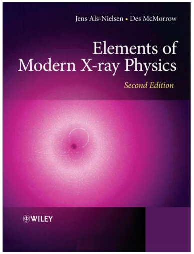
PHYS 570 - Introduction to Synchrotron Radiation



Term:	Fall 2021
Meetings:	Tuesday & Thursday 17:10-18:25
Location:	121 Pritzker Science
Instructor:	Carlo Segre
Office:	166d/172 Pritzker Science
Phone:	312.567.3498
email:	segre@iit.edu
Book:	*Elements of Modern X-Ray Physics, 2nd ed.*, J. Als-Nielsen and D. McMorrow (Wiley, 2011)
Web Site:	http://csrri.iit.edu/∼segre/phys570/21F

Course objectives

- Describe the means of production of synchrotron x-ray radiation

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- Describe the function of various components of a synchrotron beamline
- Perform calculations in support of a synchrotron experiment
- Describe the physics behind a variety of experimental techniques
- Prepare and deliver an oral presentation of a synchrotron radiation research topic
- Write a General User Proposal in the format used by the Advanced Photon Source

Course syllabus

- Focus on applications of synchrotron radiation

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  - Choose a research article which features a synchrotron technique
  - Timetable will be posted
- Final project - writing a General User Proposal
  - Start thinking about a suitable project right away
  - Synchrotron technique must differ from journal article used in final presentation
  - Make proposal and get approval before starting

Optional activities

- Visits to Advanced Photon Source
  - Because of COVID-19 the APS is on Limited Operations status
  - Only 2 people per experiment can be on site
  - Over the course of the semester, we will hold virtual beamline sessions at 10-BM for any who wish to attend

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- Hands on data analysis training
  - GSAS for Rietveld refinement of powder diffraction data https://subversion.xray.aps.anl.gov/trac/pyGSAS
  - Demeter: XAS processing and analysis https://bruceravel.github.io/demeter/
  - Larch: Data analysis tools for x-ray spectroscopy https://xraypy.github.io/xraylarch/

Course grading

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Weekly or bi-weekly

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Grading scale

A	–	80%	to	100%
B	–	65%	to	80%
C	–	50%	to	65%
E	–	0%	to	50%

Topics to be covered (at a minimum)

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Resources for the course

- Orange x-ray data booklet:
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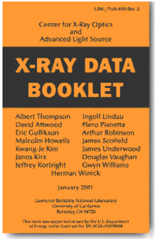

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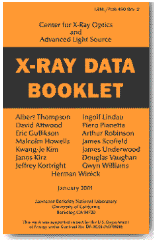

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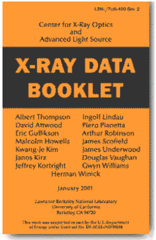

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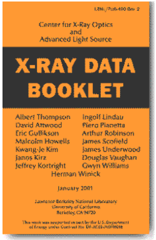

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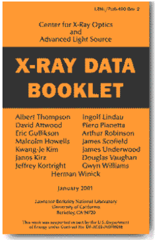
- X-ray Oriented Programs: https://www.aps.anl.gov/Science/Scientific-Software/XOP



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Reading Assignment: Chapter 1.1–1.6; 2.1–2.2

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The broad range of techniques make synchrotron x-ray sources to nearly any science or engineering field

A bit about my research. . .

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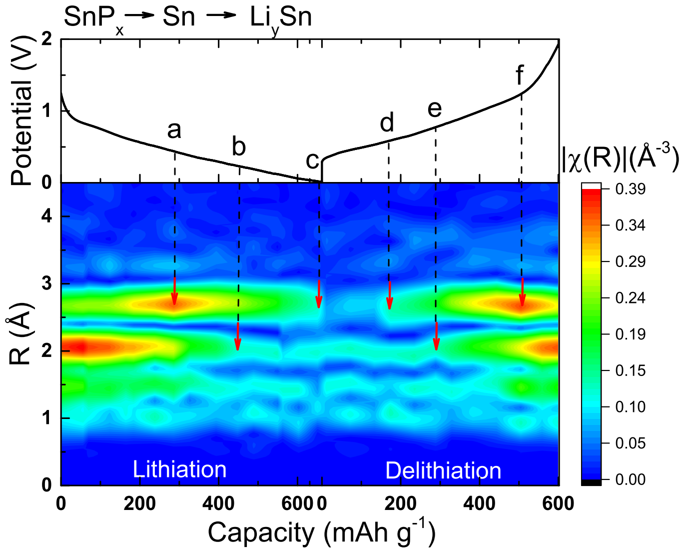

"In situ EXAFS-derived mechanism of highly reversible tin phosphide/graphite composite anode for Li-ion batteries," Y. Ding, Z. Li, E.V. Timofeeva, and C.U. Segre, *Adv. Energy Mater.* 1702134 (2017).

