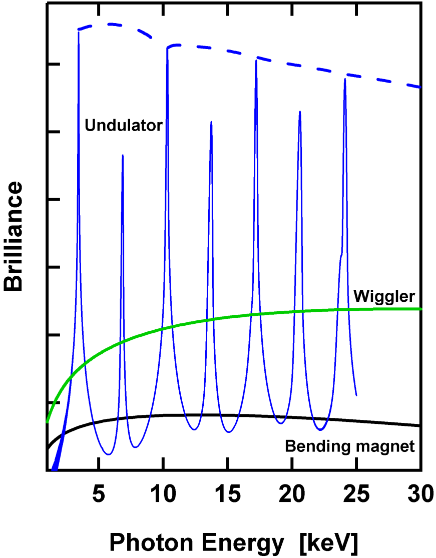
The APS pulse length in 24-bunch mode is 90 ps while the pulses come every 154 ns

Other modes include single-bunch mode for timing experiments and 324-bunch mode (inter pulse timing of 11.7 ns) for a more constant x-ray flux

Spectral comparison

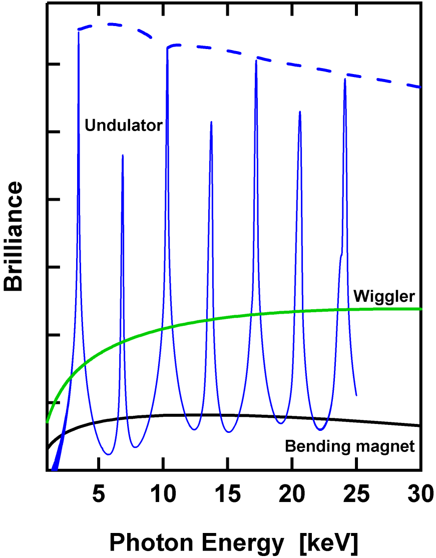


Spectral comparison



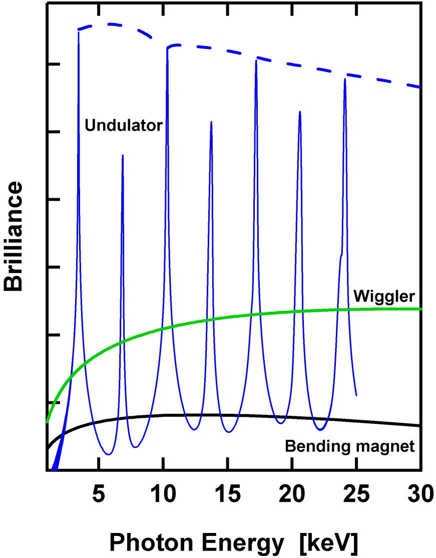
- Bending magnet

Spectral comparison



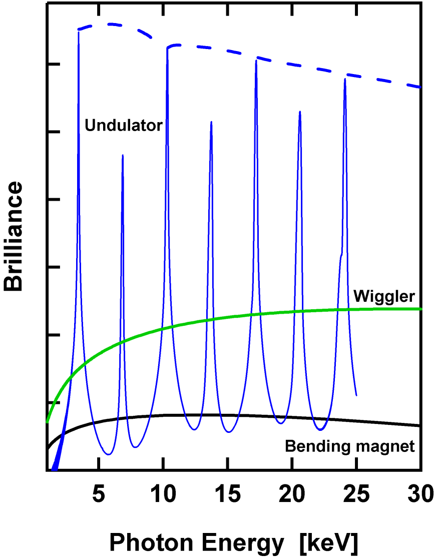
- Bending magnet
  - Broad, nearly white spectrum

Spectral comparison



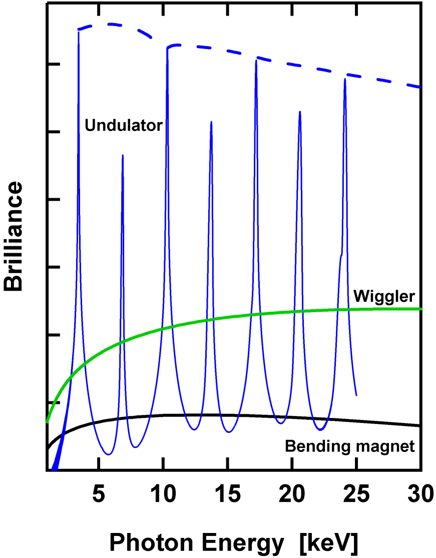
- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV

