Today's Outline - February 13, 2020

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- Elliptical lenses

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Homework Assignment #02:
Problems on Blackboard
due Tuesday, February 18, 2020

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- Elliptical lenses
- Zone plates
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- Kinematical scattering

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due Tuesday, February 18, 2020

APS Visit:
10-BM: Friday, April 24, 2020

Polycapillary optics

A polycapillary is a focusing optic made up of an array of thousands of thin-walled hollow tubes which are $> 65\%$ empty space

F.A. Hofmann et al., "Focusing of synchrotron radiation with polycapillary optics," *Nuclear Instrum. Meth. B* **133**, 145-150 (1997).

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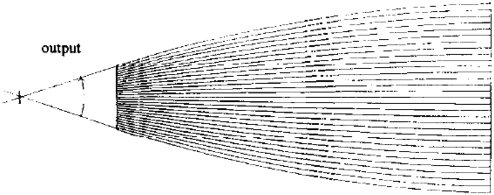

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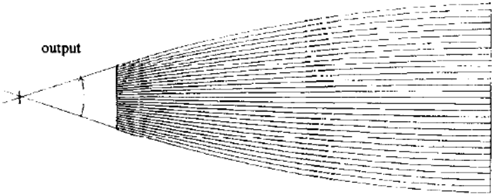

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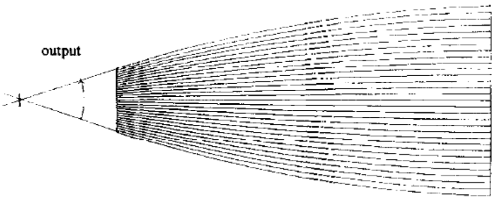

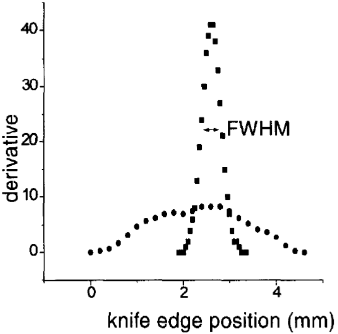

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Improving polycapillary optic performance

One drawback of a glass capillary is that the transmission at high energies is reduced because of critical angle restrictions

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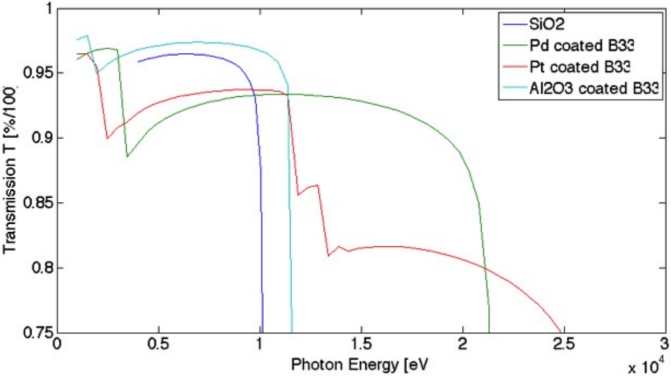

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In calculating the optimal surface profile for a refractive lens, an important approximation was made which resulted in a parabolic surface

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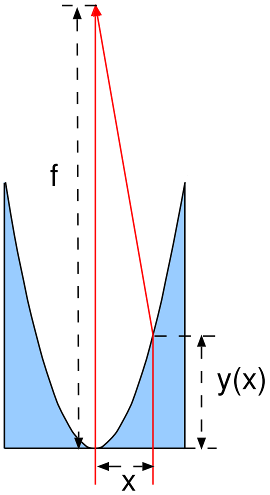

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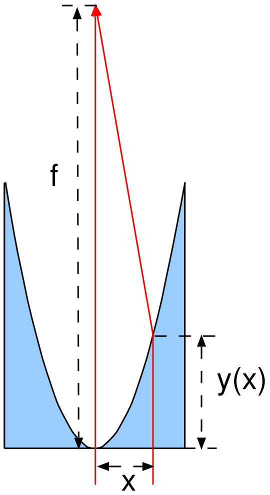

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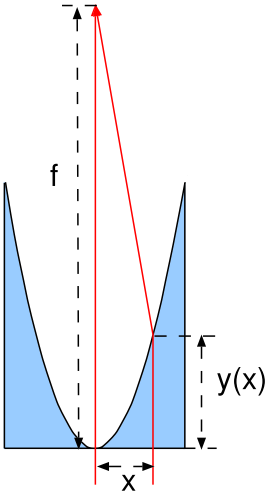
$$f = y(1-\delta) + \sqrt{(f-y)^2 + x^2}$$



