Today's outline - April 07, 2020

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- X-ray magnetic circular dichroism

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Homework Assignment #06:
Chapter 6: 1,6,7,8,9
due Tuesday, April 14, 2020

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Homework Assignment #06:
Chapter 6: 1,6,7,8,9
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Homework Assignment #07:
Chapter 7: 2,3,9,10,11
due Thursday, April 23, 2020

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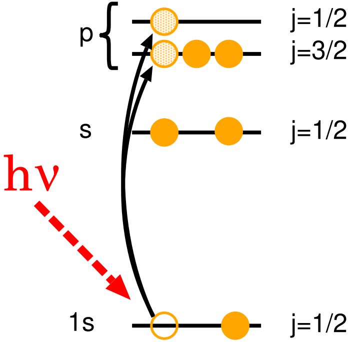

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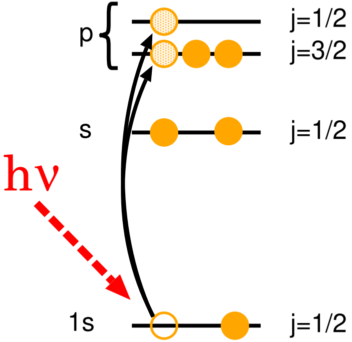

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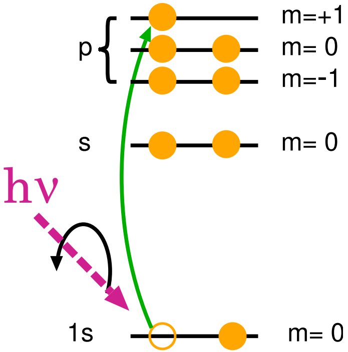

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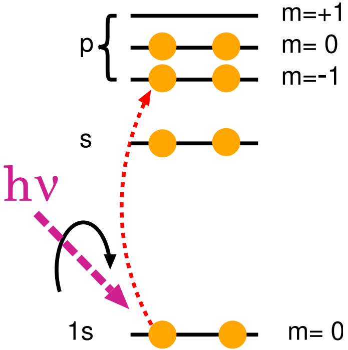

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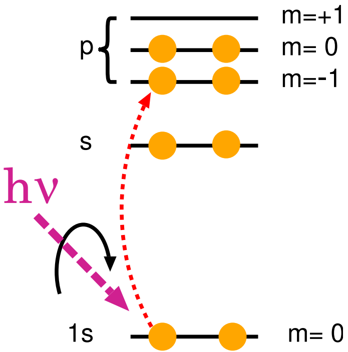

this measurement is sensitive to the internal/external magnetic fields which split the levels according to the Zeeman effect

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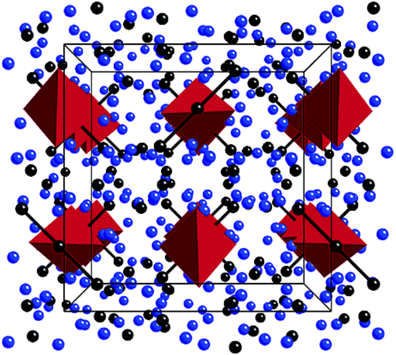
XMCD of $Yb_{14}MnSb_{11}$

The Zintl compounds exhibit interesting magnetic properties including colossal magnetoresistance which can be of value for spintronics applications



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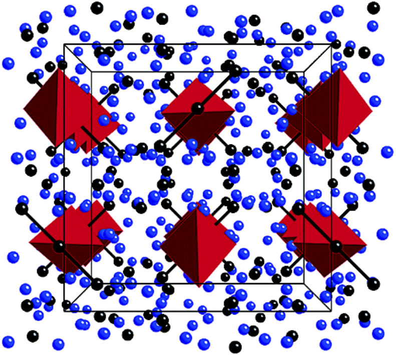

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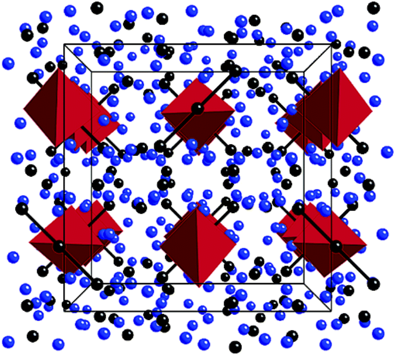

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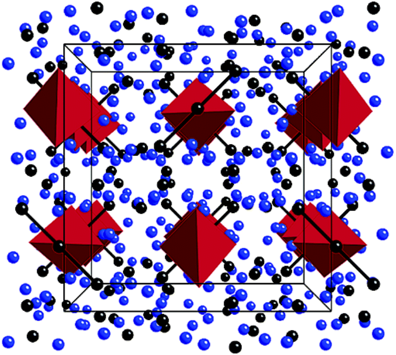

