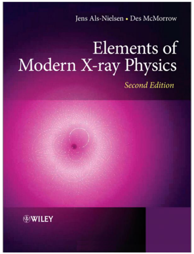
PHYS 570 - Introduction to Synchrotron Radiation

Term: Fall 2024
Meetings: Monday & Wednesday 17:00-18:15
Location: 102 Rettaliata Engineering

Instructor: Carlo Segre
Office: 166d/172 Pritzker Science
Phone: 312.567.3498
email: segre@iit.edu

Book: *Elements of Modern X-Ray Physics, 2nd ed.*,
J. Als-Nielsen and D. McMorrow (Wiley, 2011)

Web Site: http://csrri.iit.edu/∼segre/phys570/24F



Course objectives

- Describe the means of production of synchrotron x-ray radiation

Course objectives

- Describe the means of production of synchrotron x-ray radiation
- Describe the function of various components of a synchrotron beamline

Course objectives

- Describe the means of production of synchrotron x-ray radiation
- Describe the function of various components of a synchrotron beamline
- Perform calculations in support of a synchrotron experiment

Course objectives

- Describe the means of production of synchrotron x-ray radiation
- Describe the function of various components of a synchrotron beamline
- Perform calculations in support of a synchrotron experiment
- Describe the physics behind a variety of experimental techniques

Course objectives

- Describe the means of production of synchrotron x-ray radiation
- Describe the function of various components of a synchrotron beamline
- Perform calculations in support of a synchrotron experiment
- Describe the physics behind a variety of experimental techniques
- Prepare and deliver an oral presentation of a synchrotron radiation research topic

Course objectives

- Describe the means of production of synchrotron x-ray radiation
- Describe the function of various components of a synchrotron beamline
- Perform calculations in support of a synchrotron experiment
- Describe the physics behind a variety of experimental techniques
- Prepare and deliver an oral presentation of a synchrotron radiation research topic
- Write a General User Proposal in the format used by the Advanced Photon Source

Course syllabus

- Focus on applications of synchrotron radiation

Course syllabus

- Focus on applications of synchrotron radiation
- Homework assignments

Course syllabus

- Focus on applications of synchrotron radiation
- Homework assignments
- In-class student presentations on research topics
  - Choose a research article which features a synchrotron technique
  - Timetable will be posted

Course syllabus

- Focus on applications of synchrotron radiation
- Homework assignments
- In-class student presentations on research topics
  - Choose a research article which features a synchrotron technique
  - Timetable will be posted
- Final project - writing a General User Proposal
  - Start thinking about a suitable project right away
  - Synchrotron technique must differ from journal article used in final presentation
  - Make proposal and get approval before starting

Optional activities

- Visits to Advanced Photon Source
  - All students who plan to attend will need to request badges from APS
  - Go to the APS User Portal and register as a new user:
    https://beam.aps.anl.gov/pls/apsweb/ufr_main_pkg.usr_start_page
  - Use MRCAT (Sector 10) as location of experiment
  - Use Carlo Segre as local contact
  - State that your beamtime will be in the second week of March
  - Schedule to be determined

Optional activities (cont.)

- Hands on data analysis training
  - GSAS for Rietveld refinement of powder diffraction data
    https://subversion.xray.aps.anl.gov/trac/pyGSAS
  - Demeter: XAS processing and analysis
    https://bruceravel.github.io/demeter/
  - Larch: Data analysis tools for x-ray spectroscopy
    https://xraypy.github.io/xraylarch/

Course grading

33% – Homework assignments

Course grading

33% – Homework assignments

Weekly or bi-weekly

Course grading

33% – Homework assignments
- Weekly or bi-weekly
- Due at beginning of class

Course grading

33% – Homework assignments
Weekly or bi-weekly
Due at beginning of class
May be turned in via Canvas

Course grading

33% – Homework assignments

Weekly or bi-weekly

Due at beginning of class

May be turned in via Canvas

33% – General User Proposal

Course grading

33% – Homework assignments

Weekly or bi-weekly

Due at beginning of class

May be turned in via Canvas

33% – General User Proposal

33% – Final Exam Presentation

Course grading

33% – Homework assignments

Weekly or bi-weekly

Due at beginning of class

May be turned in via Canvas

33% – General User Proposal

33% – Final Exam Presentation

Grading scale

A	–	80%	to	100%
B	–	65%	to	80%
C	–	50%	to	65%
E	–	0%	to	50%

Topics to be covered (at a minimum)

Topics to be covered (at a minimum)

- X-rays and their interaction with matter

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces
- Kinematical diffraction

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces
- Kinematical diffraction
- Diffraction by perfect crystals

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces
- Kinematical diffraction
- Diffraction by perfect crystals
- Small angle scattering

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces
- Kinematical diffraction
- Diffraction by perfect crystals
- Small angle scattering
- Photoelectric absorption

Topics to be covered (at a minimum)

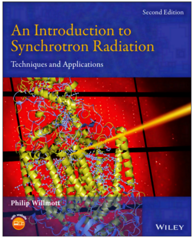
- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces
- Kinematical diffraction
- Diffraction by perfect crystals
- Small angle scattering
- Photoelectric absorption
- Resonant scattering

Topics to be covered (at a minimum)

- X-rays and their interaction with matter
- Sources of x-rays
- Refraction and reflection from interfaces
- Kinematical diffraction
- Diffraction by perfect crystals
- Small angle scattering
- Photoelectric absorption
- Resonant scattering
- Imaging

Resources for the course

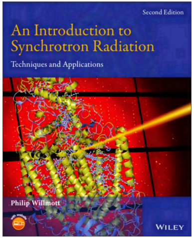
Introduction to Synchrotron Radiation: Techniques and Applications, 2nd ed., P. Willmott (Wiley, 2019)

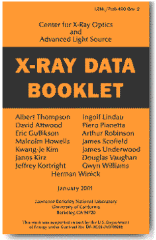


Resources for the course

Introduction to Synchrotron Radiation: Techniques and Applications, 2nd ed., P. Willmott (Wiley, 2019)

- Orange x-ray data booklet: http://xdb.lbl.gov/xdb-new.pdf

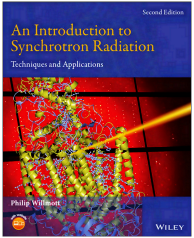


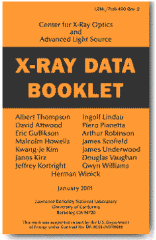


Resources for the course

Introduction to Synchrotron Radiation: Techniques and Applications, 2nd ed., P. Willmott (Wiley, 2019)

- Orange x-ray data booklet: http://xdb.lbl.gov/xdb-new.pdf
- Center for X-Ray Optics web site: http://cxro.lbl.gov

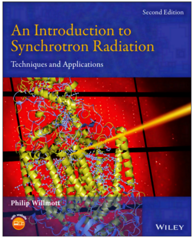


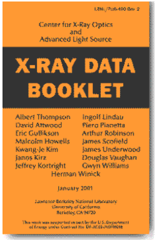


Resources for the course

Introduction to Synchrotron Radiation: Techniques and Applications, 2nd ed., P. Willmott (Wiley, 2019)

- Orange x-ray data booklet: http://xdb.lbl.gov/xdb-new.pdf
- Center for X-Ray Optics web site: http://cxro.lbl.gov
- Hephaestus from the Demeter suite: http://bruceravel.github.io/demeter