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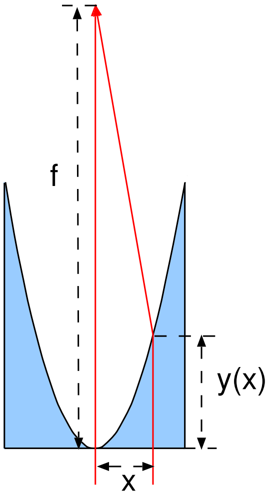
$$(f - y + \delta y)^2 = (f-y)^2 + x^2$$



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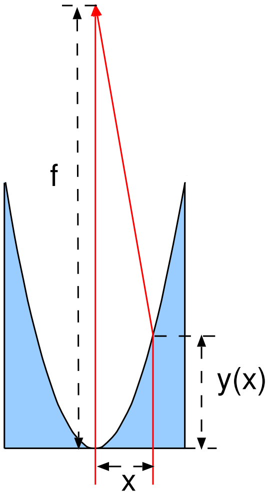
$$f = y(1-\delta) + \sqrt{(f-y)^2 + x^2}$$
$$(f - y + \delta y)^2 = (f-y)^2 + x^2$$
$$2f\delta y - (2\delta - \delta^2)y^2 = x^2$$



Elliptical lens surface

Ideal surface

Ellipse

Elliptical lens surface

Ideal surface

$$0 = x^2 + (2\delta - \delta^2)y^2 - 2f\delta y$$

Ellipse

$$1 = \frac{x^2}{a^2} + \frac{(y-b)^2}{b^2}$$

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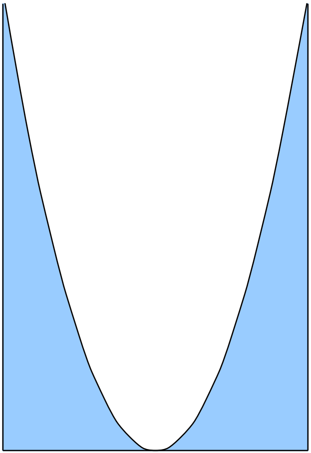
The ideal surface for a thick lens is an ellipse

How to make a Fresnel lens



The ideal refracting lens has an elliptical shape but this is impractical to make. Assuming the parabolic approximation:

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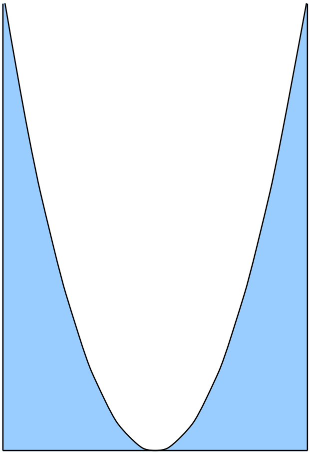

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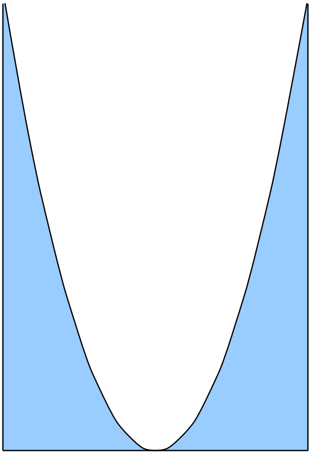

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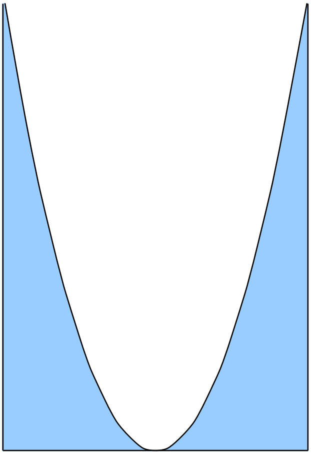

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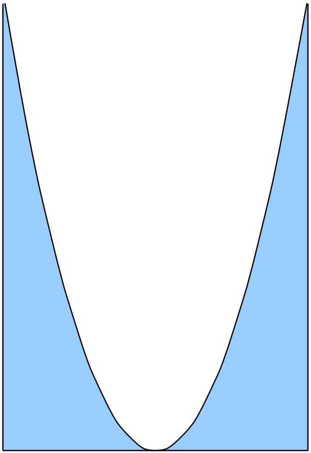

