# Today's outline - September 23, 2021

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Reading Assignment: Chapter 4.1–4.2

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Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Tuesday, October 05, 2021

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- Refractive optics
- Ideal refractive surface
- Fresnel lenses and zone plates
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Reading Assignment: Chapter 4.1–4.2

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Tuesday, October 05, 2021

Homework Assignment #04:
Chapter 4: 2,4,6,7.10
due Tuesday, October 19, 2021

# Refractive optics

Just as with visible light, it is possible to make refractive optics for x-rays

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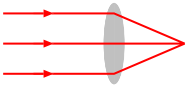
visible light:

$$n \sim 1.2 - 1.5$$
$$f \sim 0.1\text{m}$$



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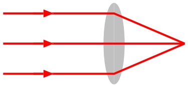

x-rays:

$$n \approx 1 - \delta,\ \delta \sim 10^{-5}$$

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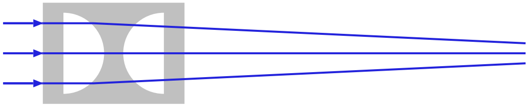

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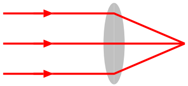

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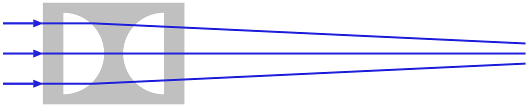

x-ray lenses are complementary to those for visible light

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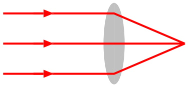

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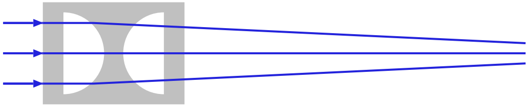
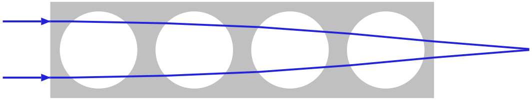
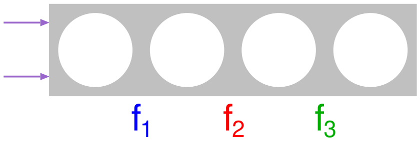


x-ray lenses are complementary to those for visible light getting manageable focal distances requires making compound lenses

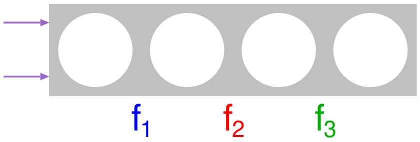


# Focal length of a compound lens




Start with a 3-element compound lens, calculate effective focal length

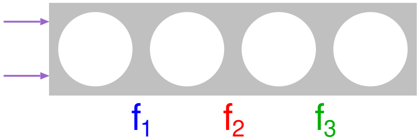
# Focal length of a compound lens




$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, $f$

# Focal length of a compound lens




$$\frac{1}{i}+\frac{1}{o}=\frac{1}{f} \quad \longrightarrow \quad \frac{1}{i}=\frac{1}{f}-\frac{1}{o}$$

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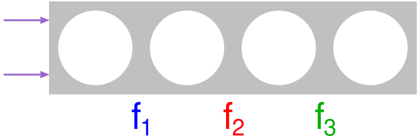


$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \quad \longrightarrow \quad \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

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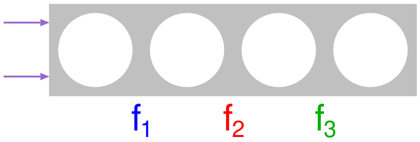


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$f_1 = f$, $o_1 = \infty$

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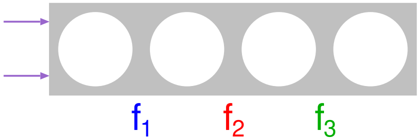


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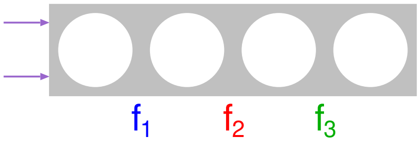


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for the second lens, the image $i_1$ is a virtual object, $o_2 = -i_1$

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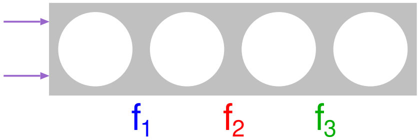


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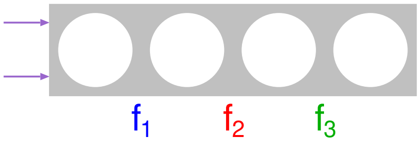


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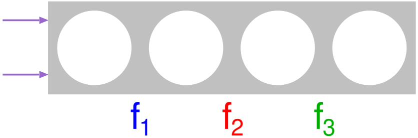


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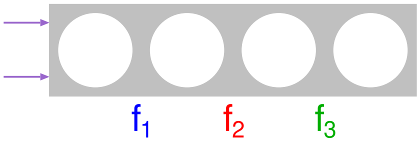


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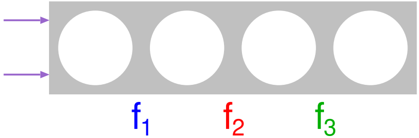


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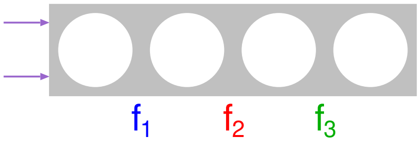


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Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, $f$

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for the second lens, the image $i_1$ is a virtual object, $o_2 = -i_1$

similarly for the third lens, $o_3 = -i_2$

so for N lenses $f_{eff} = f/N$

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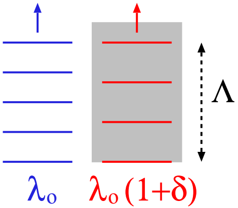


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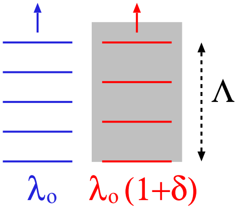


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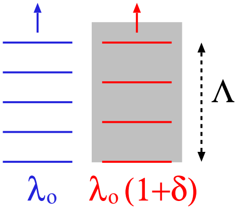


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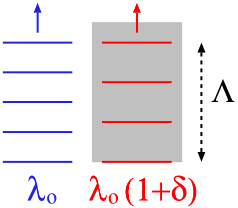


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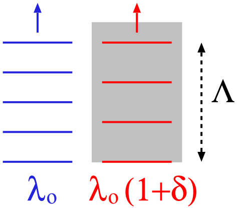


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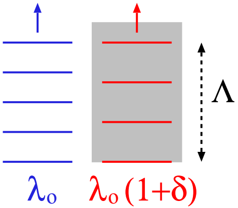


