Today's outline - September 09, 2021

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Homework Assignment #02:
Problems on Blackboard
due Tuesday, September 21, 2021

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Chapter 3: 1,3,4,6,8
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Reading Assignment: Chapter 3.4

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HW #02

1. Knowing that the photoelectric absorption of an element scales as the inverse of the energy cubed, calculate:

(a) the absorption coefficient at 10keV for copper when the value at 5keV is 1698.3 cm^{-1};

(b) The actual absorption coefficient of copper at 10keV is 1942.1 cm^{-1}, why is this so different than your calculated value?

2. A 30 cm long, ionization chamber, filled with 80% helium and 20% nitrogen gases at 1 atmosphere, is being used to measure the photon rate (photons/sec) in a synchrotron beamline at 12 keV. If a current of 10 nA is measured, what is the photon flux entering the ionization chamber?

3. A 5 cm deep ionization chamber is used to measure the fluorescence from a sample containing arsenic (As). Using any noble gases or nitrogen, determine a gas fill (at 1 atmosphere) for this chamber which absorbs at least 60% of the incident photons. How does this change if you are measuring the fluorescence from ruthenium (Ru)?

HW #02

4. Calculate the critical angle of reflection of 10 keV and 30 keV x-rays for:

(a) A slab of glass (SiO_2);

(b) A thick chromium mirror;

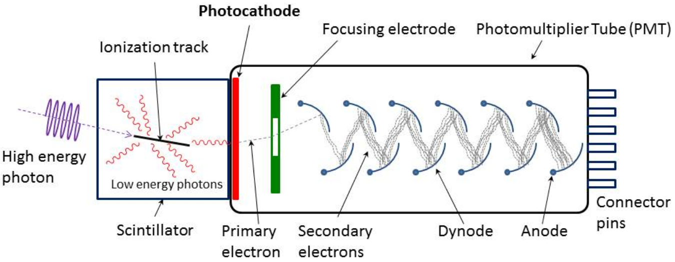
(c) A thick platinum mirror.

(d) If the incident x-ray beam is 2 mm high, what length of mirror is required to reflect the entire beam for each material?

5. Calculate the fraction of silver (Ag) fluorescence x-rays which are absorbed in a 1 mm thick silicon (Si) detector and the charge pulse expected for each absorbed photon. Repeat the calculation for a 1 mm thick germanium (Ge) detector.

Scintillation detector

Useful for photon counting experiments with rates less than $10^4/s$



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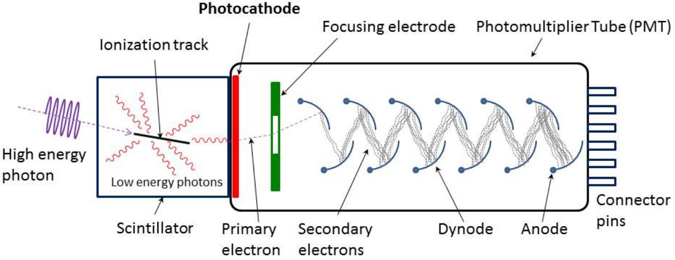

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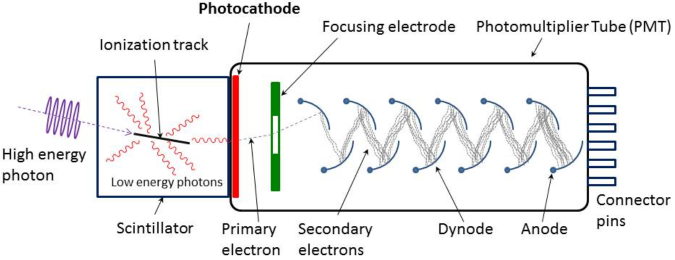

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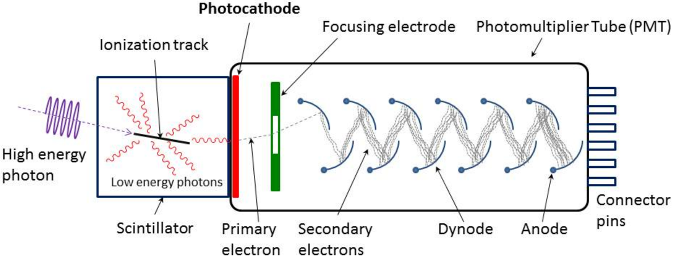

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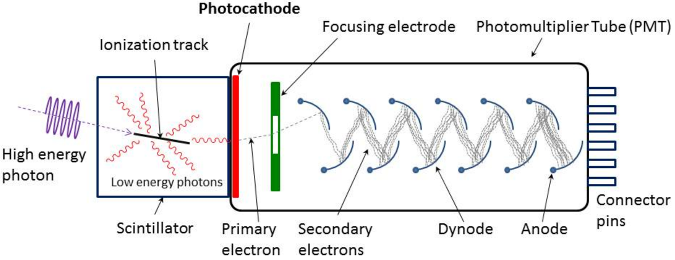

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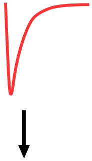
- Output voltage pulse is proportional to initial x-ray energy.

Counting a scintillator pulse



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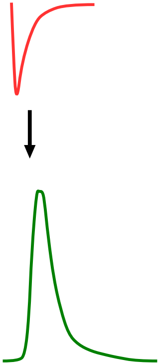

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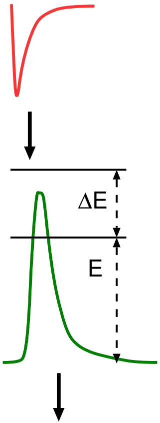

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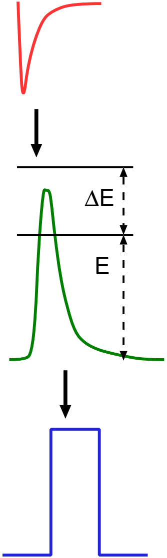

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if the voltage pulse falls within the discriminator window, a short digital pulse is output from the discriminator and into a scaler for counting

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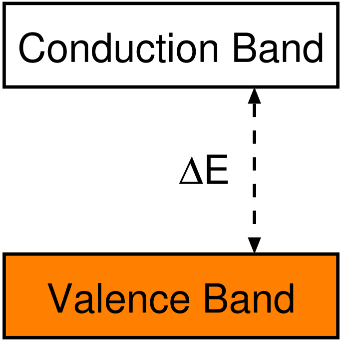

