Today's outline - April 16, 2020 (part A)

Today's outline - April 16, 2020 (part A)

- Imaging

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Homework Assignment #7:
Chapter 7: 2,3,9,10,11
due Thursday, April 23, 2020

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Final Exam, Tuesday, May 5, 2020 13:00 CDT

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Tell me what time slot you would prefer for your presentation (first come, first served!)

~~13:00~~	14:00	15:00	16:00	~~17:00~~	18:00
13:20	~~14:20~~	15:20	16:20	17:20	18:20
13:40	14:40	15:40	16:40	17:40	18:40

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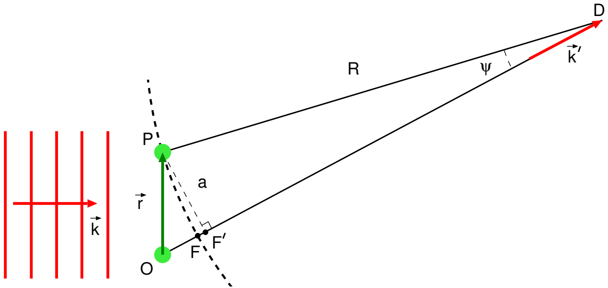
13:40	14:40	15:40	16:40	17:40	18:40

Send me your presentation in Powerpoint or PDF format before the exam

Phase difference in scattering



All imaging can be broken into a three step process

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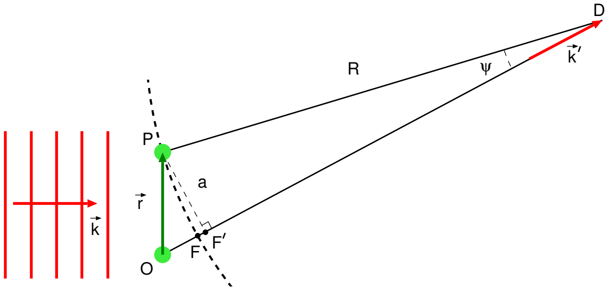

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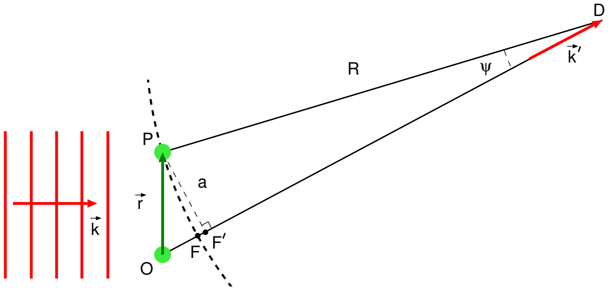

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1. x-ray interaction with sample
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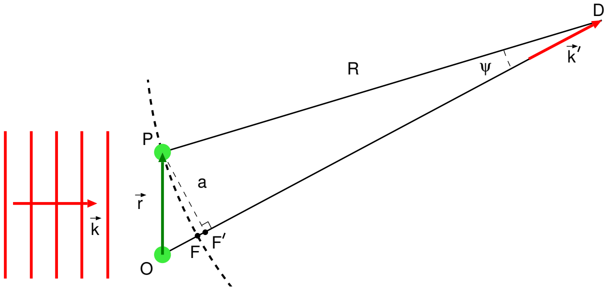

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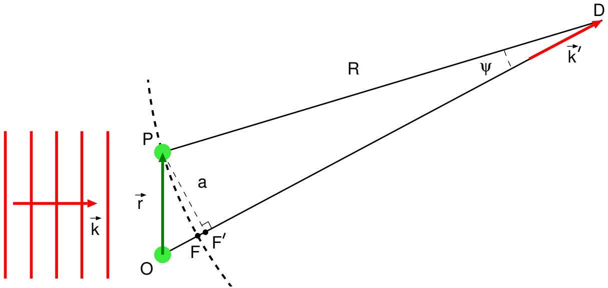

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The scattered waves from O and P will travel different distances

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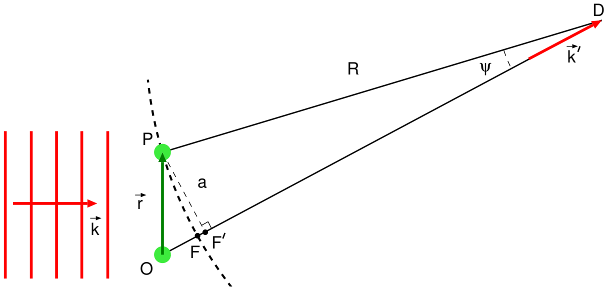

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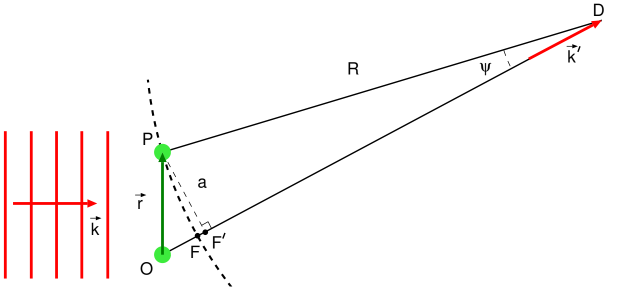

