Today's outline - September 14, 2021

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Reading Assignment: Chapter 3.5–3.8

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Reading Assignment: Chapter 3.5–3.8

Homework Assignment #02:
Problems on Blackboard
due Tuesday, September 21, 2021

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- Kiessig fringes

Reading Assignment: Chapter 3.5–3.8

Homework Assignment #02:
Problems on Blackboard
due Tuesday, September 21, 2021

Homework Assignment #03:
Chapter 3: 1,3,4,6,8
due Tuesday, October 05, 2021

Index of refraction review

By comparing the scattering and refraction approaches the index of refaction in the x-ray regime can be calculated.

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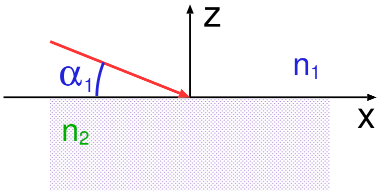
Index of refraction & critical angle

Consider an x-ray incident on an interface at angle α_1 to the surface



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Index of refraction & critical angle

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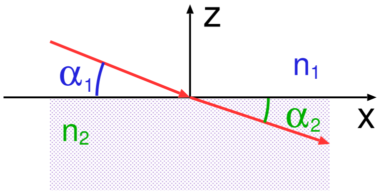

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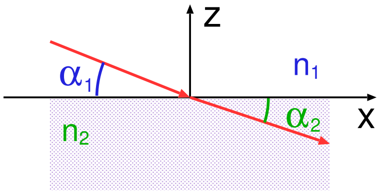

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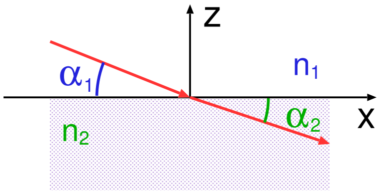

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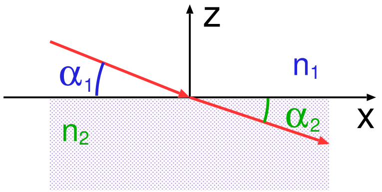

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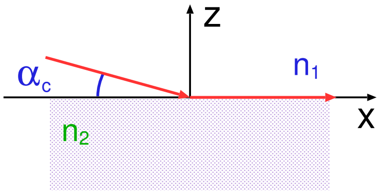

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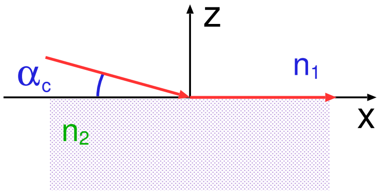

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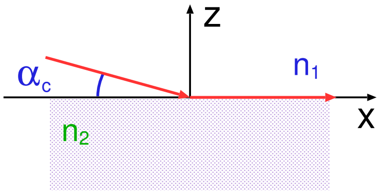

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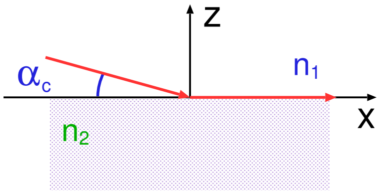

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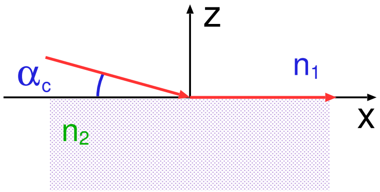

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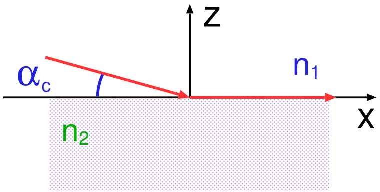

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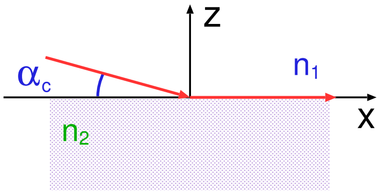

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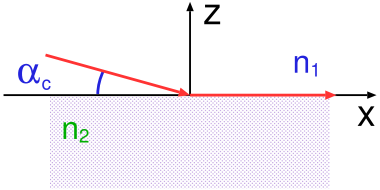

