Today's outline - September 02, 2021

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- The bending magnet source

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- The bending magnet source
    - Segmented arc approximation

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- The bending magnet source
  - Segmented arc approximation
  - Curved arc emission

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  - Characteristic energy

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  - Power and flux

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Reading Assignment: Chapter 2.5–2.6

Today's outline - September 02, 2021

- The bending magnet source
  - Segmented arc approximation
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  - Polarization
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Reading Assignment: Chapter 2.5–2.6

Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Tuesday, September 07, 2021

Today's outline - September 02, 2021

- The bending magnet source
  - Segmented arc approximation
  - Curved arc emission
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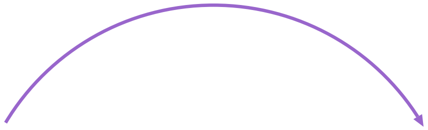
  - Power and flux
  - Polarization
- Insertion devices
- Undulator parameters

Reading Assignment: Chapter 2.5–2.6

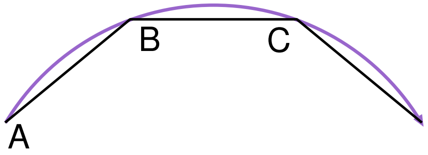
Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Tuesday, September 07, 2021

Homework Assignment #02:
Problems on Blackboard
due Tuesday, September 21, 2021

Segmented arc approximation

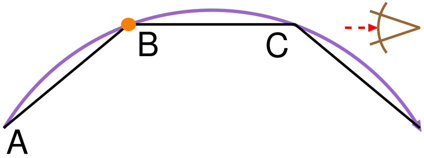


Segmented arc approximation



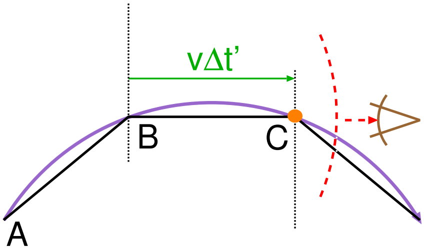
- Approximate the electron's path as a series of segments

Segmented arc approximation



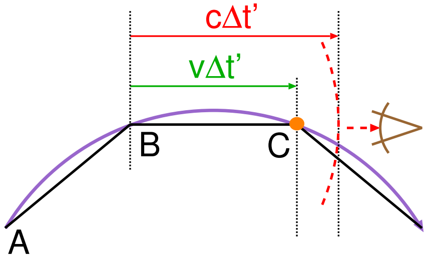
- Approximate the electron's path as a series of segments
- At each corner the electron is accelerated and emits radiation

Segmented arc approximation



- Approximate the electron's path as a series of segments
- At each corner the electron is accelerated and emits radiation
- Consider the emissions at points B and C

Segmented arc approximation



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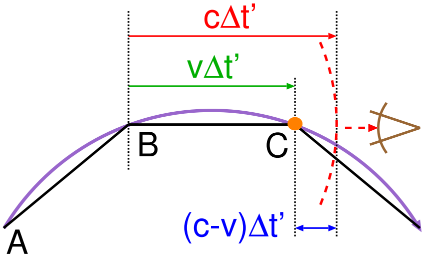
The electron travels the distance from B to C in $\Delta t'$

Segmented arc approximation



- Approximate the electron's path as a series of segments
- At each corner the electron is accelerated and emits radiation
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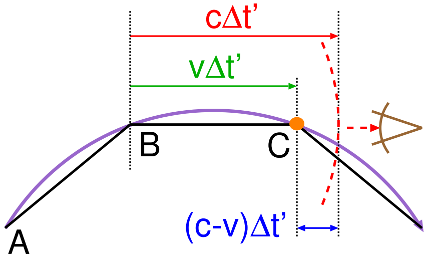
The electron travels the distance from B to C in $\Delta t'$ while the light pulse emitted at B travels further, $c\Delta t'$, in the same time.

Segmented arc approximation



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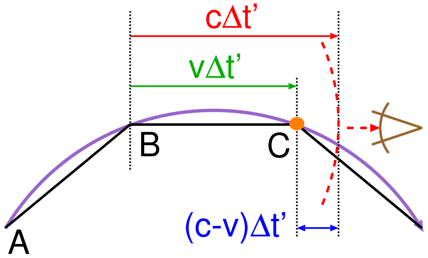
The electron travels the distance from B to C in $\Delta t'$ while the light pulse emitted at B travels further, $c\Delta t'$, in the same time.
The light pulse emitted at C is therefore, a distance $(c - v)\Delta t'$ behind the pulse emitted at B.

Segmented arc approximation



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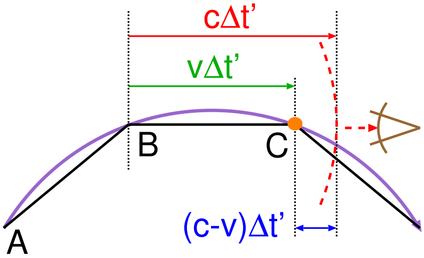
Segmented arc approximation



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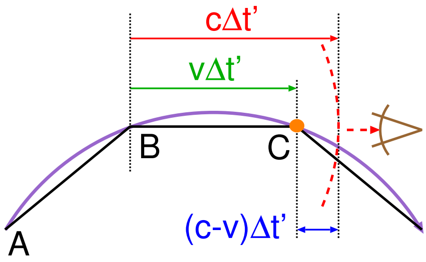
$$\Delta t = \frac{(c - v)\Delta t'}{c}$$

Segmented arc approximation



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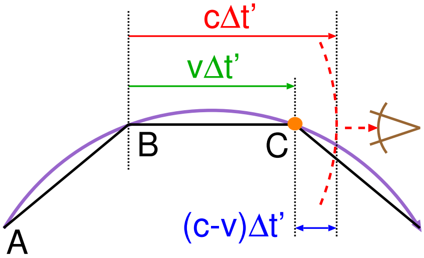
Segmented arc approximation



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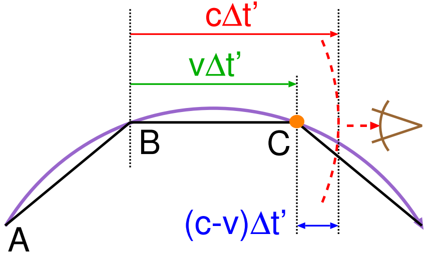
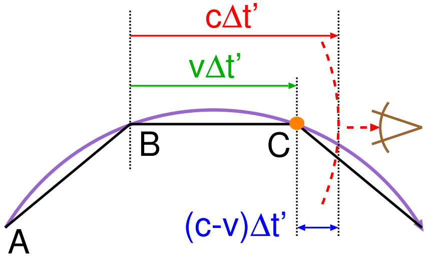
Doppler compression



$$\Delta t = (1 - \beta)\Delta t'$$

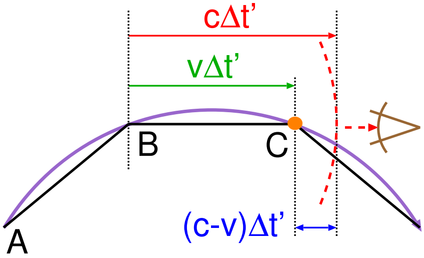
Doppler compression



$$\Delta t = (1 - \beta)\Delta t'$$

Since $0 < \beta < 1$ this translates to a Doppler compression of the emitted wavelength.