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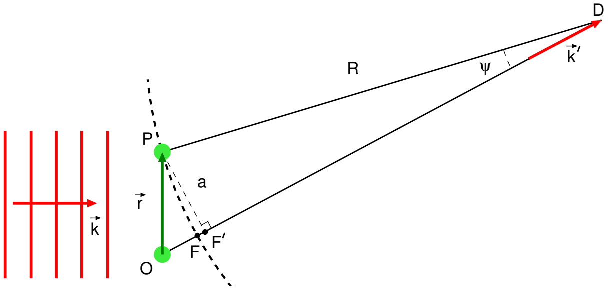

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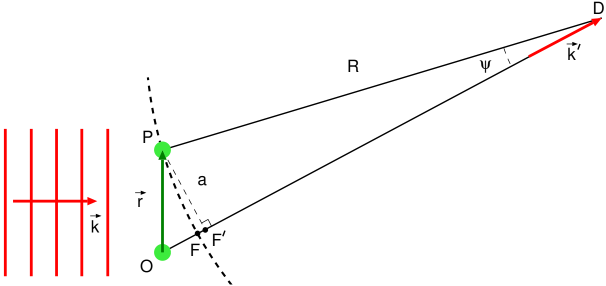
Since $\vec{k} \perp \vec{r}$, $\phi \approx \vec{Q} \cdot \vec{r} = \vec{k}' \cdot \vec{r}$

The path length difference corresponding to this phase shift is $\hat{k}' \cdot r = \overline{OF'}$

Franuhofer, Fresnel, and contact regimes



The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

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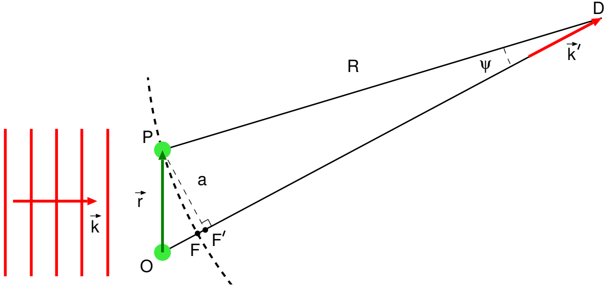

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$$\Delta = R - R\cos\psi$$

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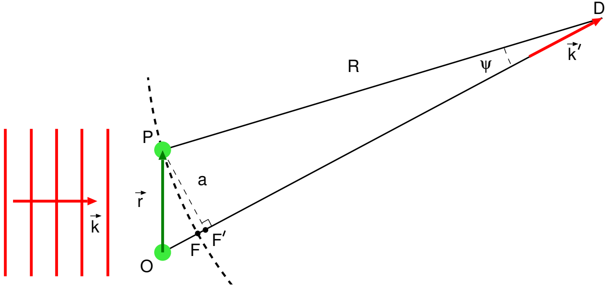

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$$\Delta = R - R\cos\psi$$
$$\approx R\left(1 - \left[1 - \frac{\psi^2}{2}\right]\right)$$

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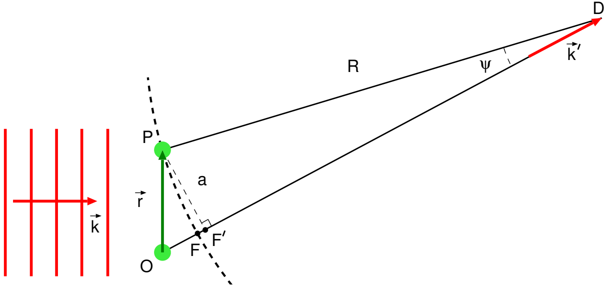

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$$
\begin{aligned}
\Delta &= R - R\cos\psi \\
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&= R\frac{a^2}{2R^2}
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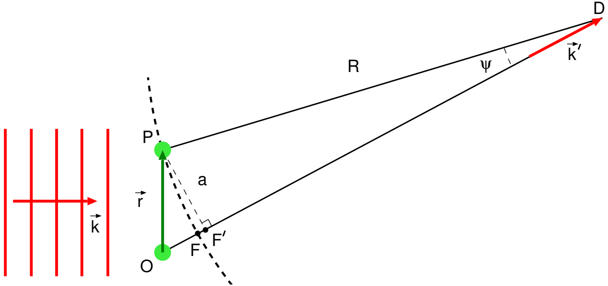

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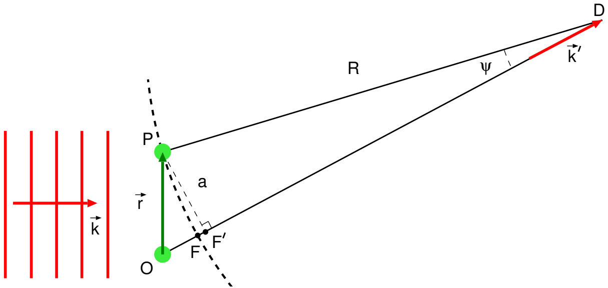