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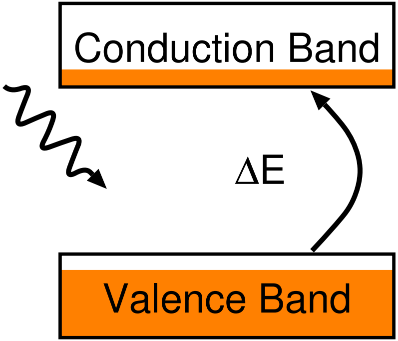

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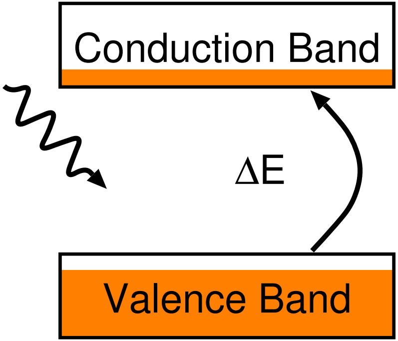

because of the small energy required to produce an electron-hole pair, one x-ray photon will create many and its energy can be detected with very high resolution

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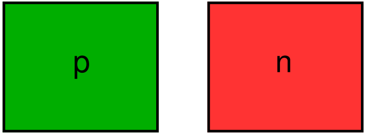

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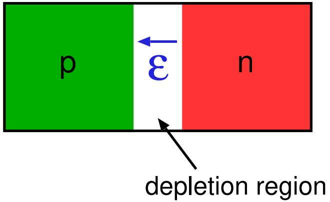

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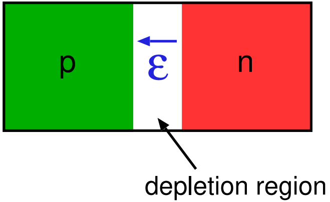

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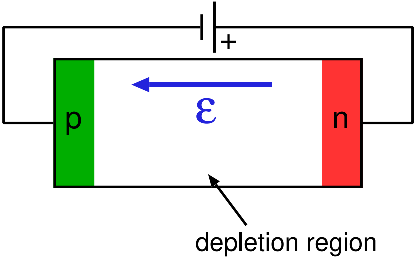
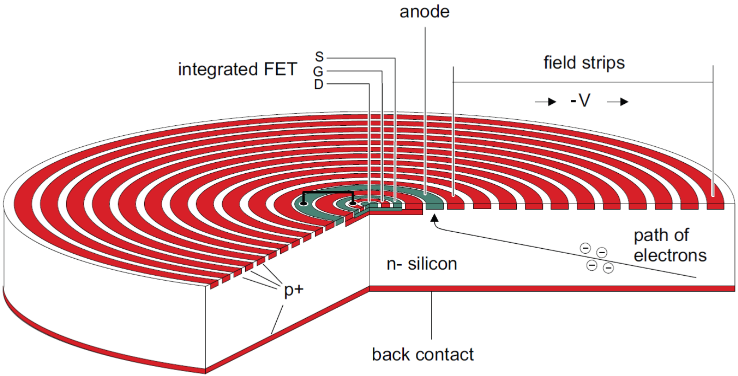


by applying a reverse bias voltage, it is possible to extend the depleted region, make the effective volume of the detector larger and increase the electric field to get faster charge collection times

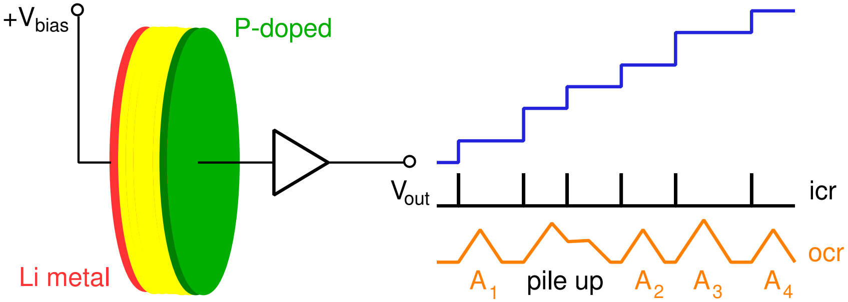
Silicon Drift Detector

Same principle as intrinsic or p-i-n detector but much more compact and operates at higher temperatures



Relatively low stopping power is a drawback

Detector operation



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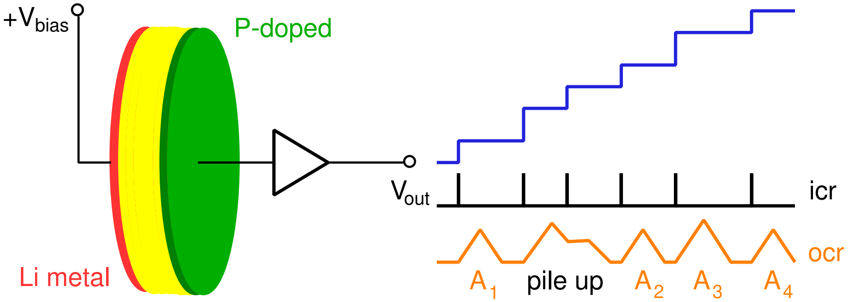

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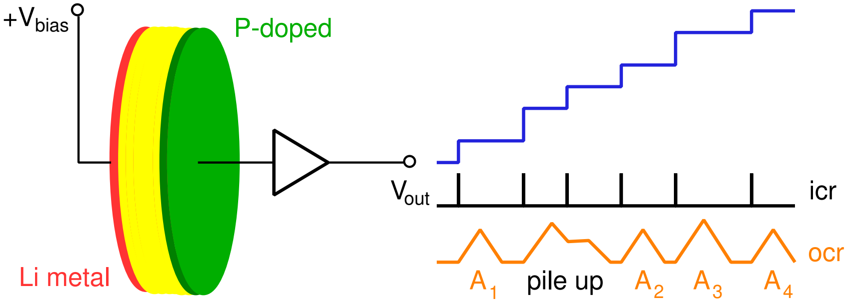