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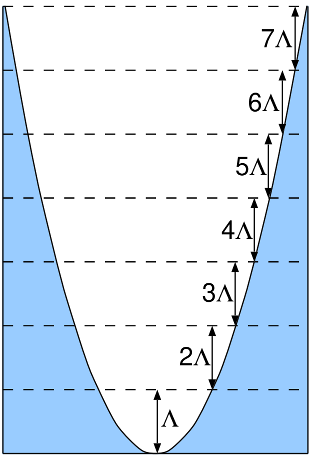
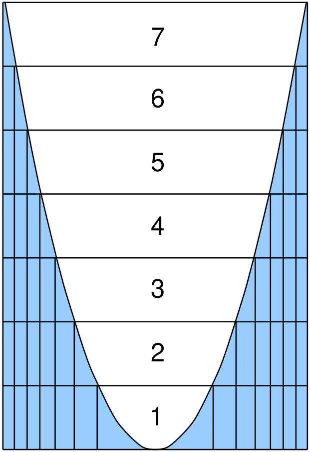
aspect ratio too large for a stable structure and absorption would be too large!

How to make a Fresnel lens



Mark off the longitudinal zones (of thickness Λ) where the waves inside and outside the material are in phase.

How to make a Fresnel lens



Mark off the longitudinal zones (of thickness Λ) where the waves inside and outside the material are in phase.

Each block of thickness Λ serves no purpose for refraction but only attenuates the wave.

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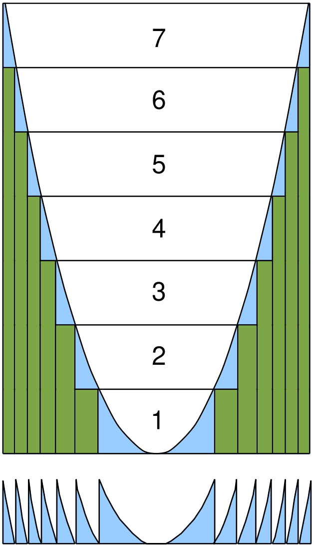

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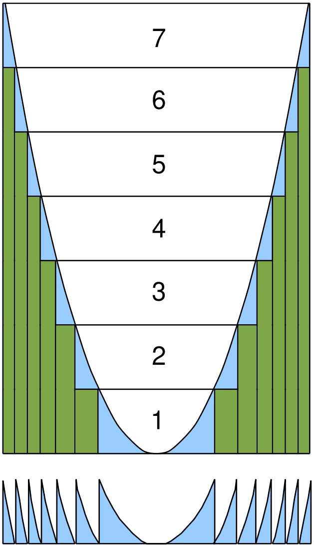
This material can be removed and the remaining material collapsed to produce a Fresnel lens which has the same optical properties as the parabolic lens as long as $f \gg N\Lambda$ where N is the number of zones.

Fresnel lens dimensions



The outermost zones become very small and thus limit the overall aperture of the zone plate. The dimensions of outermost zone, N can be calculated by first defining a scaled height and lateral dimension

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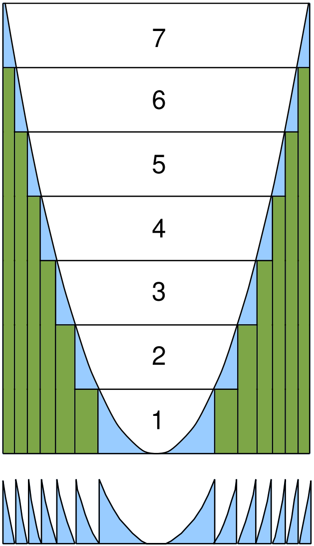

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$$\nu = \frac{h(x)}{\Lambda}$$

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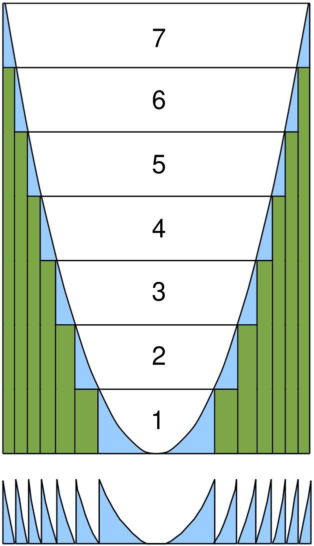

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$$\nu = \frac{h(x)}{\Lambda} \qquad \xi = \frac{x}{\sqrt{2\lambda_o f}}$$