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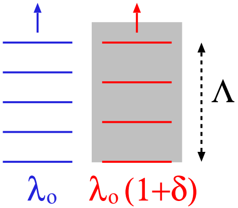


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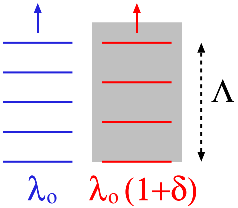


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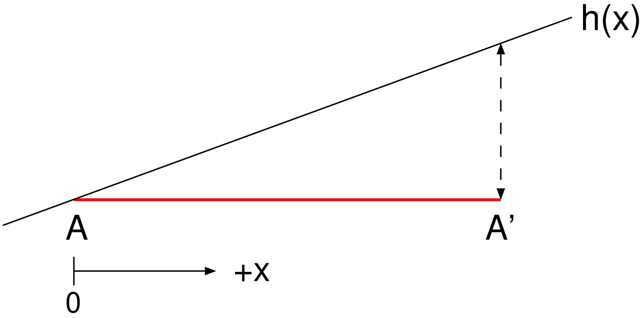
# Ideal interface profile - "thin" lens



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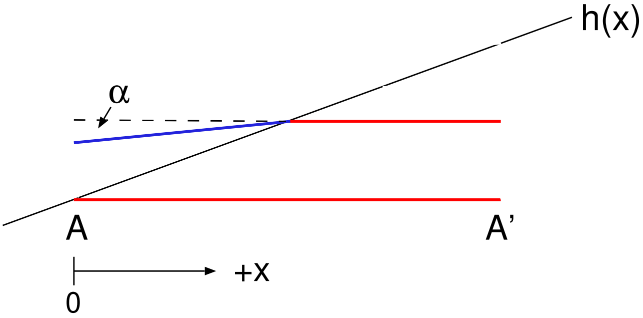


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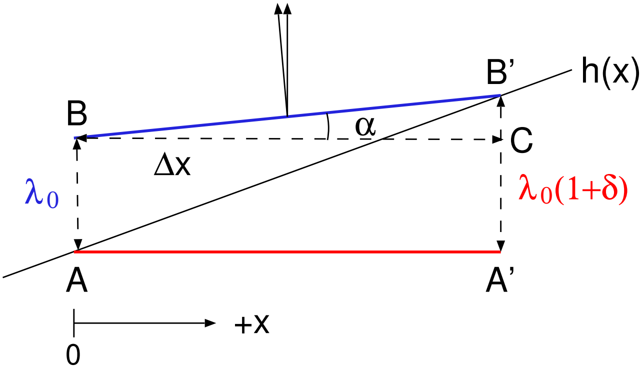


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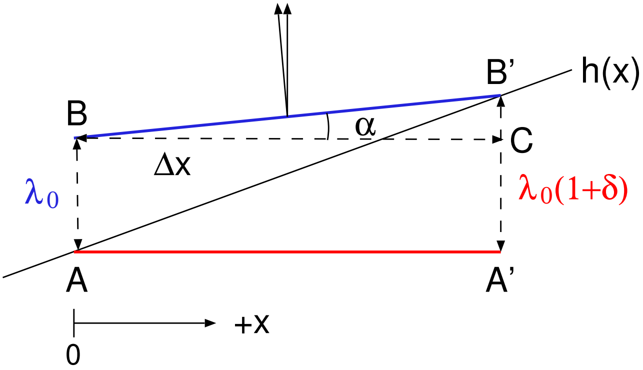


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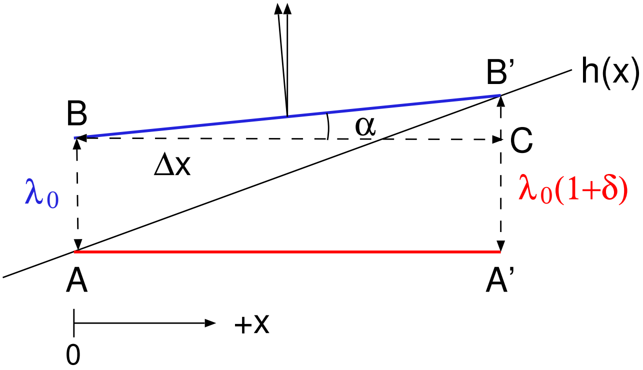


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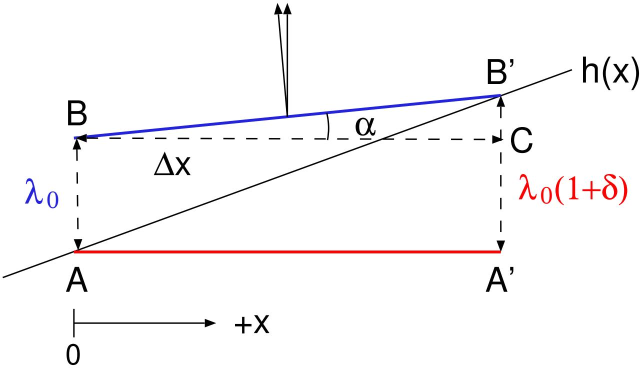


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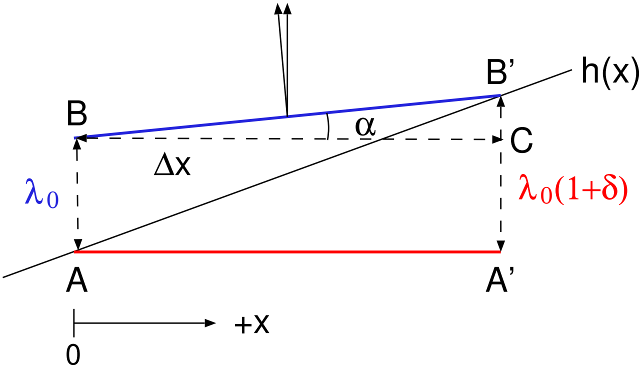


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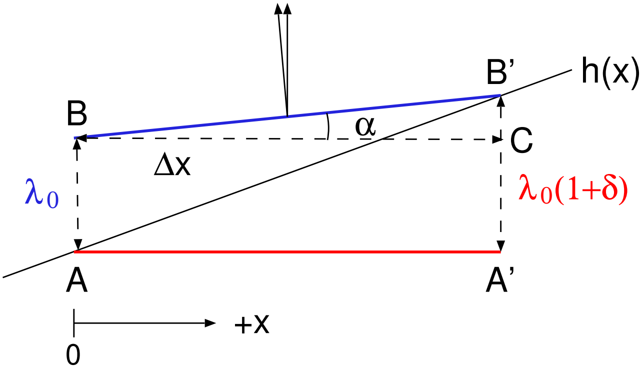