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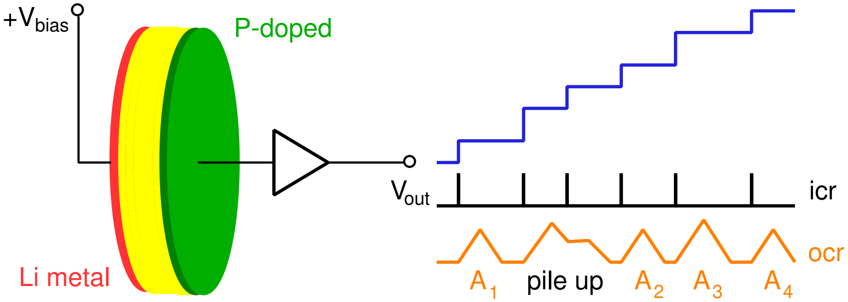

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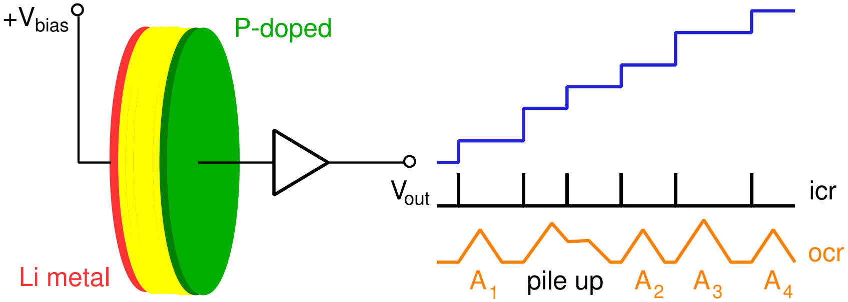

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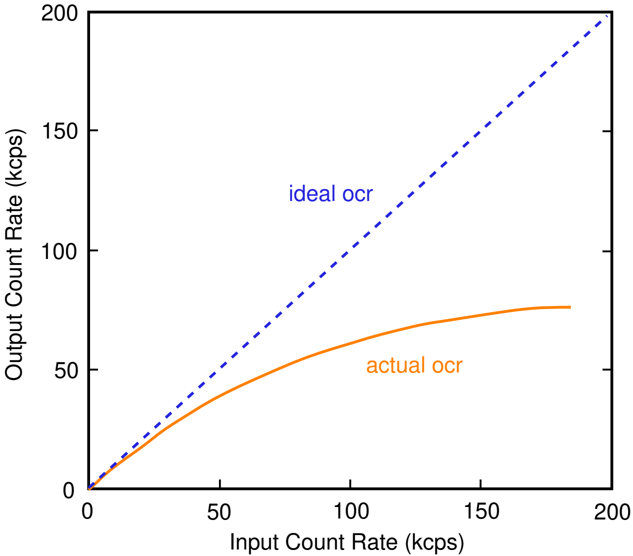
electronics outputs input count rate (icr), output count rate (ocr), and areas of integrated pulses (A_n)

Dead time correction



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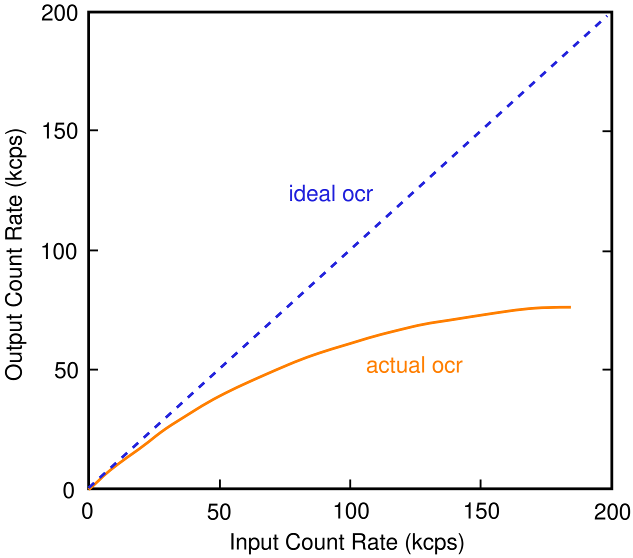

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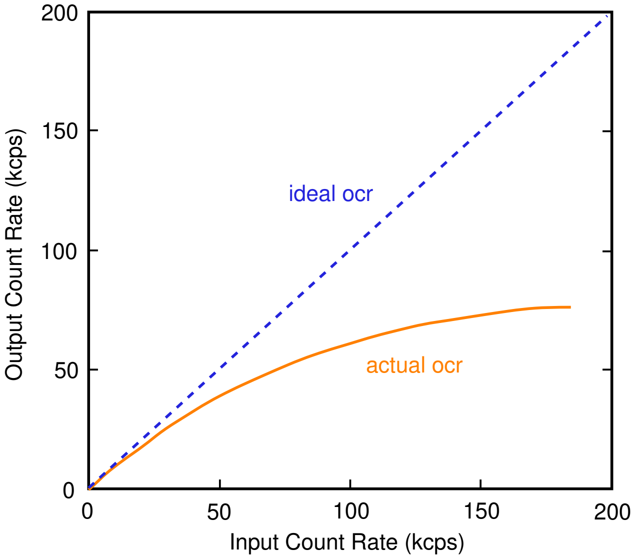

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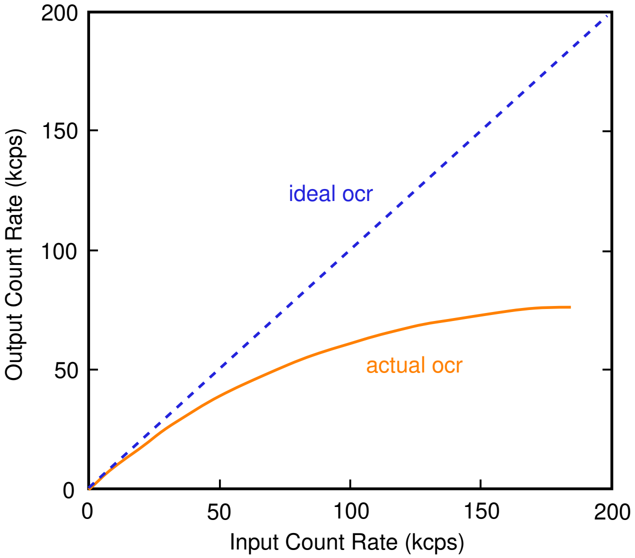