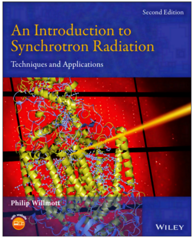


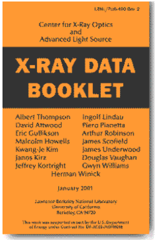


Resources for the course

Introduction to Synchrotron Radiation: Techniques and Applications, 2^{nd} ed., P. Willmott (Wiley, 2019)

- Orange x-ray data booklet: http://xdb.lbl.gov/xdb-new.pdf
- Center for X-Ray Optics web site: http://cxro.lbl.gov
- Hephaestus from the Demeter suite: http://bruceravel.github.io/demeter
- McMaster data on the Web: http://csrri.iit.edu/periodic-table.html

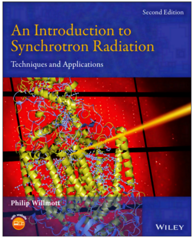


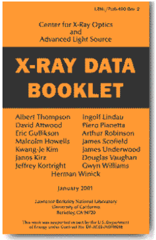


Resources for the course

Introduction to Synchrotron Radiation: Techniques and Applications, 2nd ed., P. Willmott (Wiley, 2019)

- Orange x-ray data booklet: http://xdb.lbl.gov/xdb-new.pdf
- Center for X-Ray Optics web site: http://cxro.lbl.gov
- Hephaestus from the Demeter suite: http://bruceravel.github.io/demeter
- McMaster data on the Web: http://csrri.iit.edu/periodic-table.html





- X-ray Oriented Programs: https://www.aps.anl.gov/Science/Scientific-Software/XOP

Today's outline - August 19, 2024

Today's outline - August 19, 2024

- The big picture

Today's outline - August 19, 2024

- The big picture
- History of x-ray sources

Today's outline - August 19, 2024

- The big picture
- History of x-ray sources
- X-ray interactions with matter

Today's outline - August 19, 2024

- The big picture
- History of x-ray sources
- X-ray interactions with matter
- Thomson scattering

Today's outline - August 19, 2024

- The big picture
- History of x-ray sources
- X-ray interactions with matter
- Thomson scattering
- Atomic form factor

Today's outline - August 19, 2024

- The big picture
- History of x-ray sources
- X-ray interactions with matter
- Thomson scattering
- Atomic form factor

Reading Assignment: Chapter 1.1–1.6; 2.1–2.2

Why synchrotron radiation?

X-rays are the ideal structural probe for interatomic distances with wavelengths of $\sim$1 Å

Why synchrotron radiation?

X-rays are the ideal structural probe for interatomic distances with wavelengths of $\sim$1 Å

Laboratory x-ray sources are extremely useful and have gotten much better in the past decades but some experiments can only be done at a synchrotron

Why synchrotron radiation?

X-rays are the ideal structural probe for interatomic distances with wavelengths of $\sim$1 Å

Laboratory x-ray sources are extremely useful and have gotten much better in the past decades but some experiments can only be done at a synchrotron

Synchrotron sources provide superior flux, brilliance, and tunability

Why synchrotron radiation?

X-rays are the ideal structural probe for interatomic distances with wavelengths of $\sim$1 Å

Laboratory x-ray sources are extremely useful and have gotten much better in the past decades but some experiments can only be done at a synchrotron

Synchrotron sources provide superior flux, brilliance, and tunability

Synchrotron sources and particularly FELs produce coherent beams

Why synchrotron radiation?

X-rays are the ideal structural probe for interatomic distances with wavelengths of $\sim$1 Å

Laboratory x-ray sources are extremely useful and have gotten much better in the past decades but some experiments can only be done at a synchrotron

Synchrotron sources provide superior flux, brilliance, and tunability

Synchrotron sources and particularly FELs produce coherent beams

The broad range of techniques make synchrotron x-ray sources to nearly any science or engineering field

The classical x-ray

The classical plane wave representation of x-rays is:

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r}, t) = \hat{\epsilon} E_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r}, t) = \hat{\epsilon} E_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

where $\hat{\epsilon}$ is a unit vector in the direction of the electric field

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r}, t) = \hat{\epsilon} E_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

where $\hat{\epsilon}$ is a unit vector in the direction of the electric field, $\mathbf{k}$ is the wavevector of the radiation along the propagation direction

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r}, t) = \hat{\epsilon} E_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

where $\hat{\epsilon}$ is a unit vector in the direction of the electric field, $\mathbf{k}$ is the wavevector of the radiation along the propagation direction, and ω is the angular frequency of oscillation of the radiation.

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r},t) = \hat{\epsilon} E_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

where $\hat{\epsilon}$ is a unit vector in the direction of the electric field, $\mathbf{k}$ is the wavevector of the radiation along the propagation direction, and ω is the angular frequency of oscillation of the radiation.

If the energy, $\mathcal{E}$ is in keV, the relationship among these quantities is given by:

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r}, t) = \hat{\epsilon} E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

where $\hat{\epsilon}$ is a unit vector in the direction of the electric field, $\mathbf{k}$ is the wavevector of the radiation along the propagation direction, and ω is the angular frequency of oscillation of the radiation.

If the energy, $\mathcal{E}$ is in keV, the relationship among these quantities is given by:

$$\hbar\omega = h\nu = \mathcal{E}, \lambda\nu = c$$

The classical x-ray

The classical plane wave representation of x-rays is:

$$\mathbf{E}(\mathbf{r}, t) = \hat{\epsilon} E_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

where $\hat{\epsilon}$ is a unit vector in the direction of the electric field, $\mathbf{k}$ is the wavevector of the radiation along the propagation direction, and ω is the angular frequency of oscillation of the radiation.

If the energy, $\mathcal{E}$ is in keV, the relationship among these quantities is given by:

$$\hbar\omega = h\nu = \mathcal{E}, \lambda\nu = c$$

$$\begin{aligned}
\lambda &= hc/\mathcal{E} \\
&= (4.1357 \times 10^{-15}\,\text{eV}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})/\mathcal{E} \\
&= (4.1357 \times 10^{-18}\,\text{keV}\cdot\text{s})(2.9979 \times 10^{18}\,\text{Å/s})/\mathcal{E} \\
&= 12.398\,\text{Å}\cdot\text{keV}/\mathcal{E} \qquad \text{to give units of Å}
\end{aligned}$$

Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

1. Elastic scattering

Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

1. Elastic scattering
2. Inelastic scattering

Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

1. Elastic scattering
2. Inelastic scattering
3. Absorption

Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

1. Elastic scattering
2. Inelastic scattering
3. Absorption
4. Pair production

Interactions of x-rays with matter

For the purposes of this course, we care most about the interactions of x-rays with matter.

There are four basic types of such interactions:

1. Elastic scattering
2. Inelastic scattering
3. Absorption
4. Pair production

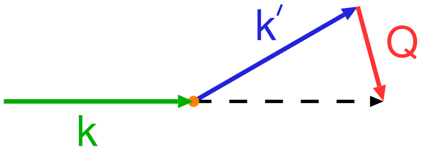
We will only discuss the first three.

Elastic scattering

Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays).