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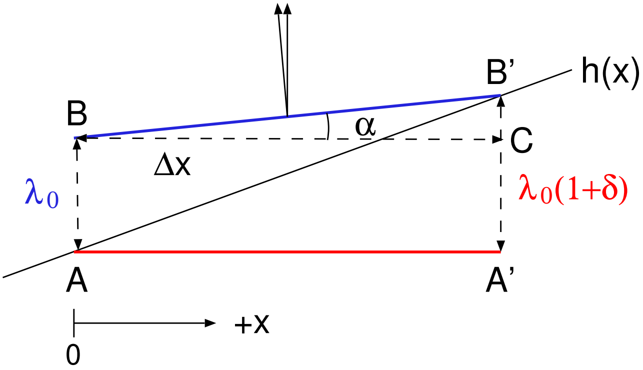


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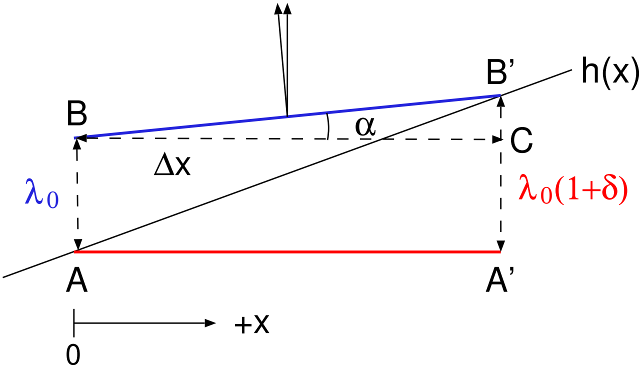


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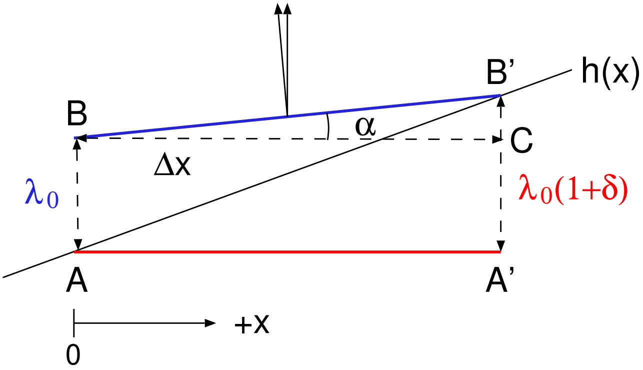


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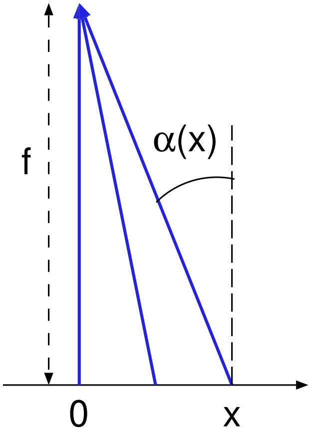

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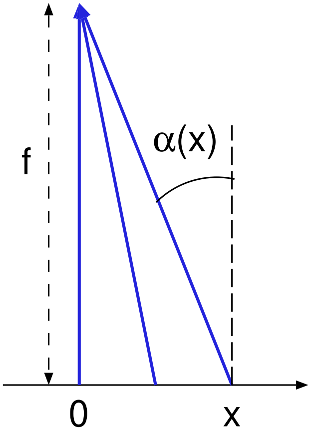

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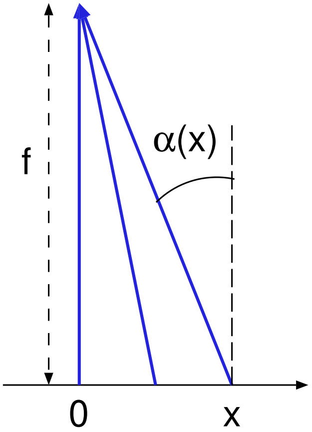

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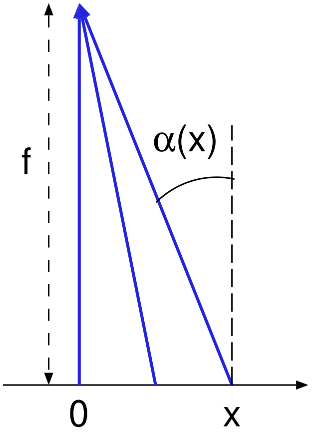

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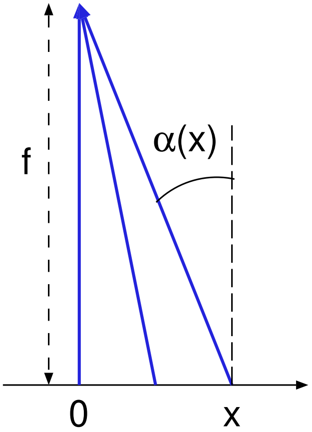

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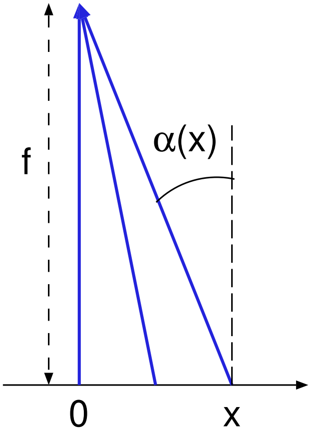

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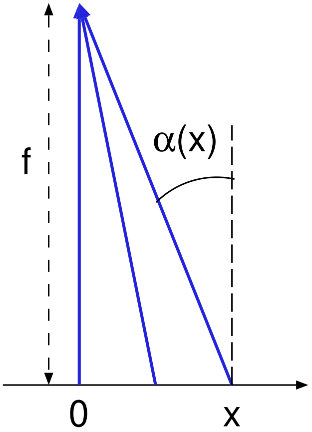

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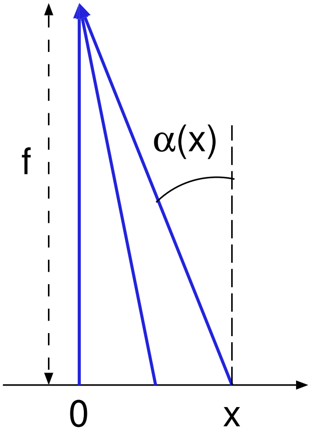


a parabola is the ideal surface shape for focusing by refraction for a "thin" lens with limited aperture

