Today's outline - August 26, 2024

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- Coherence of x-ray sources

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Reading Assignment: Chapter 2.3–2.4

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Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Wednesday, September 04, 2024

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- The x-ray tube
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Reading Assignment: Chapter 2.3–2.4

Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Wednesday, September 04, 2024

Homework Assignment #02:
Problems on Canvas
due Monday, September 16, 2024

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Real x-rays are not perfect plane waves in two ways:

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Because of these imperfections the "coherence length" of an x-ray beam is finite and we can calculate it.

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Definition: *Distance over which two waves from the same source point with slightly different wavelengths will completely dephase.*

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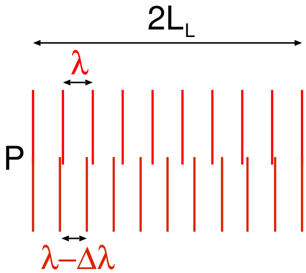

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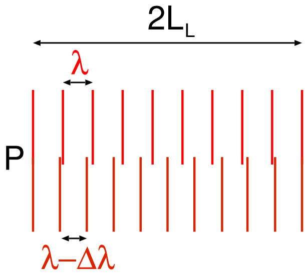

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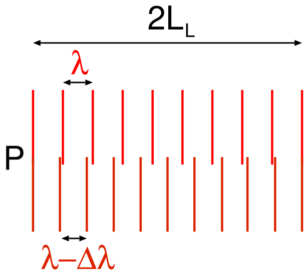

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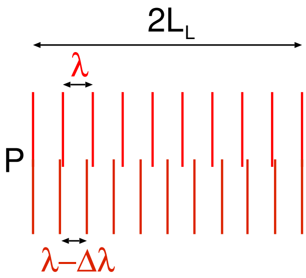

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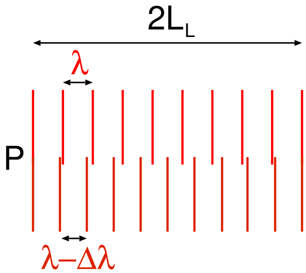

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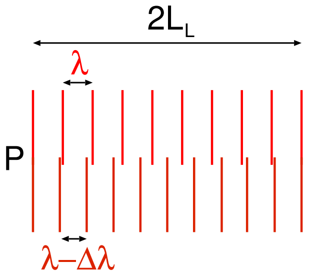

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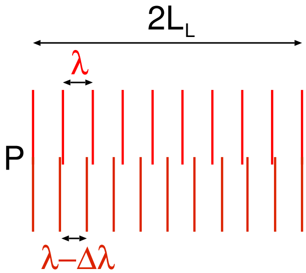

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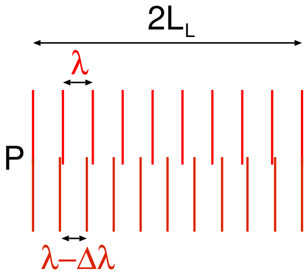

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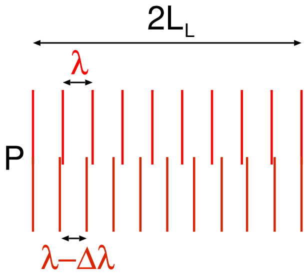

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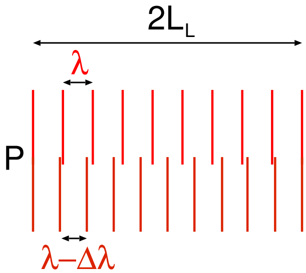

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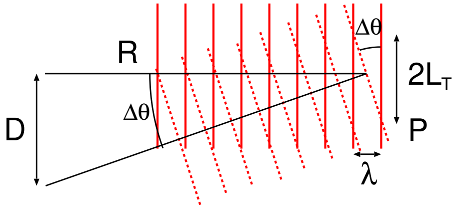

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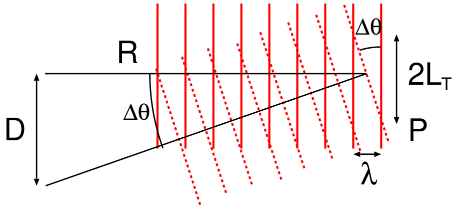

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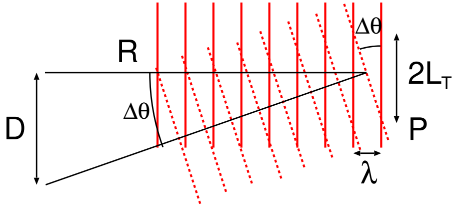

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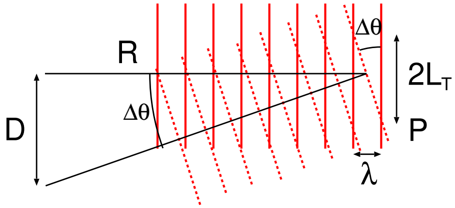

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$$L_T^v = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (3.2 \times 10^{-6})} \approx 780\mu\text{m}$$

Coherence lengths at the APS

The lateral coherence for a typical hard x-ray undulator beamline such as at the Advanced Photon Source for a wavelength of $\lambda = 1\text{Å}$, a monochromator resolution of $\Delta\lambda/\lambda = 10^{-5}$ and typically $R = 50\text{m}$ away with our experiment we have:

$$L_L = \frac{\lambda^2}{2\Delta\lambda} = \frac{\lambda}{2} \cdot \frac{\lambda}{\Delta\lambda} = \frac{1 \times 10^{-10}}{2 \cdot 10^{-5}} = 5\mu\text{m}$$

For the original Advanced Photon Source, a 3^{rd} generation undulator source, the vertical and horizontal source sizes were $D_v = 10\mu\text{m}$ and $D_h = 280\mu\text{m}$ giving transverse coherence lengths:

$$L_T^v = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (10 \times 10^{-6})} \approx 250\mu\text{m}$$

$$L_T^h = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (280 \times 10^{-6})} \approx 9\mu\text{m}$$

For the APS-U in "brightness" mode $D_v = 3.2\mu\text{m}$ and $D_v = 14.7\mu\text{m}$ so

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Coherence lengths at the APS

The lateral coherence for a typical hard x-ray undulator beamline such as at the Advanced Photon Source for a wavelength of $\lambda = 1$Å, a monochromator resolution of $\Delta\lambda/\lambda = 10^{-5}$ and typically $R = 50$m away with our experiment we have:

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For the original Advanced Photon Source, a 3rd generation undulator source, the vertical and horizontal source sizes were $D_v = 10\mu$m and $D_h = 280\mu$m giving transverse coherence lengths:

$$L_T^v = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (10 \times 10^{-6})} \approx 250\mu\text{m} \qquad L_T^h = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (280 \times 10^{-6})} \approx 9\mu\text{m}$$

For the APS-U in "brightness" mode $D_v = 3.2\mu$m and $D_v = 14.7\mu$m so

$$L_T^v = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (3.2 \times 10^{-6})} \approx 780\mu\text{m} \qquad L_T^h = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (14.7 \times 10^{-6})}$$