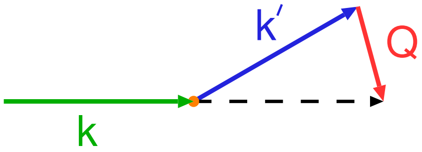
Elastic scattering

Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays). The typical scattering geometry is



Elastic scattering

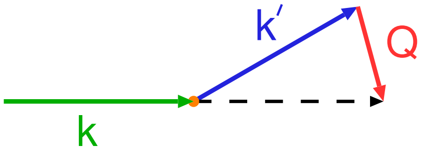
Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays). The typical scattering geometry is



where an incident x-ray of wave number **k**

Elastic scattering

Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays). The typical scattering geometry is

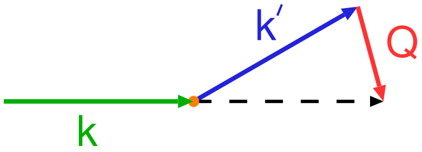


where an incident x-ray of wave number $\mathbf{k}$

scatters elastically from an electron to $\mathbf{k}'$

Elastic scattering

Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays). The typical scattering geometry is



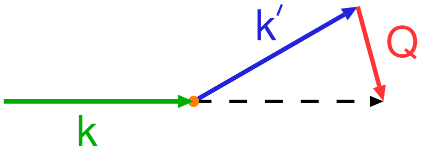
where an incident x-ray of wave number $\mathbf{k}$

scatters elastically from an electron to $\mathbf{k}'$

resulting in a scattering vector $\mathbf{Q}$

Elastic scattering

Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays). The typical scattering geometry is



where an incident x-ray of wave number $\mathbf{k}$

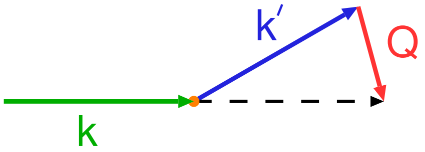
scatters elastically from an electron to $\mathbf{k}'$

resulting in a scattering vector $\mathbf{Q}$

or in terms of momentum transfer: $\hbar\mathbf{Q} = \hbar\mathbf{k} - \hbar\mathbf{k}'$

Elastic scattering

Most of the phenomena we will discuss can be treated classically as elastic scattering of electromagnetic waves (x-rays). The typical scattering geometry is



where an incident x-ray of wave number $\mathbf{k}$

scatters elastically from an electron to $\mathbf{k}'$

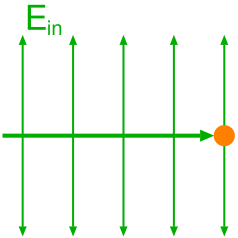
resulting in a scattering vector $\mathbf{Q}$

or in terms of momentum transfer: $\hbar\mathbf{Q} = \hbar\mathbf{k} - \hbar\mathbf{k}'$

Start with the scattering from a single electron, then build up to more complexity

Thomson scattering

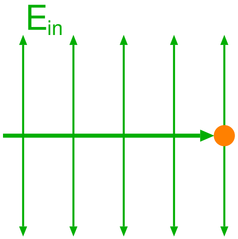
Assumptions:



Thomson scattering

Assumptions:

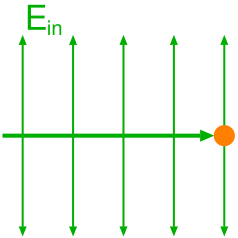
incident x-ray plane wave



Thomson scattering

Assumptions:

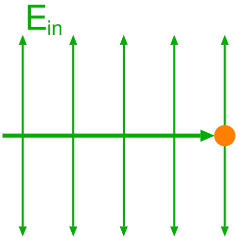
- incident x-ray plane wave
- electron is a point charge



Thomson scattering

Assumptions:

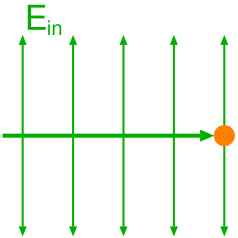
- incident x-ray plane wave
- electron is a point charge
- scattering is elastic



Thomson scattering

Assumptions:

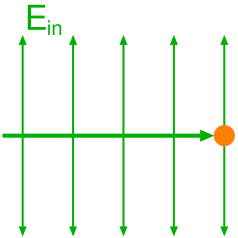
- incident x-ray plane wave
- electron is a point charge
- scattering is elastic
- scattered intensity $\propto 1/R^2$



Thomson scattering

Assumptions:

- incident x-ray plane wave
- electron is a point charge
- scattering is elastic
- scattered intensity $\propto 1/R^2$

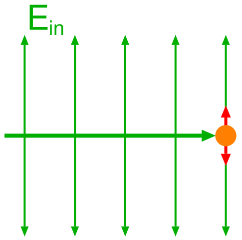


The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

Thomson scattering

Assumptions:

- incident x-ray plane wave
- electron is a point charge
- scattering is elastic
- scattered intensity $\propto 1/R^2$



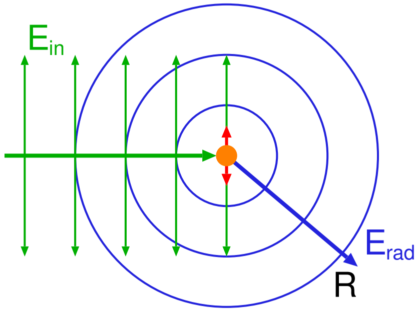
The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

Thomson scattering

Assumptions:

- incident x-ray plane wave
- electron is a point charge
- scattering is elastic
- scattered intensity $\propto 1/R^2$



The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

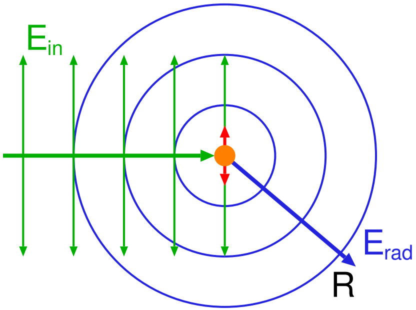
The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

The observer at R "sees" a scattered electric field $E_{rad}(R,t)$ at a later time $t = t' + R/c$

Thomson scattering

Assumptions:

- incident x-ray plane wave
- electron is a point charge
- scattering is elastic
- scattered intensity $\propto 1/R^2$



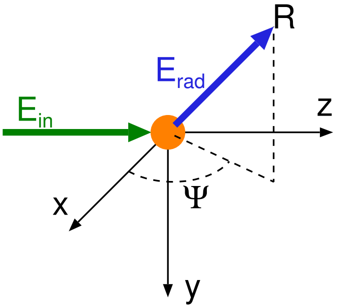
The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

The observer at R "sees" a scattered electric field $E_{rad}(R,t)$ at a later time $t = t' + R/c$

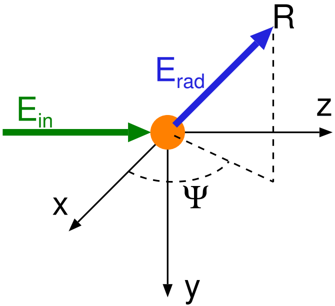
Using this, calculate the elastic scattering cross-section

Thomson scattering



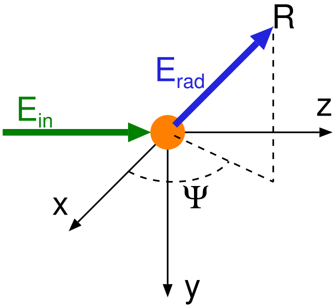
$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi,$$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

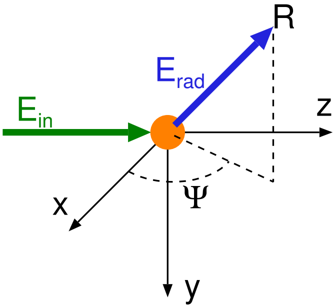
Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'}$$