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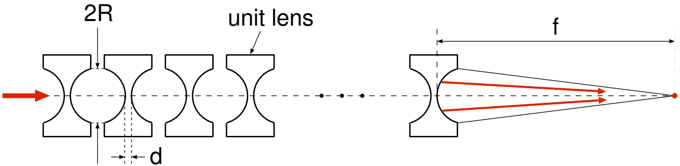
for $2N$ circular lenses we have

$$f_{2N} \approx \frac{R}{2N\delta}$$

# Focussing by a beryllium lens

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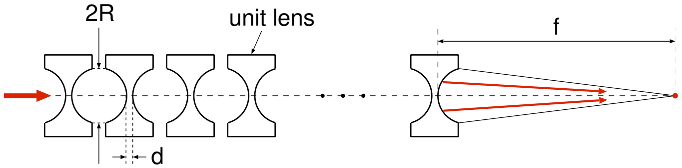


For 50 holes of radius $R = 1$mm in beryllium (Be) at $E = 10$keV, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

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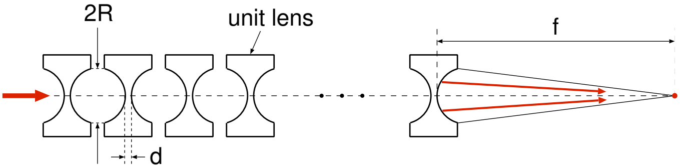


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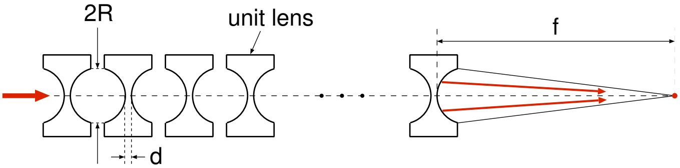


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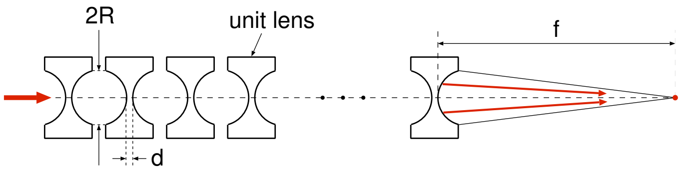


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depending on the wall thickness of the lenslets, the transmission can be up to 74%

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# Alligator-type lenses

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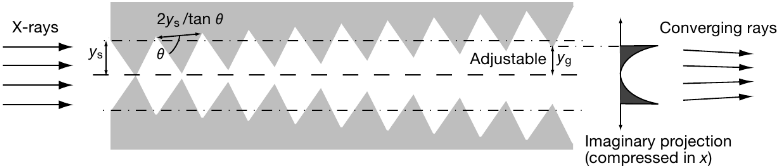


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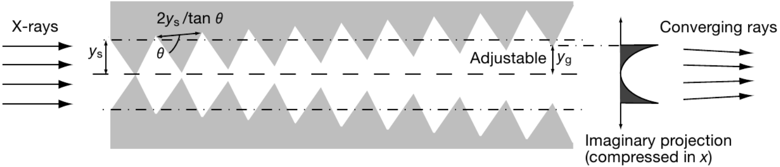


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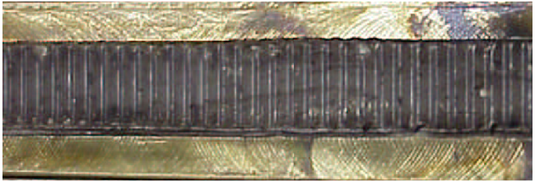

This design has also been used to make lenses out of lithium metal.

E.M. Dufresne et al., "Lithium metal for x-ray refractive optics", *Appl. Phys. Lett.* **79**, 4085 (2001).

# Extruded Al lens

The compound refractive lenses (CRL) are useful for fixed focus but are difficult to use if a variable focal distance and a long focal length is required.

A. Khounsary et al., "Fabrication, testing, and performance of a variable focus x-ray compound lens", *Proc. SPIE* **4783**, 49-54 (2002).

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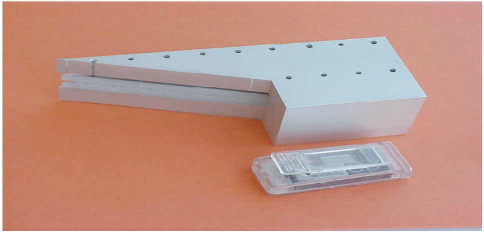
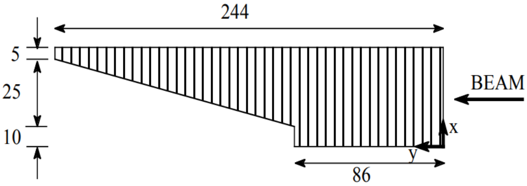
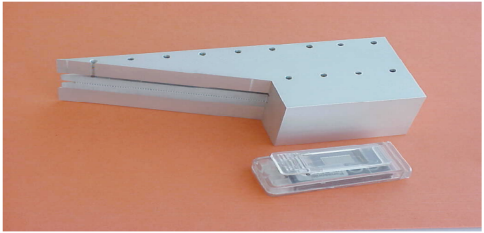


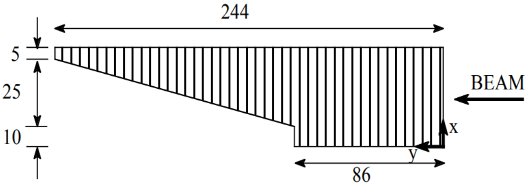



A. Khounsary et al., "Fabrication, testing, and performance of a variable focus x-ray compound lens", *Proc. SPIE* **4783**, 49-54 (2002).

# Extruded Al lens

The compound refractive lenses (CRL) are useful for fixed focus but are difficult to use if a variable focal distance and a long focal length is required.