Doppler compression



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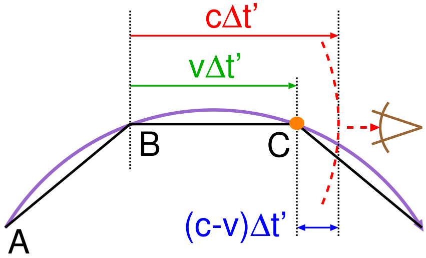
Recall that

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \ ,$$

but for synchrotron radiation, $\gamma > 1000$,

Doppler compression



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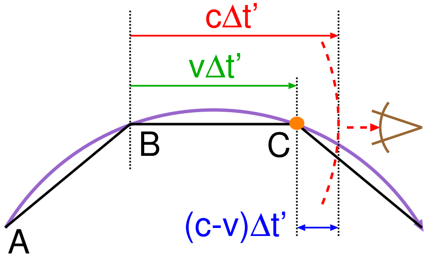

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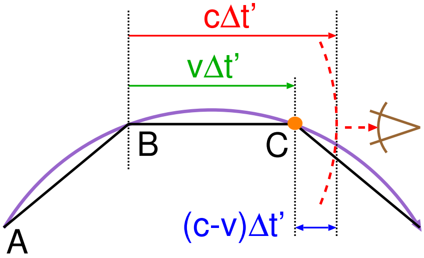
$$\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

Doppler compression



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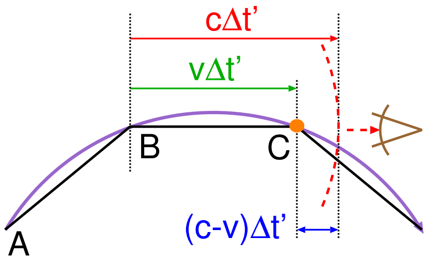
$$\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} = 1 - \frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}\frac{1}{2}\frac{1}{2!}\frac{1}{\gamma^4} + \cdots$$

Doppler compression



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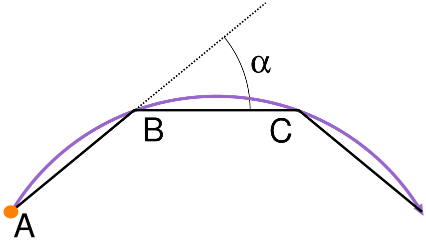
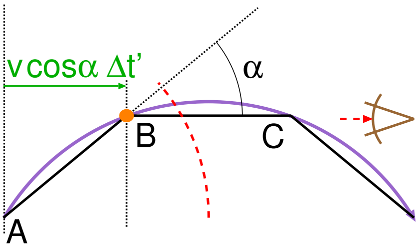
Off-axis emission



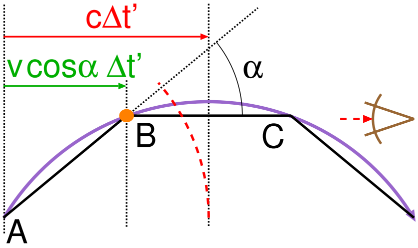
Consider the emission from segment AB, which is not along the line toward the observer.

Off-axis emission



Consider the emission from segment AB, which is not along the line toward the observer. While on the AB segment, the electron moves only a distance $v \cos\alpha \Delta t'$ in the direction of the BC segment.

Off-axis emission



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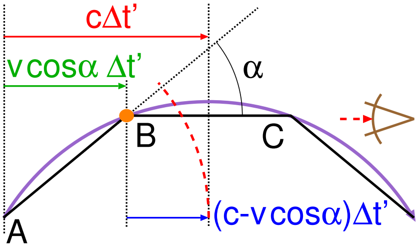
Off-axis emission



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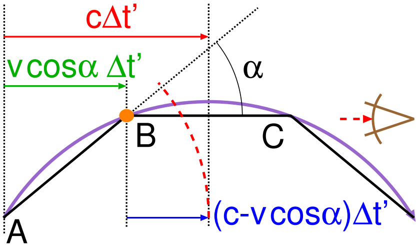

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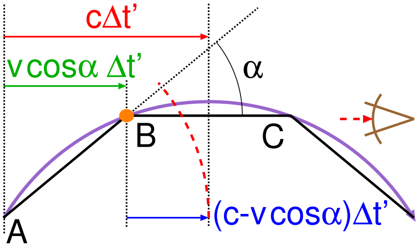

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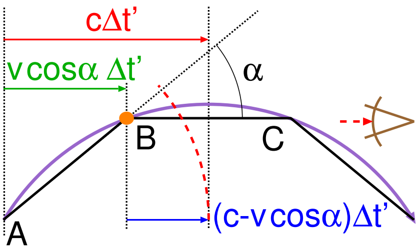

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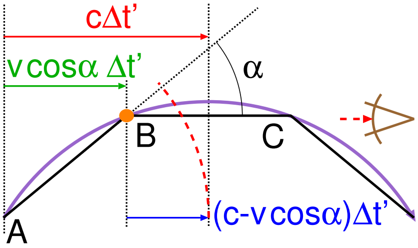

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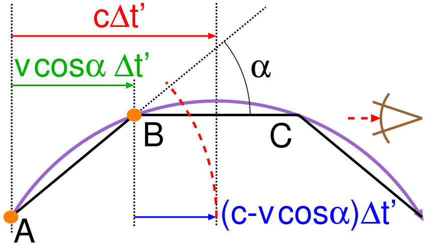
Corrected Doppler shift



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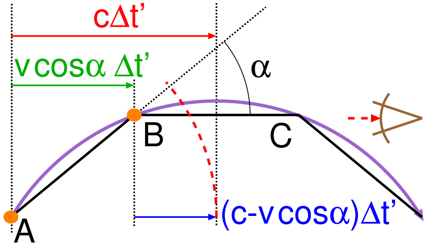

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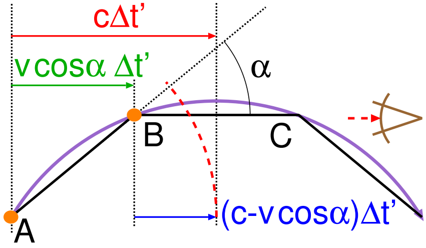
Since α is very small:

Corrected Doppler shift



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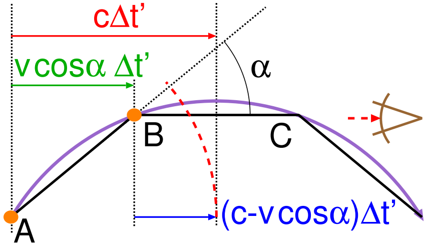
$$\cos \alpha \approx 1 - \frac{\alpha^2}{2}$$

Corrected Doppler shift



$$\Delta t = (1 - \beta \cos\alpha)\Delta t'$$

Since α is very small:

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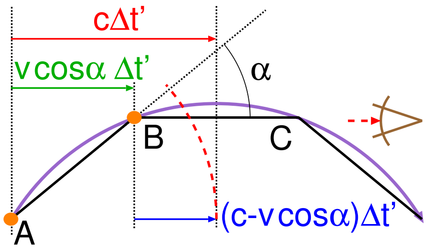
and γ is very large, we have

$$\frac{\Delta t}{\Delta t'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{\alpha^2}{2}\right)$$

Corrected Doppler shift



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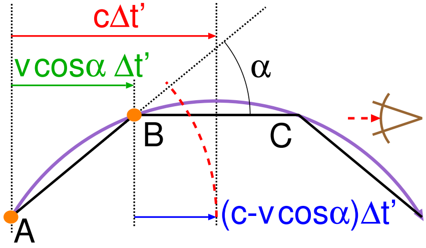
Corrected Doppler shift



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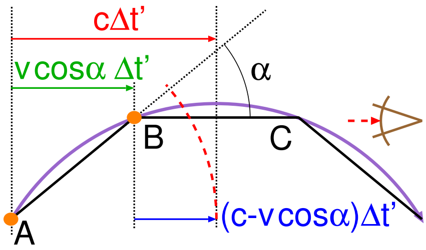
$$\frac{\Delta t}{\Delta t'} \approx \frac{\alpha^2}{2} + \frac{1}{2\gamma^2}$$

Corrected Doppler shift



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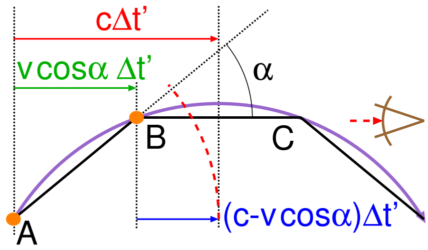
Corrected Doppler shift



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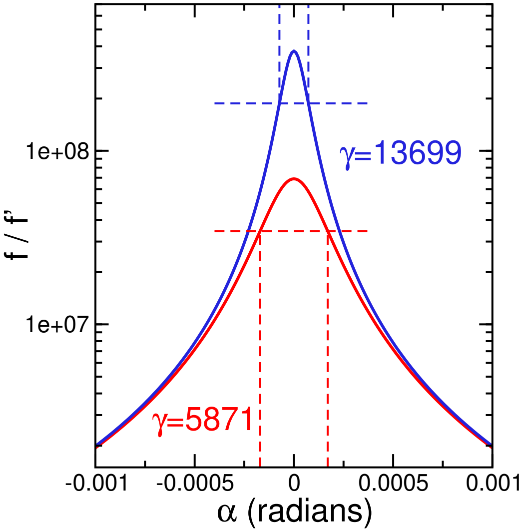
$$\frac{\Delta t}{\Delta t'} \approx \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1 + \alpha^2\gamma^2}{2\gamma^2}$$

called the time compression ratio.