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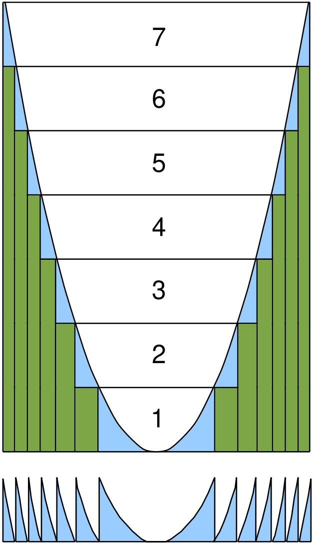

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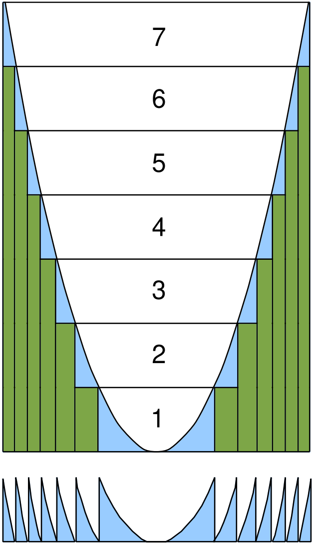
Since $\nu = \xi^2$, the position of the N^{th} zone is $\xi_N = \sqrt{N}$ and the scaled width of the N^{th} (outermost) zone is

Fresnel lens dimensions



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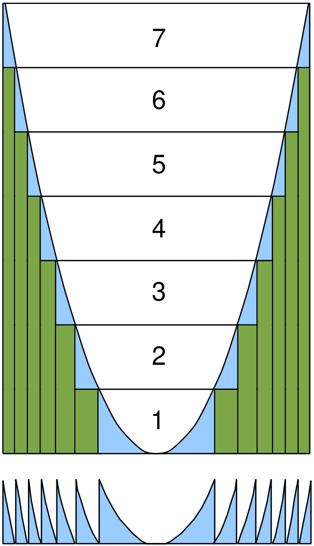

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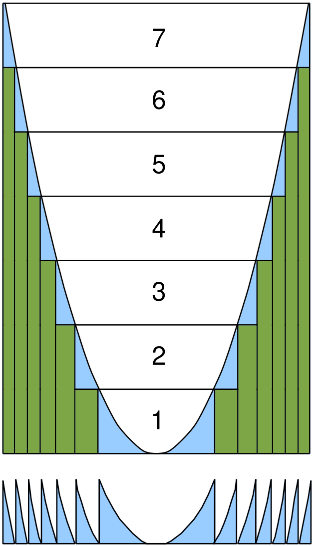

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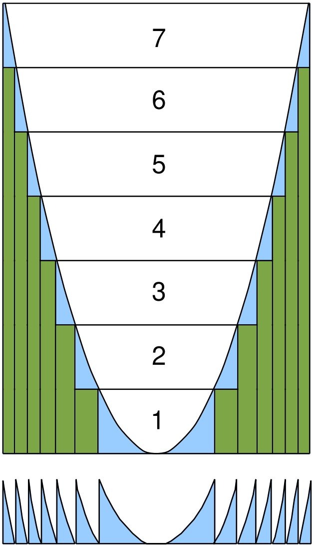

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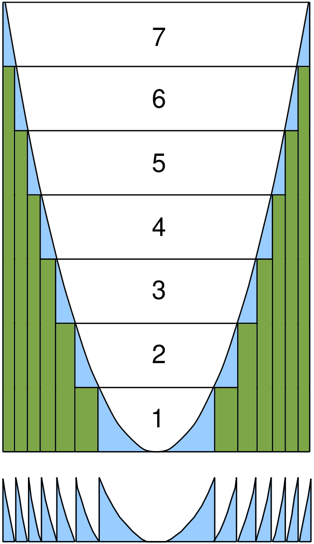

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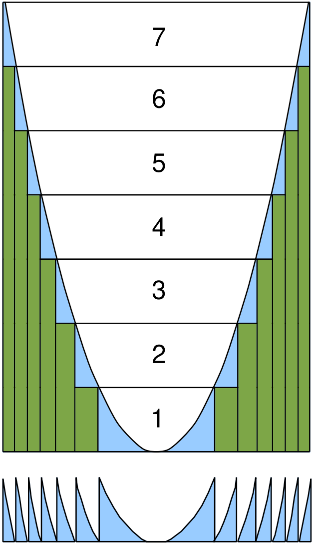
$$\Delta\xi_N \approx \frac{1}{2\sqrt{N}}$$