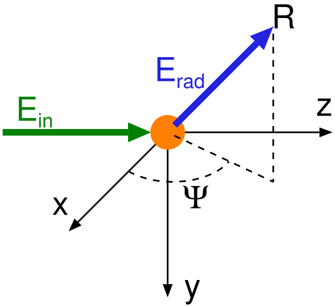
Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

Thomson scattering

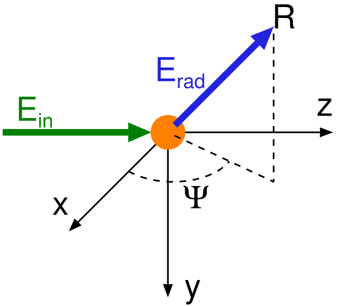


$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

Thomson scattering



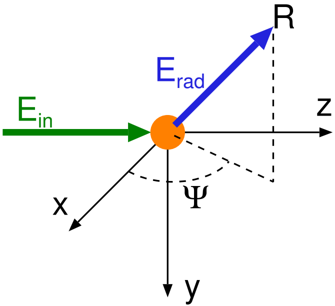
$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} \frac{-e}{m} E_{in} e^{i\omega R/c} \sin\Psi$$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

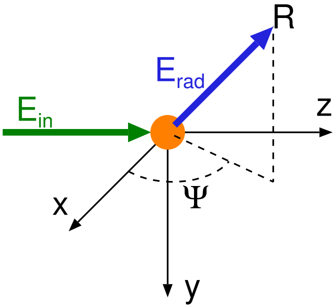
$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} \frac{-e}{m} E_{in} e^{i\omega R/c} \sin\Psi$$

$$\frac{E_{rad}(R,t)}{E_{in}} = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{i\omega R/c}}{R} \sin\Psi$$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

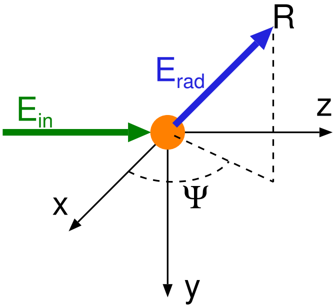
$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} \frac{-e}{m} E_{in} e^{i\omega R/c} \sin\Psi$$

$$\frac{E_{rad}(R,t)}{E_{in}} = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{i\omega R/c}}{R} \sin\Psi = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{ikR}}{R} \sin\Psi$$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

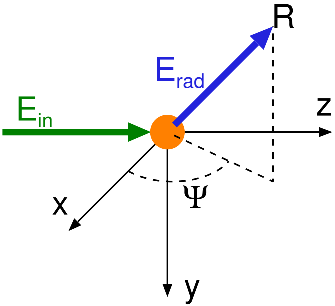
$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} \frac{-e}{m} E_{in} e^{i\omega R/c} \sin\Psi$$

$$\frac{E_{rad}(R,t)}{E_{in}} = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{i\omega R/c}}{R} \sin\Psi = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{ikR}}{R} \sin\Psi = -r_0 \frac{e^{ikR}}{R} \sin\Psi$$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

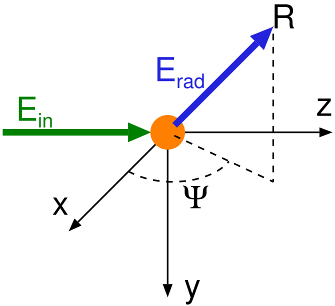
$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} \frac{-e}{m} E_{in} e^{i\omega R/c} \sin\Psi$$

$$\frac{E_{rad}(R,t)}{E_{in}} = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{i\omega R/c}}{R} \sin\Psi = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{ikR}}{R} \sin\Psi = -r_0 \frac{e^{ikR}}{R} \sin\Psi$$

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} = 2.82 \times 10^{-5} \text{Å}$$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi, \qquad t' = t - R/c$$

$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'} = -\frac{e}{m} E_{x0} e^{-i\omega t} e^{i\omega R/c}$$

$$a_x(t') = -\frac{e}{m} E_{in} e^{i\omega R/c}$$

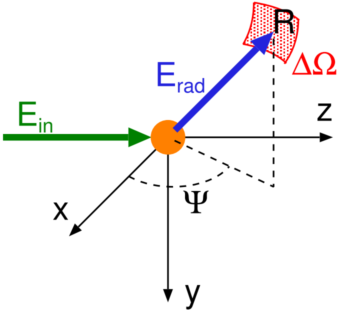
$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} \frac{-e}{m} E_{in} e^{i\omega R/c} \sin\Psi$$

$$\frac{E_{rad}(R,t)}{E_{in}} = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{i\omega R/c}}{R} \sin\Psi = -\frac{e^2}{4\pi\epsilon_0 mc^2} \frac{e^{ikR}}{R} \sin\Psi = -r_0 \frac{e^{ikR}}{R} \sin\Psi$$

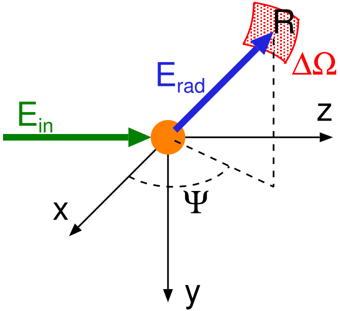
$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} = 2.82 \times 10^{-5}\text{Å}$$

r_0 is called the Thomson scattering length or the “classical” radius of the electron

Scattering cross-section

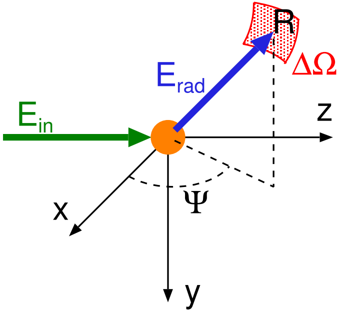


Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

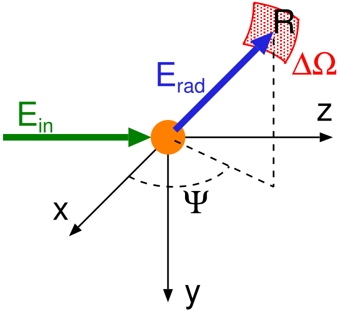
Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

Scattering cross-section

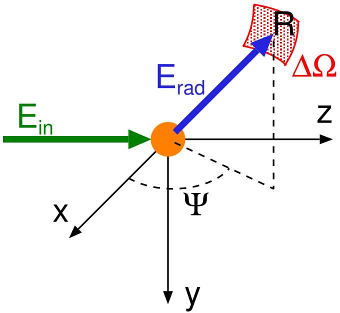


detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

Scattering cross-section



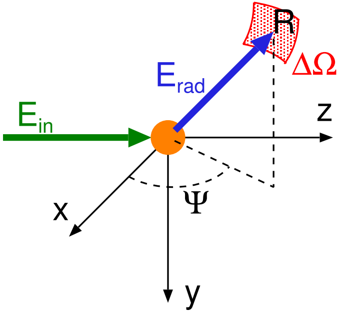
$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega}$$

detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

Scattering cross-section



$$I_{sc} \propto c A_{sc} \frac{|E_{rad}|^2}{\hbar\omega}$$

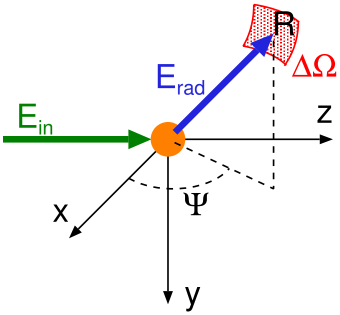
detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