Coherence lengths at the APS

The lateral coherence for a typical hard x-ray undulator beamline such as at the Advanced Photon Source for a wavelength of $\lambda = 1\text{Å}$, a monochromator resolution of $\Delta\lambda/\lambda = 10^{-5}$ and typically $R = 50\text{m}$ away with our experiment we have:

$$L_L = \frac{\lambda^2}{2\Delta\lambda} = \frac{\lambda}{2} \cdot \frac{\lambda}{\Delta\lambda} = \frac{1 \times 10^{-10}}{2 \cdot 10^{-5}} = 5\mu\text{m}$$

For the original Advanced Photon Source, a 3^{rd} generation undulator source, the vertical and horizontal source sizes were $D_v = 10\mu\text{m}$ and $D_h = 280\mu\text{m}$ giving transverse coherence lengths:

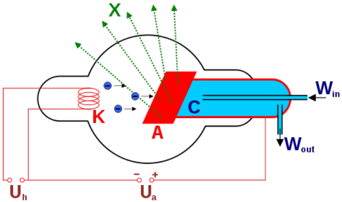
$$L_T^v = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (10 \times 10^{-6})} \approx 250\mu\text{m} \qquad L_T^h = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (280 \times 10^{-6})} \approx 9\mu\text{m}$$

For the APS-U in "brightness" mode $D_v = 3.2\mu\text{m}$ and $D_v = 14.7\mu\text{m}$ so

$$L_T^v = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (3.2 \times 10^{-6})} \approx 780\mu\text{m} \qquad L_T^h = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (14.7 \times 10^{-6})} \approx 170\mu\text{m}$$

Lab x-ray source schematics

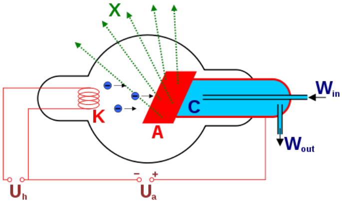
Fixed anode tube



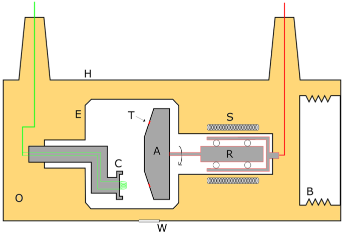
- low power
- low maintenance

Lab x-ray source schematics

Fixed anode tube



Rotating anode tube

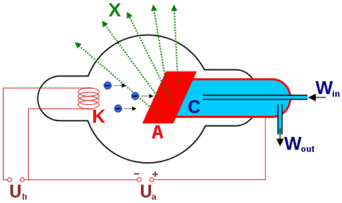


- low power
- low maintenance

- high power
- high maintenance

Lab x-ray source schematics

Fixed anode tube



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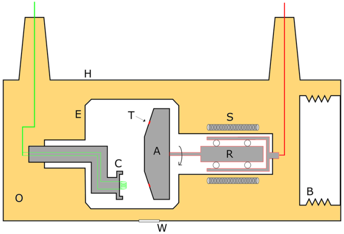
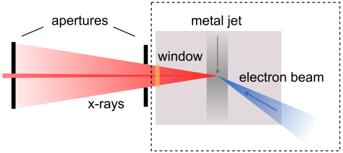


Liquid metal jet

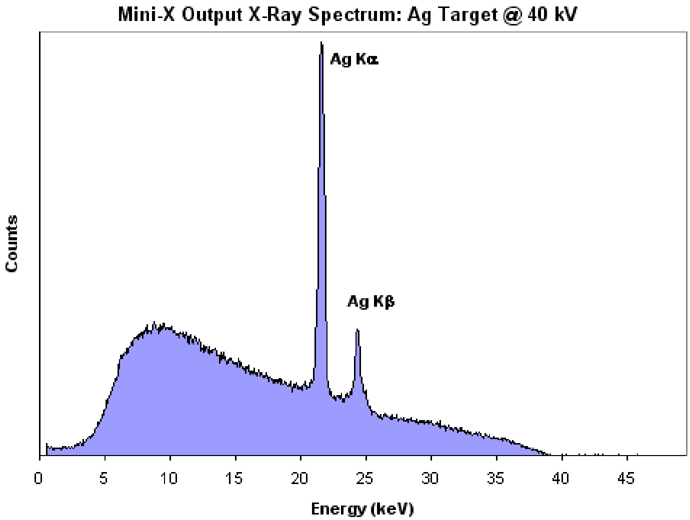


- low power
- low maintenance

- high power
- high maintenance

- high brightness
- small spot size

X-ray tube spectrum



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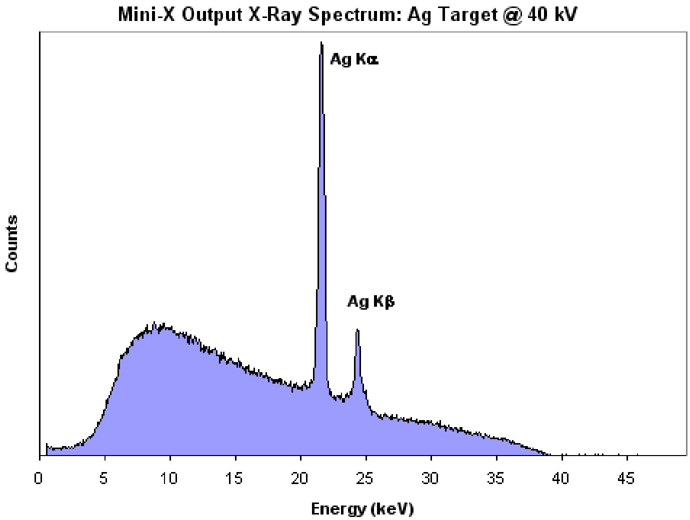
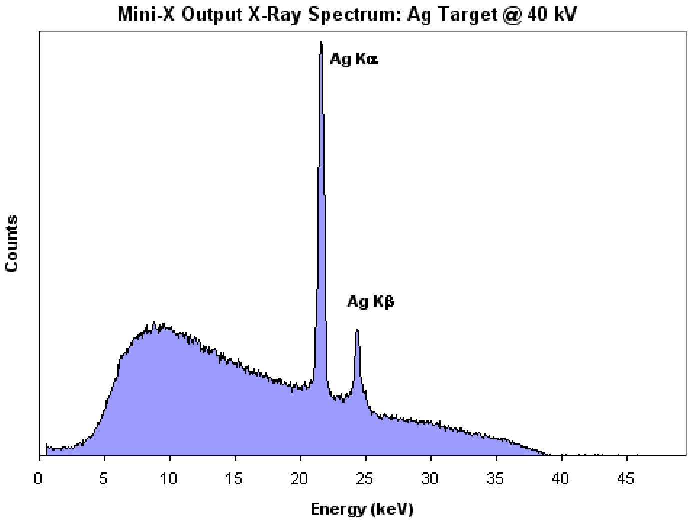


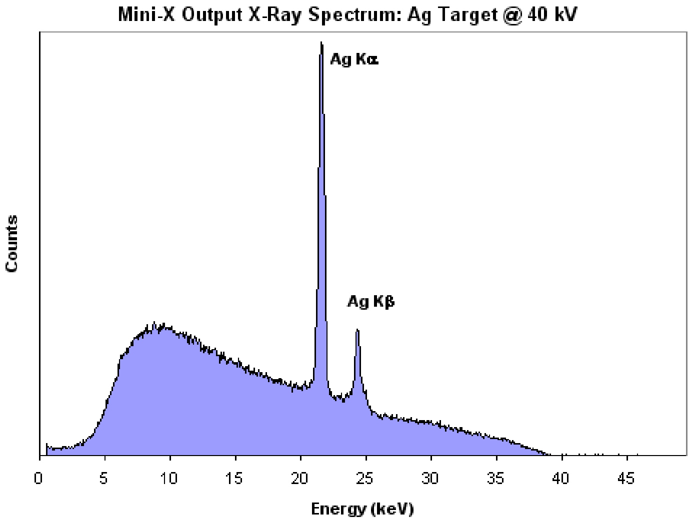
- Minimum wavelength (maximum energy) set by accelerating potential