Fresnel lens dimensions



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$$= \sqrt{N}\left(1 - \sqrt{1 - \frac{1}{N}}\right)$$
$$\approx \sqrt{N}\left(1 - \left[1 - \frac{1}{2N}\right]\right)$$
$$\Delta\xi_N \approx \frac{1}{2\sqrt{N}}$$

The diameter of the entire lens is thus

$$2\xi_N = 2\sqrt{N} = \frac{1}{\Delta\xi_N}$$

Fresnel lens example

In terms of the unscaled variables

$$\Delta x_N = \Delta \xi_N \sqrt{2\lambda_o f}$$

Fresnel lens example

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$$\Delta x_N = \Delta \xi_N \sqrt{2\lambda_o f} = \sqrt{\frac{\lambda_o f}{2N}}$$

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$$d_N = 2\xi_N = \frac{\sqrt{2\lambda_o f}}{\Delta\xi_N}$$

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$$d_N = 2\xi_N = \frac{\sqrt{2\lambda_o f}}{\Delta \xi_N} = 2\sqrt{N}\sqrt{2\lambda_o f} = \sqrt{2N\lambda_o f}$$

Fresnel lens example

In terms of the unscaled variables

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$$d_N = 2\xi_N = \frac{\sqrt{2\lambda_o f}}{\Delta\xi_N} = 2\sqrt{N}\sqrt{2\lambda_o f} = \sqrt{2N\lambda_o f}$$

If we take

$$\lambda_o = 1\text{Å} = 1\times 10^{-10}\text{m}$$
$$f = 50\text{cm} = 0.5\text{m}$$
$$N = 100$$

Fresnel lens example

In terms of the unscaled variables

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If we take

$$\lambda_o = 1\text{Å} = 1\times10^{-10}\text{m}$$
$$f = 50\text{cm} = 0.5\text{m}$$
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$$\Delta x_N = 5\times10^{-7}\text{m} = 500\text{nm}$$

Fresnel lens example

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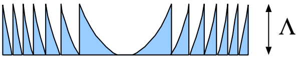
Making a Fresnel zone plate



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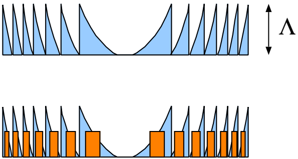

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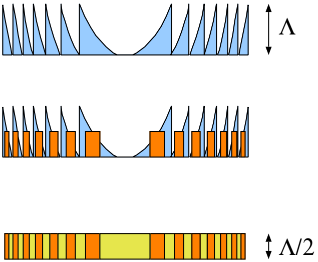

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This kind of zone plate is not as efficient as a true Fresnel lens would be in the x-ray regime. Nevertheless, efficiencies up to 35% have been achieved.

Zone plate fabrication

Making high aspect ratio zone plates is challenging but a new process has been developed to make plates with an aspect ratio as high as 25.

M. Wojick et al., "X-ray zone plates with 25 aspect ratio using a 2-μm-thick ultrananocrystalline diamond mold," *Microsyst. Technol.* **20**, 2045-2050 (2014).

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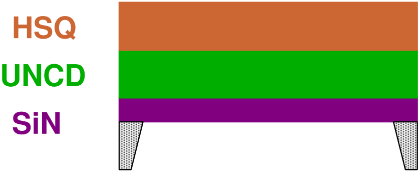

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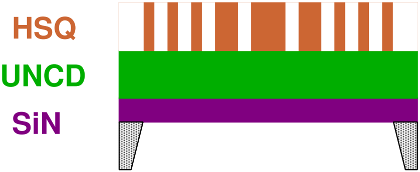

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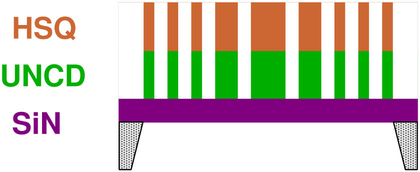

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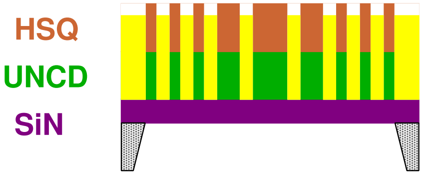

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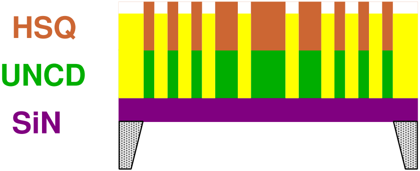
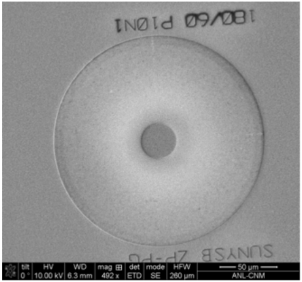
The whole 150nm diameter zone plate