The classical x-ray

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$$\begin{aligned}
\lambda &= hc/\mathcal{E} \\
&= (4.1357 \times 10^{-15}\,\text{eV}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})/\mathcal{E} \\
&= (4.1357 \times 10^{-18}\,\text{keV}\cdot\text{s})(2.9979 \times 10^{18}\,\text{Å/s})/\mathcal{E} \\
&= 12.398\,\text{Å}\cdot\text{keV}/\mathcal{E} \quad \text{to give units of Å}
\end{aligned}$$

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We will only discuss the first three.

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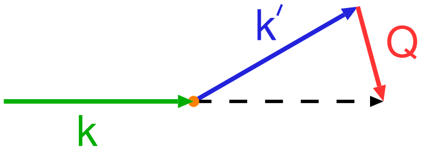

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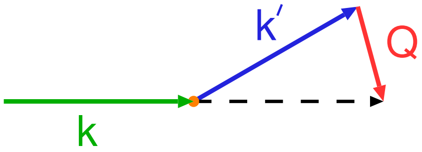

where an incident x-ray of wave number **k**

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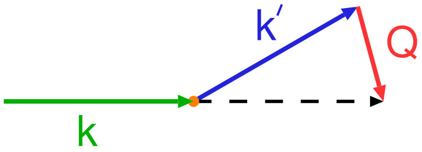

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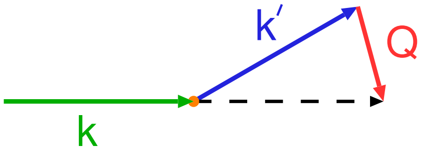

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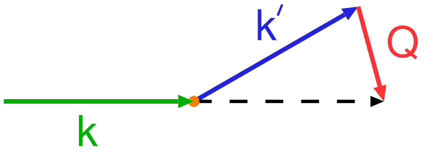

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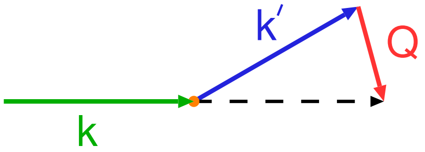

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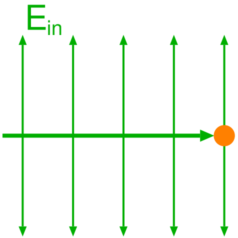
Start with the scattering from a single electron, then build up to more complexity

Thomson scattering

Assumptions:



Thomson scattering

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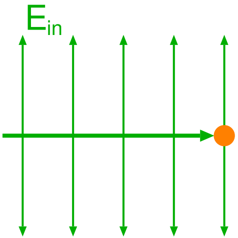

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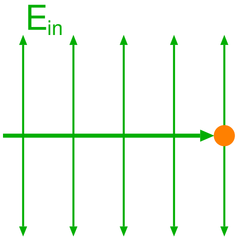

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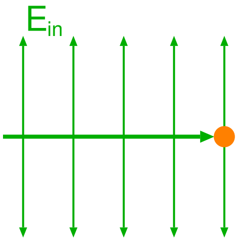

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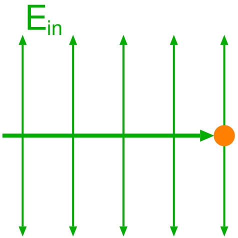

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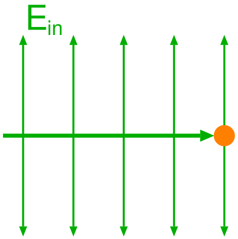

The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

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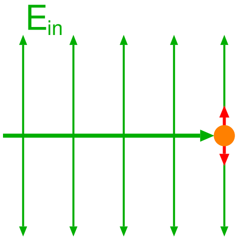

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The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

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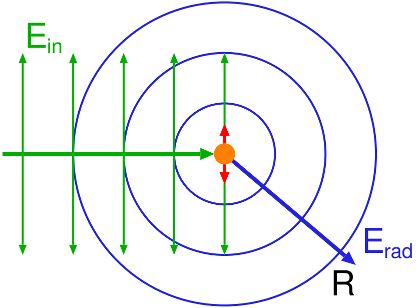