Spectral comparison



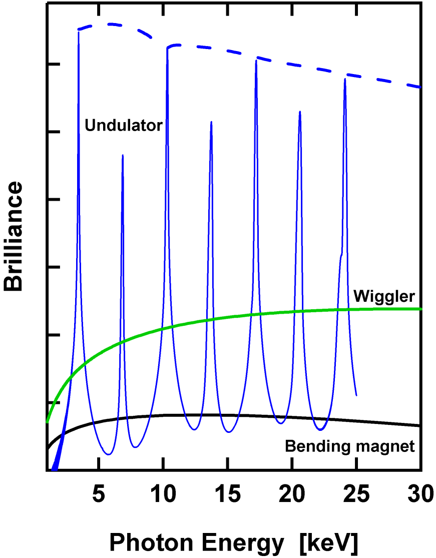
- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler

Spectral comparison



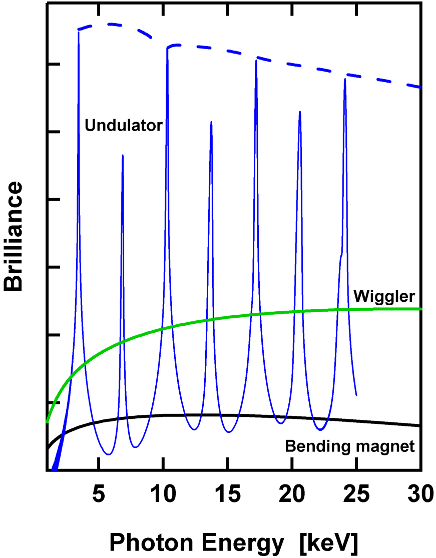
- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler
  - Shifts critical energy higher than bending magnet

Spectral comparison



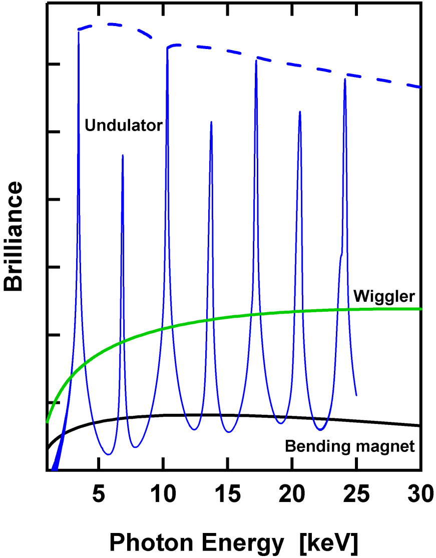
- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler
  - Shifts critical energy higher than bending magnet
  - Brilliance is more than an order of magnitude greater than a bending magnet

Spectral comparison



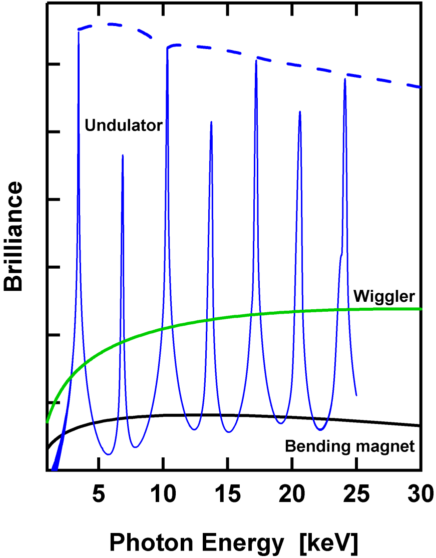
- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler
  - Shifts critical energy higher than bending magnet
  - Brilliance is more than an order of magnitude greater than a bending magnet
- Undulator

Spectral comparison



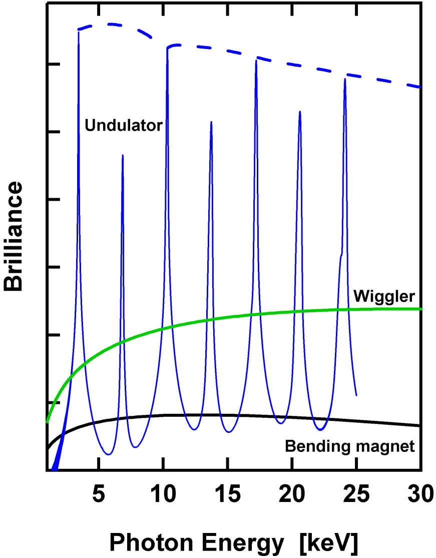
- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler
  - Shifts critical energy higher than bending magnet
  - Brilliance is more than an order of magnitude greater than a bending magnet
- Undulator
  - Brilliance is 6 orders larger than a bending magnet

Spectral comparison



- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler
  - Shifts critical energy higher than bending magnet
  - Brilliance is more than an order of magnitude greater than a bending magnet
- Undulator
  - Brilliance is 6 orders larger than a bending magnet
  - Both odd and even harmonics appear

Spectral comparison



- Bending magnet
  - Broad, nearly white spectrum
  - Energies extend to 100's of keV
- Wiggler
  - Shifts critical energy higher than bending magnet
  - Brilliance is more than an order of magnitude greater than a bending magnet
- Undulator
  - Brilliance is 6 orders larger than a bending magnet
  - Both odd and even harmonics appear
  - Harmonics can be tuned in energy (dashed lines)

Emittance

Is there a limit to the brightness of an undulator source at a synchrotron?