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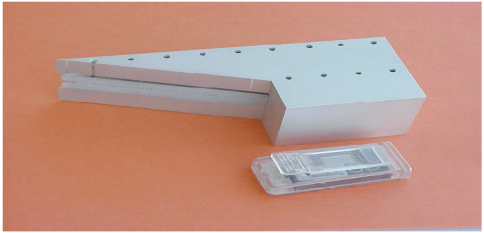

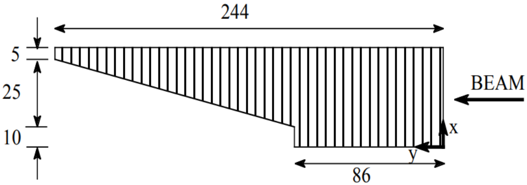


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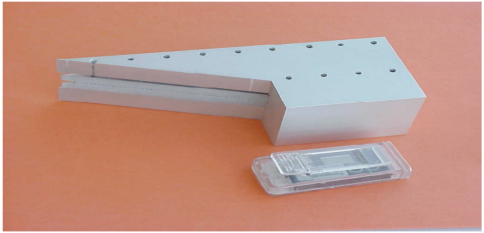

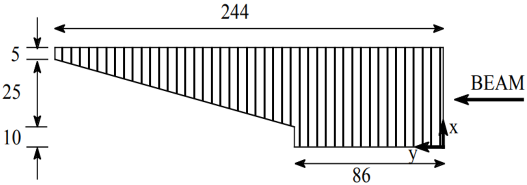


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Cut diagonally to expose variable number of "lenses" to a horizontal beam

Horizontal translation allows change in focal length but it is quantized, not continuous

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# Variable focal length CRL

A continuously variable focal length is very important for two specific reasons: tracking sample position, and keeping the focal length constant as energy is changed.

B. Adams and C. Rose-Petruck, "X-ray focusing scheme with continuously variable lens," *J. Synchrotron Radiation* **22**, 16-22 (2015).

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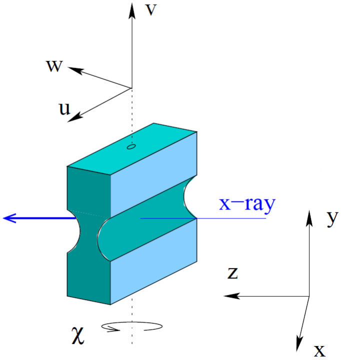


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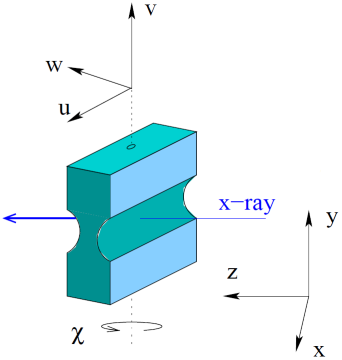


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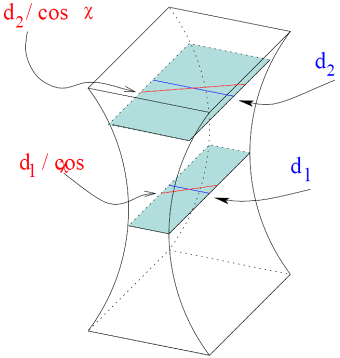


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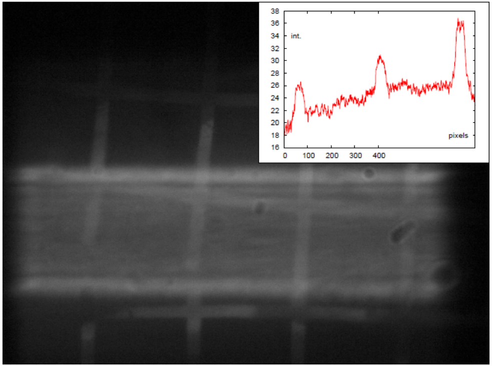


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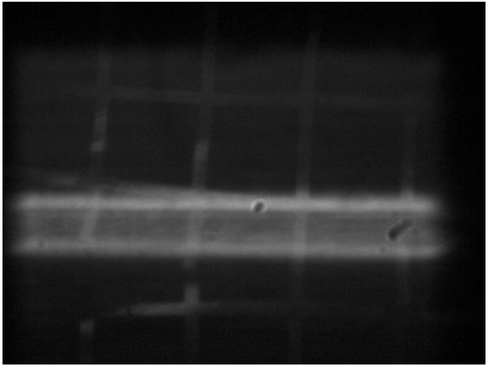

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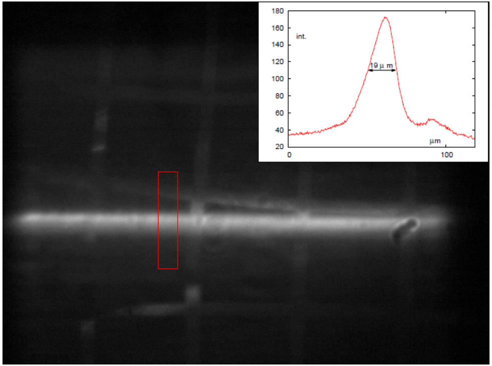
Optimal focus is $20\mu$m at $\chi = 40^\circ$

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A polycapillary is a focusing optic made up of an array of thousands of thin-walled hollow tubes which are $> 65\%$ empty space

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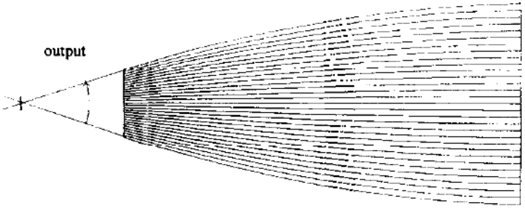


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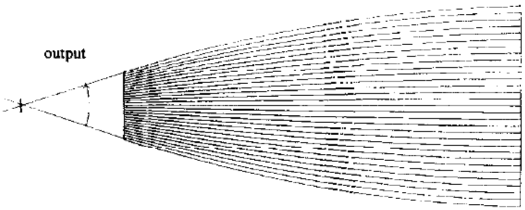


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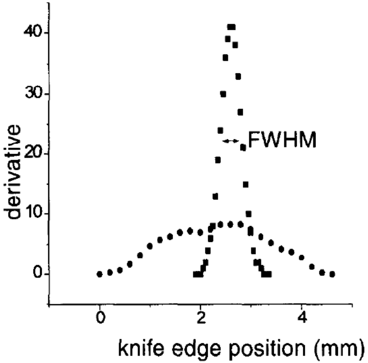

# Improving polycapillary optic performance

One drawback of a glass capillary is that the transmission at high energies is reduced because of critical angle restrictions