The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

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The observer at R "sees" a scattered electric field $E_{rad}(R, t)$ at a later time $t = t' + R/c$

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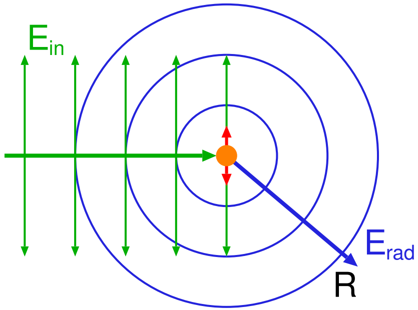

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Using this, calculate the elastic scattering cross-section

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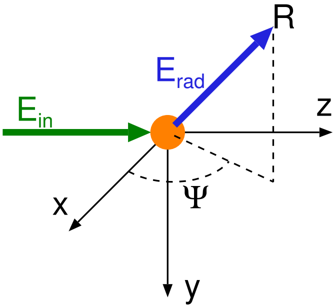

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi,$$

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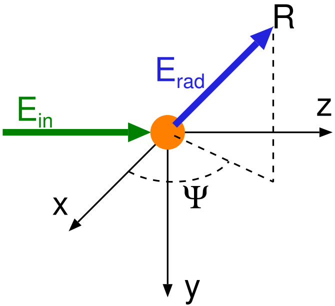

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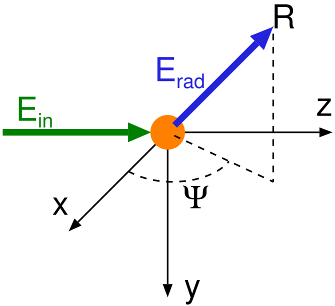

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$$a_x(t') = -\frac{e}{m} E_{x0} e^{-i\omega t'}$$

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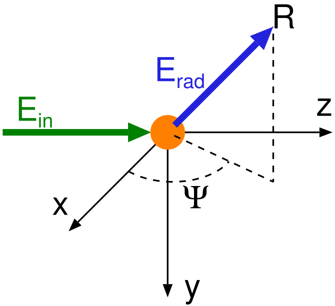

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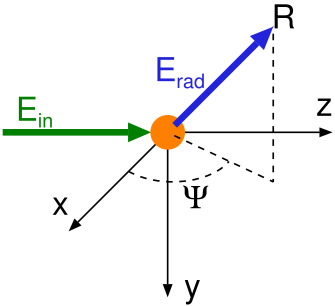

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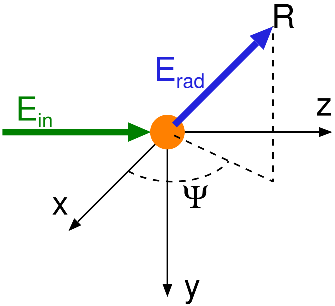

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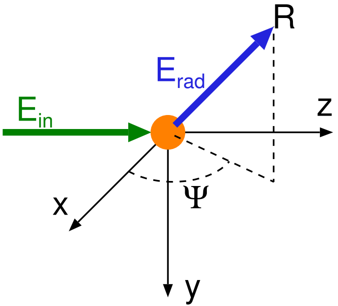

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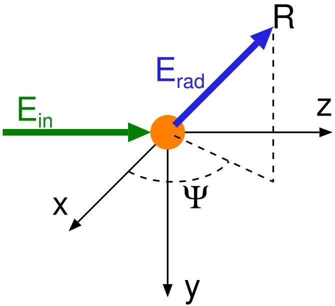

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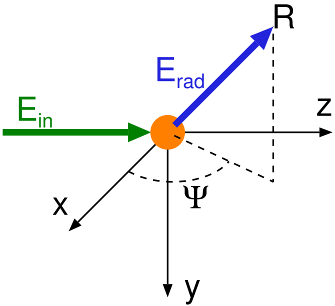

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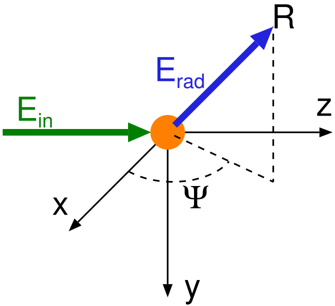