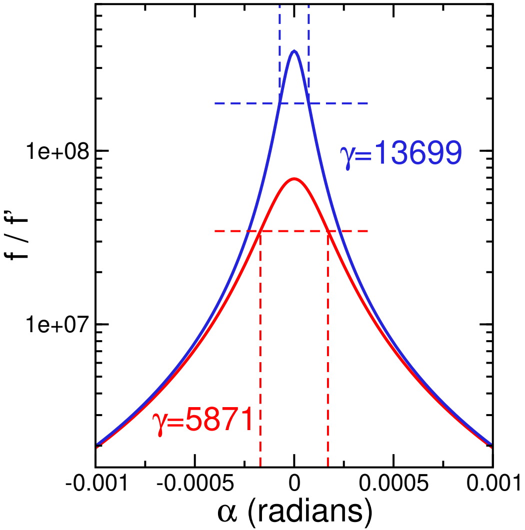
Radiation opening angle



The Doppler shift is defined in terms of the time compression ratio

$$\frac{f}{f'} = \frac{\Delta t'}{\Delta t} = \frac{2\gamma^2}{1+\alpha^2\gamma^2}$$

Radiation opening angle



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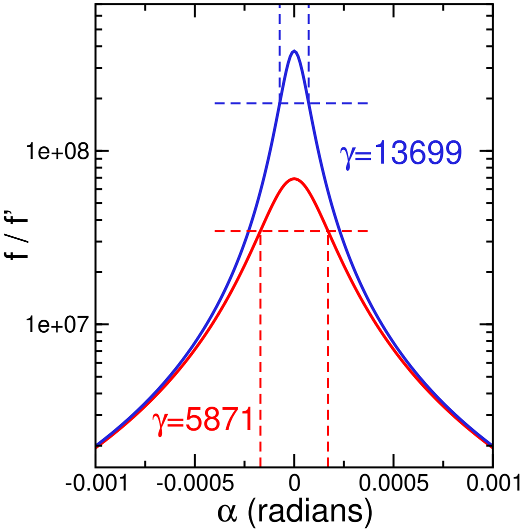
- For APS and NSLS II the Doppler blue shift is between 10^7 and 10^9

Radiation opening angle



The Doppler shift is defined in terms of the time compression ratio

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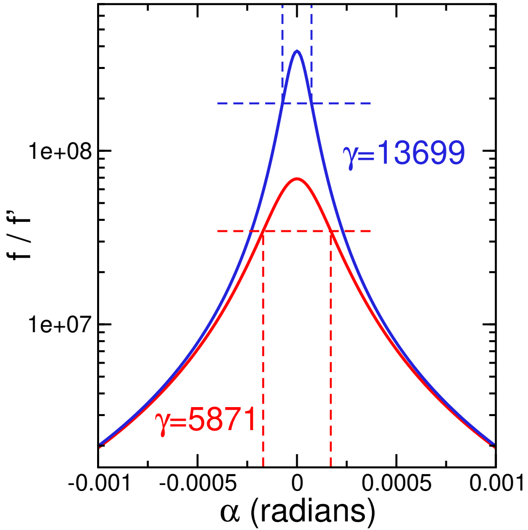

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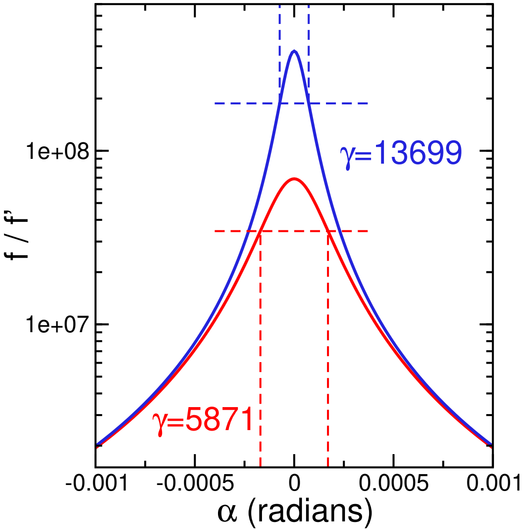

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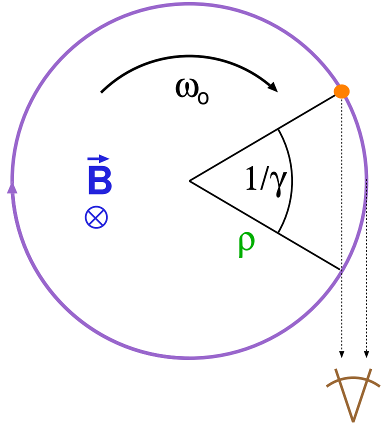

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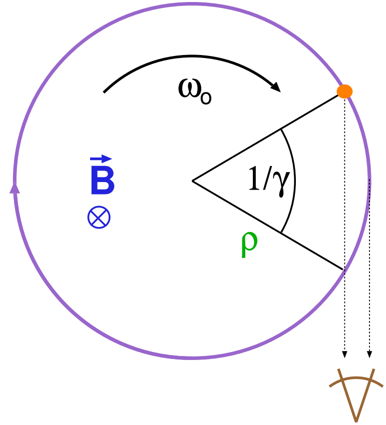

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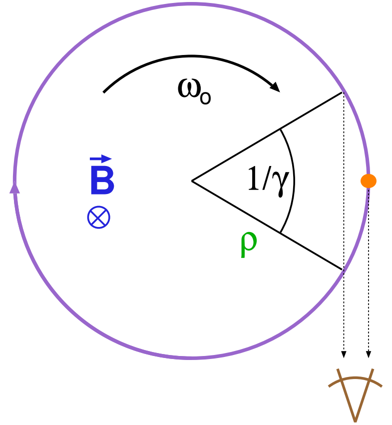

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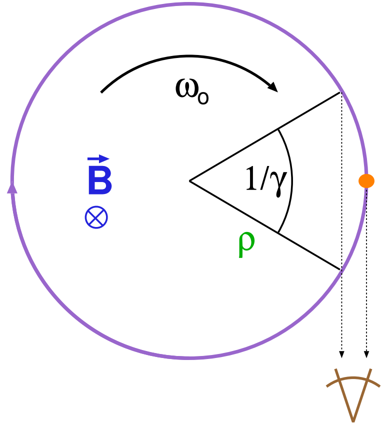

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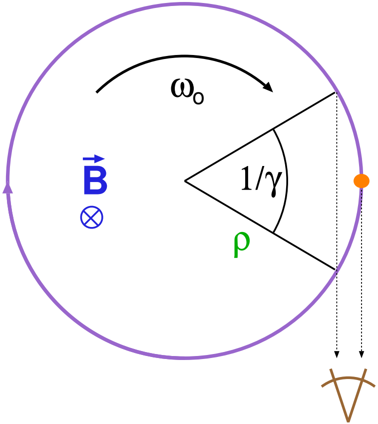

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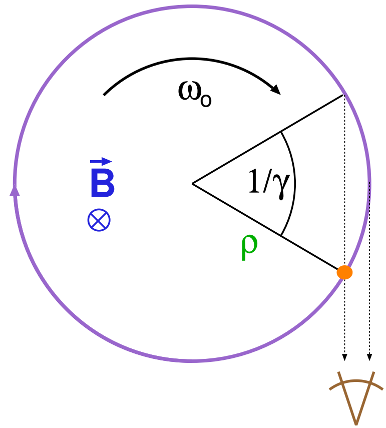

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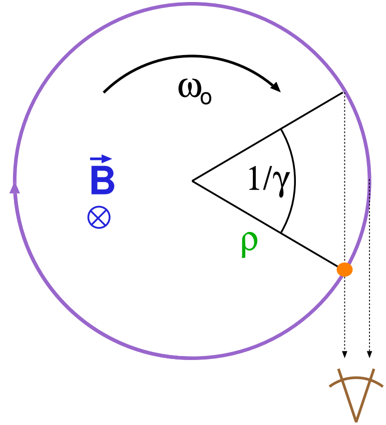