X-ray tube spectrum



- Minimum wavelength (maximum energy) set by accelerating potential
- Bremßtrahlung radiation provides smooth background (charged particle deceleration)

X-ray tube spectrum



- Minimum wavelength (maximum energy) set by accelerating potential
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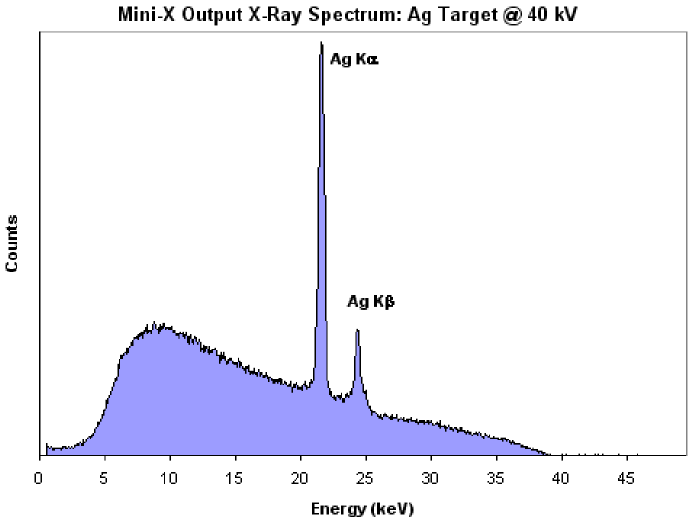
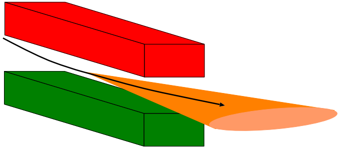
X-ray tube spectrum



- Minimum wavelength (maximum energy) set by accelerating potential
- Bremßtrahlung radiation provides smooth background (charged particle deceleration)
- Highest intensity at the characteristic fluorescence emission energy of the anode material
- Unpolarized, incoherent x-rays emitted in all directions from anode surface, must be collimated with slits

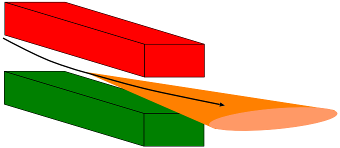
Synchrotron sources

Bending magnet



Synchrotron sources

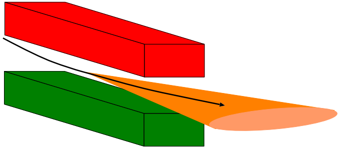
Bending magnet



- Wide horizontal beam

Synchrotron sources

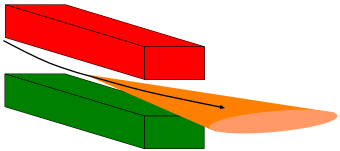
Bending magnet



- Wide horizontal beam
- Broad spectrum

Synchrotron sources

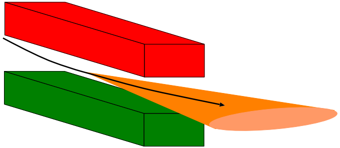
Bending magnet



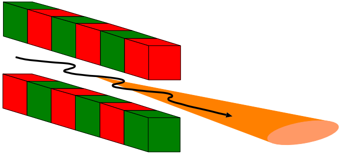
- Wide horizontal beam
- Broad spectrum
- Low brilliance

Synchrotron sources

Bending magnet



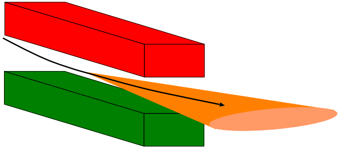
Wiggler



- Wide horizontal beam
- Broad spectrum
- Low brilliance

Synchrotron sources

Bending magnet



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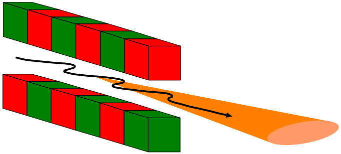
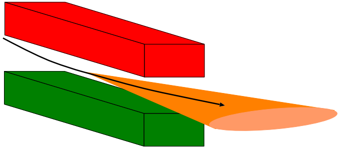


- Wide horizontal beam

Synchrotron sources

Bending magnet



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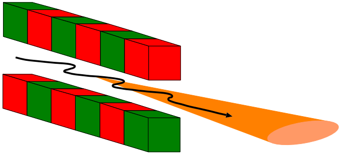

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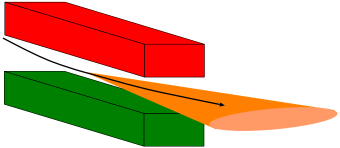
Synchrotron sources

Bending magnet



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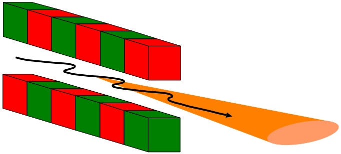
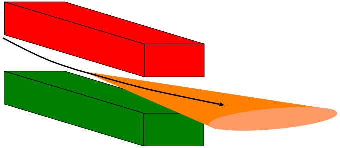


- Wide horizontal beam
- Broad spectrum
- Higher critical energy

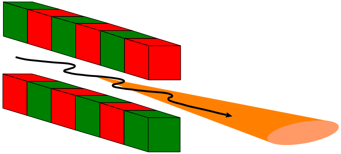
Synchrotron sources

Bending magnet



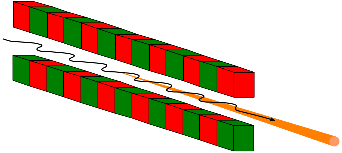
- Wide horizontal beam
- Broad spectrum
- Low brilliance

Wiggler



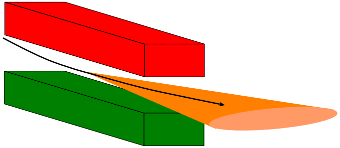
- Wide horizontal beam
- Broad spectrum
- Higher critical energy

Undulator



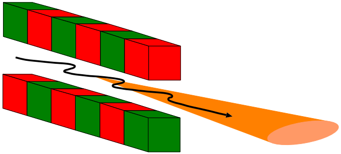
Synchrotron sources

Bending magnet



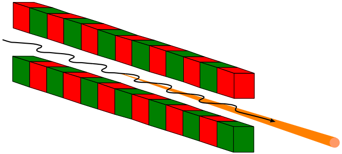
- Wide horizontal beam
- Broad spectrum
- Low brilliance