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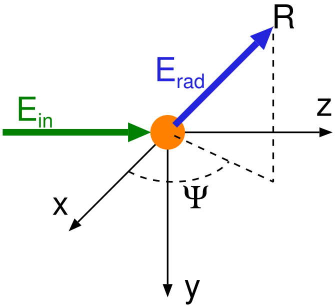

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r_0 is called the Thomson scattering length or the "classical" radius of the electron

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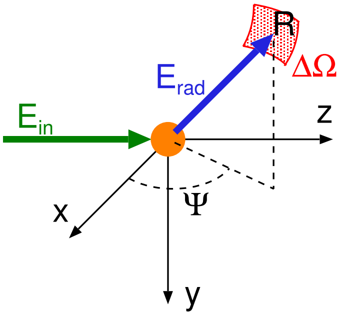

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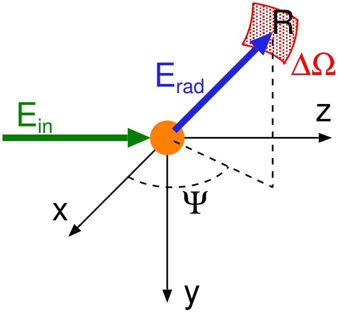

detector of solid angle $\Delta\Omega$ located a distance R from electron

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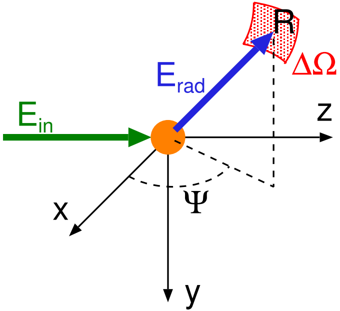

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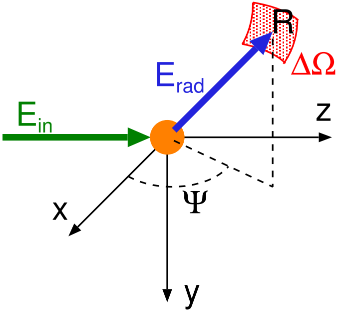

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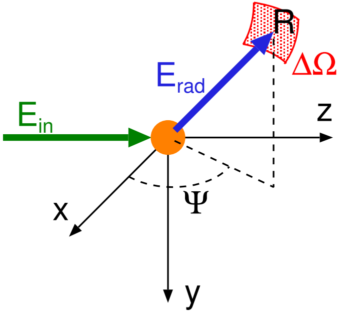

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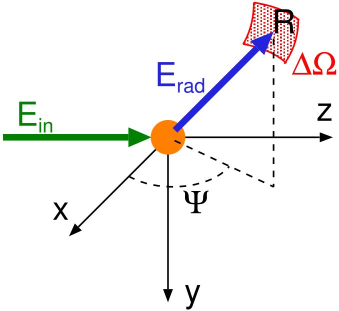

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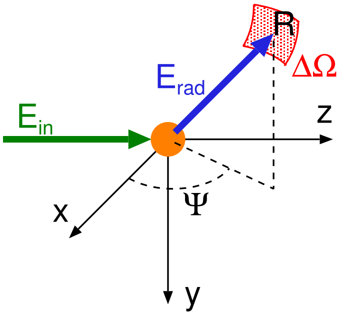

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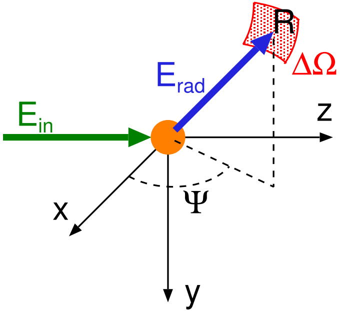

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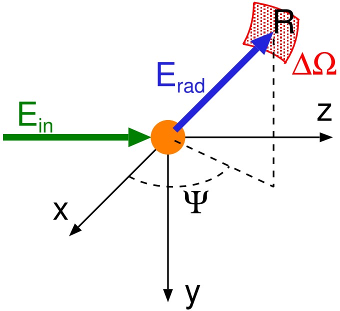

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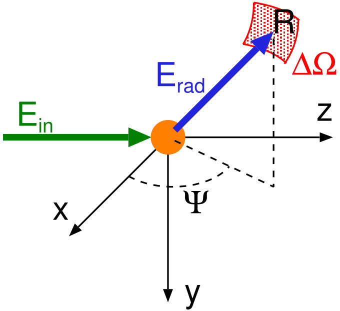