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Connection to atomic scattering

So far, we have made the assumption that the charge distribution is uniform. We know that this is not correct, and that usually electron charge distributions are those of the atoms making up the solid.

$$\psi^P = \psi_0^P \left[1 - i\frac{2\pi\rho b\Delta}{k} \right]$$

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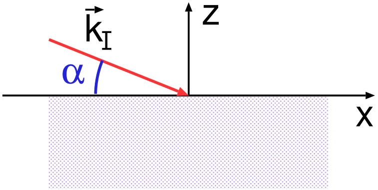
Electromagnetic boundary conditions

Maxwell's equations require that an electromagnetic wave and its derivative be continuous in all directions at any interface. This condition places restrictions on the waves which exist at any interface:



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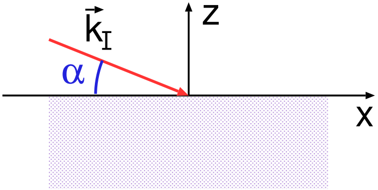

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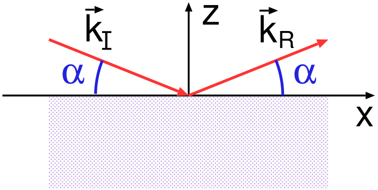

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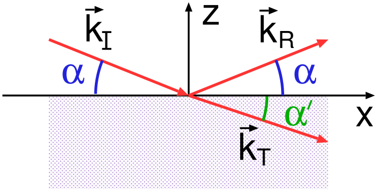

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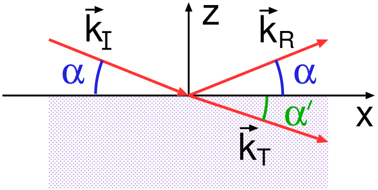

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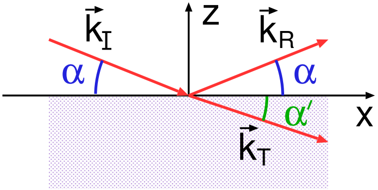

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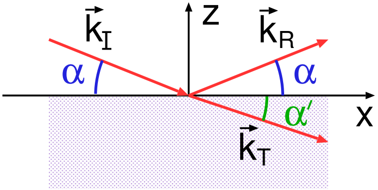

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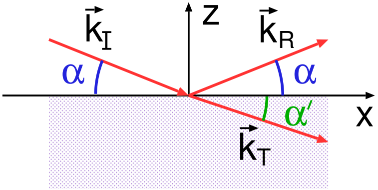

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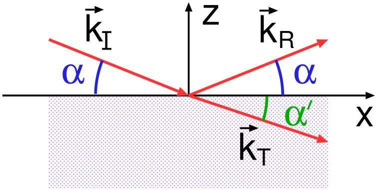

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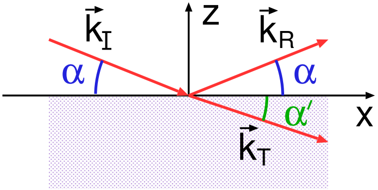

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Parallel projection & Snell's Law

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Recalling that $\alpha_c = \sqrt{2\delta}$

$$1 - \frac{\alpha^2}{2} = (1 - \delta + i\beta)\left(1 - \frac{\alpha'^2}{2}\right) \quad \longrightarrow \quad \alpha^2 = \alpha'^2 + \alpha_c^2 - 2i\beta$$

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The Fresnel Equations can now be derived

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Perpendicular projection & Fresnel equations

Taking the perpendicular projection, substituting for the wave vectors and using the amplitude equation

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q is a convenient parameter to use because it is a combination of two parameters which are often varied in experiments, the angle of incidence α and the wavenumber (energy) of the x-ray, k.