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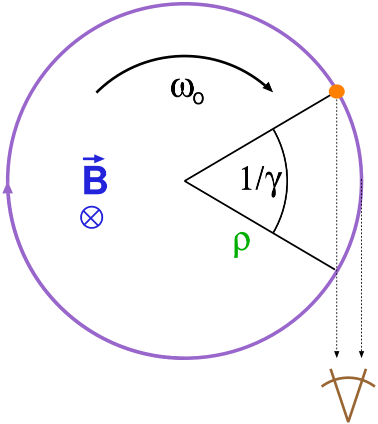

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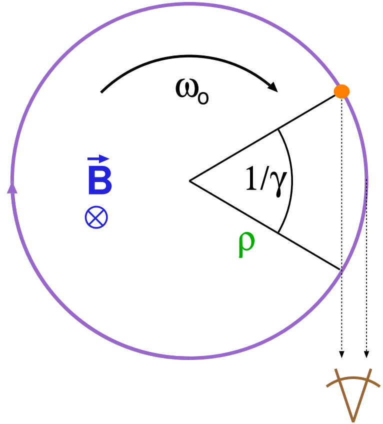

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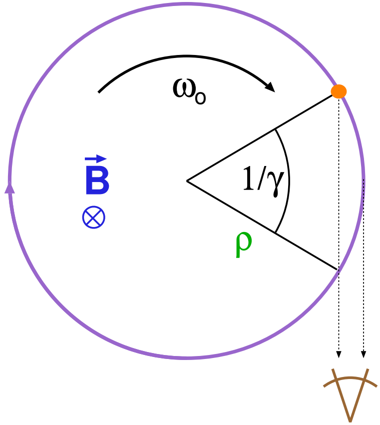

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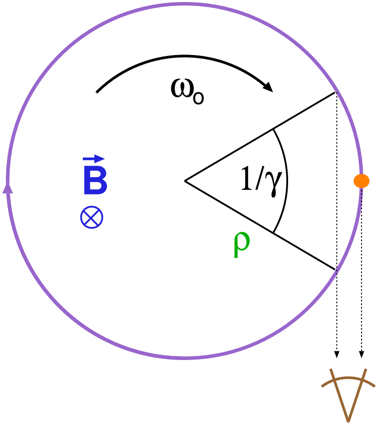

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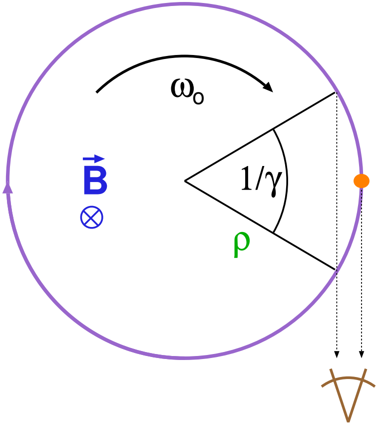

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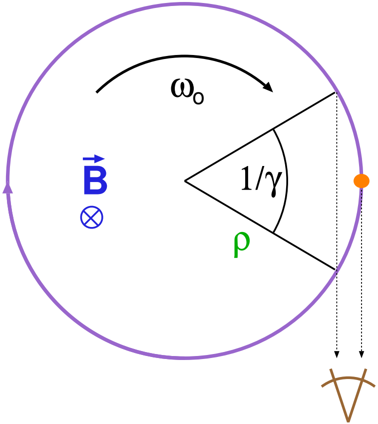

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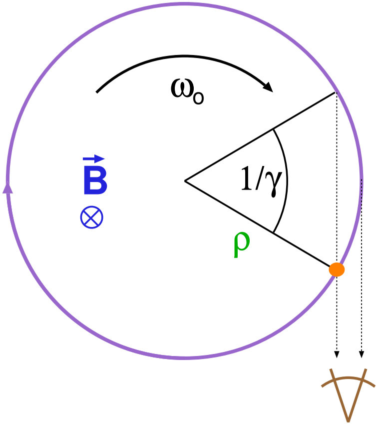

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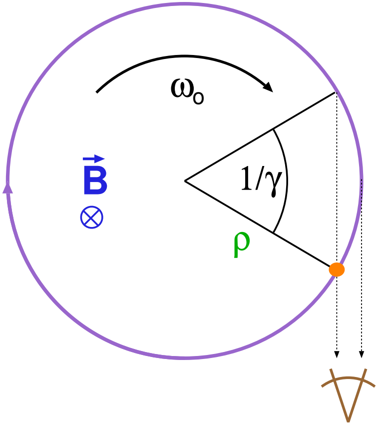

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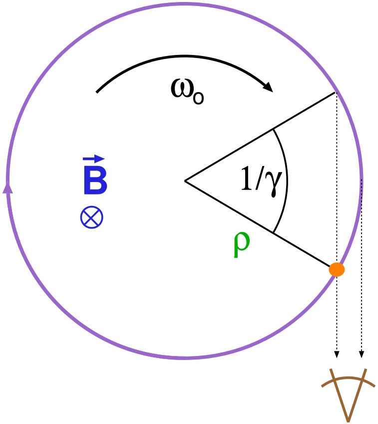

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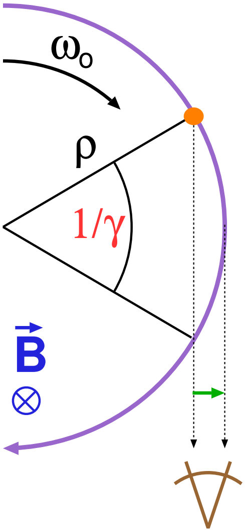

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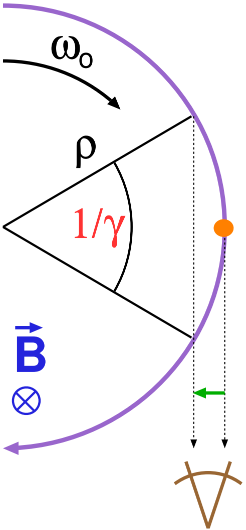

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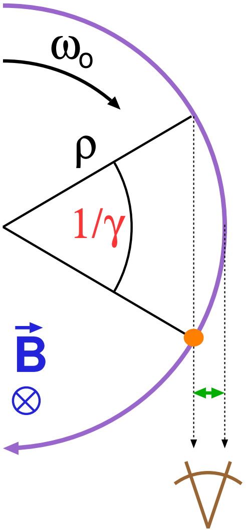

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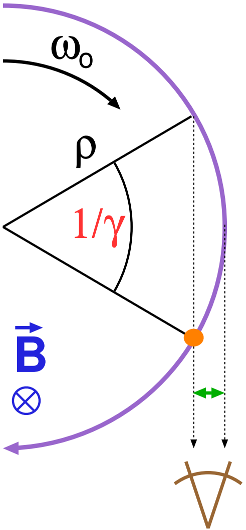

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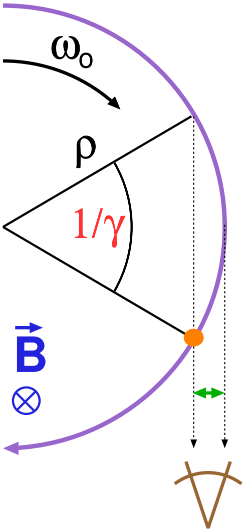

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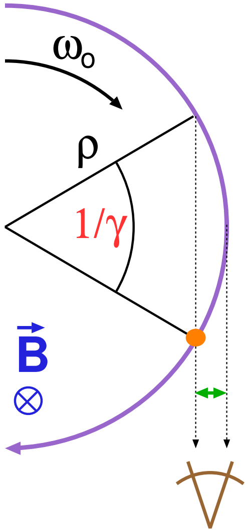

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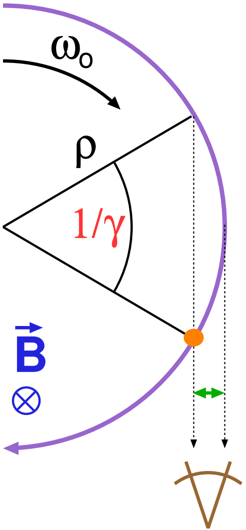

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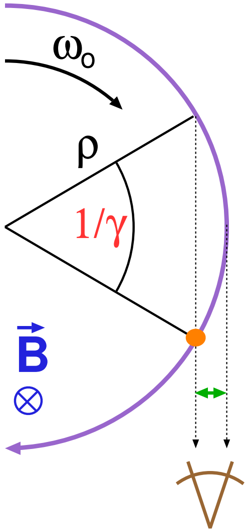

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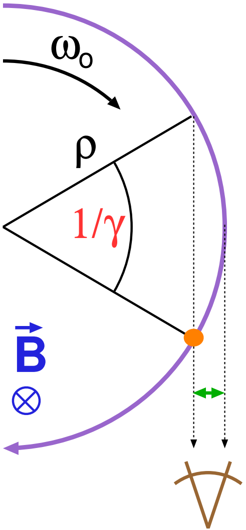

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The Fourier transform of this pulse is the spectrum of the radiation from the bending magnet.