Wiggler



- Wide horizontal beam
- Broad spectrum
- Higher critical energy

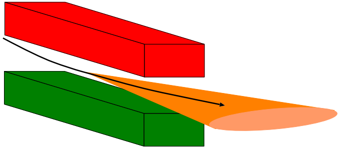
Undulator



- Highly collimated beam

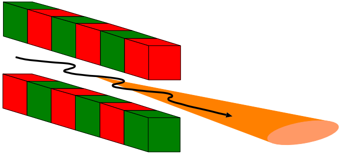
Synchrotron sources

Bending magnet



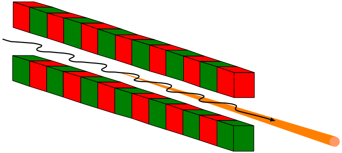
- Wide horizontal beam
- Broad spectrum
- Low brilliance

Wiggler



- Wide horizontal beam
- Broad spectrum
- Higher critical energy

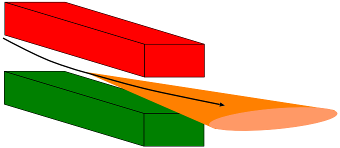
Undulator



- Highly collimated beam
- Highly peaked spectrum

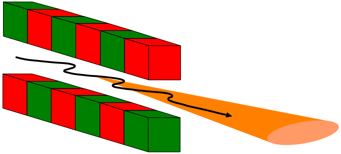
Synchrotron sources

Bending magnet



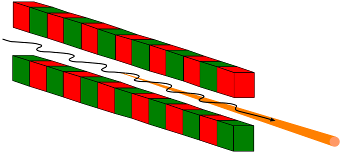
- Wide horizontal beam
- Broad spectrum
- Low brilliance

Wiggler



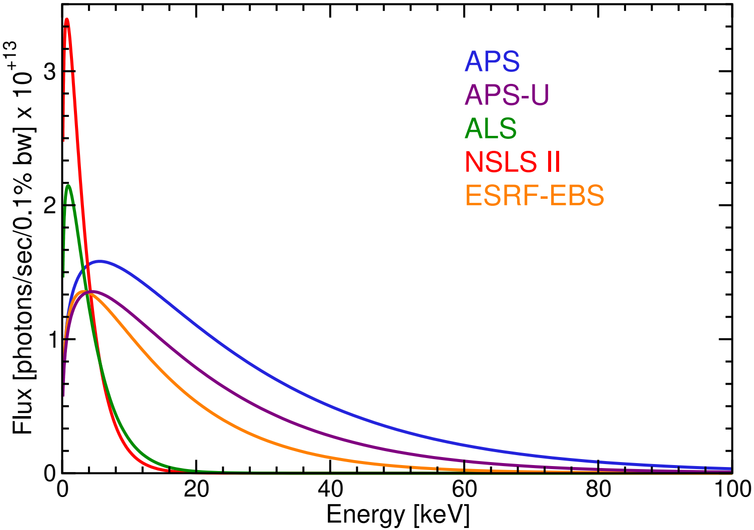
- Wide horizontal beam
- Broad spectrum
- Higher critical energy

Undulator

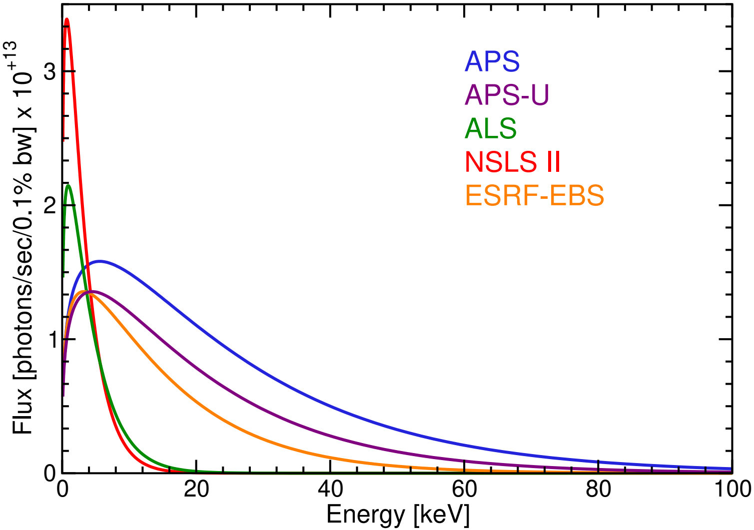


- Highly collimated beam
- Highly peaked spectrum
- High brightness

Bending magnet spectra



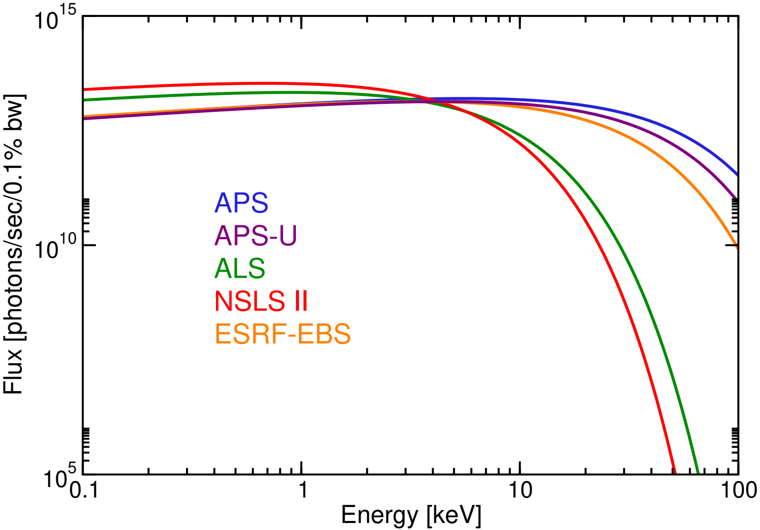
Bending magnet spectra



Lower energy sources, such as NSLS have lower peak energy and higher intensity at the peak.

Logarithmic scale shows clearly how much more energetic and intense the bending magnet sources at the 6 GeV and 7 GeV sources are.

Bending magnet spectra

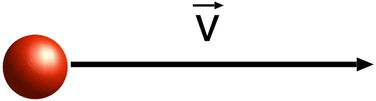


Lower energy sources, such as NSLS have lower peak energy and higher intensity at the peak.

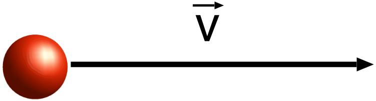
Higher energy sources, such as APS have higher energy spectrum and are only off by a factor of 2 intensity at low energy.

Logarithmic scale shows clearly how much more energetic and intense the bending magnet sources at the 6 GeV and 7 GeV sources are.