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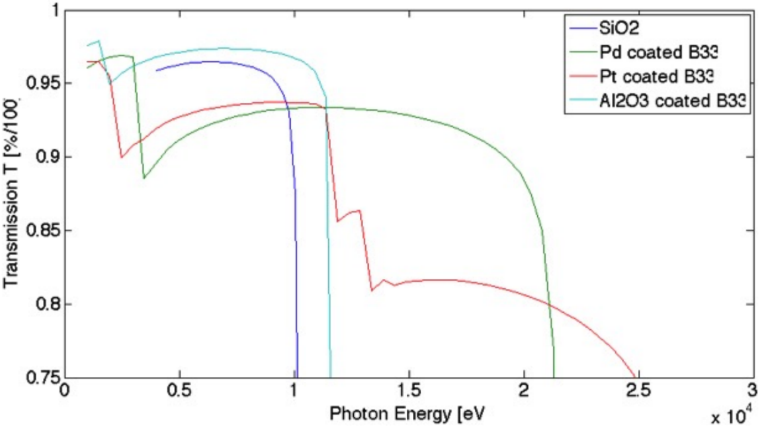


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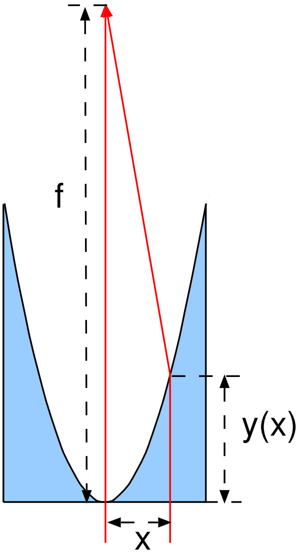

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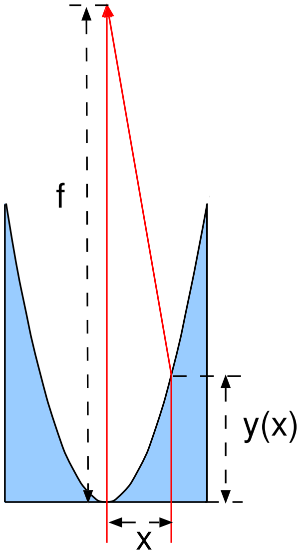

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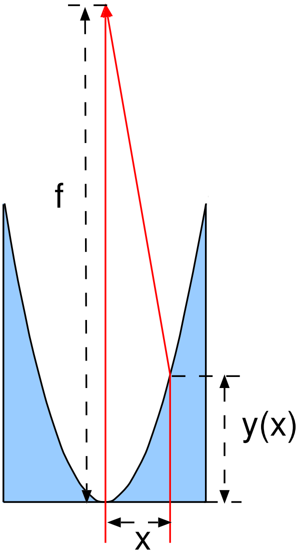

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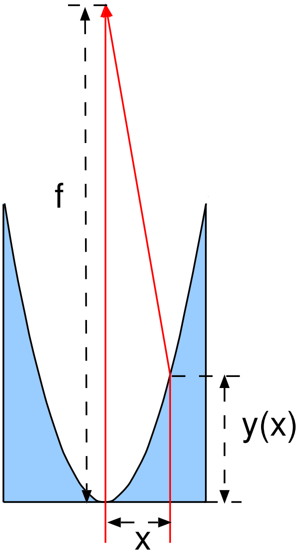

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\begin{aligned}
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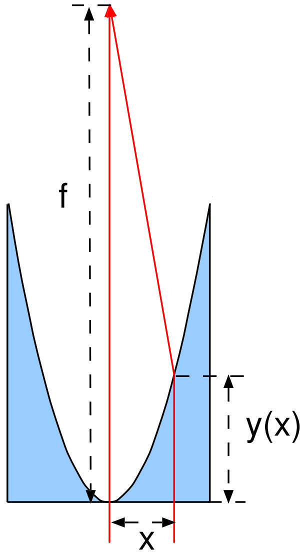
2f\delta y - (2\delta - \delta^2)y^2 &= x^2
\end{aligned}
$$



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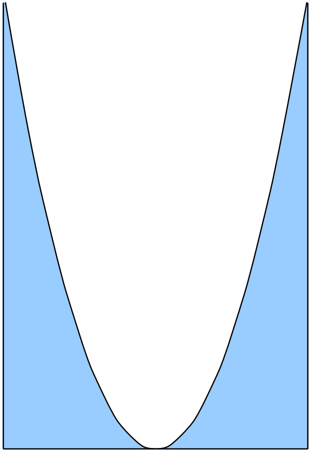
The ideal surface for a thick lens is an ellipse

# How to make a Fresnel lens



The ideal refracting lens has an elliptical shape but this is impractical to make. Assuming the parabolic approximation:

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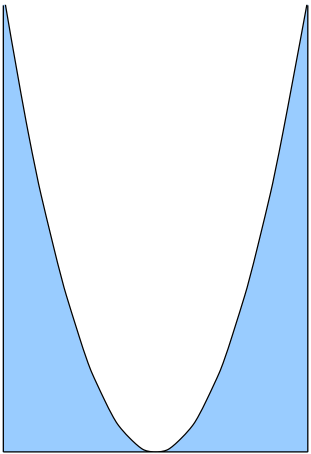

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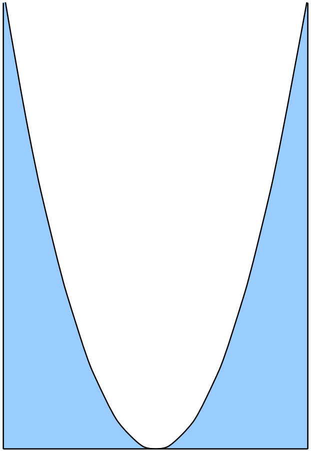

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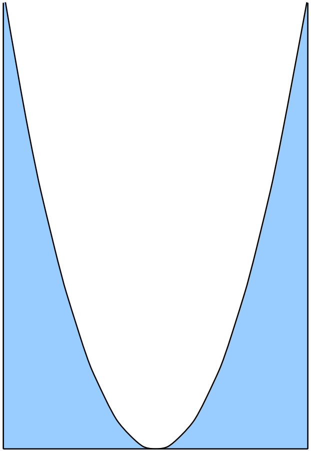

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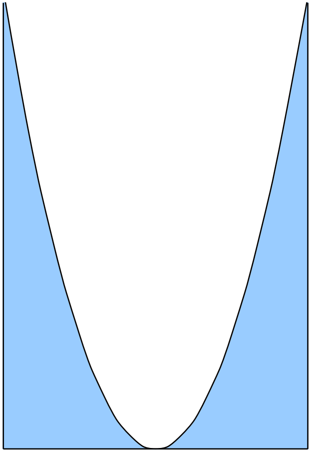

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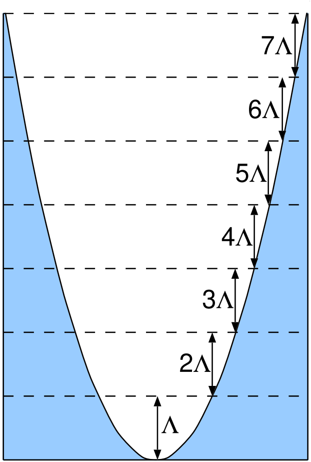
aspect ratio too large for a stable structure and absorption would be too large!