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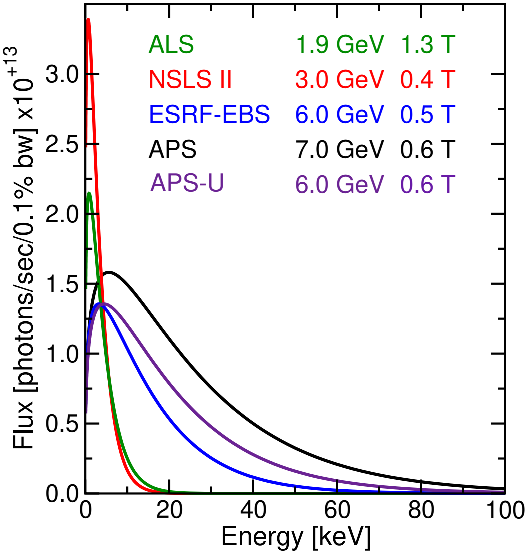
Bending magnet spectrum

When the radiation pulse time is Fourier transformed, we obtain the spectrum of a bending magnet.



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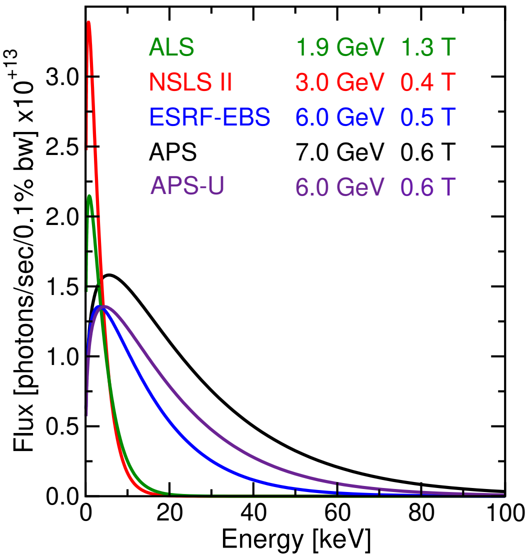
Scaling by the characteristic energy, gives a universal curve



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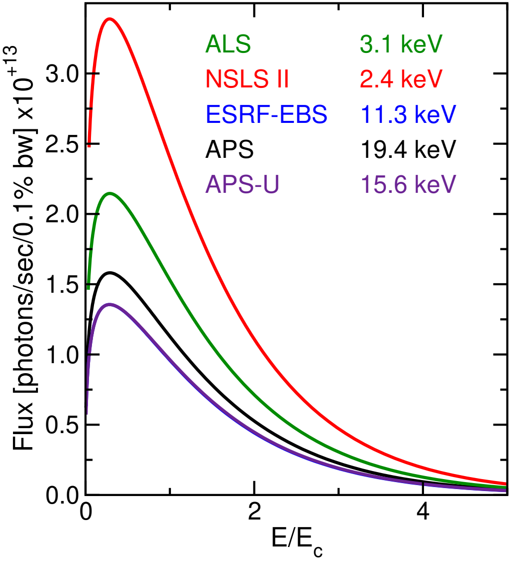

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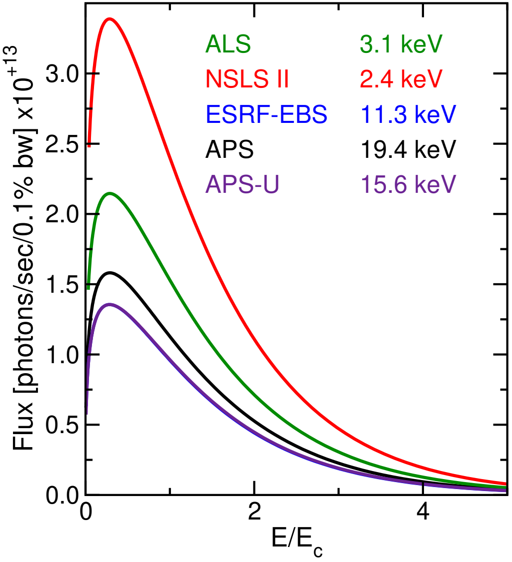
$$1.33 \times 10^{13} \mathcal{E}^2 I \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2 \left(\frac{\omega}{2\omega_c}\right)$$

where $K_{2/3}$ is a modified Bessel function of the second kind.



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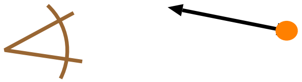
$$P = 1.266(6\text{GeV})^2(0.8\text{T})^2(1.24 \times 10^{-3}\text{m})(0.2\text{A}) = 7.3\text{W}$$

Polarization

A bending magnet also produces circularly polarized radiation



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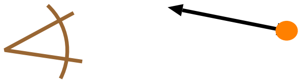

- If the observer is in the plane of the electron orbit, the electron motion looks like a half period of linear sinusoidal motion

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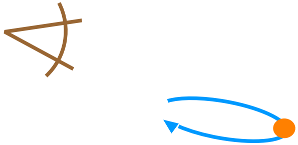

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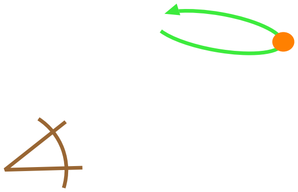

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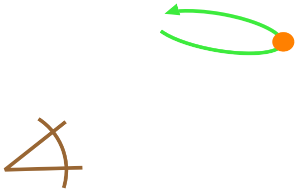

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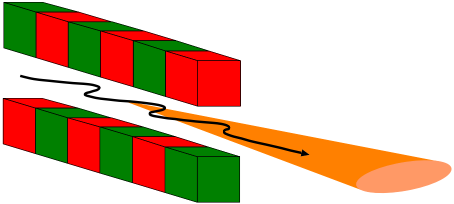
The result is circularly polarized radiation above and below the on-axis radiation.

Wigglers and undulators

Wiggler



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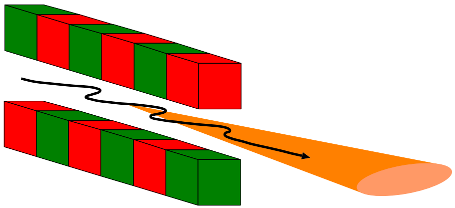

Like bending magnet except:

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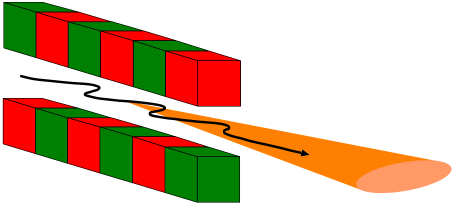

Like bending magnet except:

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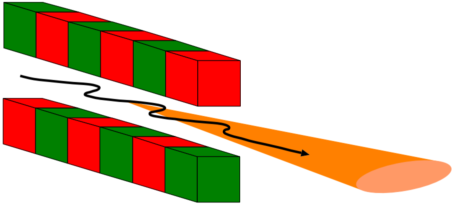

Like bending magnet except:

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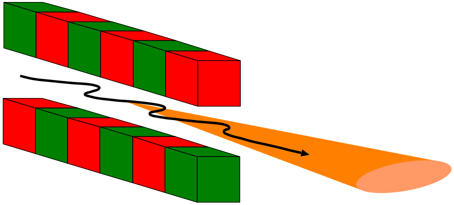