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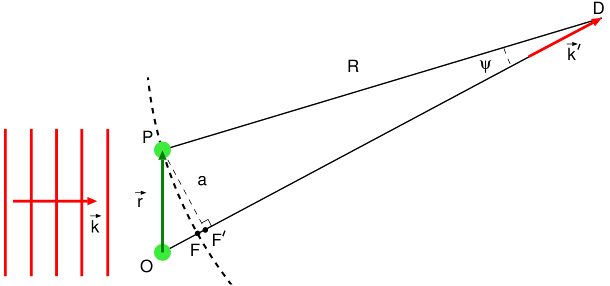
$$R \gg \frac{a^2}{\lambda} \qquad \text{Fraunhofer}$$

Franuhofer, Fresnel, and contact regimes



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$R \gg \dfrac{a^2}{\lambda}$ Fraunhofer

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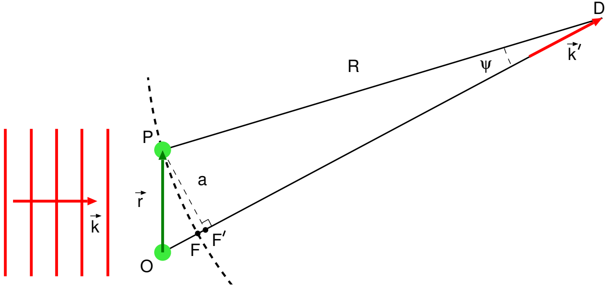

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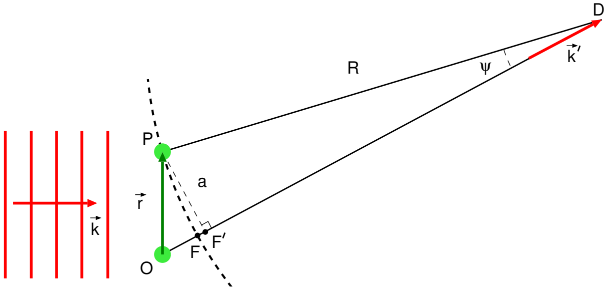
$R \gg \dfrac{a^2}{\lambda}$ Fraunhofer

$R \approx \dfrac{a^2}{\lambda}$ Fresnel

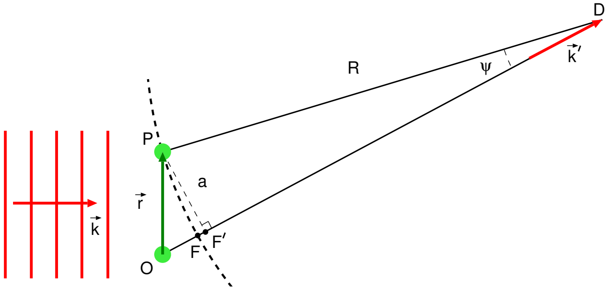
$R \ll \dfrac{a^2}{\lambda}$ Contact

Detector placement



If $\lambda = 1$ Å and the distance to be resolved is $a = 1$ Å, then $a^2/\lambda = 1$ Å and *any* detector placement is in the Fraunhofer (far field) regime

Detector placement



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if $a = 1\mu$m, then $a^2/\lambda = 10$ mm and the imaging regime can be selected by detector placement

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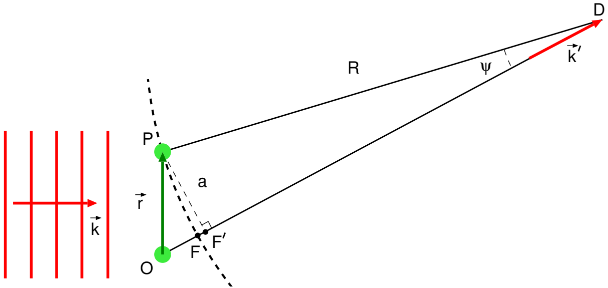

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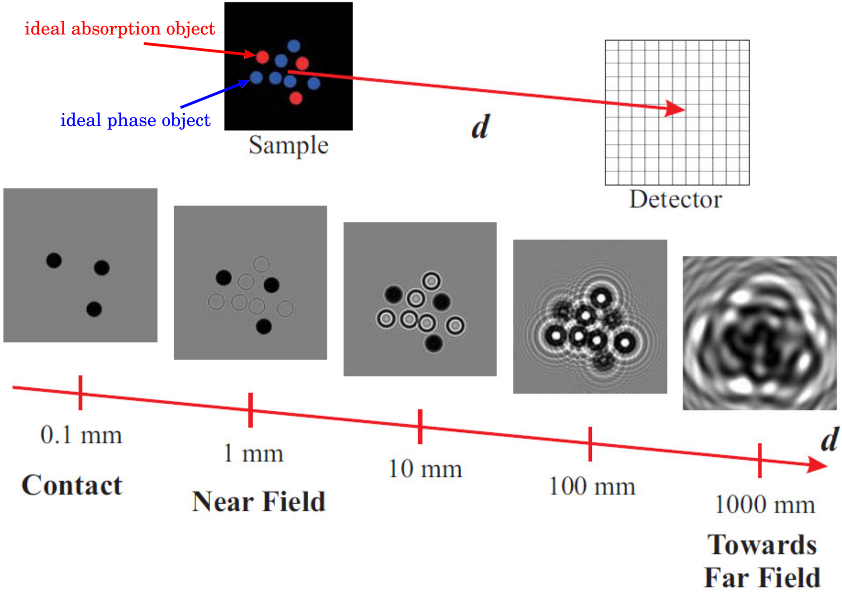
if $a = 1\mu$m, then $a^2/\lambda = 10$ mm and the imaging regime can be selected by detector placement