# How to make a Fresnel lens




Mark off the longitudinal zones (of thickness $\Lambda$) where the waves inside and outside the material are in phase.

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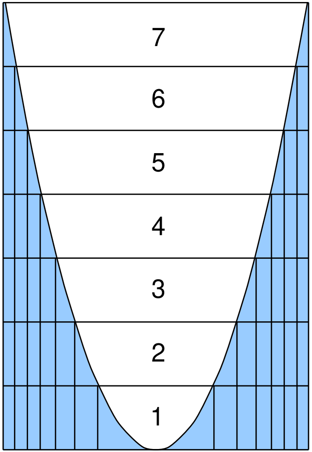


Mark off the longitudinal zones (of thickness $\Lambda$) where the waves inside and outside the material are in phase.

Each block of thickness $\Lambda$ serves no purpose for refraction but only attenuates the wave.

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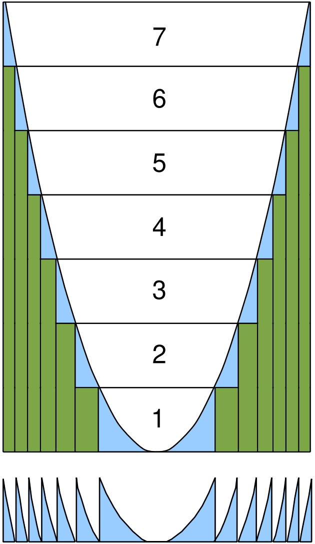


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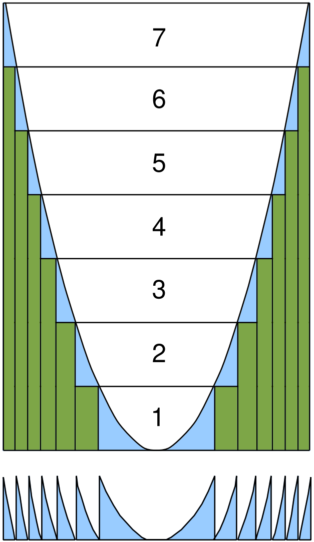
Each block of thickness $\Lambda$ serves no purpose for refraction but only attenuates the wave.

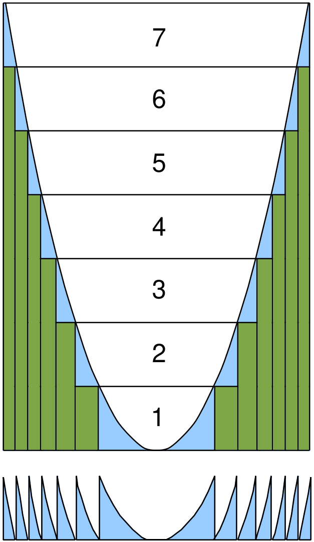
This material can be removed and the remaining material collapsed to produce a Fresnel lens which has the same optical properties as the parabolic lens as long as $f \gg N\Lambda$ where $N$ is the number of zones.

# Fresnel lens dimensions




The outermost zones become very small and thus limit the overall aperture of the zone plate. The dimensions of outermost zone, $N$ can be calculated by first defining a scaled height and lateral dimension

# Fresnel lens dimensions




The outermost zones become very small and thus limit the overall aperture of the zone plate. The dimensions of outermost zone, $N$ can be calculated by first defining a scaled height and lateral dimension

$$\nu = \frac{h(x)}{\Lambda}$$

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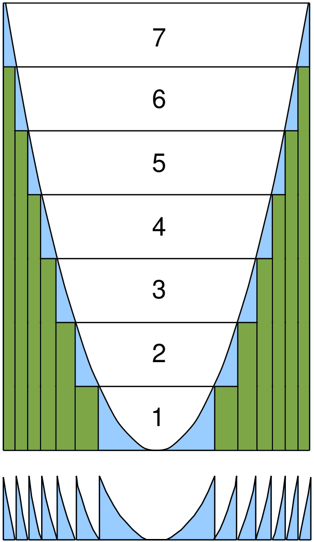


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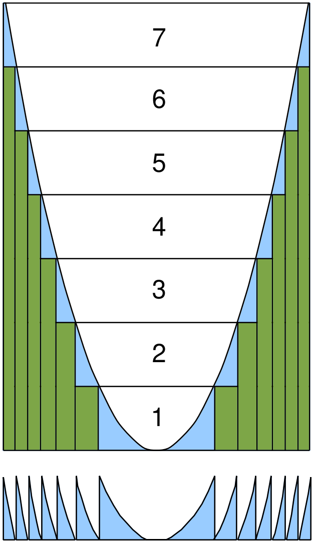


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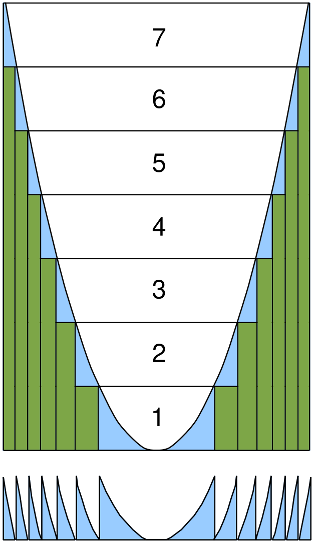


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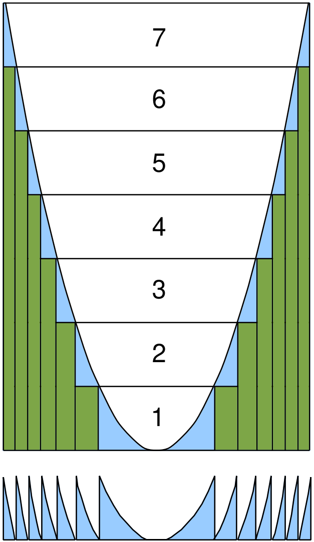


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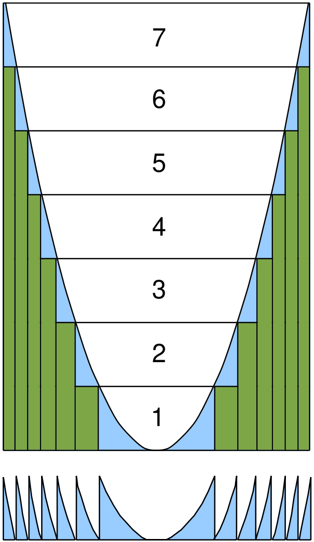