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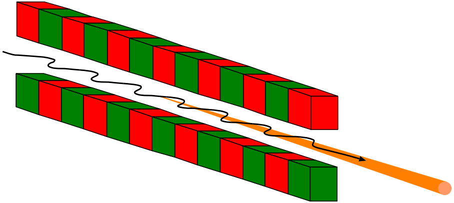

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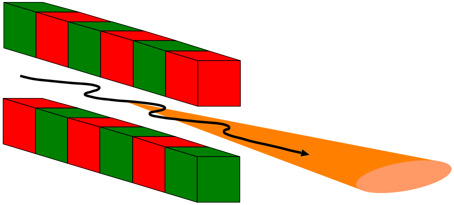

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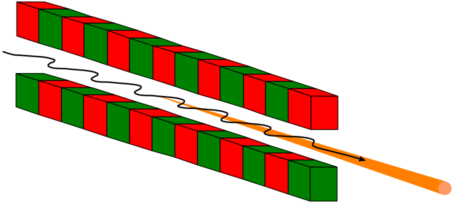

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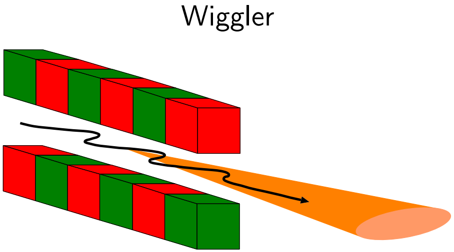
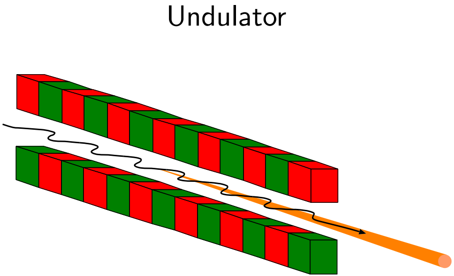
Different from bending magnet:

Wigglers and undulators





Like bending magnet except:

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Different from bending magnet:

- shallow bends $\rightarrow$ smaller source

Wigglers and undulators

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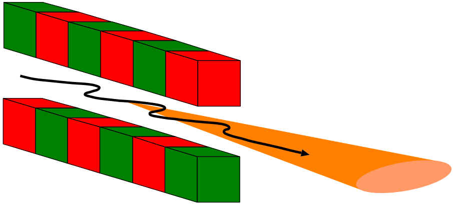

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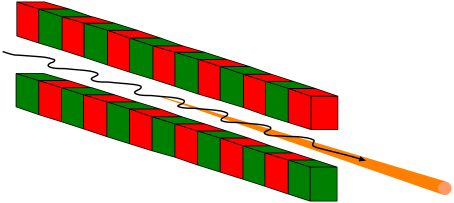

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Different from bending magnet:

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- interference $\rightarrow$ peaked spectrum

Wiggler radiation

- The electron's trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet

$$Power[\text{kW}] = 1.266\mathcal{E}_e^2[\text{GeV}]B^2[\text{T}]L[\text{m}]I[\text{A}]$$

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- The magnetic field varies along the length of the wiggler and is higher than that in a bending magnet, having an average value of $B_{rms} = B_0/\sqrt{2}$
- This results in a significantly higher power load on all downstream components

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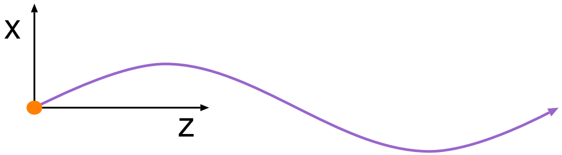
Undulator characterization



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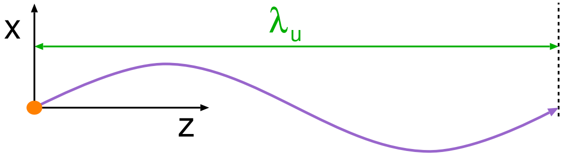

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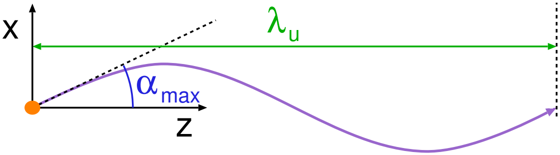

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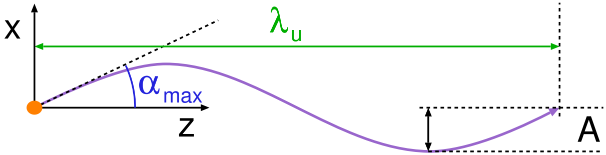

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From the electron trajectory:

$$x = A\sin(k_u z)$$

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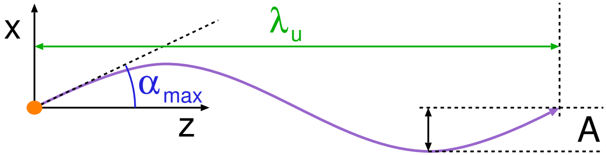

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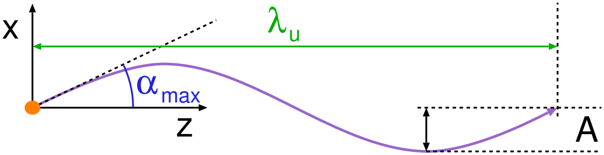

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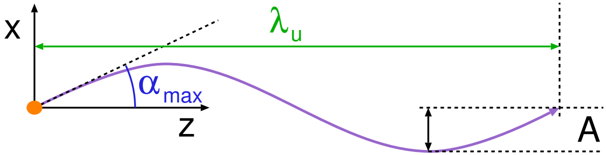

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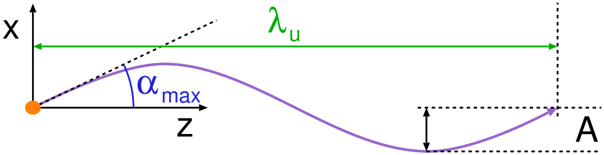
Undulator characterization



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- The energy of the electrons, γmc^2
- The wavelength, $\lambda_u = 2\pi/k_u$, of its magnetic field
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From the electron trajectory:

$$x = A\sin(k_u z)$$

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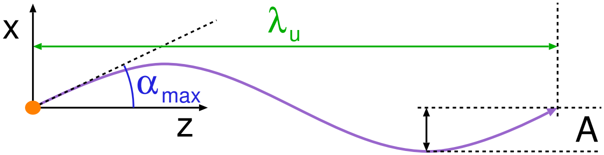
Undulator characterization



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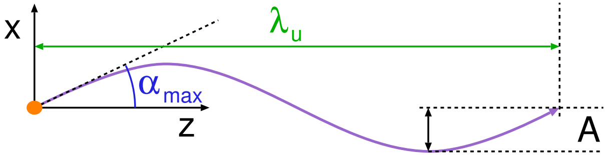

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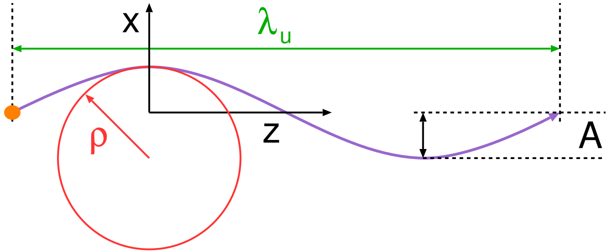
Circular path approximation



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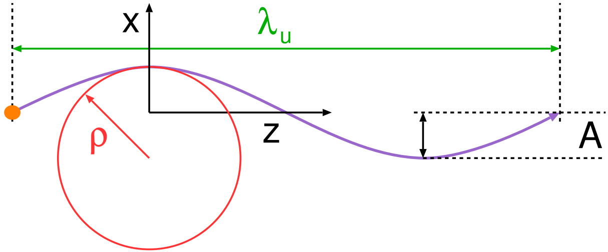
Circular path approximation



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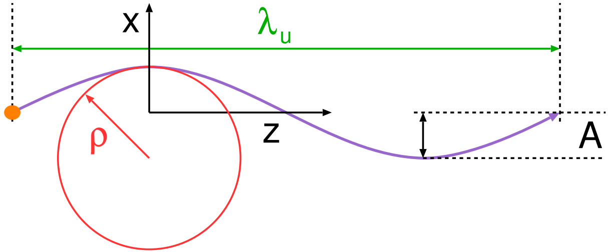