Emittance

Is there a limit to the brightness of an undulator source at a synchrotron?

the brightness is inversely proportional to the square of the product of the linear source size and the angular divergence

$$brightness = \frac{flux\ [\text{photons/s}]}{divergence\ \left[\text{mrad}^2\right] \cdot source\ size\ [\text{mm}^2]\ [0.1\%\ \text{bandwidth}]}$$

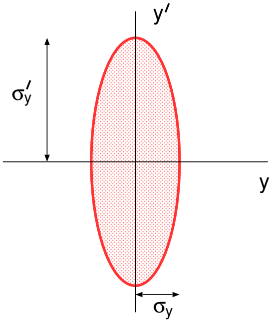
Emittance

Is there a limit to the brightness of an undulator source at a synchrotron?

the brightness is inversely proportional to the square of the product of the linear source size and the angular divergence

$$\textit{brightness} = \frac{\textit{flux}\ [\text{photons/s}]}{\textit{divergence}\ \left[\text{mrad}^2\right] \cdot \textit{source size}\ [\text{mm}^2]\ [0.1\%\ \text{bandwidth}]}$$

the product of the source size (σ) and divergence (σ') is called the emittance, ϵ



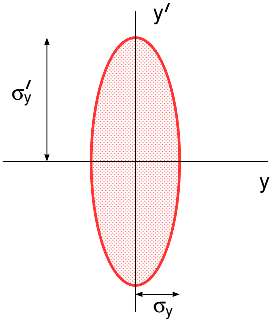
Emittance

Is there a limit to the brightness of an undulator source at a synchrotron?

the brightness is inversely proportional to the square of the product of the linear source size and the angular divergence

$$\textit{brightness} = \frac{\textit{flux}\,[\text{photons/s}]}{\textit{divergence}\left[\text{mrad}^2\right] \cdot \textit{source size}\,[\text{mm}^2]\,[0.1\%\,\text{bandwidth}]}$$

the product of the source size (σ) and divergence (σ') is called the emittance, ϵ and the brightness is limited by the product of the emittance of the radiation in the horizontal and vertical directions $\epsilon_x\epsilon_y = (\sigma_x\sigma'_x)(\sigma_y\sigma'_y)$



Emittance

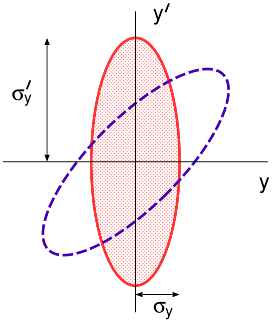
Is there a limit to the brightness of an undulator source at a synchrotron?

the brightness is inversely proportional to the square of the product of the linear source size and the angular divergence

$$\textit{brightness} = \frac{\textit{flux}\ [\text{photons/s}]}{\textit{divergence}\left[\text{mrad}^2\right] \cdot \textit{source size}\ [\text{mm}^2]\ [0.1\%\ \text{bandwidth}]}$$

the product of the source size (σ) and divergence (σ') is called the emittance, ϵ and the brightness is limited by the product of the emittance of the radiation in the horizontal and vertical directions $\epsilon_x\epsilon_y = (\sigma_x\sigma'_x)(\sigma_y\sigma'_y)$

this emittance cannot be changed but it can be rotated



Emittance

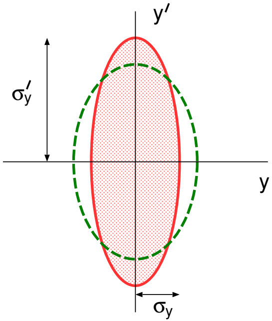
Is there a limit to the brightness of an undulator source at a synchrotron?