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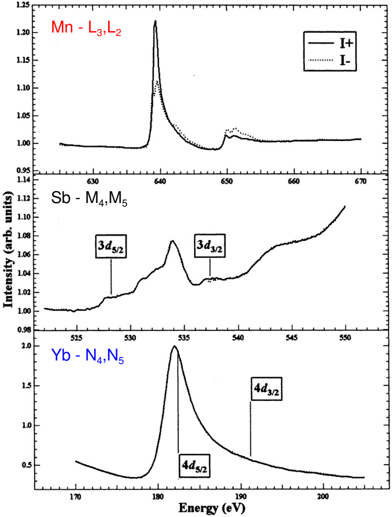

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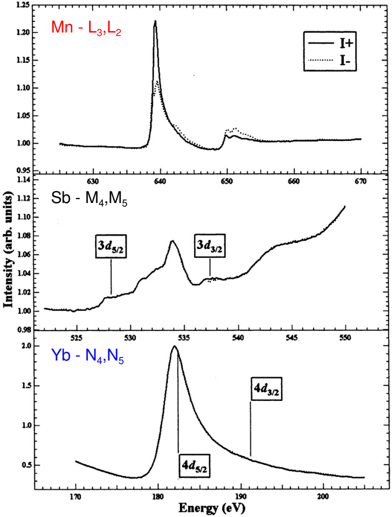

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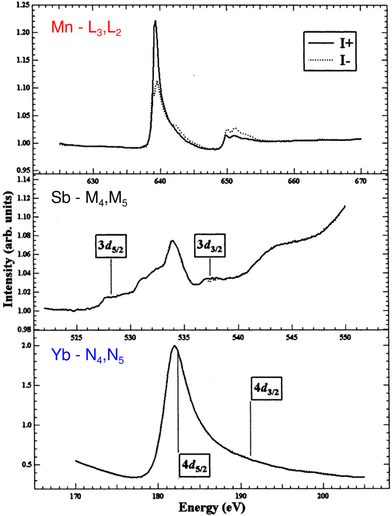

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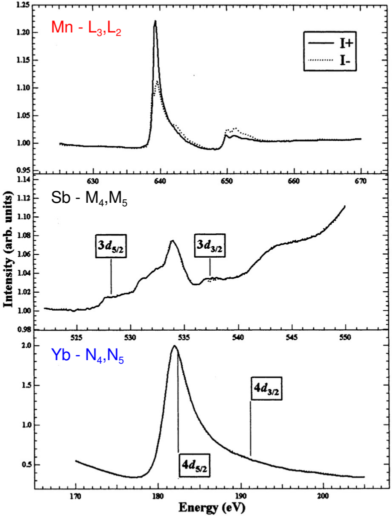

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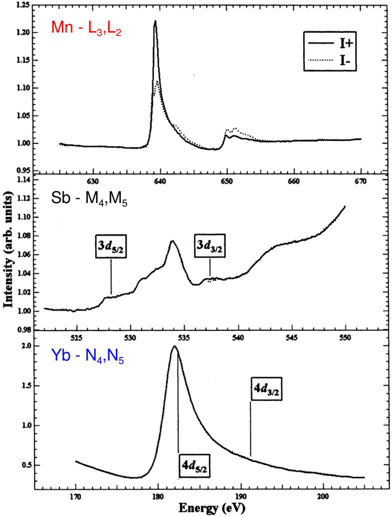

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Mn provides the bulk of the magnetic moment and appears to be in the divalent state. Sb provides a small antiferromagnetic component to the overall magnetic moment

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- Resonant Scattering

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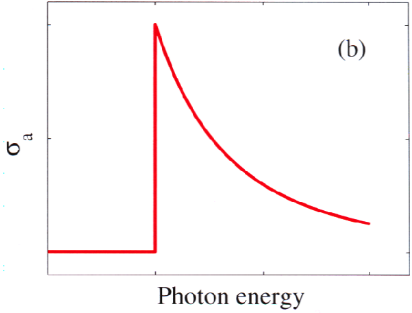

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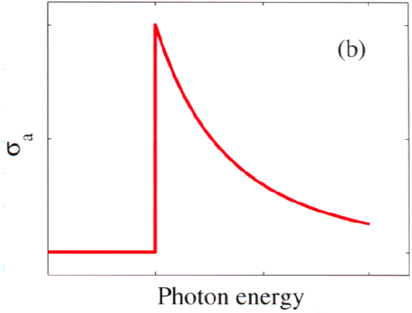

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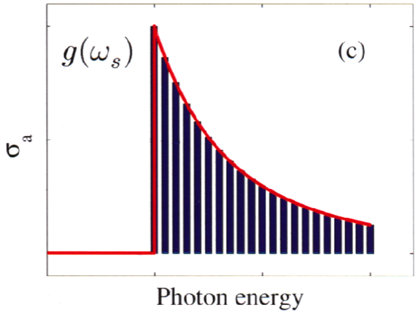

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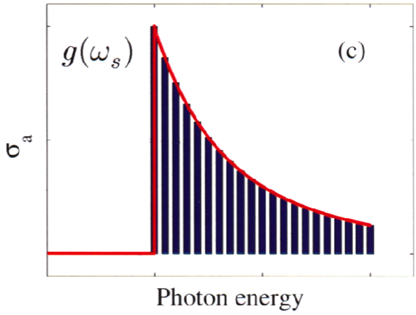