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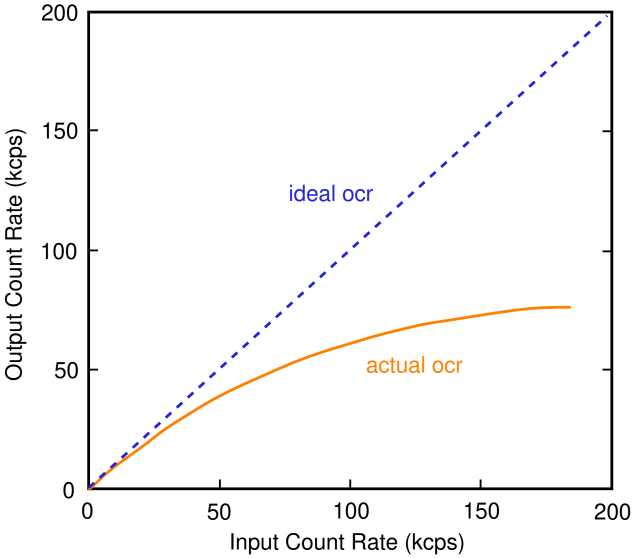

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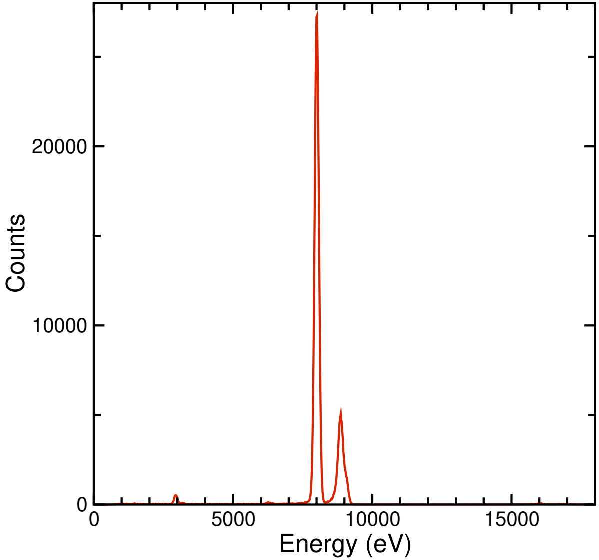
When dead time is too large, correction will not be accurate!

SDD spectrum



Fluorescence spectrum of Cu foil in air using 9200 eV x-rays

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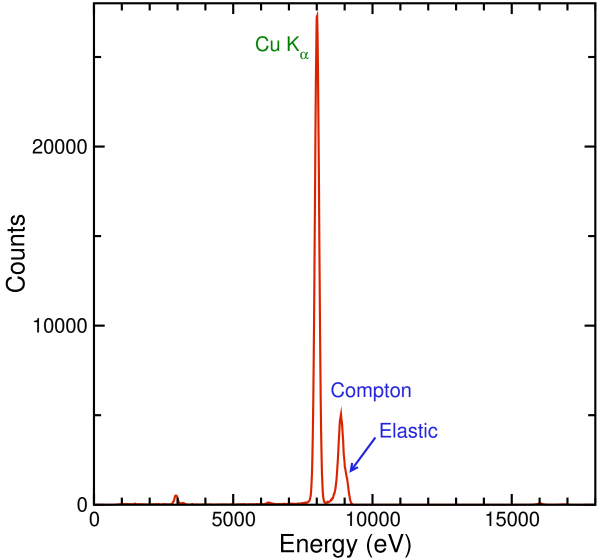

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Compton and elastic peaks are visible just above the Cu K_α fluorescence line

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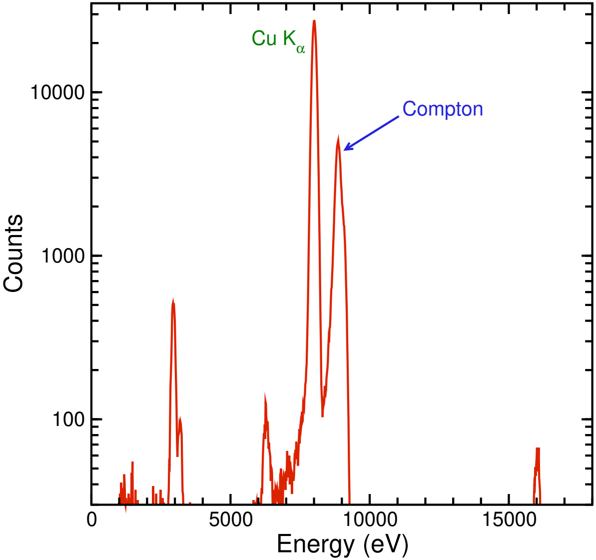

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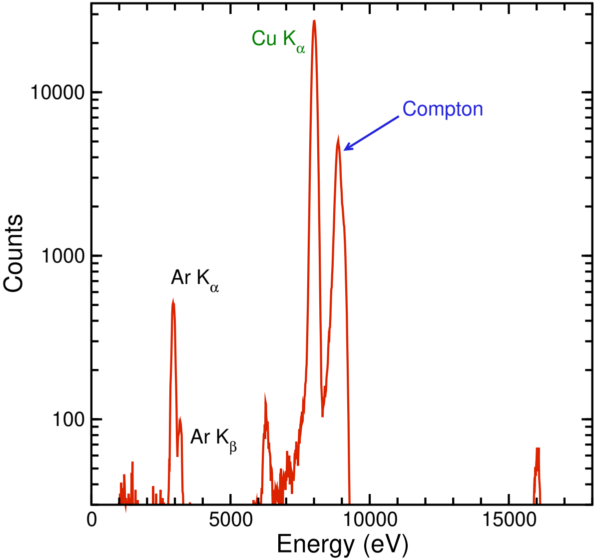

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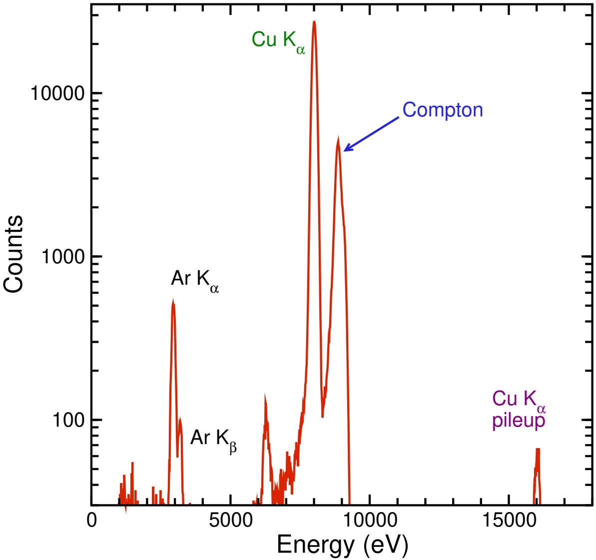