the brightness is inversely proportional to the square of the product of the linear source size and the angular divergence

$$\textit{brightness} = \frac{\textit{flux}\,[\text{photons/s}]}{\textit{divergence}\left[\text{mrad}^2\right] \cdot \textit{source size}\,[\text{mm}^2]\,[0.1\%\,\text{bandwidth}]}$$

the product of the source size (σ) and divergence (σ') is called the emittance, ϵ and the brightness is limited by the product of the emittance of the radiation in the horizontal and vertical directions $\epsilon_x \epsilon_y = (\sigma_x \sigma'_x)(\sigma_y \sigma'_y)$

this emittance cannot be changed but it can be rotated or deformed by magnetic fields as the electron beam travels around the storage ring as long as the area is kept constant



APS emittance

For photon emission from a single electron in a 2m undulator at 1Å

APS emittance

For photon emission from a single electron in a 2m undulator at 1Å

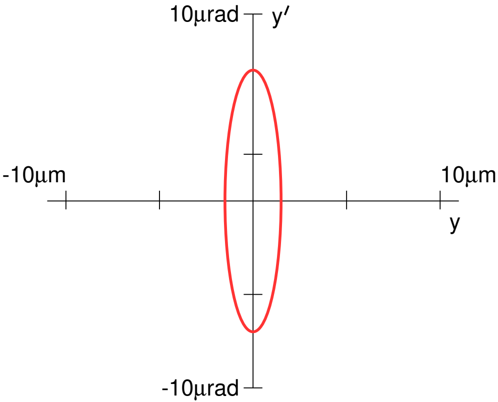
$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

APS emittance

For photon emission from a single electron in a 2m undulator at 1Å

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$



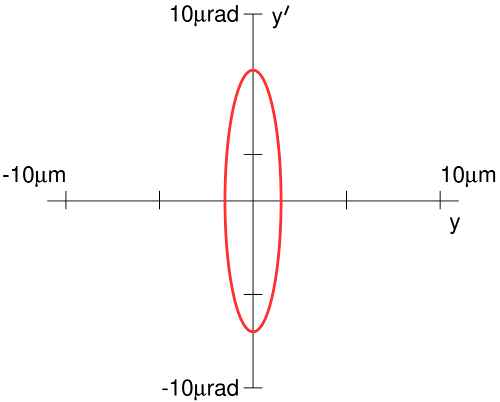
APS emittance

For photon emission from a single electron in a 2m undulator at 1Å

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$

current APS electron beam parameters are



APS emittance

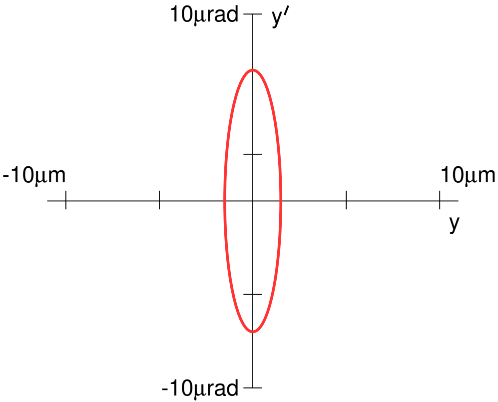
For photon emission from a single electron in a 2m undulator at 1Å

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$

current APS electron beam parameters are

$$\sigma_y = 9.1\mu\text{m}$$



APS emittance

For photon emission from a single electron in a 2m undulator at 1Å

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

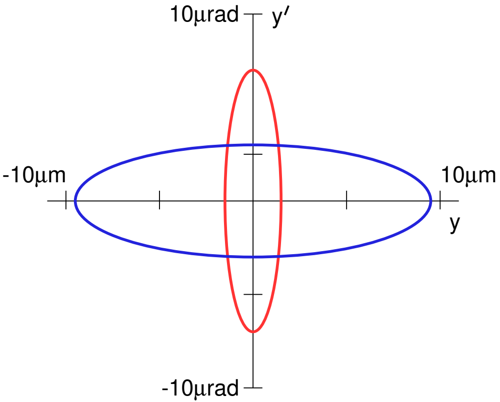
$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$

current APS electron beam parameters are

$$\sigma_y = 9.1\mu\text{m}$$

$$\sigma'_y = 3.0\mu\text{rad}$$

must convolute to get photon emission from entire beam (in vertical direction)