Review of special relativity

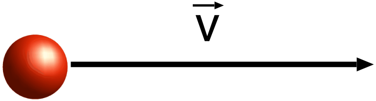


Review of special relativity



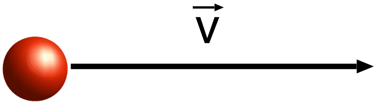
$$\beta = \frac{v}{c}$$

Review of special relativity



$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

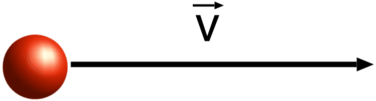
Review of special relativity



$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

$$E = \gamma m c^2$$

Review of special relativity

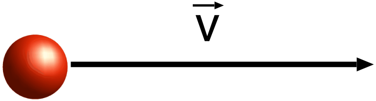


$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

$$E = \gamma m c^2$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Review of special relativity



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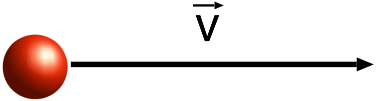
$$E = \gamma m c^2$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad \longrightarrow \quad \beta \approx 1 - \frac{1}{2}\frac{1}{\gamma^2}$$

use binomial expansion since $1/\gamma^2 << 1$

Review of special relativity



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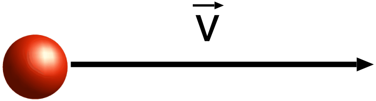
Let's calculate these quantities for an electron at NSLS and APS

Review of special relativity



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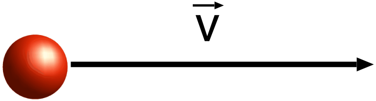
$$m_e = 0.511 \text{ MeV}/c^2$$

Review of special relativity



$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

$$E = \gamma m c^2$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \longrightarrow \beta \approx 1 - \frac{1}{2}\frac{1}{\gamma^2}$$

use binomial expansion since $1/\gamma^2 << 1$

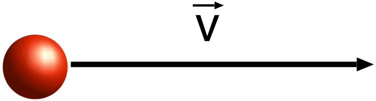
Let's calculate these quantities for an electron at NSLS and APS

$$m_e = 0.511 \text{ MeV}/c^2$$

NSLS II: $E = 3.0$ GeV

$$\gamma = \frac{3.0 \times 10^9}{0.511 \times 10^6} = 5871$$

Review of special relativity



$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

$$E = \gamma m c^2$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad \longrightarrow \quad \beta \approx 1 - \frac{1}{2}\frac{1}{\gamma^2}$$

use binomial expansion since $1/\gamma^2 << 1$

Let's calculate these quantities for an electron at NSLS and APS

$$m_e = 0.511 \text{ MeV}/\text{c}^2$$

NSLS II: $E = 3.0$ GeV

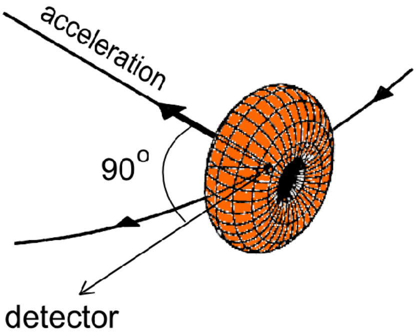
$$\gamma = \frac{3.0 \times 10^9}{0.511 \times 10^6} = 5871$$

APS: $E = 7.0$ GeV

$$\gamma = \frac{7.0 \times 10^9}{0.511 \times 10^6} = 13700$$

"Headlight" effect

In electron rest frame:



emission is symmetric about the axis of the acceleration vector

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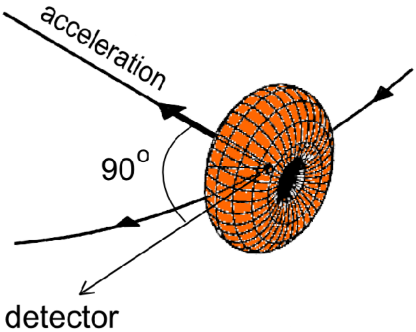
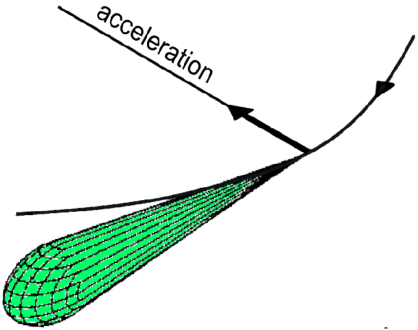


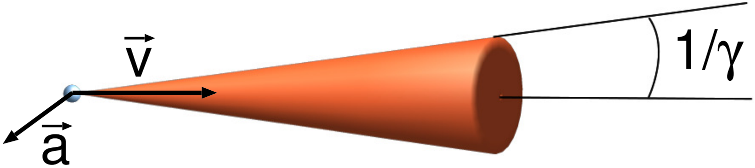
emission is symmetric about the axis of the acceleration vector

In lab frame:



emission is pushed into the direction of motion of the electron

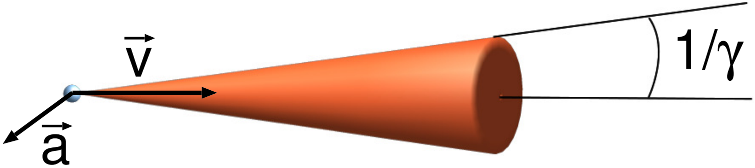
Relativistic emission



the electron is in constant transverse acceleration due to the Lorentz force from the magnetic field of the bending magnet

$$\vec{F} = e\vec{v} \times \vec{B} = m_e\vec{a}$$

Relativistic emission



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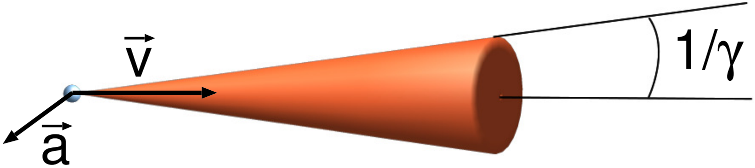

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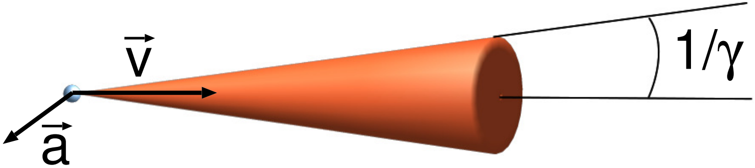

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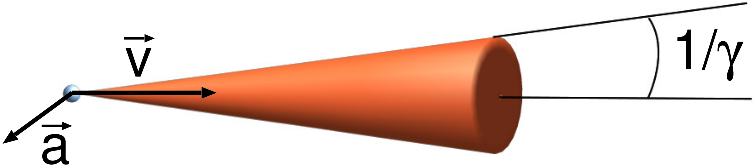

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for the APS, with $\gamma \approx 10^4$ we have

$$E_{max} \approx (10^4)^3 \cdot 10^6 = 10^{18}$$

Flux and brightness

There are a number of important quantities which are relevant to the quality of an x-ray source:

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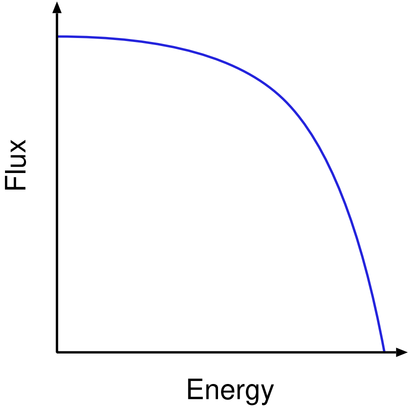

For a specific photon flux distribution, we would normally integrate to get the total flux.