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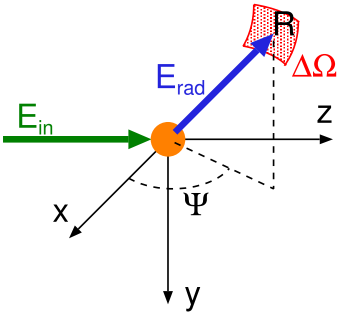

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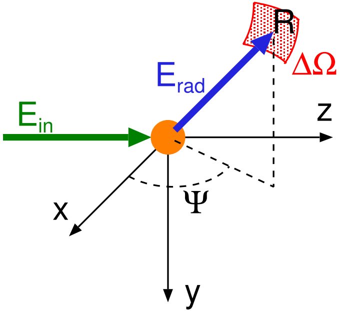

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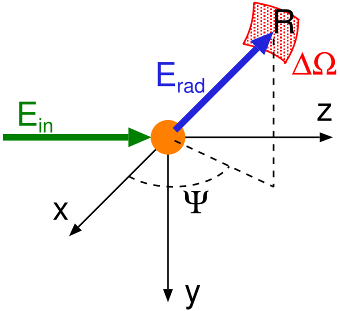

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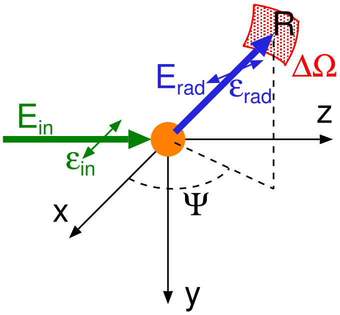

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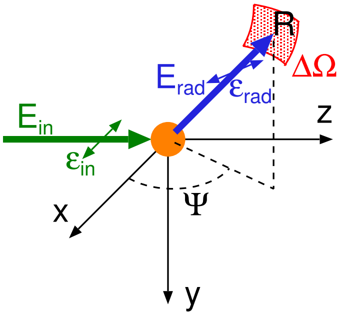

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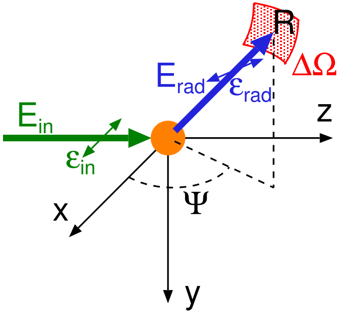

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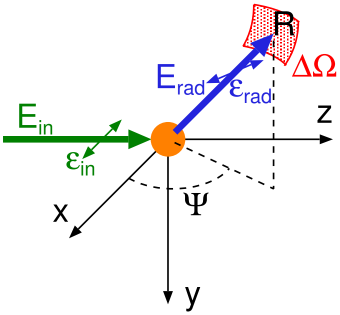

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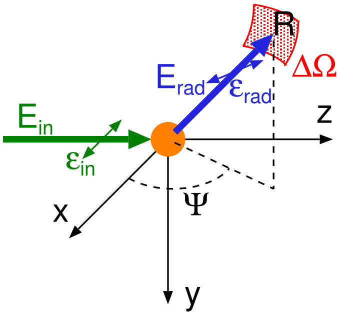

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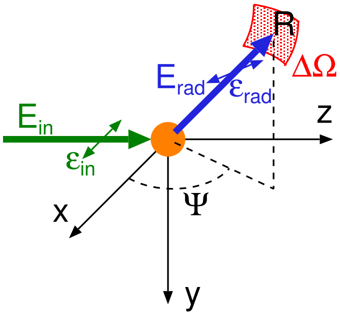

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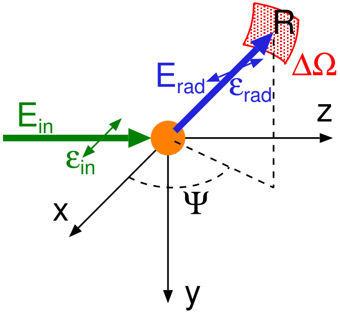

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Integrate to obtain the total Thomson scattering cross-section from an electron.