APS emittance

For photon emission from a single electron in a 2m undulator at 1Å

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

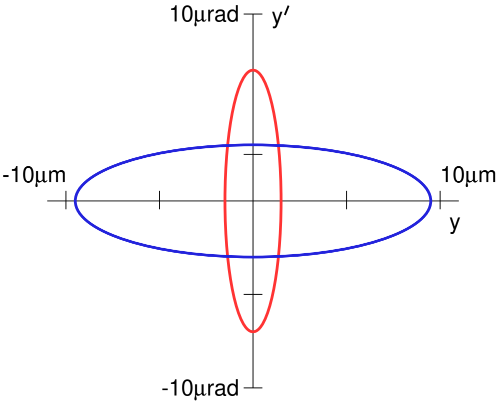
$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$

current APS electron beam parameters are

$$\sigma_y = 9.1\mu\text{m}$$

$$\sigma'_y = 3.0\mu\text{rad}$$

must convolute to get photon emission from entire beam (in vertical direction)



$$\sigma_{radiation} = 9.1\mu\text{m}$$

APS emittance

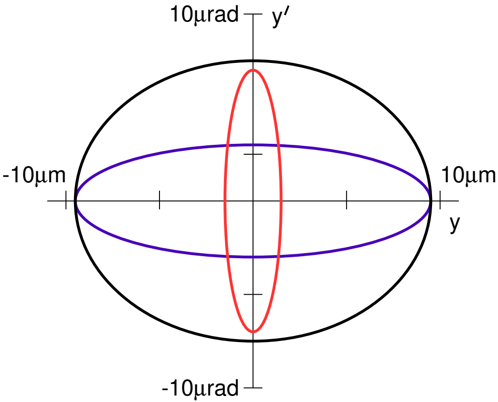
For photon emission from a single electron in a 2m undulator at 1Å

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$
$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$

current APS electron beam parameters are

$$\sigma_y = 9.1\mu\text{m}$$
$$\sigma'_y = 3.0\mu\text{rad}$$

must convolute to get photon emission from entire beam (in vertical direction)



$$\sigma_{radiation} = 9.1\mu\text{m}$$
$$\sigma'_{radiation} = 7.7\mu\text{rad}$$

Evolution of APS parameters

Parameter	APS 1995	APS 2001	APS 2005
Bunches		24 & 324	
σ_x	334 μm	352 μm	280 μm
σ'_x	24 μrad	22 μrad	11.6 μrad
σ_y	89 μm	18.4 μm	9.1 μm
σ'_y	8.9 μrad	4.2 μrad	3.0 μrad

Evolution of APS parameters

Parameter	APS 1995	APS 2001	APS 2005
Bunches		24 & 324	
σ_x	334 μm	352 μm	280 μm
σ'_x	24 μrad	22 μrad	11.6 μrad
σ_y	89 μm	18.4 μm	9.1 μm
σ'_y	8.9 μrad	4.2 μrad	3.0 μrad

When first commissioned in 1995, the APS electron beam size and divergence was relatively large, particularly in the horizontal, x direction

Evolution of APS parameters

Parameter	APS 1995	APS 2001	APS 2005
Bunches		24 & 324	
σ_x	334 μm	352 μm	280 μm
σ'_x	24 μrad	22 μrad	11.6 μrad
σ_y	89 μm	18.4 μm	9.1 μm
σ'_y	8.9 μrad	4.2 μrad	3.0 μrad

When first commissioned in 1995, the APS electron beam size and divergence was relatively large, particularly in the horizontal, x direction

By the end of the first decade of operation, the horizontal source size decreased by about 16% and its horizontal divergence by more than 50%

Evolution of APS parameters

Parameter	APS 1995	APS 2001	APS 2005
Bunches		24 & 324	
σ_x	334 μm	352 μm	280 μm
σ'_x	24 μrad	22 μrad	11.6 μrad
σ_y	89 μm	18.4 μm	9.1 μm
σ'_y	8.9 μrad	4.2 μrad	3.0 μrad

When first commissioned in 1995, the APS electron beam size and divergence was relatively large, particularly in the horizontal, x direction

By the end of the first decade of operation, the horizontal source size decreased by about 16% and its horizontal divergence by more than 50% while the vertical source size decreased by over 90% and the vertical divergence by nearly 67%

Evolution of APS parameters

Parameter	APS 1995	APS 2001	APS 2005	APS-U Timing	APS-U Brightness
Bunches		24 & 324		48	324
σ_x	334 μm	352 μm	280 μm	18.1 μm	21.8 μm
σ'_x	24 μrad	22 μrad	11.6 μrad	2.6 μrad	3.1 μrad
σ_y	89 μm	18.4 μm	9.1 μm	10.6 μm	4.1 μm
σ'_y	8.9 μrad	4.2 μrad	3.0 μrad	4.2 μrad	1.7 μrad

When first commissioned in 1995, the APS electron beam size and divergence was relatively large, particularly in the horizontal, x direction

By the end of the first decade of operation, the horizontal source size decreased by about 16% and its horizontal divergence by more than 50% while the vertical source size decreased by over 90% and the vertical divergence by nearly 67%

The APS-U will make the beam smaller and more square in space making for higher performance insertion device beam lines.