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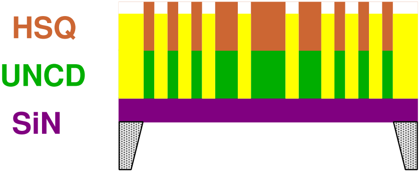

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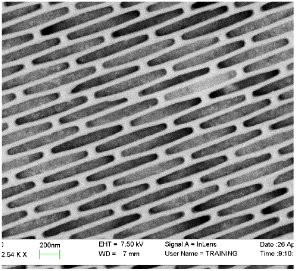
Detail view of outer zones



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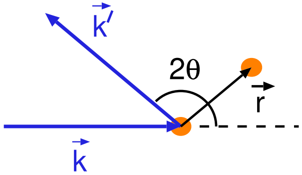

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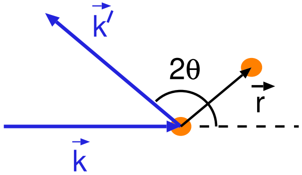

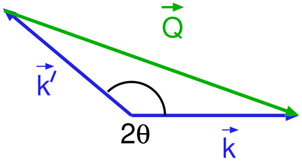

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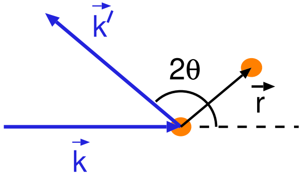

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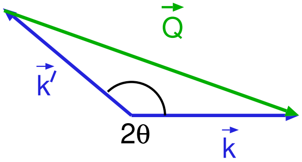

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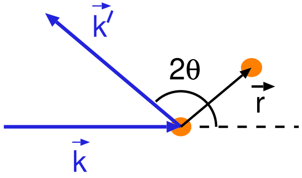

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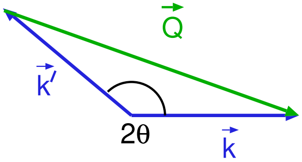

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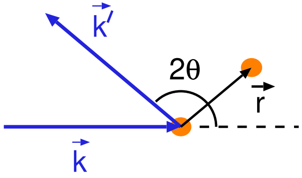

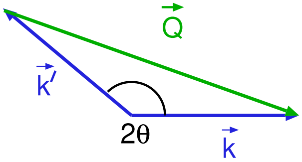

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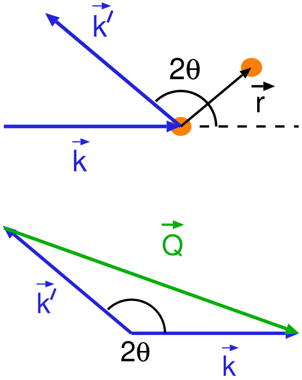

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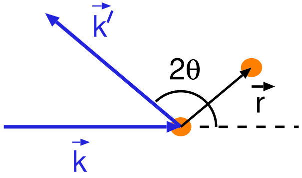

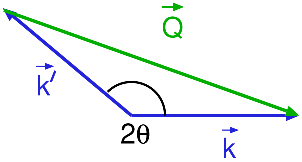

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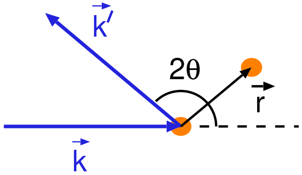

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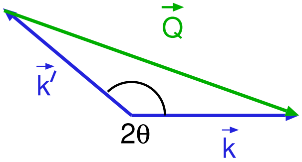

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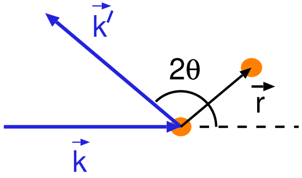

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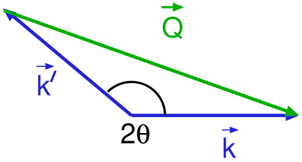

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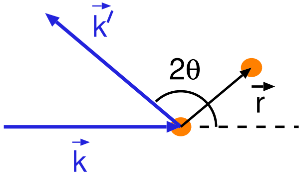

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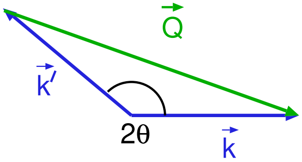

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Scattering from many electrons

for many electrons

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generalizing to a crystal

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We will now look at the consequences of this orientation and generalize to more than two electrons

Two electrons — fixed orientation

The expression

$$I(\vec{Q}) = 2r_0^2 \left(1 + \cos(\vec{Q} \cdot \vec{r})\right)$$

assumes that the two electrons have a specific, fixed orientation. In this case the intensity as a function of Q is.