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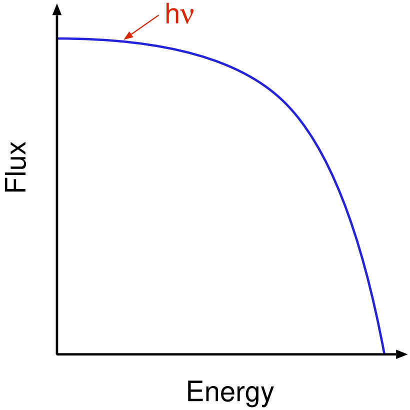

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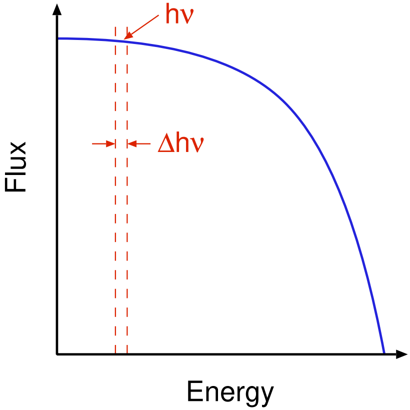

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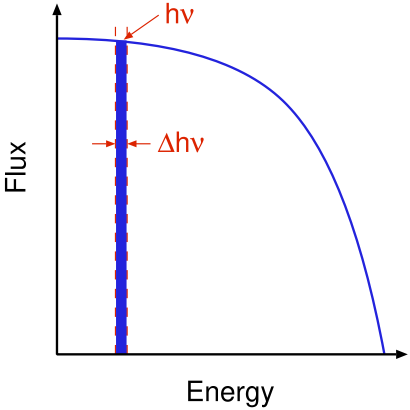

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Compute the integrated photon flux in that bandwidth.

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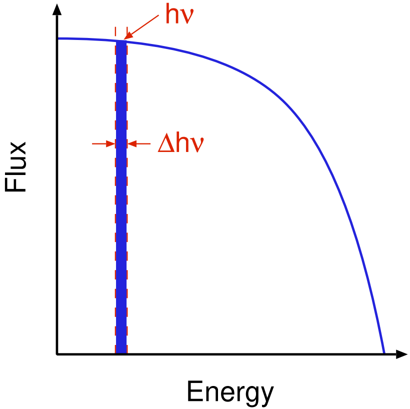

The source size depends on the electron beam size, its excursion, and any slits which define how much of the source is visible by the observer.

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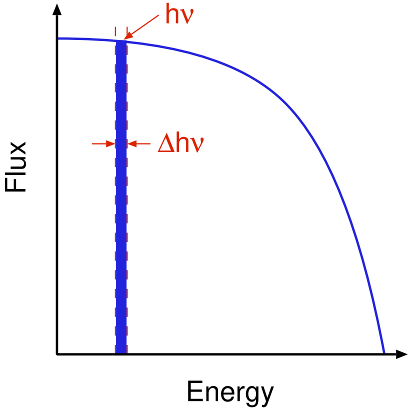

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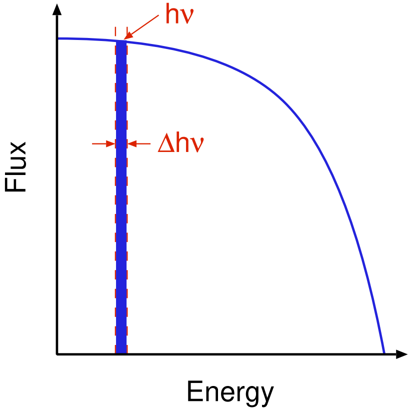

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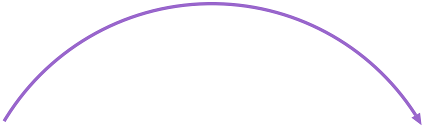
The divergence is the angular spread the x-ray beam in the x and y directions.

$$\alpha \approx x/z \qquad \beta \approx y/z,$$

where z is the distance from the source over which there is a lateral spread x and y in each direction

Segmented arc approximation



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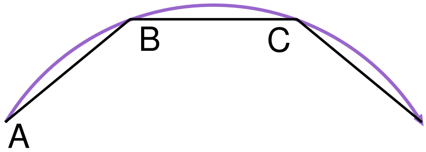
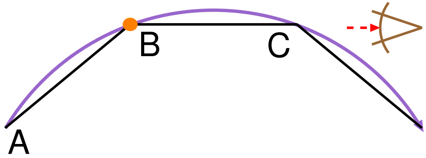


- Approximate the electron's path as a series of segments

Segmented arc approximation



- Approximate the electron's path as a series of segments
- At each corner the electron is accelerated and emits radiation

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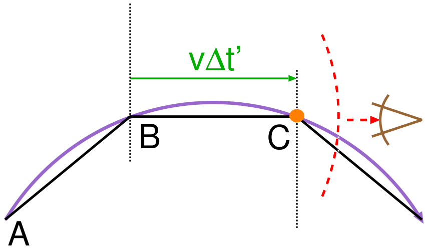

- Approximate the electron's path as a series of segments
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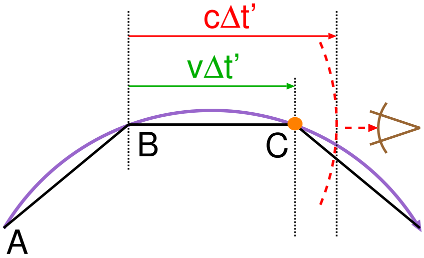

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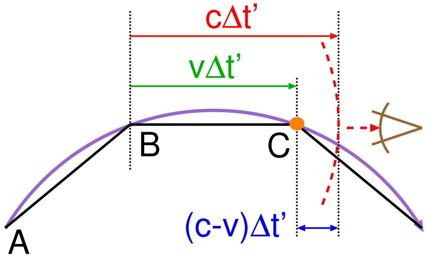
The electron travels the distance from B to C in $\Delta t'$

Segmented arc approximation



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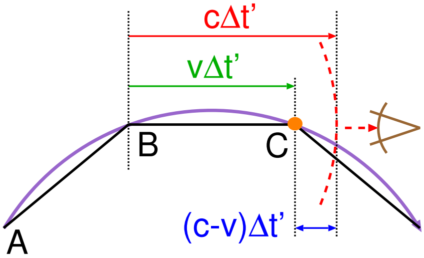
The electron travels the distance from B to C in $\Delta t'$ while the light pulse emitted at B travels further, $c\Delta t'$, in the same time.

Segmented arc approximation



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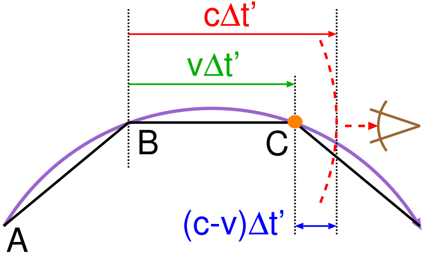

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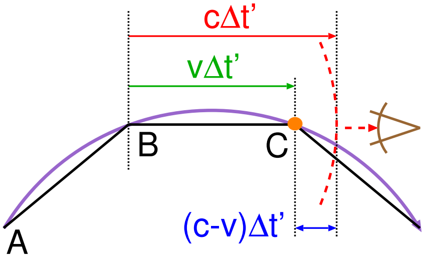

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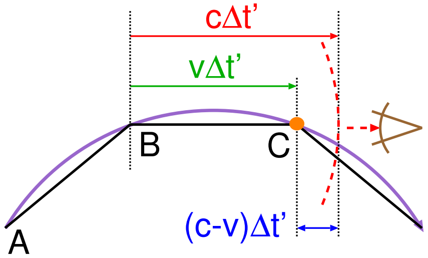