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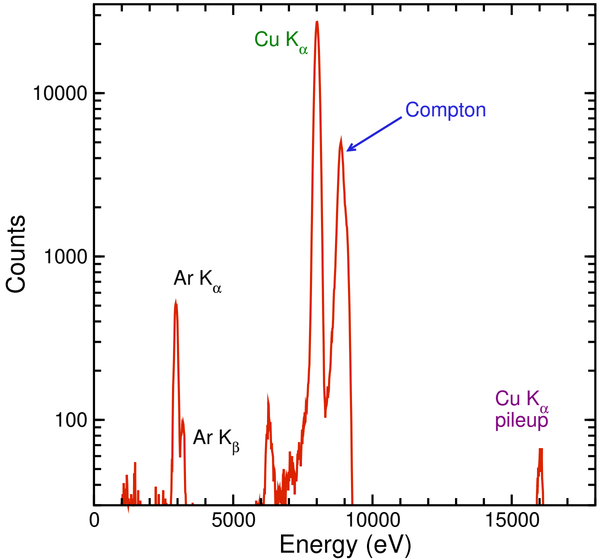

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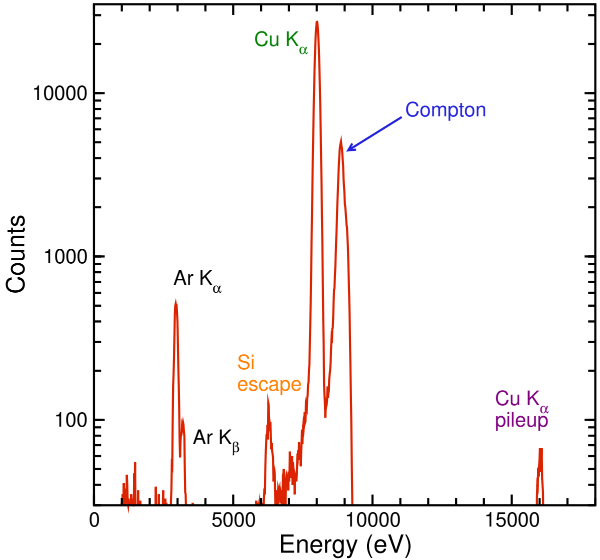

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Si escape peak
$E_{esc} = 8046 - 1839 = 6207$ eV

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The most advanced detectors can easily cost over a million dollars!

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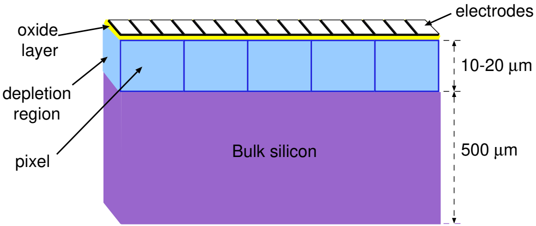

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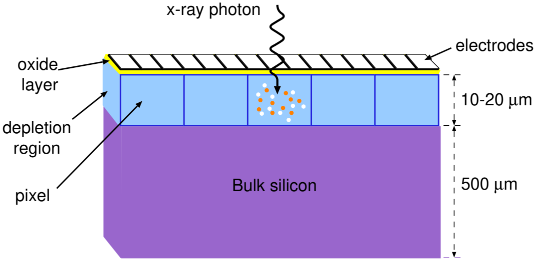

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the x-ray deposits its energy directly in the depletion region, creating electron-hole pairs

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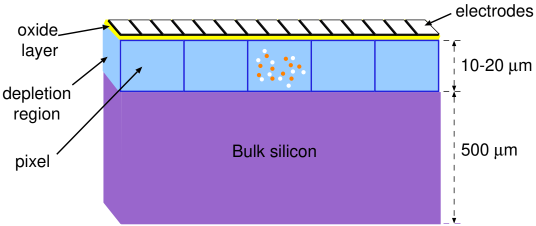

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the electrons and holes are trapped and accumulate during the exposure time

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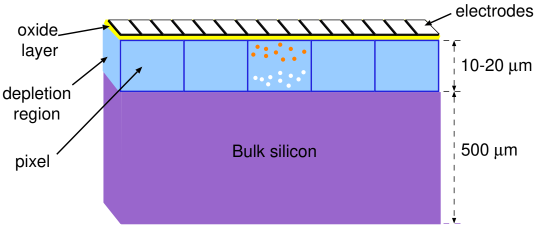

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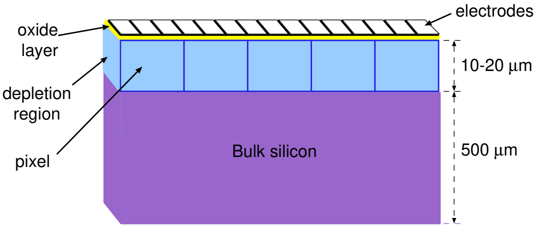

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expensive to make very large, limited sensitivity to high energies

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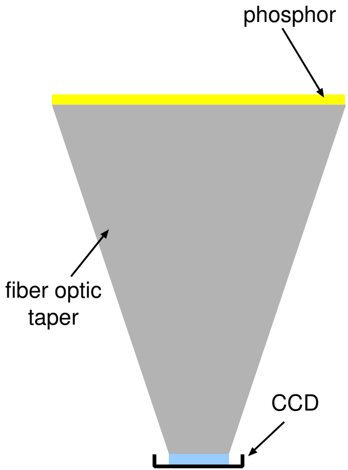

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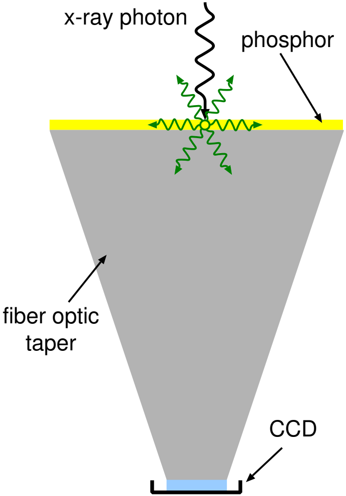