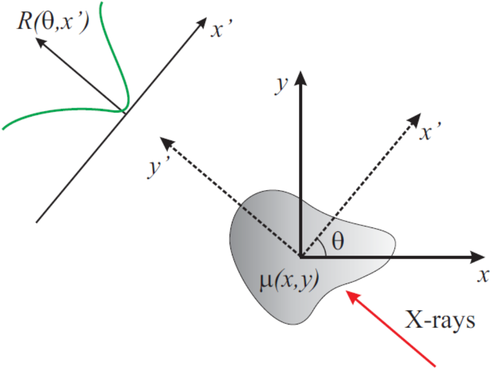
if $a = 1$ mm, then $a^2/\lambda = 10$ km and the detector will always be in the contact regime

Contact to far-field imaging



Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895.



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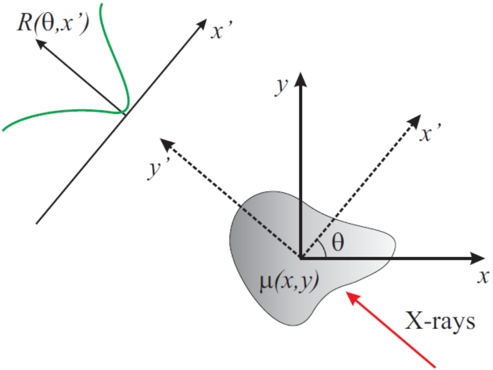

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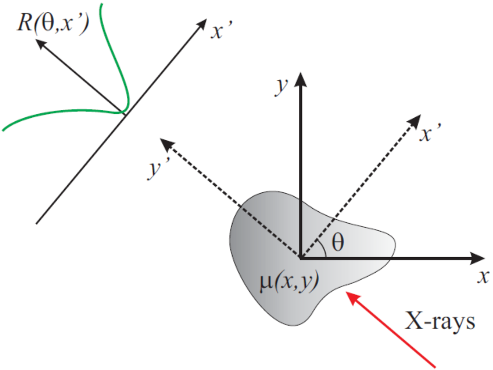
Assume the object to be imaged has a non uniform absorption coefficient $\mu(x, y)$



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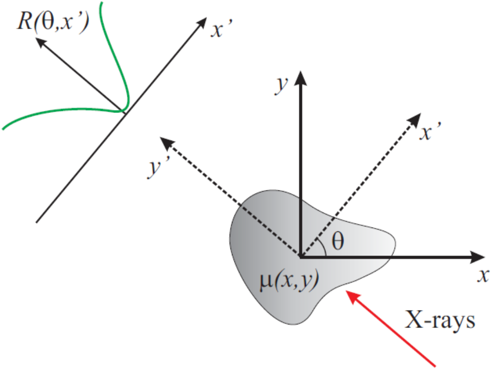
Assume the object to be imaged has a non uniform absorption coefficient $\mu(x, y)$ The line integral of the absorption coefficient at a particular value of x' is measured as the ratio of the transmitted to the incident beam



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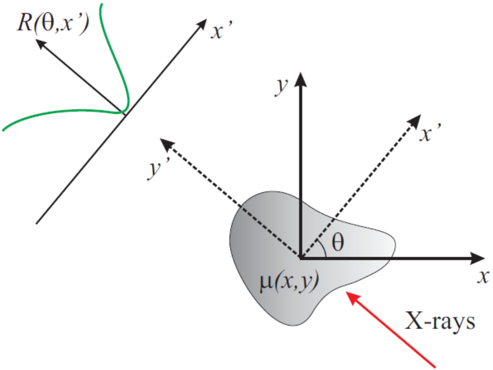

$$I = I_0 e^{-\int \mu(x,y) dy'}$$

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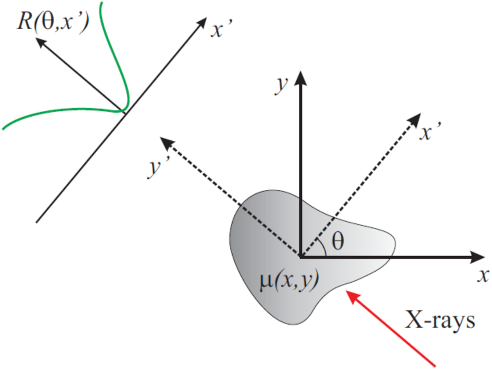

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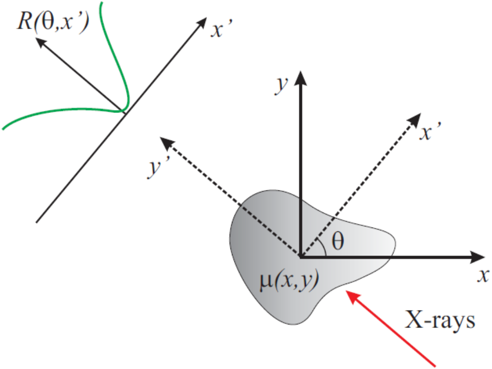

$$I = I_0 e^{-\int \mu(x,y) dy'}$$

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The radon transform $R(\theta, x')$ is used to reconstruct the 3D absorption image of the object numerically.