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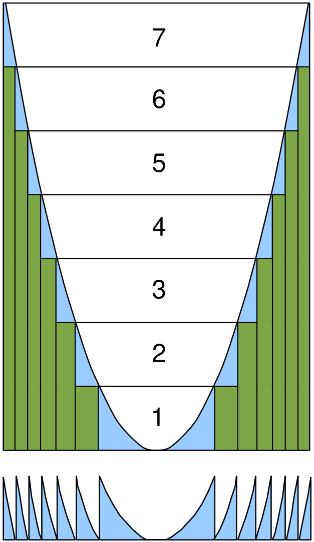


$$
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\Delta\xi_N = \xi_N - \xi_{N-1} &= \sqrt{N} - \sqrt{N-1} \\
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\end{aligned}
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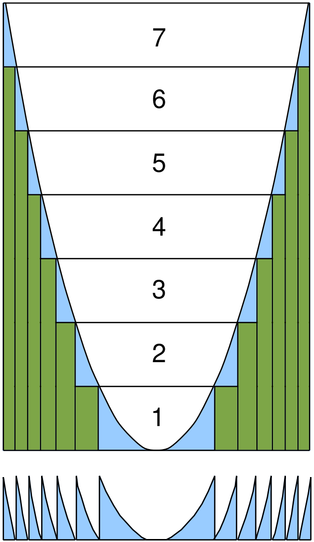


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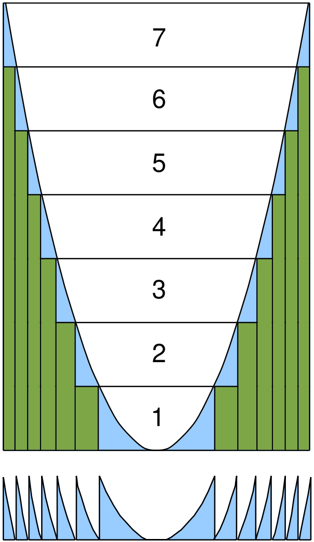


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$$
\Delta\xi_N \approx \frac{1}{2\sqrt{N}}
$$

The diameter of the entire lens is thus

$$
2\xi_N = 2\sqrt{N} = \frac{1}{\Delta\xi_N}
$$

# Fresnel lens example

In terms of the unscaled variables

$$\Delta x_N = \Delta\xi_N \sqrt{2\lambda_o f}$$

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$$\begin{aligned} \lambda_o &= 1\text{Å} = 1\times 10^{-10}\text{m} \\ f &= 50\text{cm} = 0.5\text{m} \\ N &= 100 \end{aligned}$$

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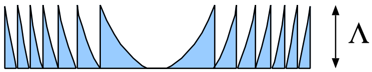
# Making a Fresnel zone plate

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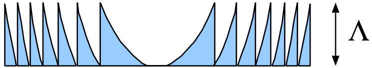


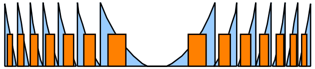

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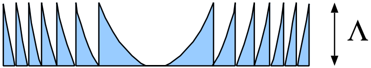
In practice, since the outermost zones are very small, zone plates are generally fabricated as alternating zones (rings for 2D) of materials with a large Z-contrast, such as Au/Si or W/C.

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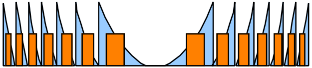

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This kind of zone plate is not as efficient as a true Fresnel lens would be in the x-ray regime. Nevertheless, efficiencies up to 35% have been achieved.

# Zone plate fabrication

Making high aspect ratio zone plates is challenging but a new process has been developed to make plates with an aspect ratio as high as 25.

M. Wojick et al., "X-ray zone plates with 25 aspect ratio using a 2-$\mu$m-thick ultra-nanocrystalline diamond mold," *Microsyst. Technol.* **20**, 2045-2050 (2014).

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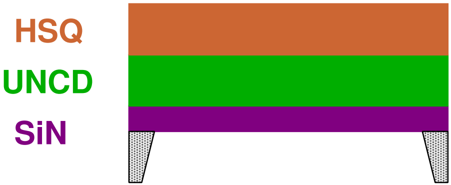


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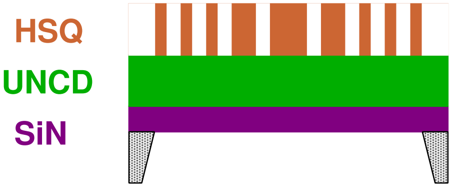


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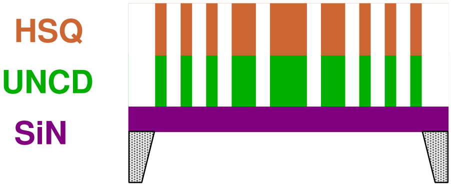


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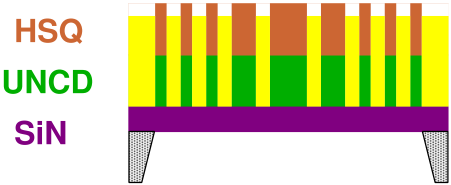


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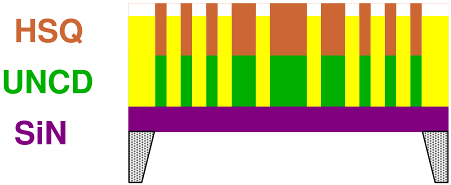
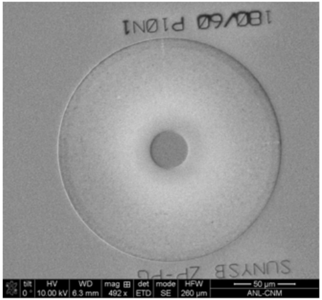
The whole 150nm diameter zone plate



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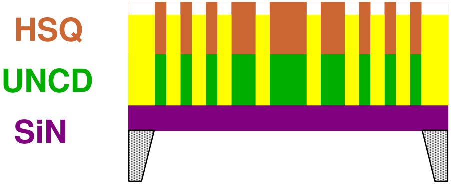


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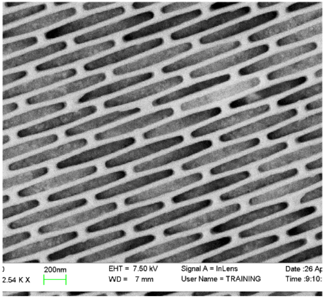


Detail view of outer zones

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