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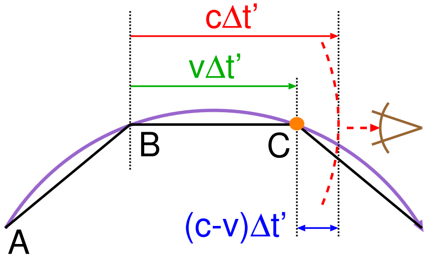

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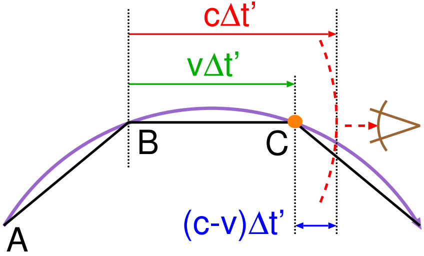
Doppler compression



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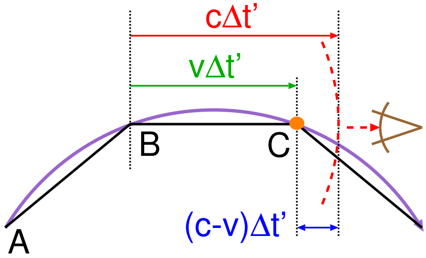

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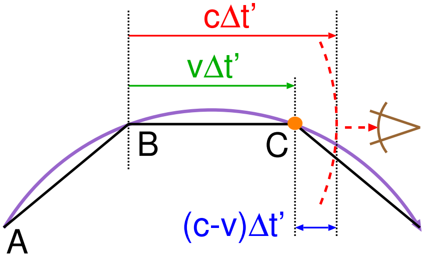
Since $0 < \beta < 1$ this translates to a Doppler compression of the emitted wavelength.

Doppler compression



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Recall that

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \ ,$$

but for synchrotron radiation, $\gamma > 1000$,

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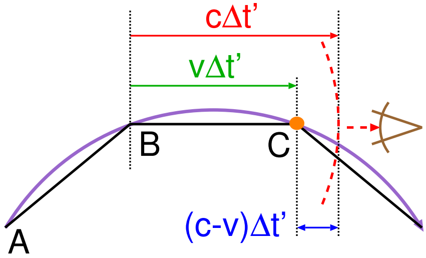

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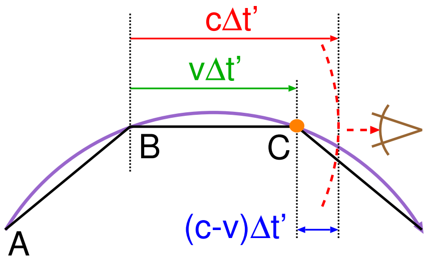

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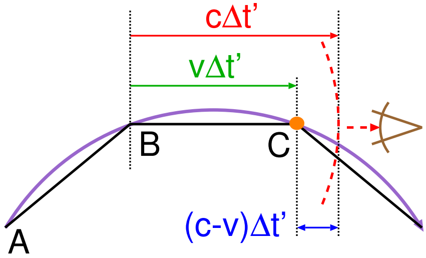
Doppler compression



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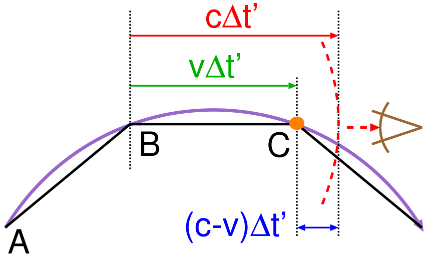
$$\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} = 1 - \frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}\frac{1}{2}\frac{1}{2!}\frac{1}{\gamma^4} + \cdots$$

Doppler compression



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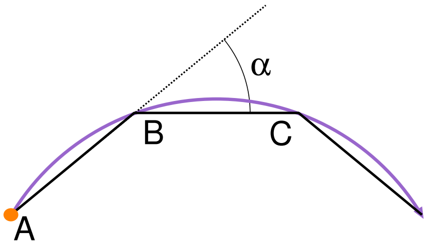
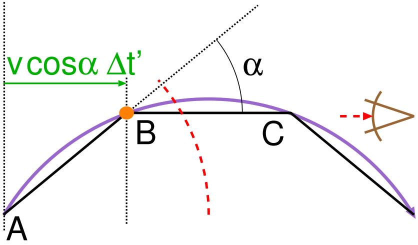
Off-axis emission



Consider the emission from segment AB, which is not along the line toward the observer.

Off-axis emission



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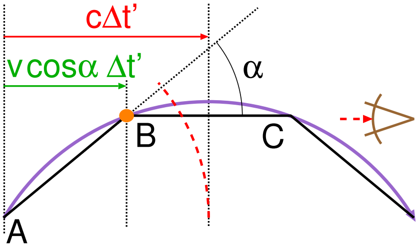

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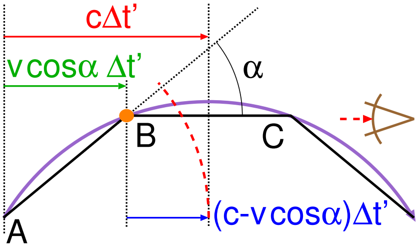

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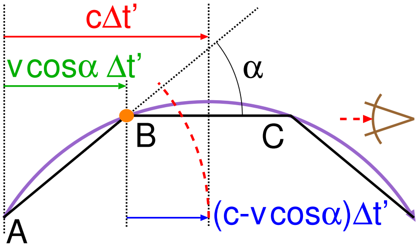

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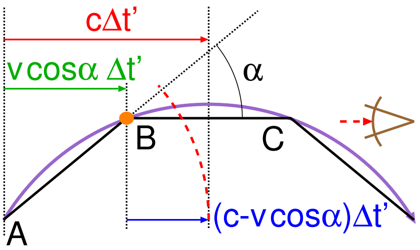

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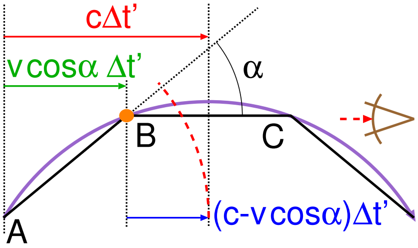

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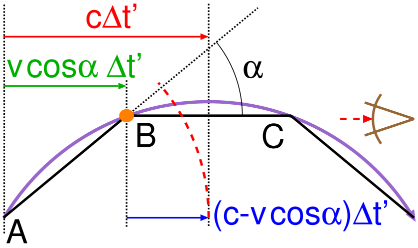

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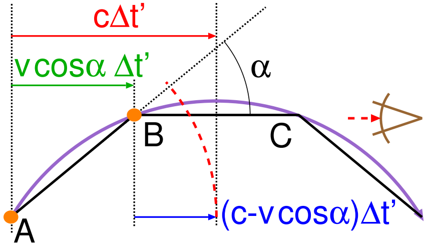
Corrected Doppler shift



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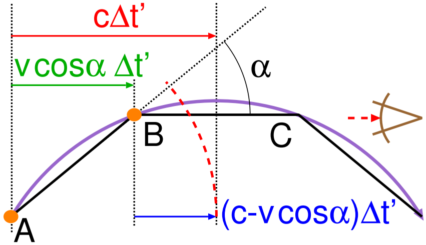