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

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$$F(q_x, q_y = 0) = \int \left[\int f(x, y) dy \right] e^{iq_x x} dx$$

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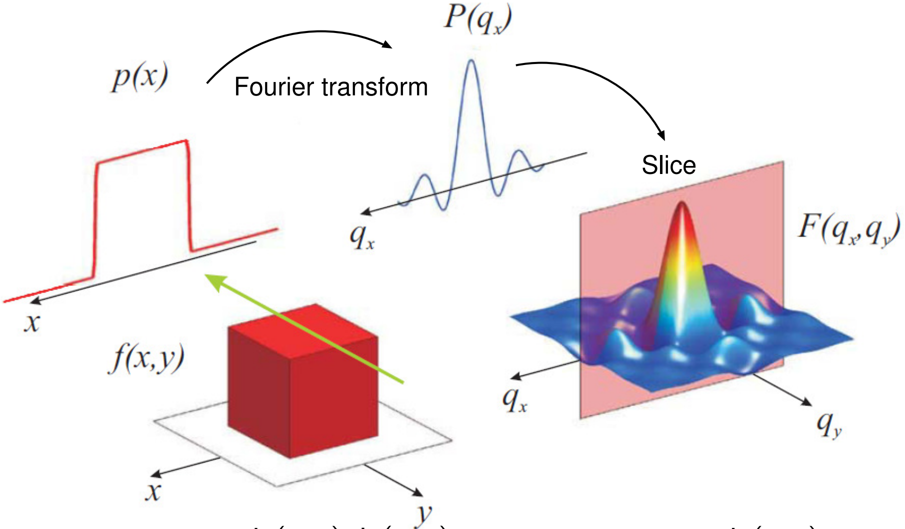
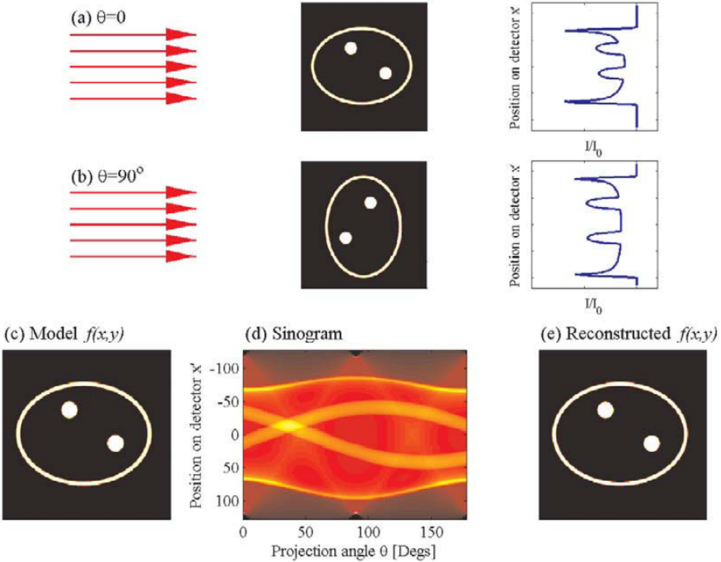
The Fourier transform of the projection is equal to a slice through the Fourier transform of the object at the origin in the direction of propagation

Fourier transform reconstruction



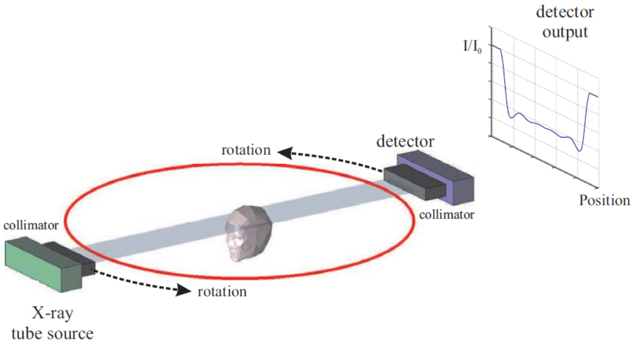
$$F(q_x, q_y) = \frac{\sin(q_x x)}{q_x x}\frac{\sin(q_y y)}{q_y y} \qquad P(q_x) = \frac{\sin(q_x x)}{q_x x}$$

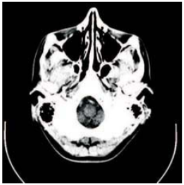
Sinograms



(a) θ=0
Position on detector x'
I/I0
(b) θ=90°
Position on detector x'
I/I0
(c) Model f(x,y)
(d) Sinogram
Position on detector x'
-100
-50
0
50
100
0
50
100
150
Projection angle θ [Degs]
(e) Reconstructed f(x,y)

Medical tomography





Today's outline - April 16, 2020 (part B)

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- Fresnel zone plate review

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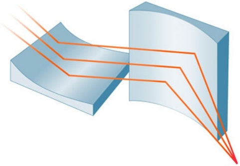

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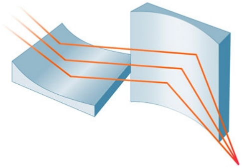