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$$r = \frac{(q-q')(q+q')}{(q+q')(q+q')} = \frac{q^2 - q'^{\,2}}{(q+q')^2} \approx \frac{1}{(2q)^2}\,, \quad t = \frac{2q}{q+q'} \approx 1\,, \quad \Lambda \approx \frac{\alpha}{\mu}$$

the reflected wave is in phase with the incident wave, almost total transmission

Limiting cases - $q \ll 1$

Starting with Snell's Law again

$$q^2 = q'^2 + 1 - 2ib_\mu$$

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The reflected wave is out of phase with the incident wave, there is only small transmission in the form of an evanescent wave, and the penetration depth is very short.

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Using Snell's Law, with $q \sim 1$,

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adding and subtracting b_μ,

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1)$$

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The reflected wave is in phase with the incident, there is significant (larger amplitude than the reflection) transmission with a large penetration depth.

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We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium.

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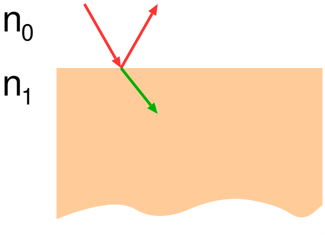

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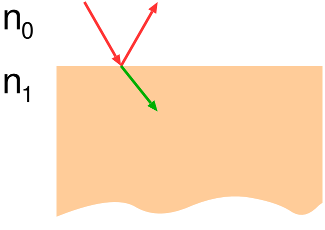
We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.



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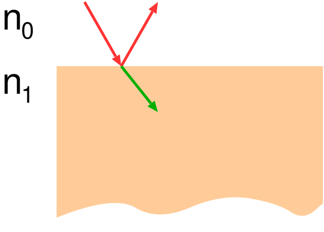

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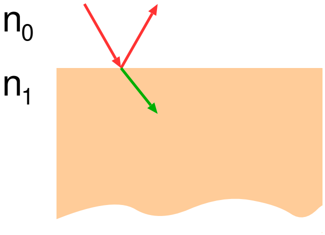

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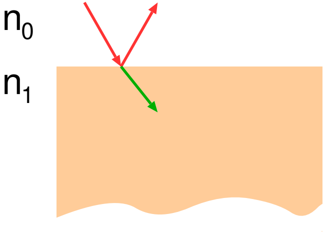

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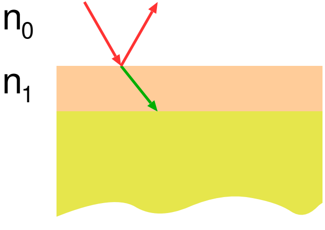

$$r = \frac{Q - Q'}{Q + Q'}$$

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We have assumed that the transmitted wave eventually attenuates to zero in all cases due to absorption. We now consider what happens if there is a second interface encountered by the transmitted wave before it dies away. That is, a thin slab of material on top of an infinite substrate

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:

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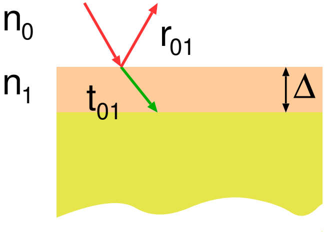

r_{01} – reflection in n_0 off n_1
t_{01} – transmission from n_0 into n_1

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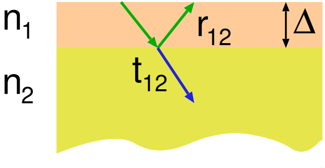

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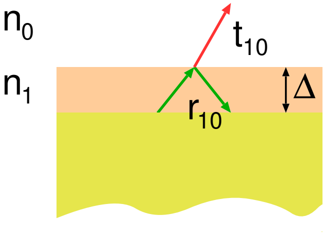

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t_{10} – transmission from n_1 into n_0

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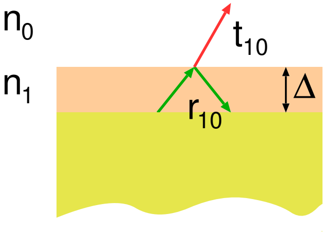

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Build the composite reflection coefficient from all possible events