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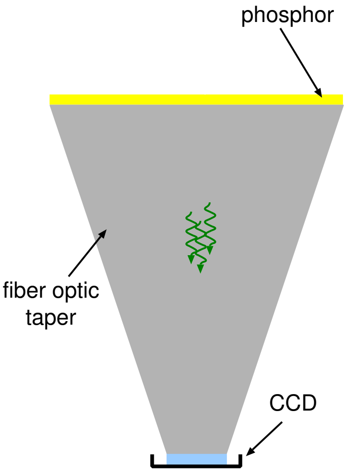

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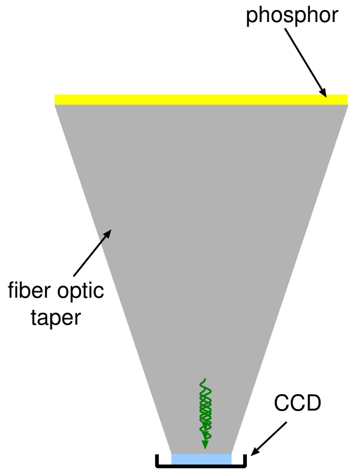

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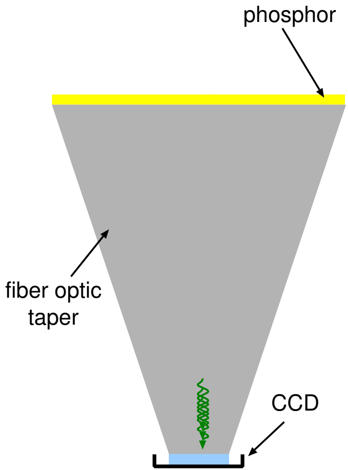
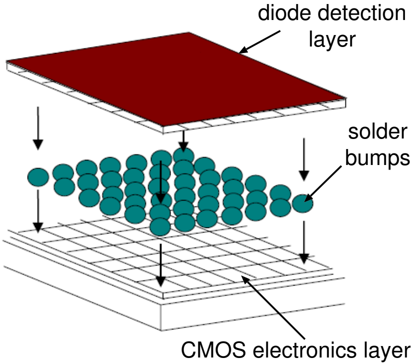
Pixel sizes are usually rather large (50 μm $\times$ 50 μm)



Pixel array detectors - schematic

The Pixel Array Detector combines area detection with on-board electronics for fast signal processing

Pixel array detectors - schematic



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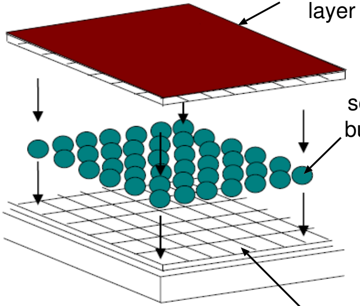

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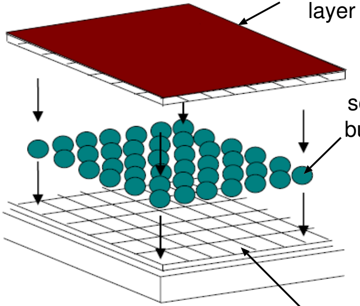

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This permits fast processing and possibly energy discrimination on a per-pixel level

Pixel array detectors - Pilatus

Pixel array detector with 1,000,000 pixels.

Each pixel has energy resolving capabilities & high speed readout.

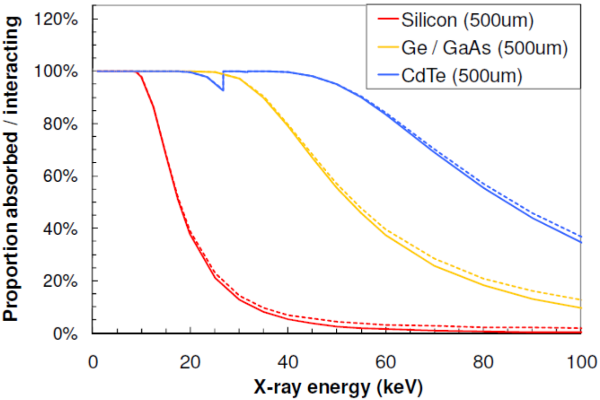
Silicon sensor limits energy range of operation.

from Swiss Light Source

High energy solutions

One of the major problems with pixel array detectors and SDDs is the low absorption cross section at high energies

High energy solutions



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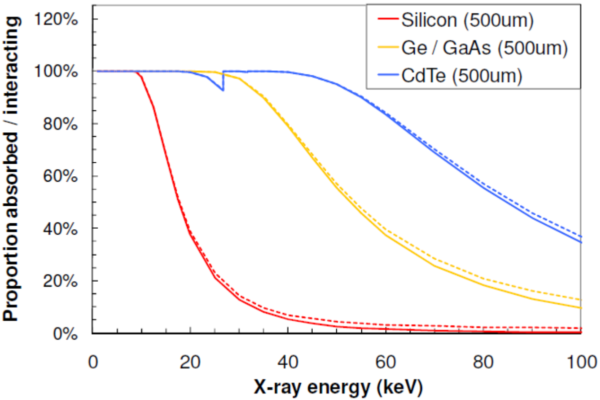

One of the major problems with pixel array detectors and SDDs is the low absorption cross section at high energies

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The absorption can be significantly enhanced with these higher Z elements while maintaining good energy discrimination capabilities.

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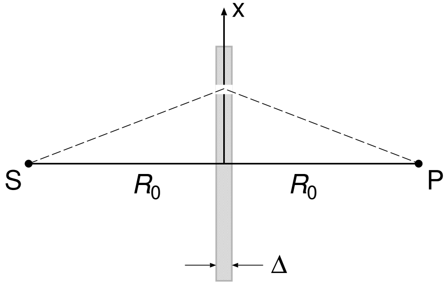
Initially assume that all interfaces are perfectly flat and ignore all absorption processes.

Thin plate response - scattering approach

Consider a thin plate of thickness Δ onto which x-rays are incident from a point source S a perpendicular distance R_0 away.