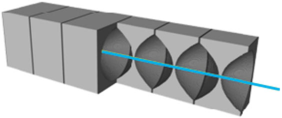

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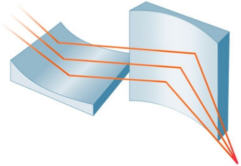
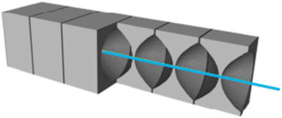
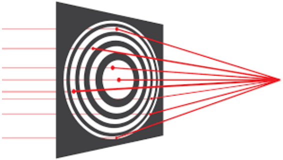

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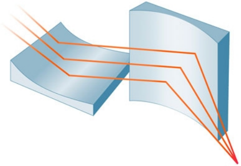

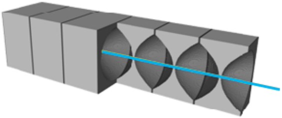

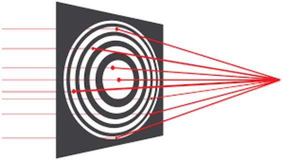

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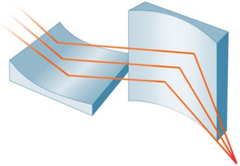
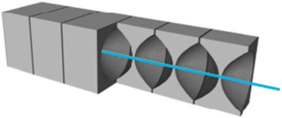
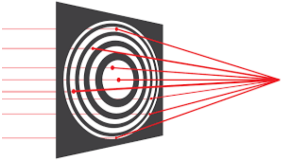

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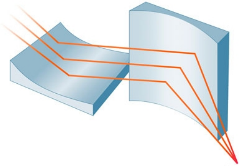

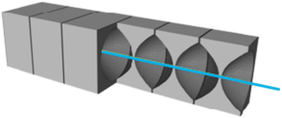

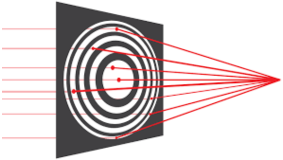

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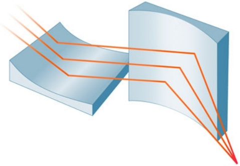

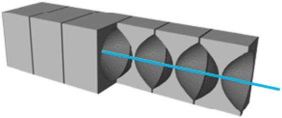

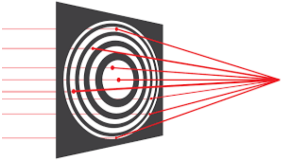

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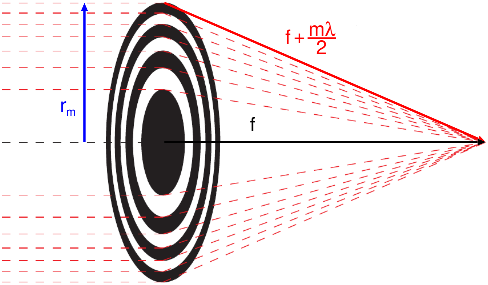

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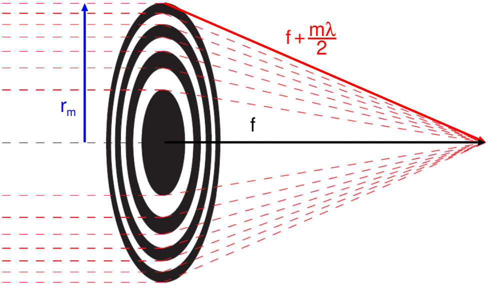

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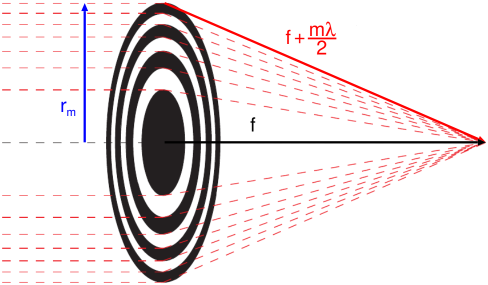

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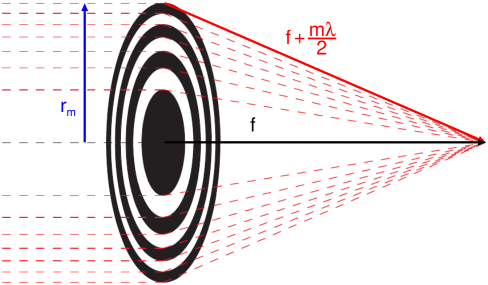

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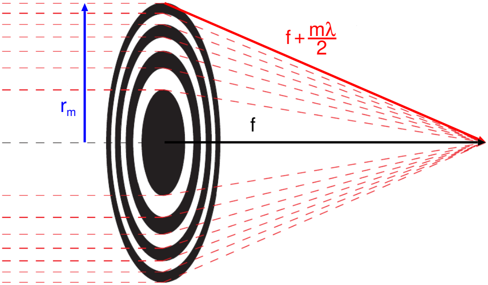