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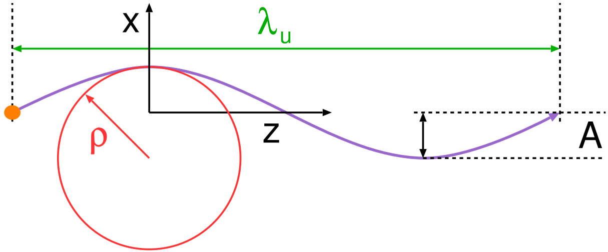

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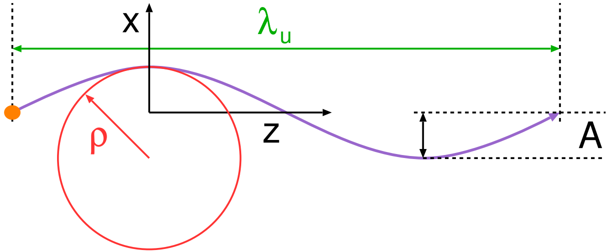

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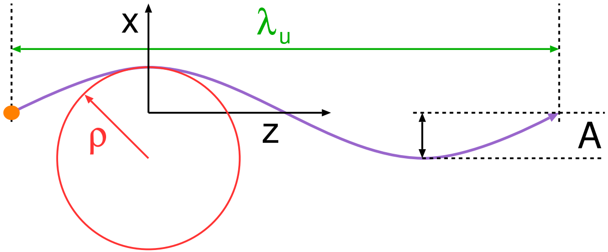
Radius of curvature



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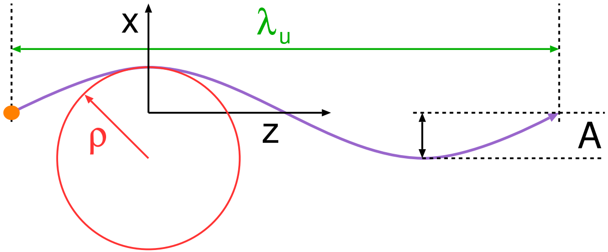

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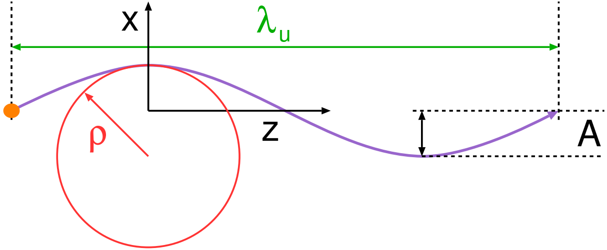

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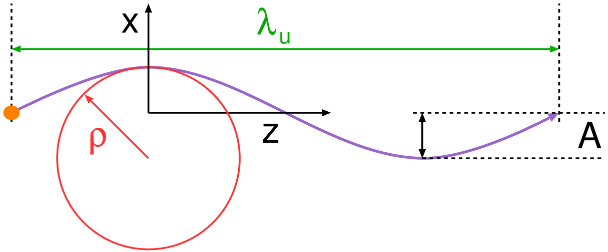

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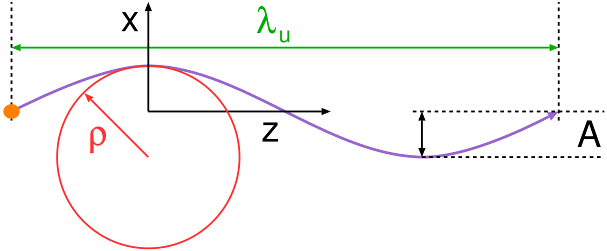

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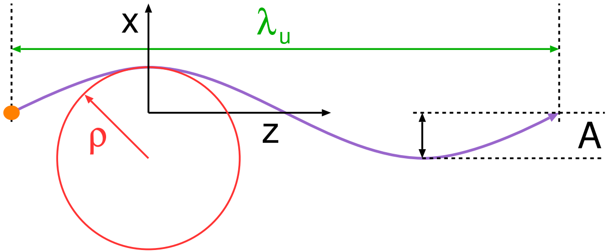

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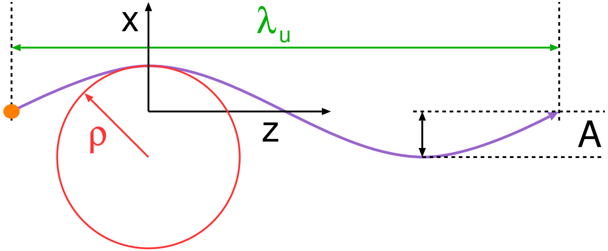

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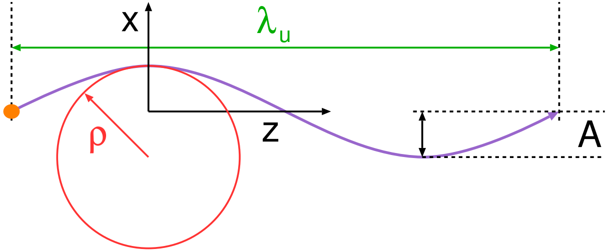

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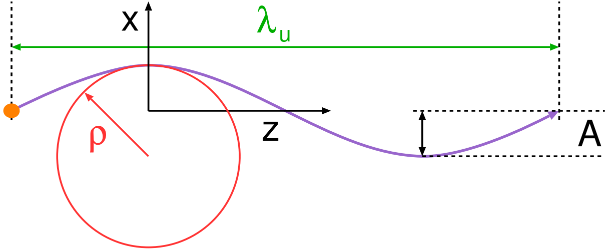

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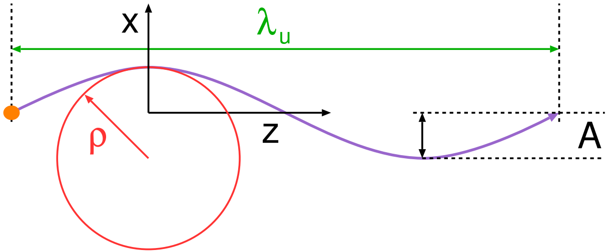

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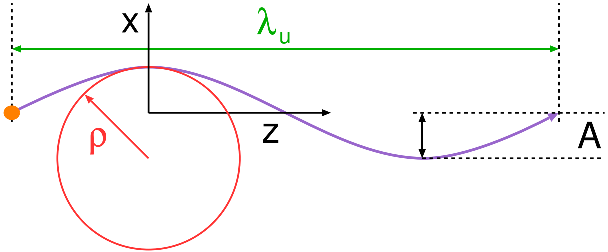

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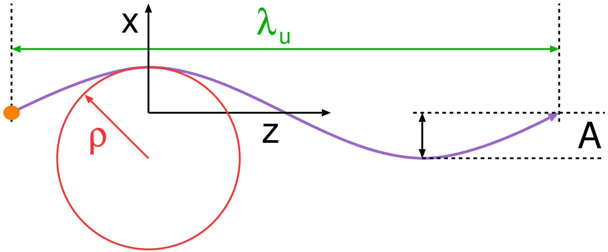

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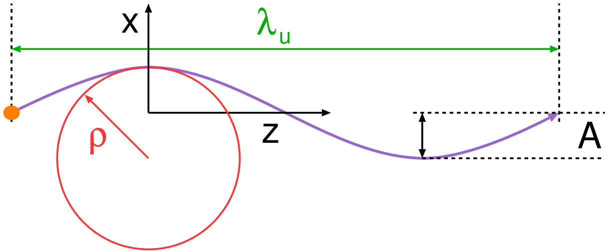

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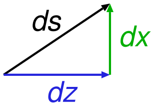
Electron path length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



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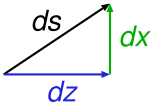

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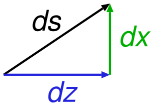

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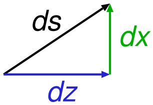

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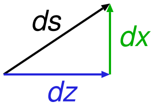

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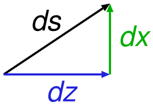

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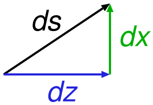

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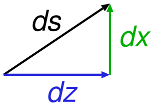