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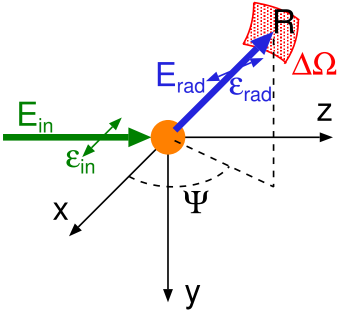

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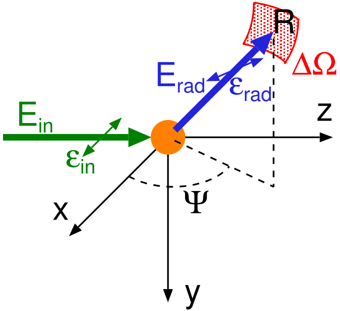

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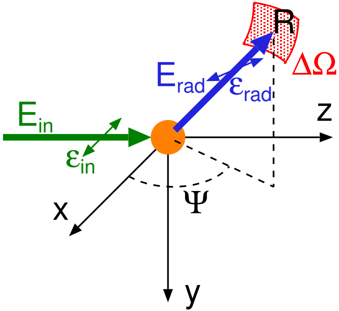

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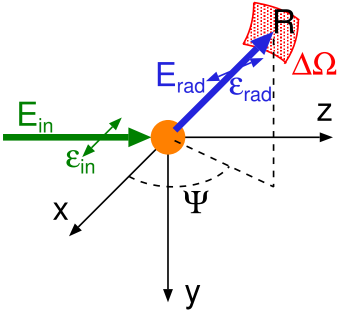

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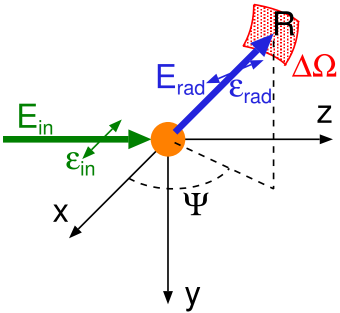

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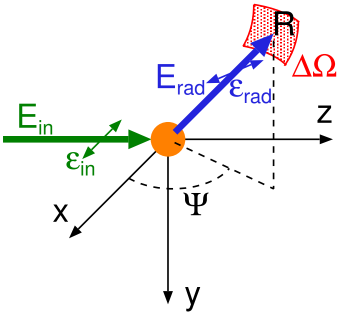

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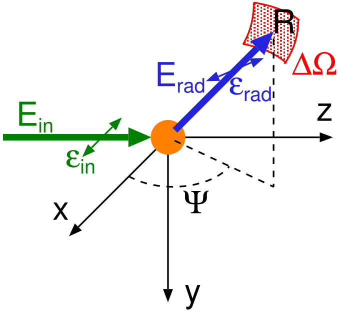

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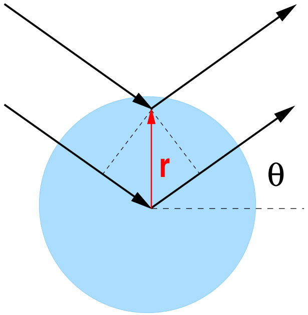
Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex



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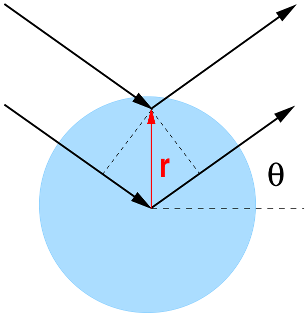

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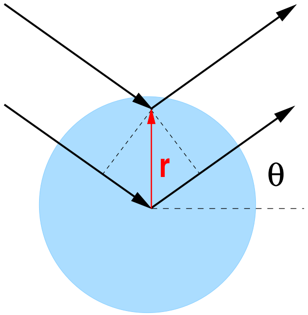

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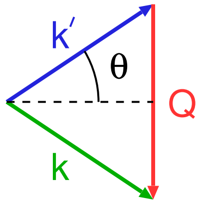

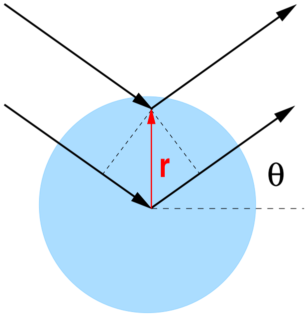

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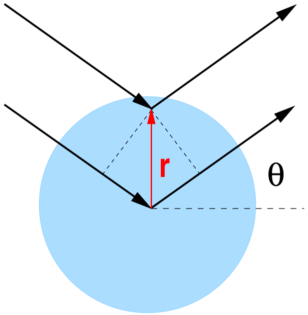