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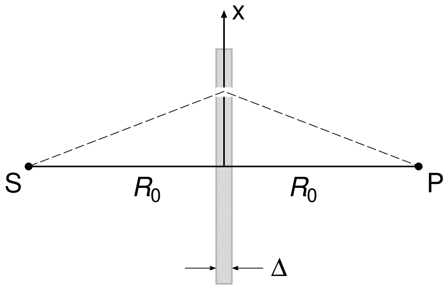

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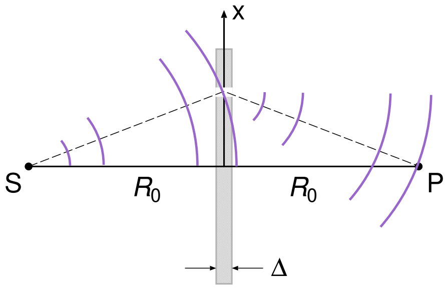

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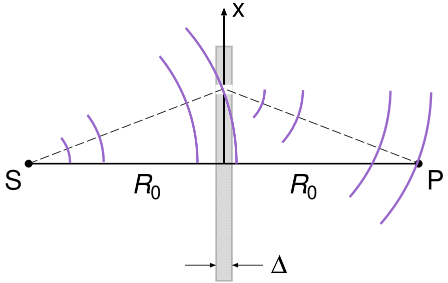

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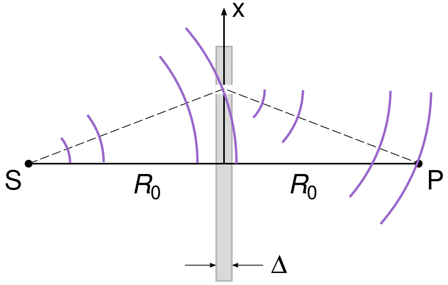

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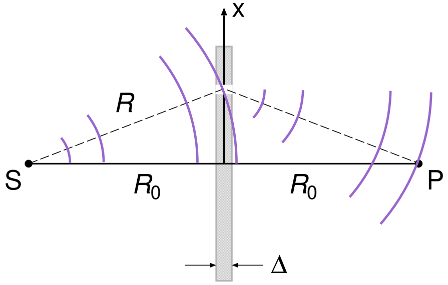

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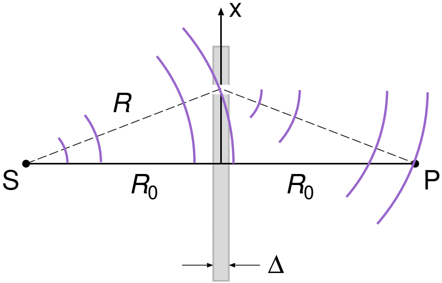

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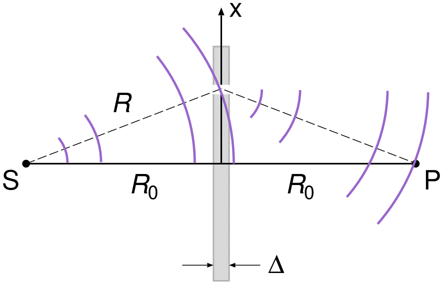

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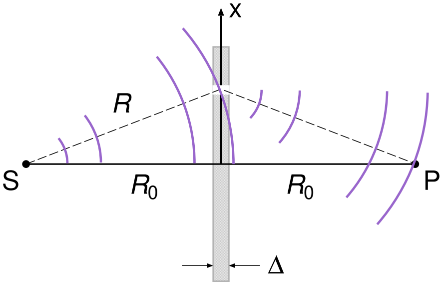
Thin plate response - scattering approach

R is also the distance between the scattering volume and P so, a wave (x-ray) which travels from $S \rightarrow P$ through the scattering volume will have an extra phase shift



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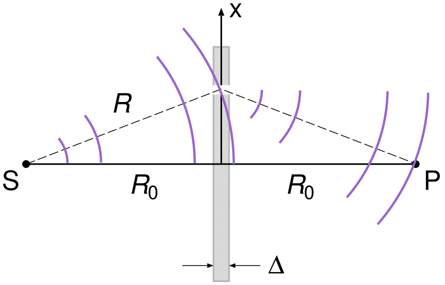

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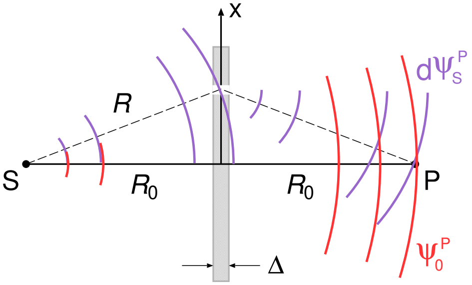

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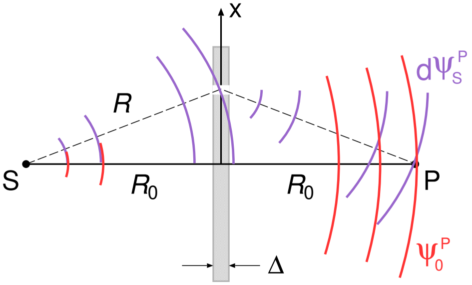

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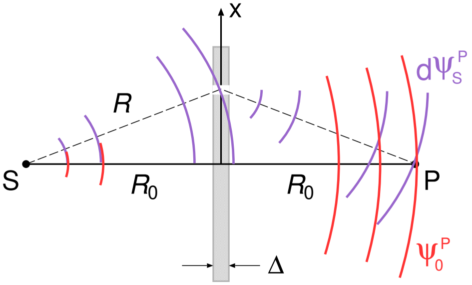

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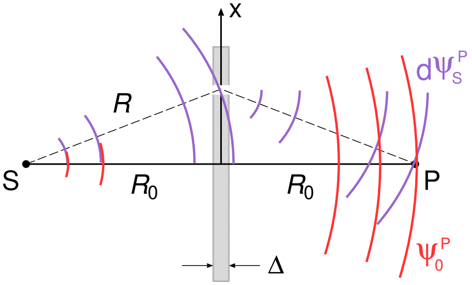

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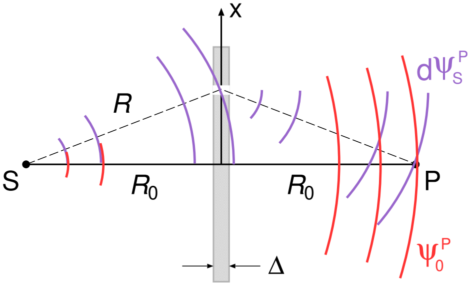