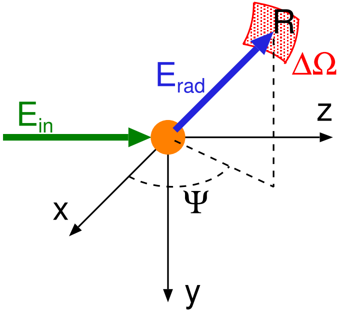
incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega}$$

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

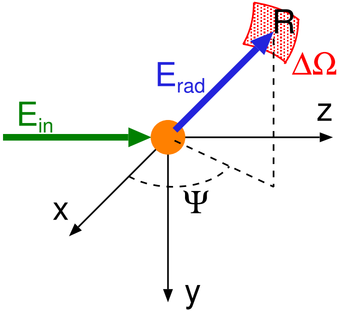
incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega} \quad \longrightarrow \quad \frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2\Delta\Omega$$

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

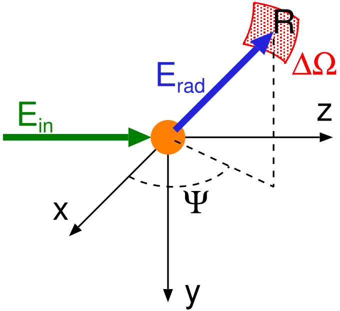
$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega} \quad \longrightarrow \quad \frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2\Delta\Omega$$

the differential cross-section is obtained by normalizing

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

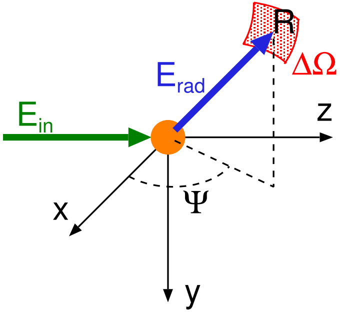
cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega} \quad \longrightarrow \quad \frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2\Delta\Omega$$

the differential cross-section is obtained by normalizing

$$\frac{d\sigma}{d\Omega}$$

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

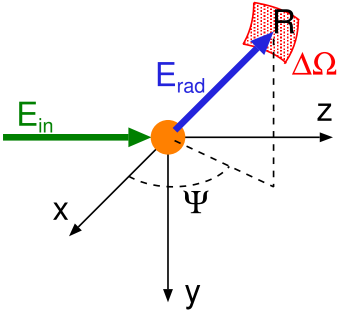
cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega} \quad \longrightarrow \quad \frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2\Delta\Omega$$

the differential cross-section is obtained by normalizing

$$\frac{d\sigma}{d\Omega} = \frac{I_{sc}}{\Phi_0\Delta\Omega}$$

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

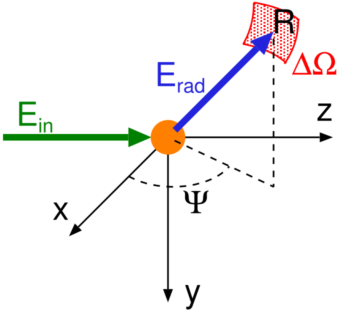
cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega} \quad \longrightarrow \quad \frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2\Delta\Omega$$

the differential cross-section is obtained by normalizing

$$\frac{d\sigma}{d\Omega} = \frac{I_{sc}}{\Phi_0\Delta\Omega} = \frac{I_{sc}}{(I_0/A_0)\,\Delta\Omega}$$

Scattering cross-section



detector of solid angle $\Delta\Omega$ located a distance R from electron

incoming beam has cross-section A_0 so the flux, Φ_0 is

$$\Phi_0 \equiv \frac{I_0}{A_0} = c\frac{|E_{in}|^2}{\hbar\omega}$$

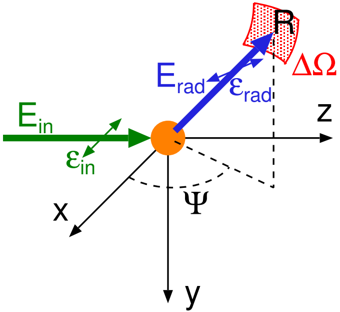
cross section of the scattered beam (into detector) is $A_{sc} = R^2\Delta\Omega$

$$I_{sc} \propto cA_{sc}\frac{|E_{rad}|^2}{\hbar\omega} = c(R^2\Delta\Omega)\frac{|E_{rad}|^2}{\hbar\omega} \longrightarrow \frac{I_{sc}}{I_0} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2\Delta\Omega$$

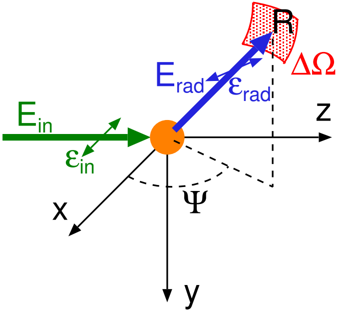
the differential cross-section is obtained by normalizing

$$\frac{d\sigma}{d\Omega} = \frac{I_{sc}}{\Phi_0\Delta\Omega} = \frac{I_{sc}}{(I_0/A_0)\,\Delta\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2$$

Total cross-section

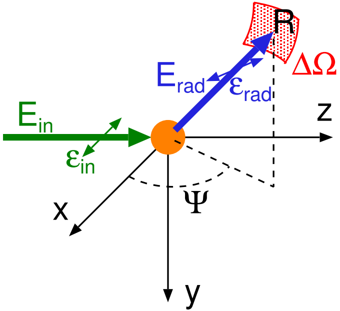


Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2$$

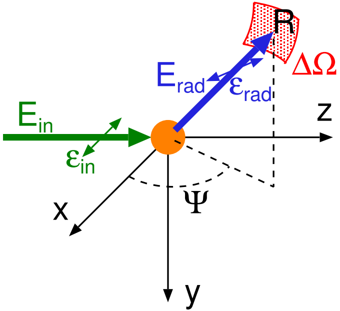
Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

Total cross-section

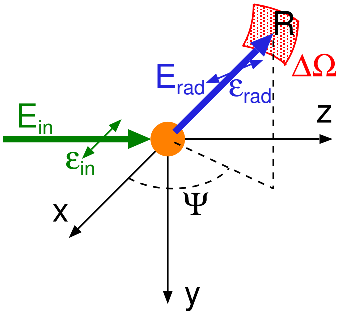


$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left| \cos\left(\frac{\pi}{2} - \Psi\right) \right|$$

Total cross-section

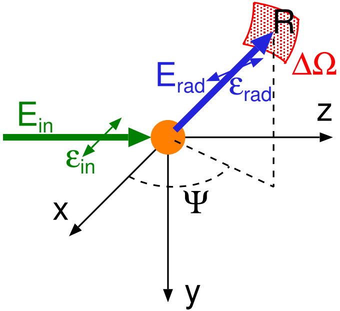


$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left|\hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}}\right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left|\cos\left(\frac{\pi}{2} - \Psi\right)\right| = -r_0 \frac{e^{ikR}}{R} \sin\Psi$$