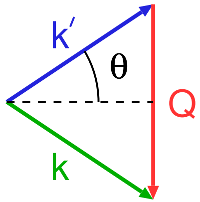

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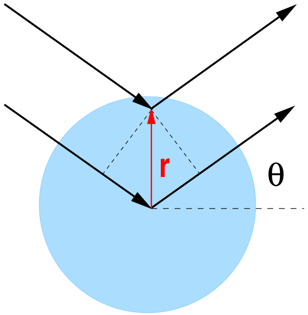

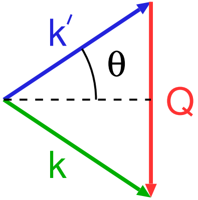

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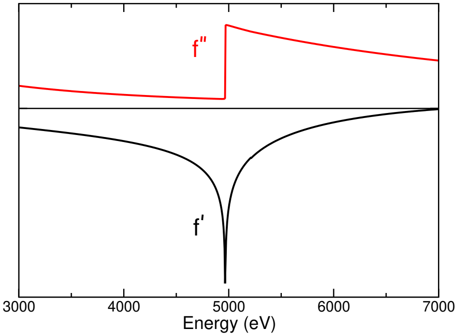

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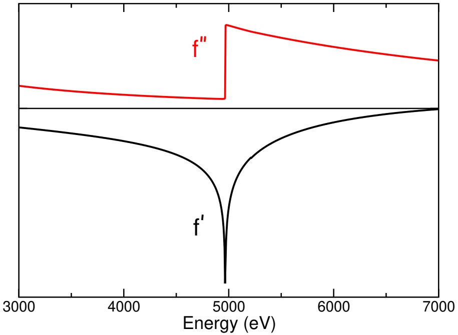

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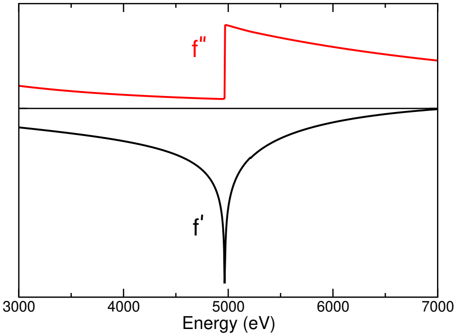

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$$f(\mathbf{Q}, \hbar\omega) = f^0(\mathbf{Q}) + f'(\hbar\omega) + if''(\hbar\omega)$$

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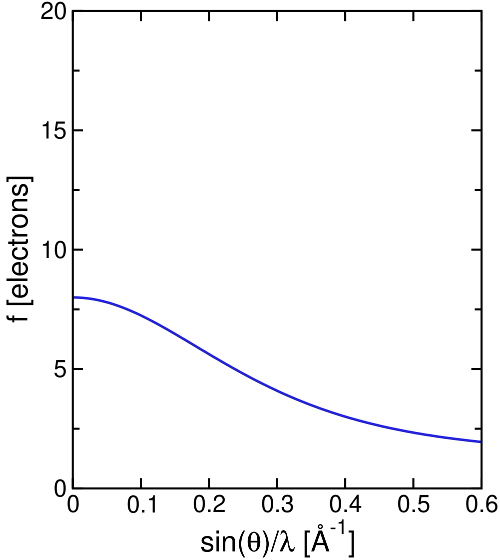

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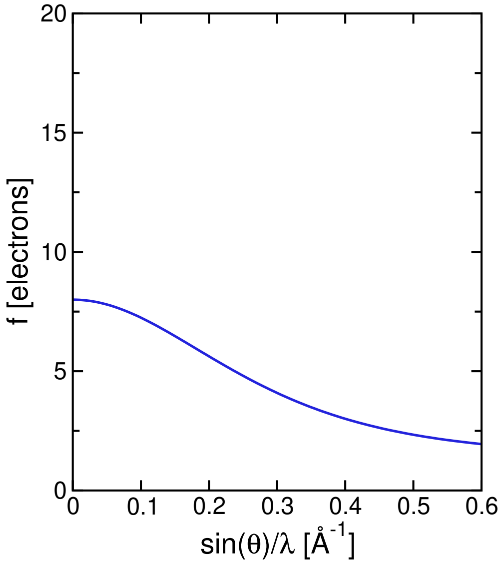