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed

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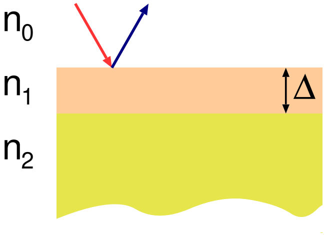

r_{01}

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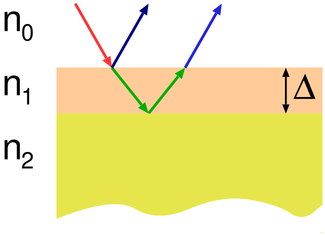

$$r_{01}$$

$$+$$

$$t_{01}\, r_{12}\, t_{10}$$

Overall reflection from a slab

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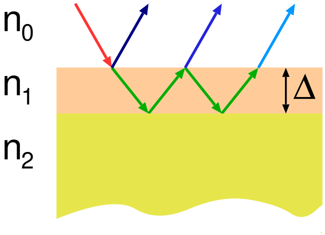

$$r_{01}$$
$$+$$
$$t_{01}r_{12}t_{10}$$
$$+$$
$$t_{01}r_{12}r_{10}r_{12}t_{10}$$

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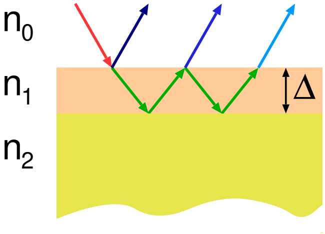

$$r_{01}$$
$$+$$
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Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta}$$

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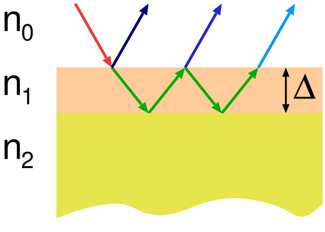

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$$+$$
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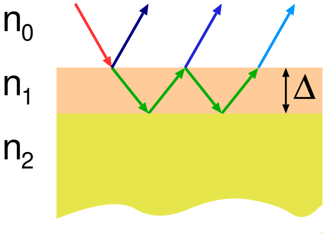

$$r_{01}$$
$$+$$
$$t_{01} r_{12} t_{10} \cdot p^2$$
$$+$$
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Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin \alpha_1)\Delta} = e^{iQ_1 \Delta}$$

which multiplies the reflection coefficient

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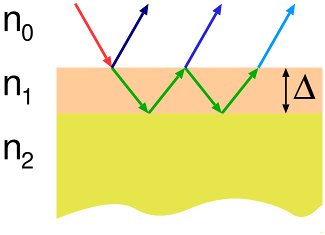

$$r_{01}$$
$$+$$
$$t_{01} r_{12} t_{10} \cdot p^2$$
$$+$$
$$t_{01} r_{12} r_{10} r_{12} t_{10} \cdot p^4$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta} = e^{iQ_1\Delta}$$

which multiplies the reflection coefficient with each pass through the slab

Composite reflection coefficient

The composite reflection coefficient can now be expressed as a sum

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$$r_{slab} = r_{01} + t_{01} r_{12} t_{10} p^2 + t_{01} r_{10} r_{12}^2 t_{10} p^4 + t_{01} r_{10}^2 r_{12}^3 t_{10} p^6 + \cdots$$

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factoring out the second term from all the rest

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$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \sum_{m=0}^{\infty} \left(r_{10} r_{12} p^2 \right)^m$$

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summing the geometric series as previously

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summing the geometric series as previously

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The composite reflection coefficient can now be expressed as a sum

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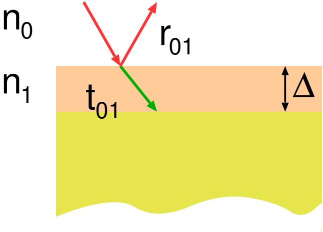
Fresnel equation identity

Applying the Fresnel equations to the top interface

Fresnel equation identity

Applying the Fresnel equations to the top interface



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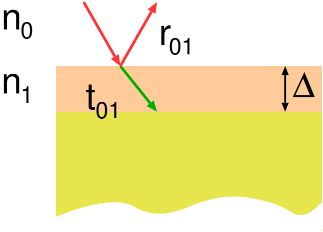