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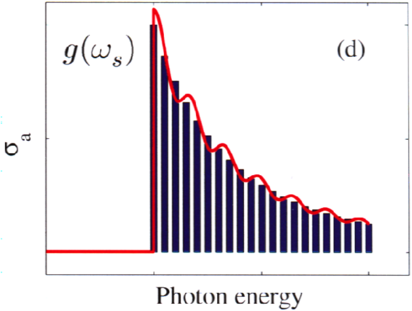

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Forced charged oscillator

Consider an electron under the influence of an oscillating electric field $\vec{E}_{in} = \hat{x} E_0 e^{-i\omega t}$.

$$\ddot{x} + \Gamma \dot{x} + \omega_s^2 x = -\left(\frac{eE_0}{m}\right) e^{-i\omega t}$$

where Γ is the damping constant, ω_s is the resonant frequency of the oscillator, and $\Gamma \ll \omega_s$.

assuming a solution of the form

$$x = x_0 e^{-i\omega t}$$
$$\dot{x} = -i\omega x_0 e^{-i\omega t}$$
$$\ddot{x} = -\omega^2 x_0 e^{-i\omega t}$$

$$(-\omega^2 - i\omega\Gamma + \omega_s^2) x_0 e^{-i\omega t} = -\left(\frac{eE_0}{m}\right) e^{-i\omega t}$$
$$x_0 = -\left(\frac{eE_0}{m}\right) \frac{1}{(\omega_s^2 - \omega^2 - i\omega\Gamma)}$$

The amplitude of the response has a resonance and dissipation

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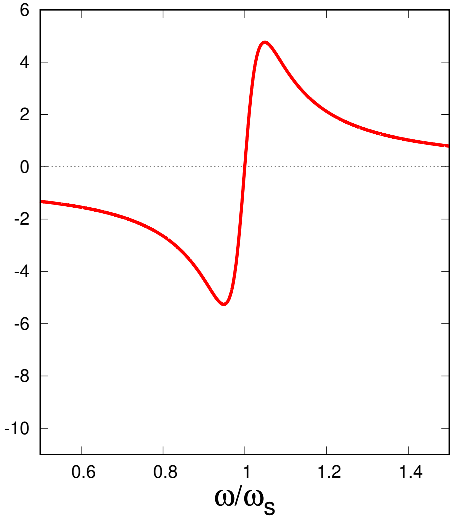

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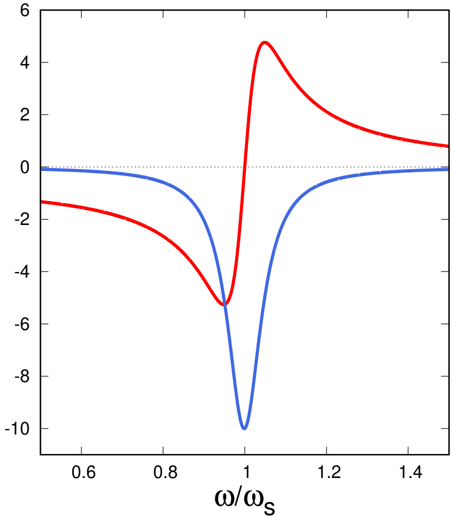

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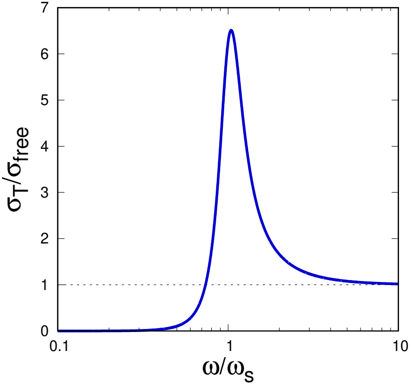

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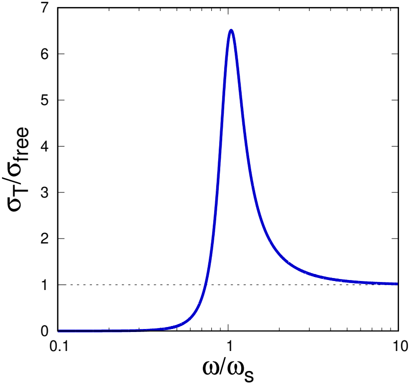

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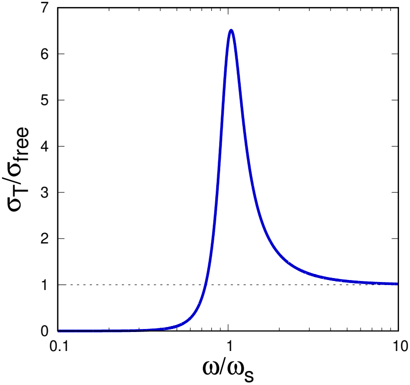

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