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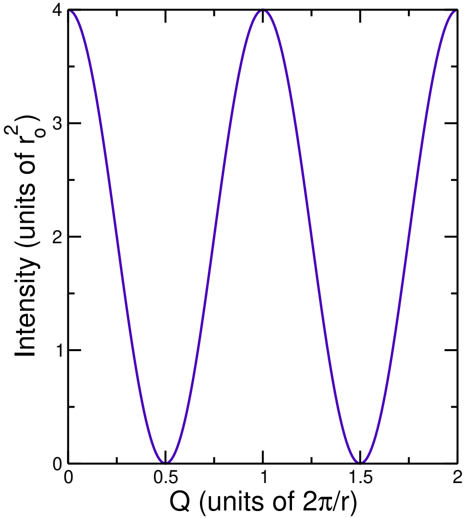

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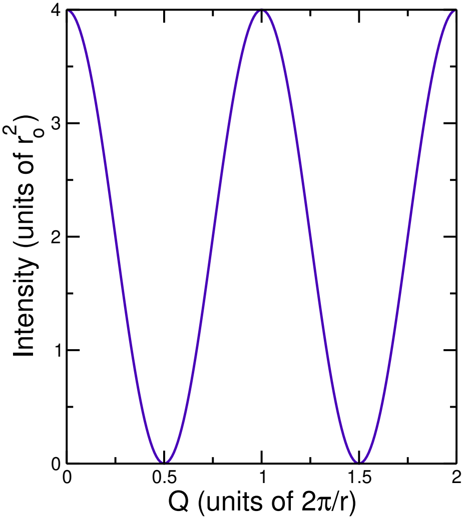
Fixed orientation is not the usual case, particularly for solution and small-angle scattering.



Orientation averaging

Consider scattering from two arbitrary electron distributions, f_1 and f_2. $A(\vec{Q})$, is given by

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Consider scattering from two arbitrary electron distributions, f_1 and f_2. $A(\vec{Q})$, is given by

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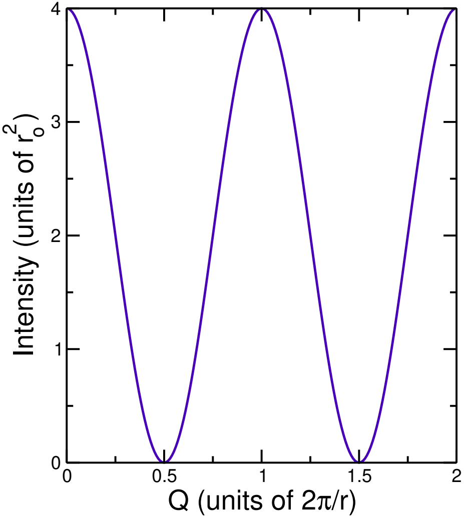

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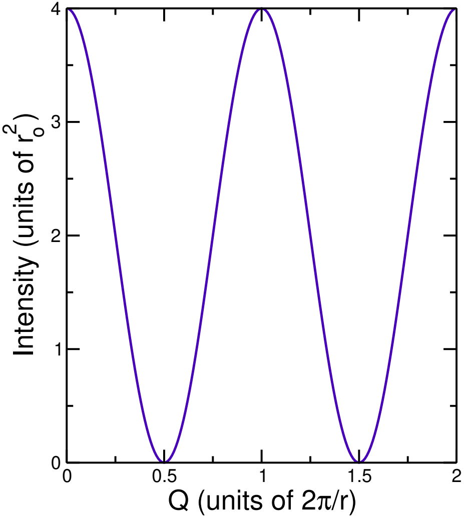

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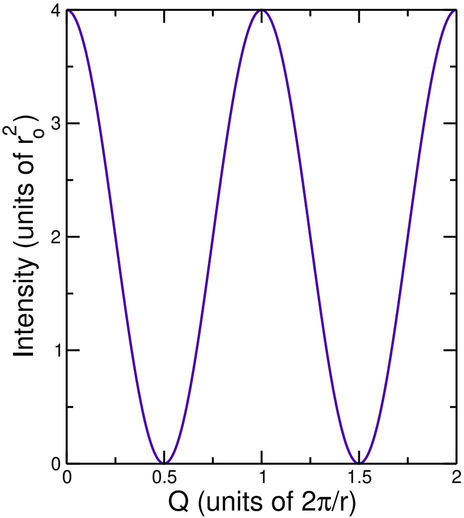