Total cross-section

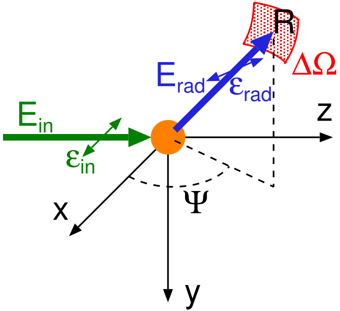


$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left|\hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}}\right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left|\cos\left(\frac{\pi}{2} - \Psi\right)\right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

Total cross-section



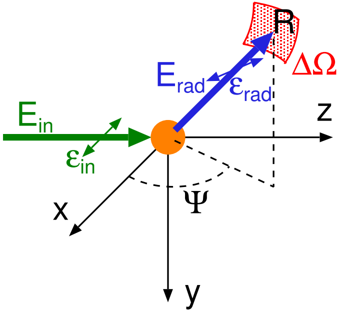
$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left| \cos\left(\frac{\pi}{2} - \Psi\right) \right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

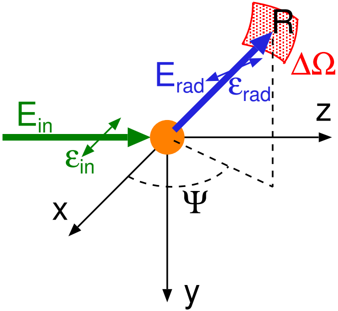
$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left| \cos\left(\frac{\pi}{2} - \Psi\right) \right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

$$\sigma = \int r_0^2 \sin^2 \Psi \, d\Omega$$

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

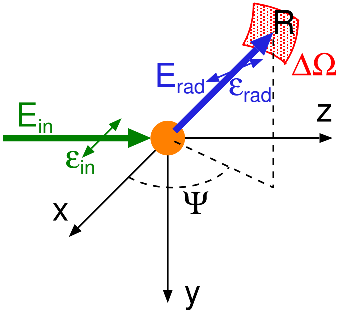
$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left| \cos\left(\frac{\pi}{2} - \Psi\right) \right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

$$\sigma = \int r_0^2 \sin^2 \Psi \, d\Omega = \frac{2}{3} 4\pi r_0^2$$

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

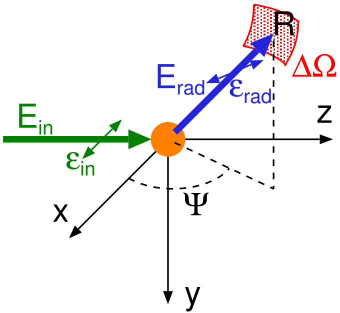
$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left| \cos\left(\frac{\pi}{2} - \Psi\right) \right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

$$\sigma = \int r_0^2 \sin^2 \Psi \, d\Omega = \frac{2}{3} 4\pi r_0^2 = \frac{8\pi}{3} r_0^2$$

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2}R^2 = r_0^2 \sin^2\Psi$$

$$\frac{E_{rad}}{E_{in}} = -r_0\frac{e^{ikR}}{R}\left|\hat{\epsilon_{in}}\cdot\hat{\epsilon_{rad}}\right|$$

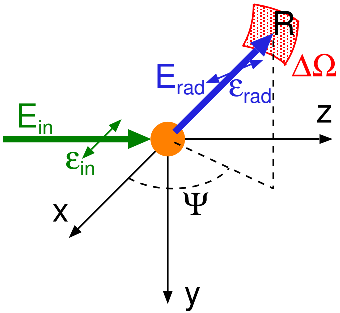
$$= -r_0\frac{e^{ikR}}{R}\left|\cos\left(\frac{\pi}{2}-\Psi\right)\right| = -r_0\frac{e^{ikR}}{R}\sin\Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

$$\sigma = \int r_0^2 \sin^2\Psi\, d\Omega = \frac{2}{3}4\pi r_0^2 = \frac{8\pi}{3}r_0^2$$

$$= 0.665 \times 10^{-24}\ cm^2 = 0.665\ barn$$

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left| \cos\left(\frac{\pi}{2} - \Psi\right) \right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

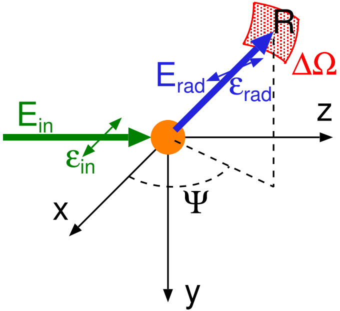
Integrate to obtain the total Thomson scattering cross-section from an electron.

$$\sigma = \int r_0^2 \sin^2 \Psi \, d\Omega = \frac{2}{3} 4\pi r_0^2 = \frac{8\pi}{3} r_0^2$$

$$= 0.665 \times 10^{-24} \; cm^2 = 0.665 \; barn$$

$$P = \left\langle \left| \hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}} \right|^2 \right\rangle = \begin{cases} \left\langle \sin^2 \Psi \right\rangle = \frac{2}{3} \end{cases}$$

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2 \Psi$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left|\hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}}\right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left|\cos\left(\frac{\pi}{2} - \Psi\right)\right| = -r_0 \frac{e^{ikR}}{R} \sin \Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

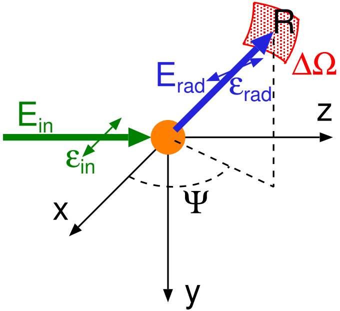
If displacement is in vertical direction, $\sin \Psi$ term is replaced by unity

$$\sigma = \int r_0^2 \sin^2 \Psi \, d\Omega = \frac{2}{3} 4\pi r_0^2 = \frac{8\pi}{3} r_0^2$$

$$= 0.665 \times 10^{-24} \ cm^2 = 0.665 \ barn$$

$$P = \left\langle \left|\hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}}\right|^2 \right\rangle = \begin{cases} 1 \\ \left\langle \sin^2 \Psi \right\rangle = \frac{2}{3} \end{cases}$$

Total cross-section



$$\frac{d\sigma}{d\Omega} = \frac{|E_{rad}|^2}{|E_{in}|^2} R^2 = r_0^2 \sin^2\Psi$$

$$\frac{E_{rad}}{E_{in}} = -r_0 \frac{e^{ikR}}{R} \left|\hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}}\right|$$

$$= -r_0 \frac{e^{ikR}}{R} \left|\cos\left(\frac{\pi}{2} - \Psi\right)\right| = -r_0 \frac{e^{ikR}}{R} \sin\Psi$$

Integrate to obtain the total Thomson scattering cross-section from an electron.

If displacement is in vertical direction, $\sin\Psi$ term is replaced by unity and if the source is unpolarized, it is a combination.