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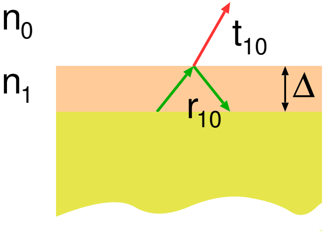

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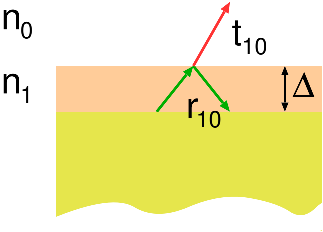

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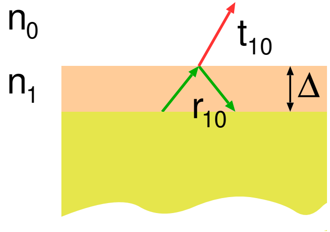

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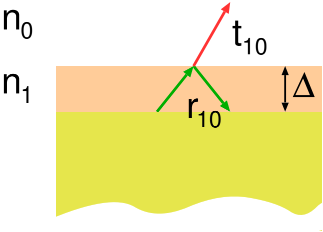

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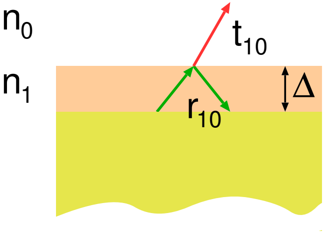

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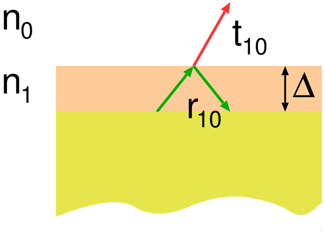

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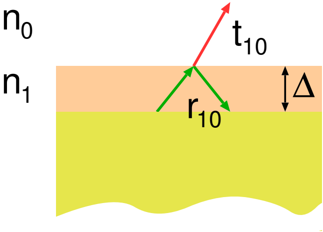

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Starting with the reflection coefficient of the slab obtained earlier

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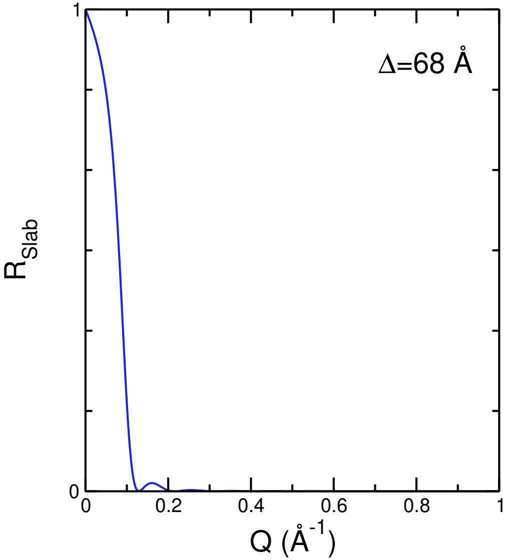

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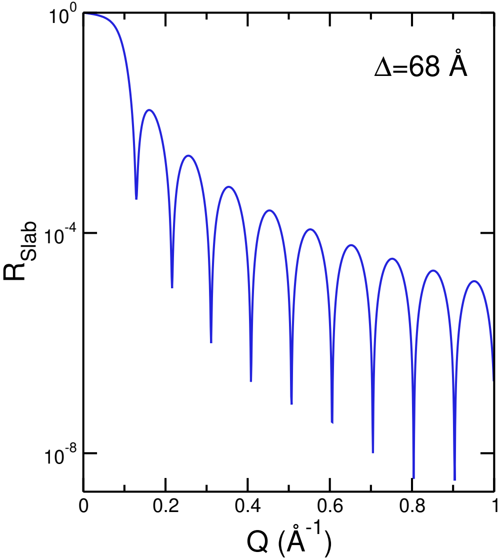

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