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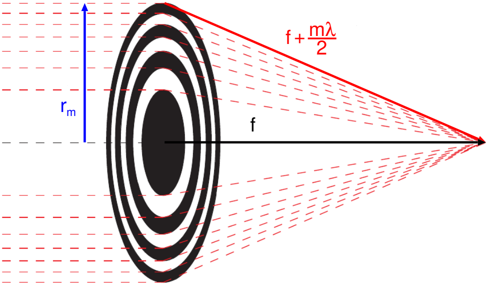

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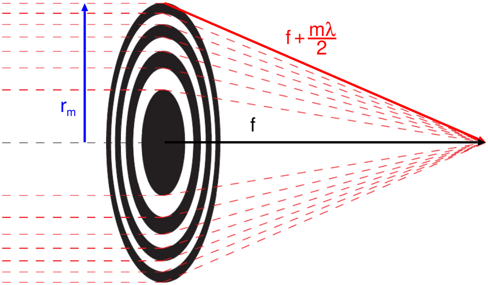

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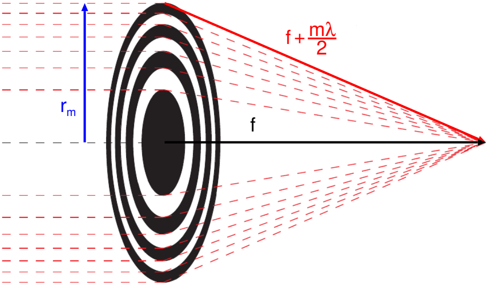

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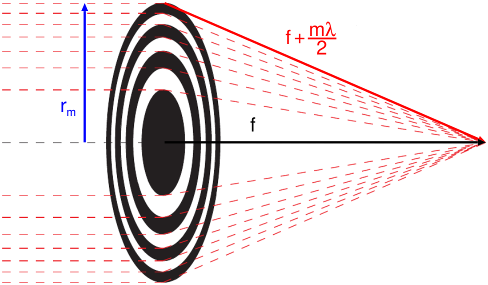

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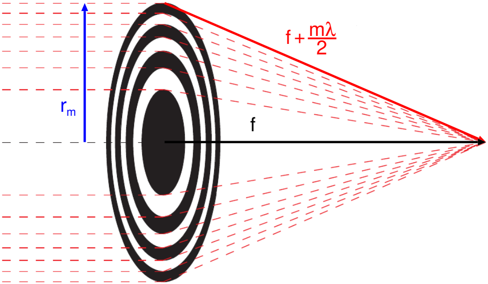

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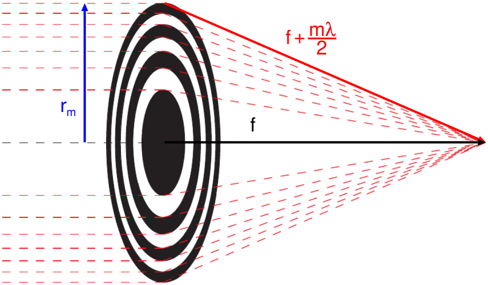

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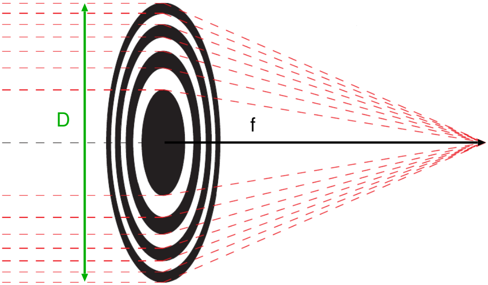

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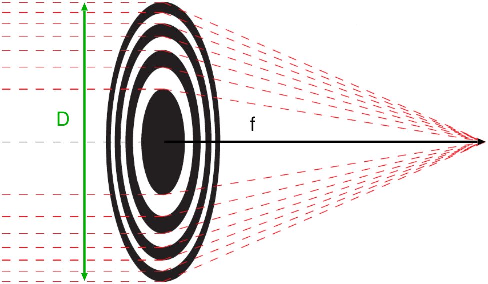

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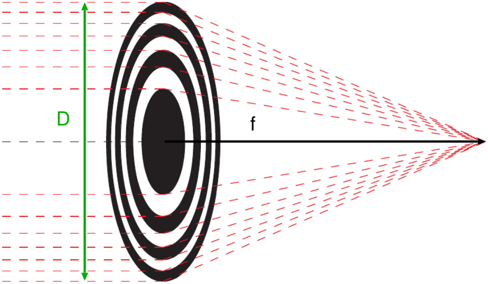

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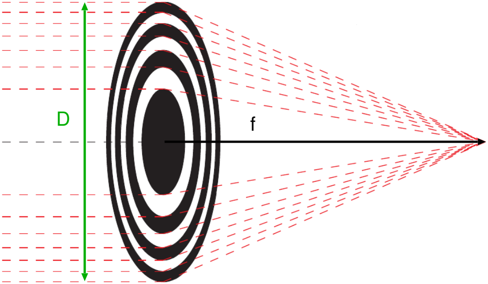

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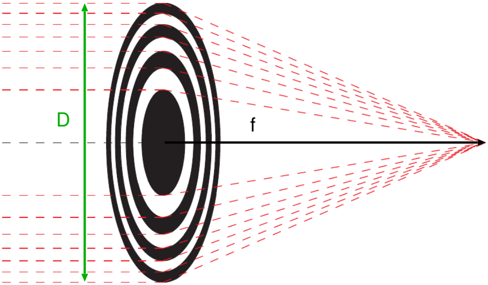