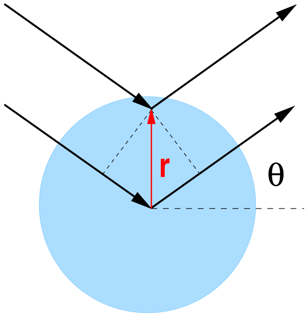
$$\sigma = \int r_0^2 \sin^2\Psi \, d\Omega = \frac{2}{3} 4\pi r_0^2 = \frac{8\pi}{3} r_0^2$$

$$= 0.665 \times 10^{-24} \; cm^2 = 0.665 \; barn$$

$$P = \left\langle \left|\hat{\epsilon_{in}} \cdot \hat{\epsilon_{rad}}\right|^2 \right\rangle = \begin{cases} 1 \\ \left\langle \sin^2\Psi \right\rangle = \frac{2}{3} \\ \frac{1}{2}\left(1 + \left\langle \sin^2\Psi \right\rangle\right) = \frac{5}{6} \end{cases}$$

Atomic scattering

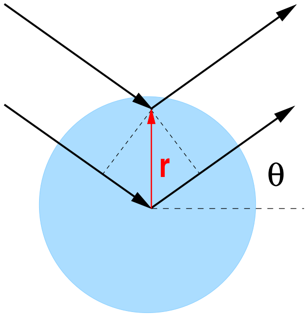
If we have a charge distribution instead of a single electron, the scattering is more complex



Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex

A phase shift arises because of scattering from different portions of extended electron distribution

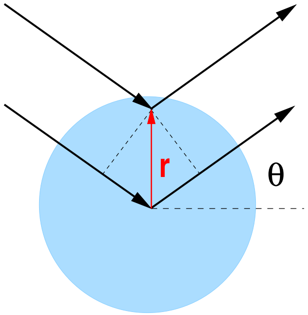


Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex

A phase shift arises because of scattering from different portions of extended electron distribution

$$\Delta\phi(\mathbf{r}) = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{r} = \mathbf{Q} \cdot \mathbf{r}$$

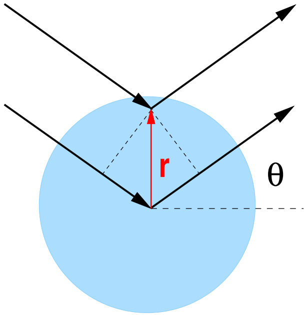


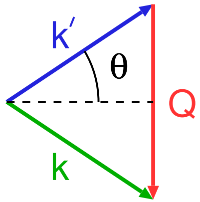
Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex

A phase shift arises because of scattering from different portions of extended electron distribution

$$\Delta\phi(\mathbf{r}) = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{r} = \mathbf{Q} \cdot \mathbf{r}$$



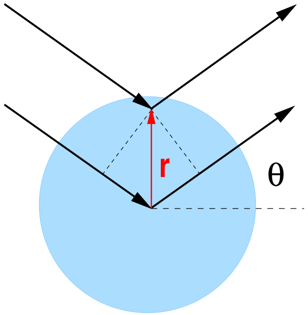


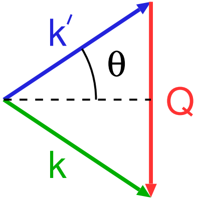
Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex

A phase shift arises because of scattering from different portions of extended electron distribution

$$\Delta\phi(\mathbf{r}) = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{r} = \mathbf{Q} \cdot \mathbf{r}$$





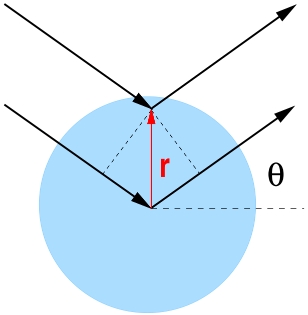
where the scattering vector, $\mathbf{Q}$ is given by

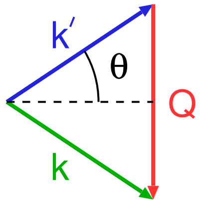
Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex

A phase shift arises because of scattering from different portions of extended electron distribution

$$\Delta\phi(\mathbf{r}) = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{r} = \mathbf{Q} \cdot \mathbf{r}$$





where the scattering vector, **Q** is given by

$$|\mathbf{Q}| = 2\,|\mathbf{k}|\,sin\theta = \frac{4\pi}{\lambda} sin\theta$$

Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$

Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

$$-r_0 f^0(\mathbf{Q}) = -r_0 \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d^3r$$

Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

$$-r_0 f^0(\mathbf{Q}) = -r_0 \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d^3r$$

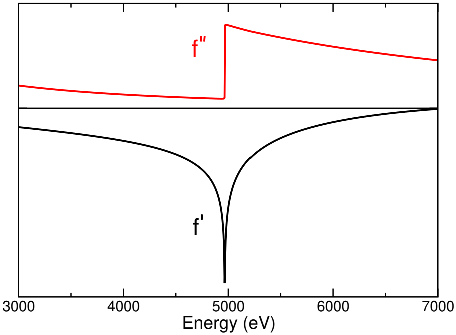
Electrons which are tightly bound cannot respond like a free electron. This results in a depression of the atomic form factor, called f' and a lossy term near an ionization energy, called f''. Together these are the "anomalous" corrections to the atomic form factor.

Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

$$-r_0 f^0(\mathbf{Q}) = -r_0 \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d^3r$$

Electrons which are tightly bound cannot respond like a free electron. This results in a depression of the atomic form factor, called f' and a lossy term near an ionization energy, called f''. Together these are the "anomalous" corrections to the atomic form factor.

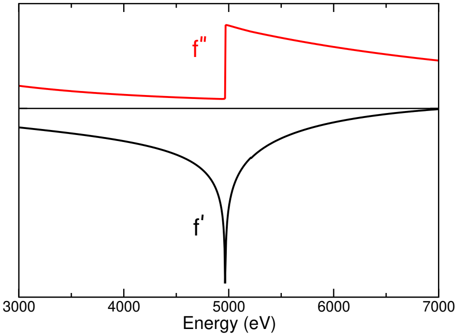


Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

$$-r_0 f^0(\mathbf{Q}) = -r_0 \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d^3r$$

Electrons which are tightly bound cannot respond like a free electron. This results in a depression of the atomic form factor, called f' and a lossy term near an ionization energy, called f''. Together these are the "anomalous" corrections to the atomic form factor.



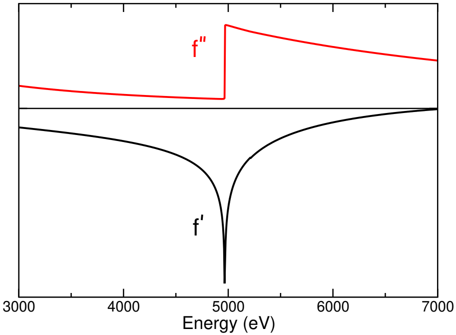
the total atomic scattering factor is

Atomic form factor

The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

$$-r_0 f^0(\mathbf{Q}) = -r_0 \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d^3r$$

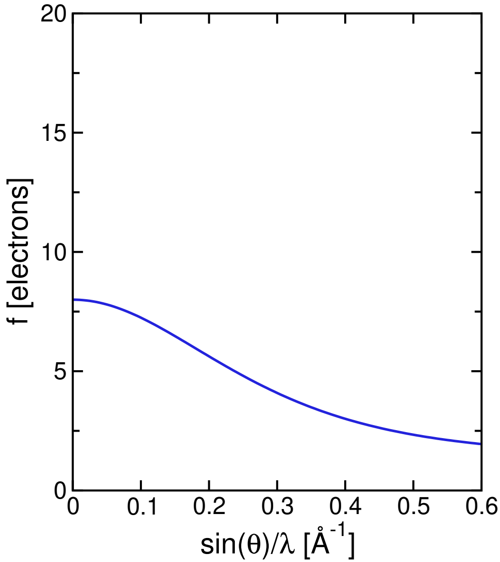
Electrons which are tightly bound cannot respond like a free electron. This results in a depression of the atomic form factor, called f' and a lossy term near an ionization energy, called f''. Together these are the "anomalous" corrections to the atomic form factor.