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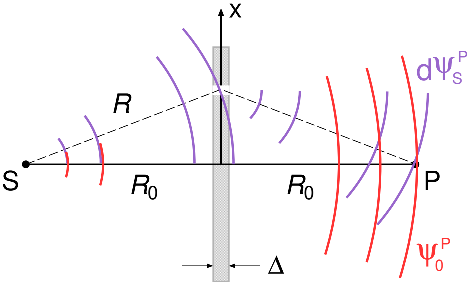

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$$\psi_S^P = \int d\psi_S^P = -\rho b\Delta\frac{e^{i2kR_0}}{R_0^2}\int_{-\infty}^{\infty} e^{i\frac{x^2+y^2}{R_0^2}k}dxdy$$

$$= -\rho b\Delta\frac{e^{i2kR_0}}{R_0^2}\left(i\frac{\pi R_0}{k}\right)$$

Integrate the scattered wave over the entire plate. This integral is basically a Gaussian integral squared with an imaginary (instead of real) constant in the exponent and it gives

$$\int_{-\infty}^{\infty} e^{i\frac{x^2+y^2}{R_0^2}k}dxdy = i\frac{\pi R_0}{k}$$

Thus the total wave (electric field) at P can be written

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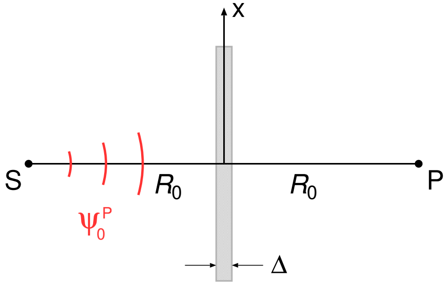
Now let's look at this phenomenon from a different point of view, that of refraction.



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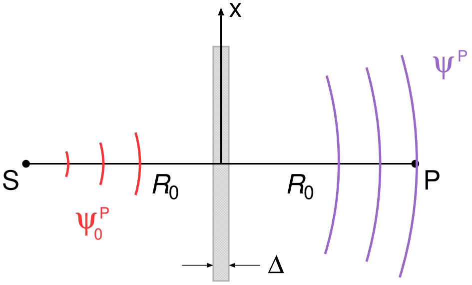

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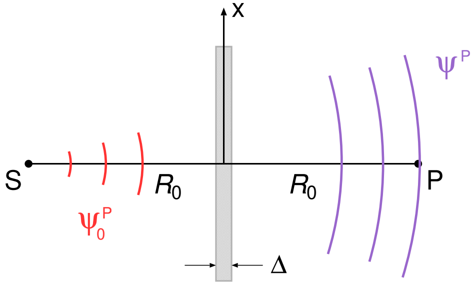

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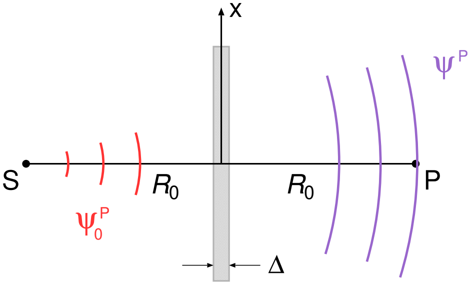

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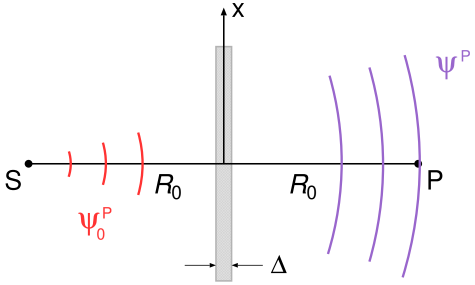

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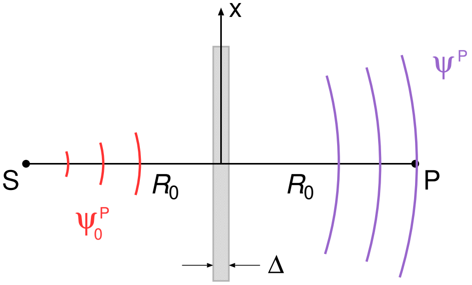

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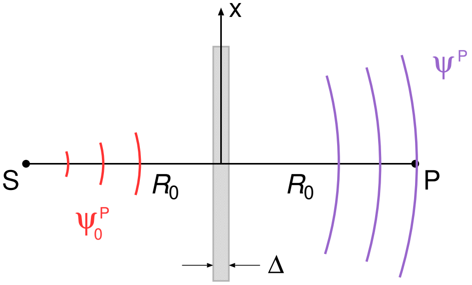

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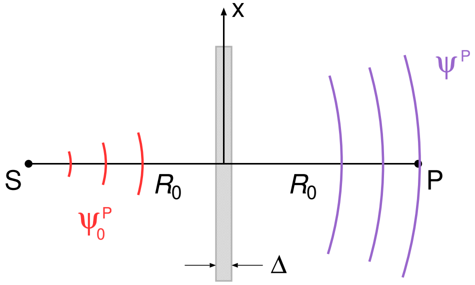

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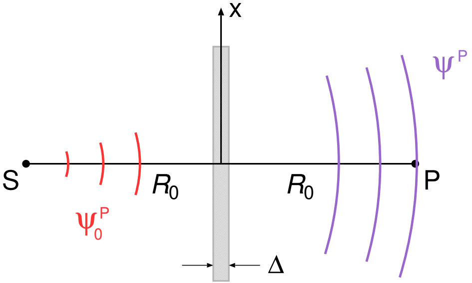

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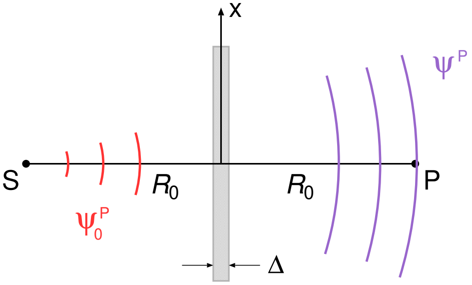

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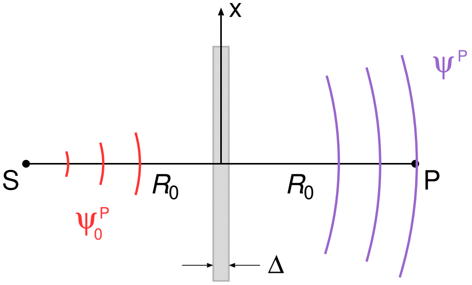

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