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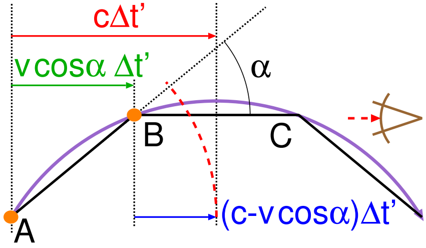
Corrected Doppler shift



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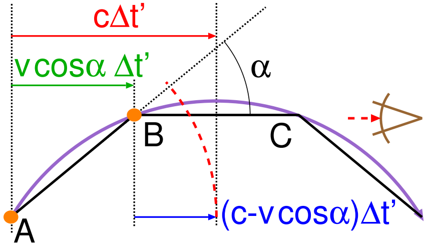

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and γ is very large, we have

$$\frac{\Delta t}{\Delta t'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{\alpha^2}{2}\right)$$

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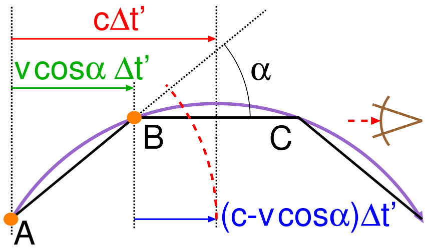

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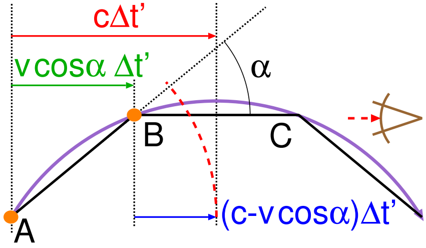

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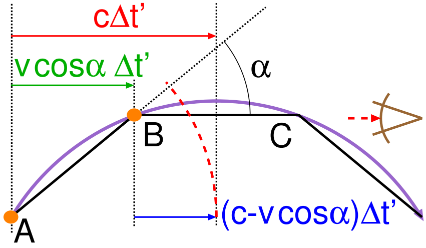

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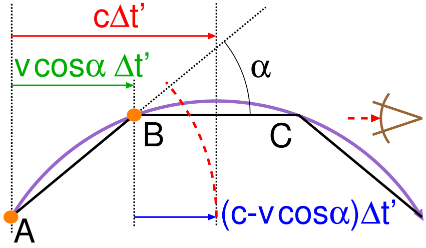

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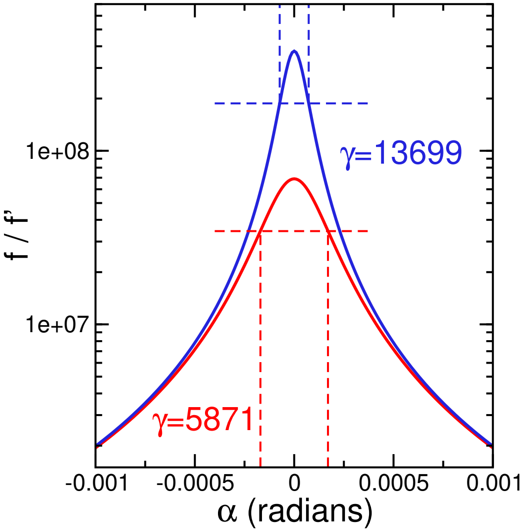
called the time compression ratio.

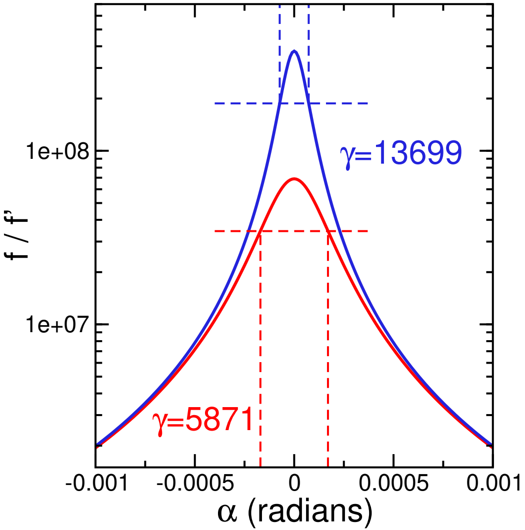
Radiation opening angle



The Doppler shift is defined in terms of the time compression ratio

$$\frac{f}{f'} = \frac{\Delta t'}{\Delta t} = \frac{2\gamma^2}{1+\alpha^2\gamma^2}$$

Radiation opening angle



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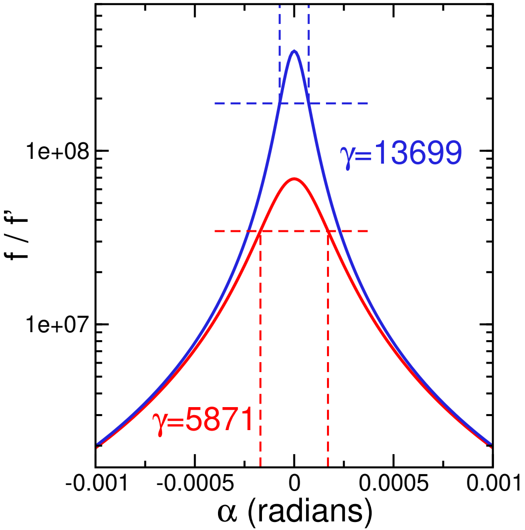

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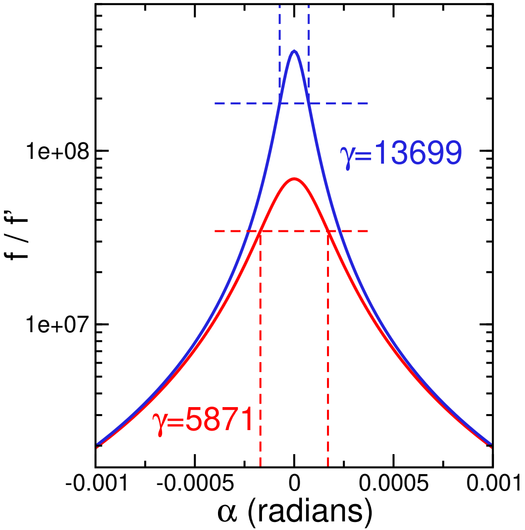

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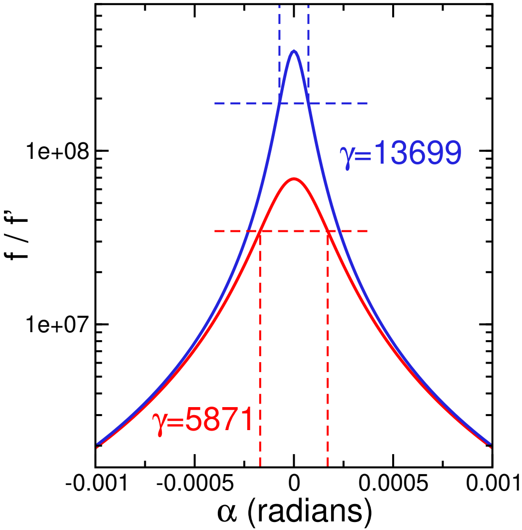

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- The dashed lines indicate where $\alpha = \pm 1/\gamma$ and f/f' is half it's maximum
- The highest energy emitted radiation appears within a cone of half angle $1/\gamma$
- Lower energies appear above and below the plane of the electron orbit