the total atomic scattering factor is

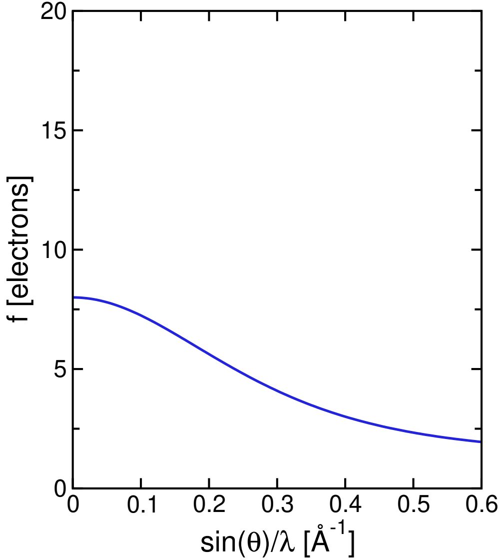
$$f(\mathbf{Q},\hbar\omega) = f^0(\mathbf{Q}) + f'(\hbar\omega) + if''(\hbar\omega)$$

Atomic form factor



The atomic form factor has an angular dependence

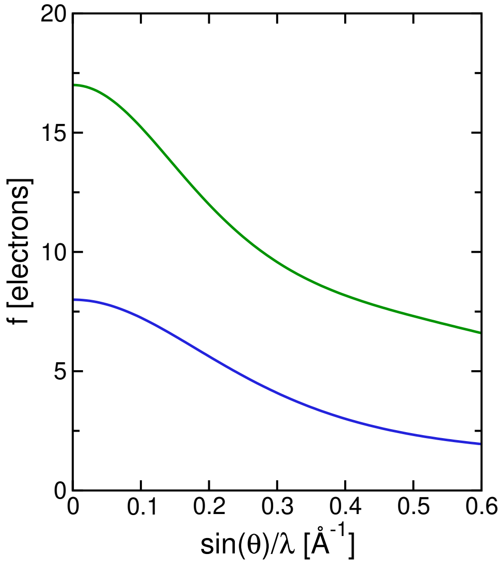
Atomic form factor



The atomic form factor has an angular dependence

$$\mathbf{Q} = \frac{4\pi}{\lambda} \sin\theta$$

Atomic form factor

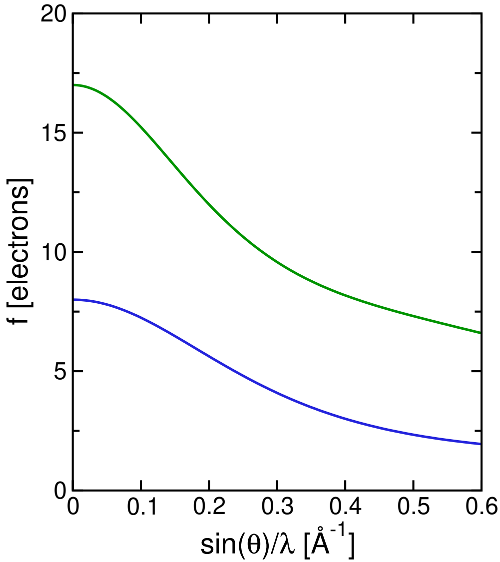


The atomic form factor has an angular dependence

$$\mathbf{Q} = \frac{4\pi}{\lambda} \sin\theta$$

lighter atoms have a broader form factor

Atomic form factor



The atomic form factor has an angular dependence

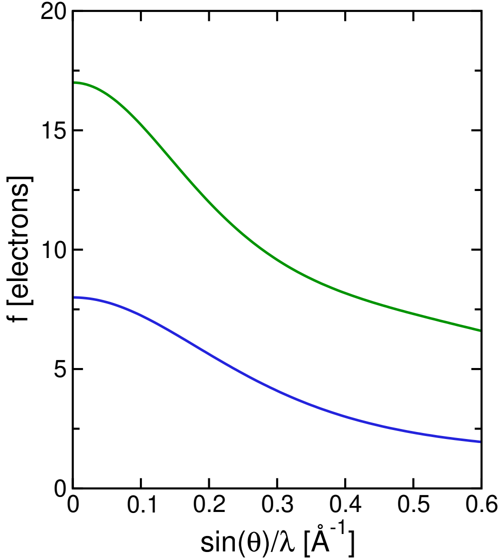
$$\mathbf{Q} = \frac{4\pi}{\lambda} \sin\theta$$

lighter atoms have a broader form factor

forward scattering counts electrons

$$f(0) = Z$$

Atomic form factor



The atomic form factor has an angular dependence

$$\mathbf{Q} = \frac{4\pi}{\lambda}\sin\theta$$

lighter atoms have a broader form factor

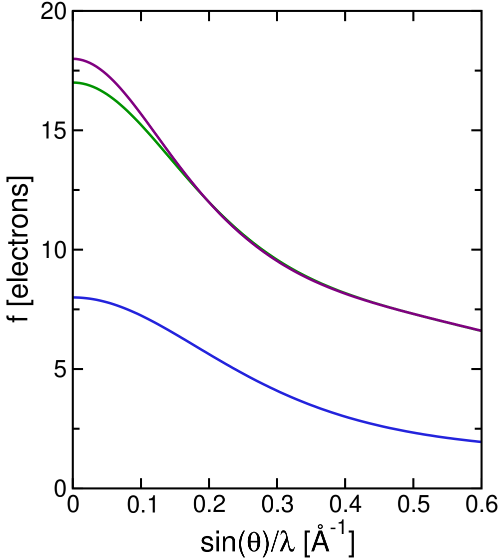
forward scattering counts electrons

$$f(0) = Z$$

$$Z_O = 8$$

$$Z_{Cl} = 17$$

Atomic form factor



The atomic form factor has an angular dependence

$$\mathbf{Q} = \frac{4\pi}{\lambda}\sin\theta$$

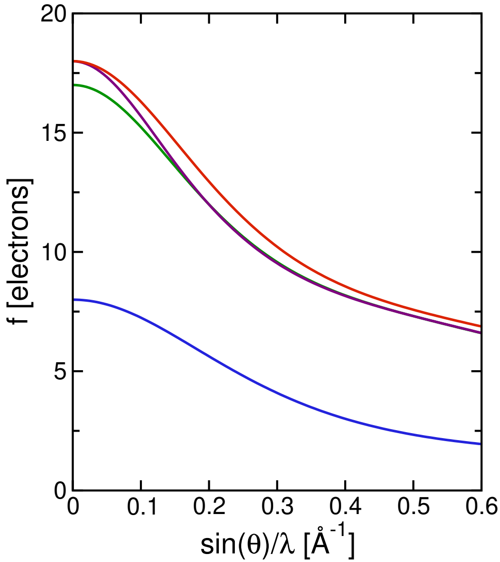
lighter atoms have a broader form factor

forward scattering counts electrons

$$f(0) = Z$$

$$Z_O = 8$$
$$Z_{Cl} = 17$$
$$Z_{Cl^-} = 18$$

Atomic form factor



The atomic form factor has an angular dependence

$$\mathbf{Q} = \frac{4\pi}{\lambda}\sin\theta$$

lighter atoms have a broader form factor

forward scattering counts electrons

$$f(0) = Z$$

$$Z_O = 8$$

$$Z_{Cl} = 17$$

$$Z_{Cl^-} = 18$$

$$Z_{Ar} = 18$$