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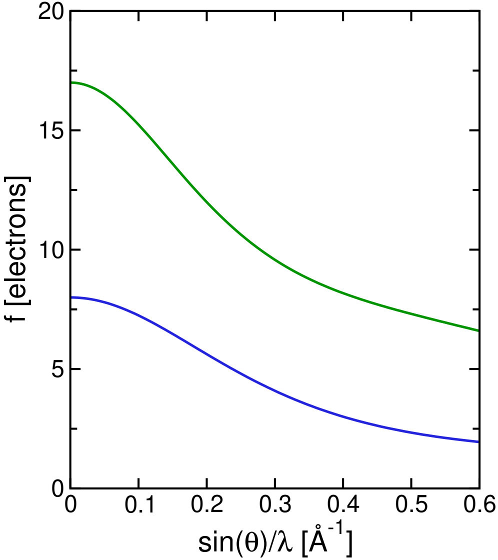

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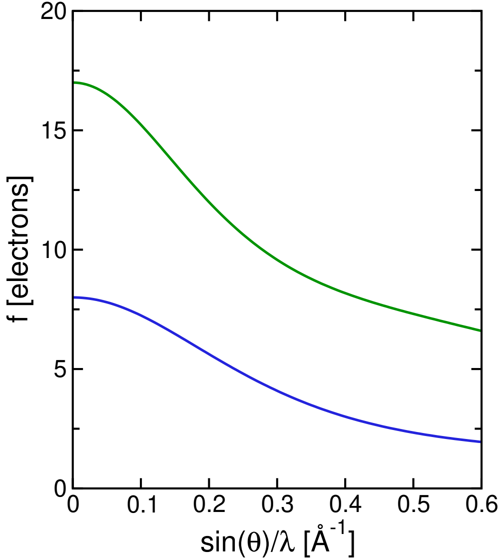

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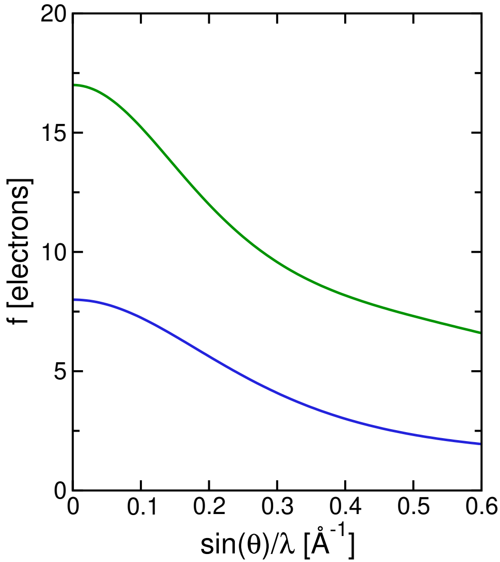

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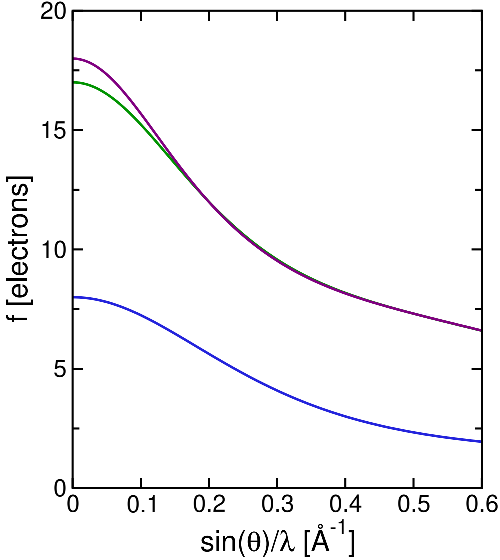

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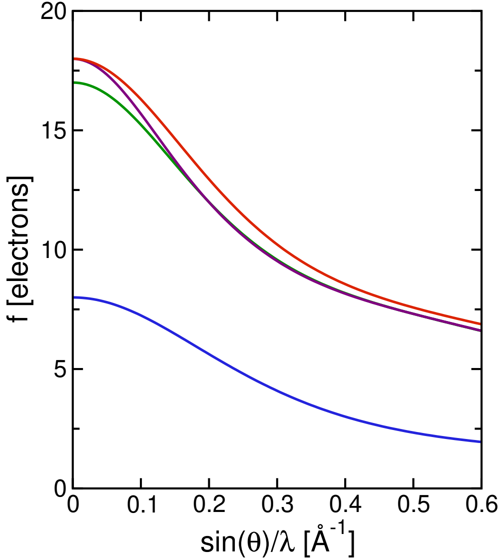

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