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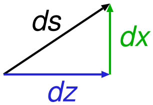

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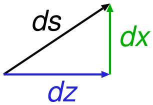

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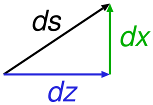

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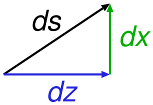

$$ds = \sqrt{(dx)^2 + (dz)^2} = \sqrt{1 + \left(\frac{dx}{dz}\right)^2} dz$$

$$\frac{dx}{dz} = \frac{d}{dz} A \cos k_u z = -A k_u \sin k_u z$$

Now calculate the length of the path traveled by the electron over one period of the undulator

$$S\lambda_u = \int_0^{\lambda_u} \sqrt{1 + \left(\frac{dx}{dz}\right)^2}\, dz \approx \int_0^{\lambda_u} \left[1 + \frac{1}{2}\left(\frac{dx}{dz}\right)^2\right] dz = \int_0^{\lambda_u} \left[1 + \frac{A^2 k_u^2}{2} \sin^2 k_u z\right] dz$$

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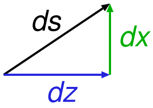

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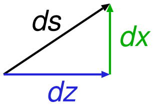

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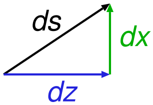

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$$= \lambda_u\left(1 + \frac{A^2k_u^2}{4}\right) = \lambda_u\left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right) \qquad K = \gamma A k_u$$

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Combining the above expressions yields

$$K = \frac{e B_0}{m c k_u} = \frac{e}{2\pi m c} \lambda_u B_0 = 0.934 \lambda_u[\text{cm}] B_0[\text{T}]$$

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For APS Undulator A, $\lambda_u = 3.3\text{cm}$ and $B_0 = 0.6\text{T}$ at closed gap, so

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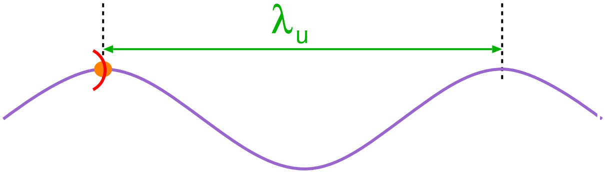
Undulator wavelength

Consider an electron traveling through the undulator and emitting radiation at the first maximum excursion from the center.



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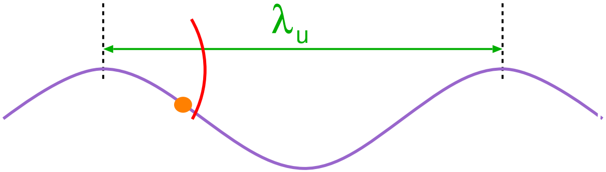

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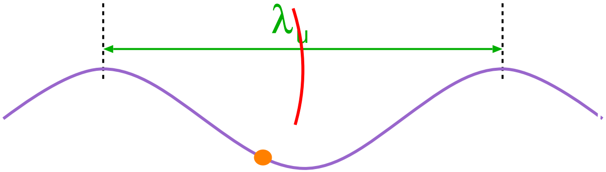

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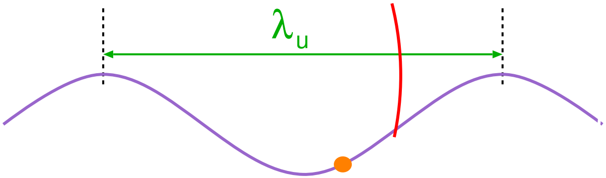

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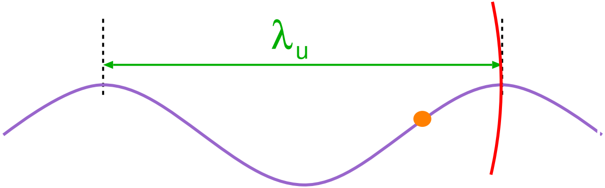

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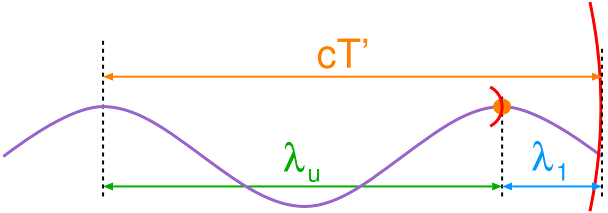

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moving cT' in the time the electron travels a distance λ_u along the undulator

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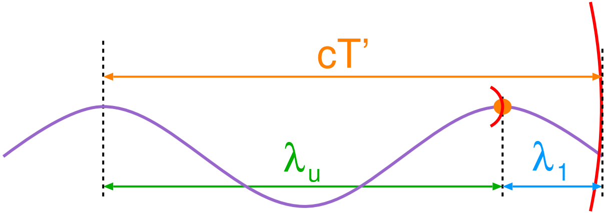

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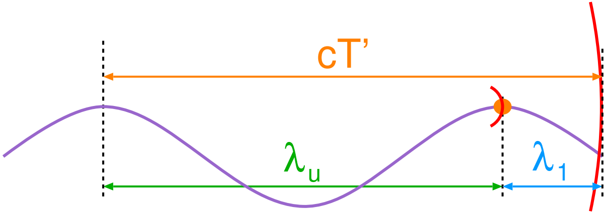

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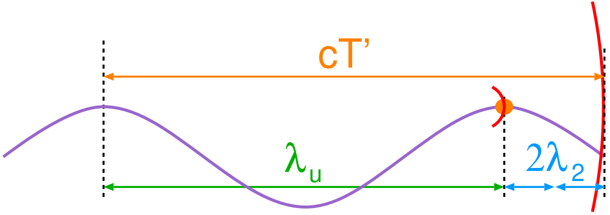

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The observer sees radiation with a compressed wavelength, along with harmonics which satisfy the same condition.

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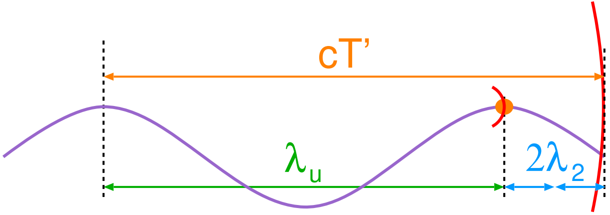

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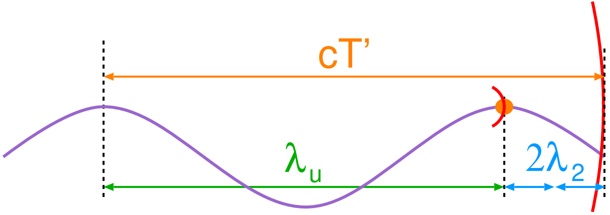

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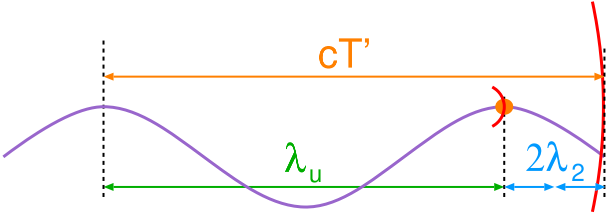

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$$\lambda_1 = cT' - \lambda_u \cos\theta$$

The fundamental wavelength

The fundamental wavelength emitted from the undulator depends on the photon propagation time, T'

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In a time T' the electron travels a distance $S\lambda_u$, so $T' = S\lambda_u/v$

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In a time T' the electron travels a distance $S\lambda_u$, so $T' = S\lambda_u/v$ and we know that

$$S \approx 1 + \frac{K^2}{4\gamma^2}$$

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$$\begin{aligned}
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&= \lambda_u \left(S\frac{c}{v} - \cos\theta \right) = \lambda_u \left(\frac{S}{\beta} - \cos\theta \right) \\
&= \lambda_u \left(\left[1 + \frac{K^2}{4\gamma^2} \right] \frac{1}{\beta} - \cos\theta \right)
\end{aligned}$$

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Since γ is large, the maximum observation angle θ is small so

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$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right)$$

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Regrouping and substituting . . .

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Regrouping and substituting ...

The fundamental wavelength

The fundamental wavelength emitted from the undulator depends on the photon propagation time, T'

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If we assume that $\beta \sim 1$ for these highly relativistic electrons

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