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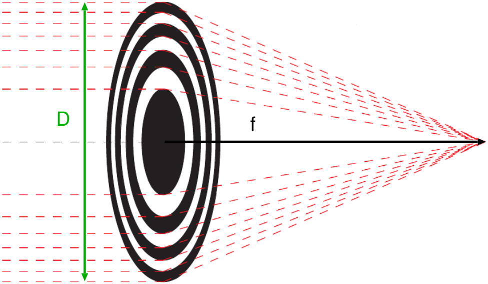

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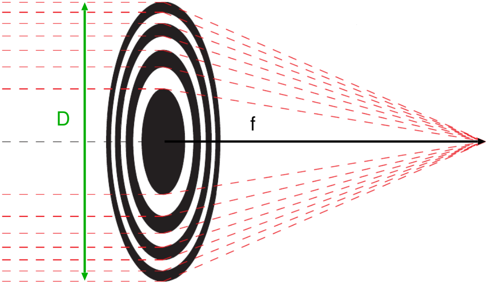
using the Rayleigh criterion for resolvability



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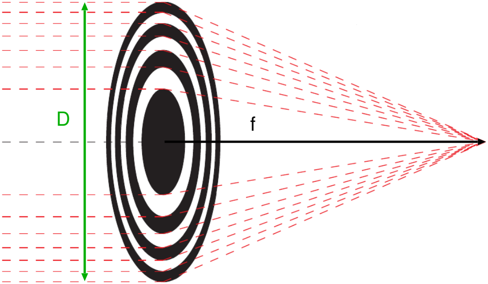

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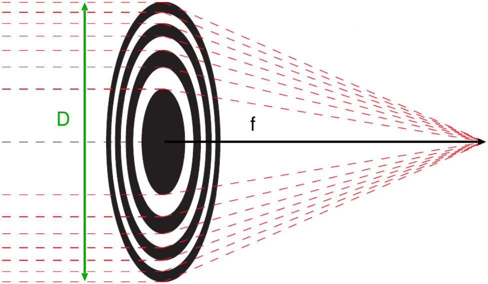

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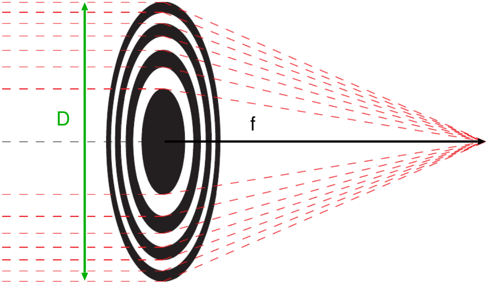

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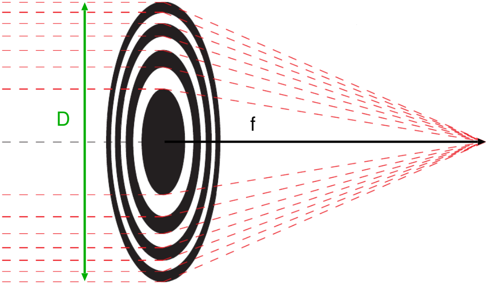

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Fresnel zone plate review

$$r_m \approx \sqrt{m\lambda f}, \quad \Delta r_M \approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}, \quad f \approx 4M\frac{(\Delta r_M)^2}{\lambda}$$

The diameter of the zone plate is

$$D = 2r_M = 2\sqrt{M\lambda f}$$
$$= 2\sqrt{M}\sqrt{\lambda f} = 4M\Delta r_M$$

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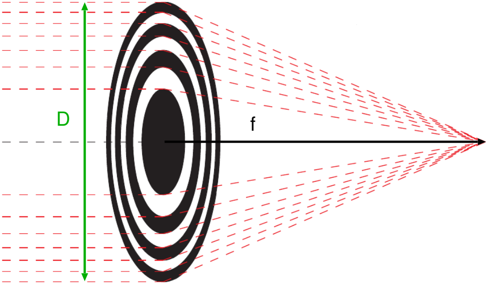

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The resolution of a zone plate is determined by the width of the outermost ring and fabrication considerations limit this to $\sim$ 20 nm

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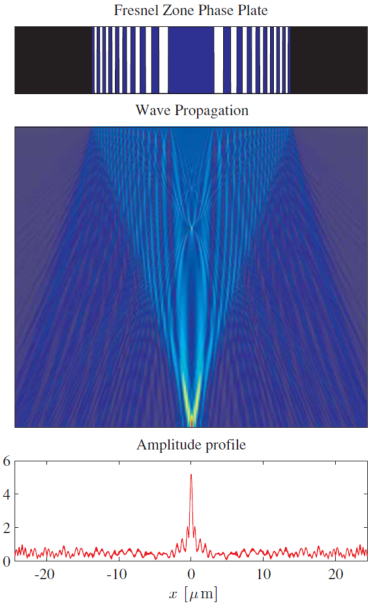
One example is the comparison between a phase Fresnel zone plate and an absorption Fresnel zone plate

Fresnel zone plates



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