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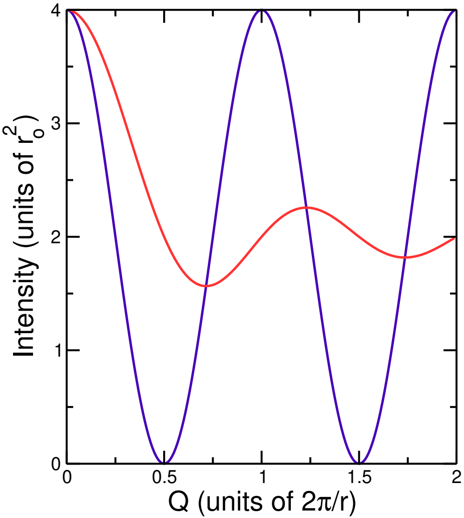

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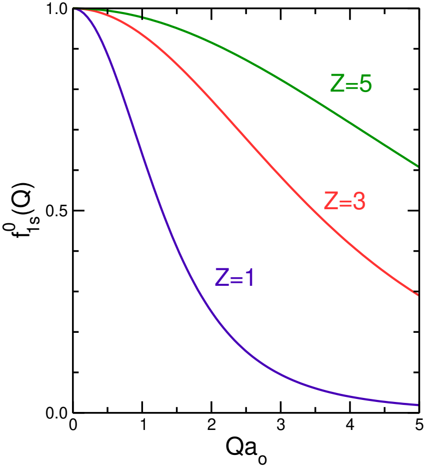

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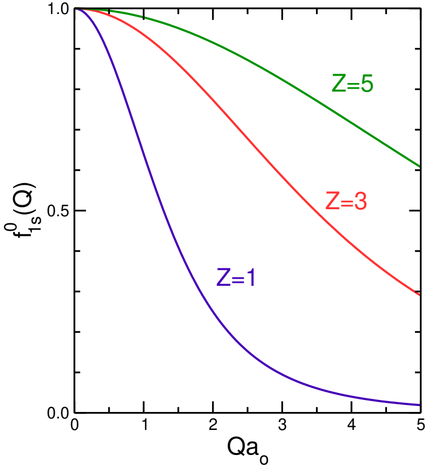

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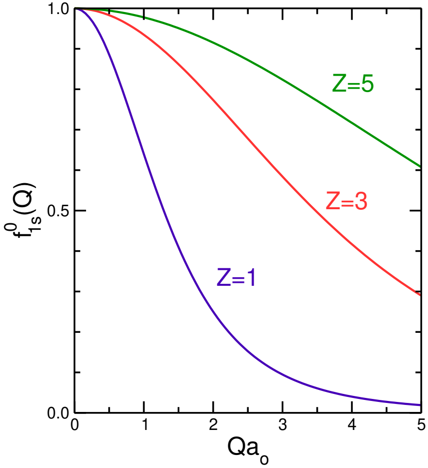

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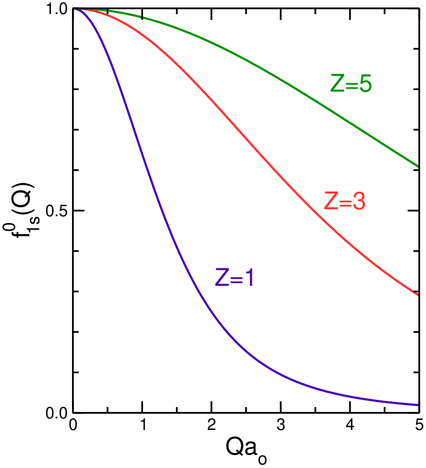

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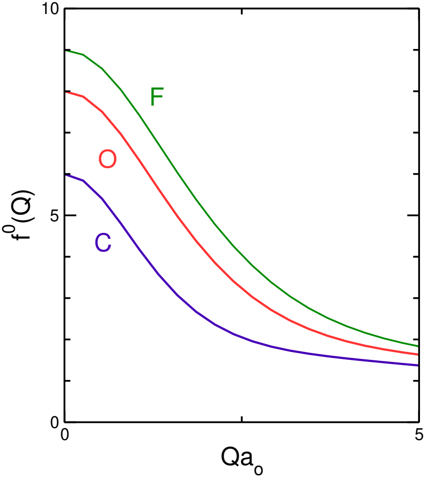

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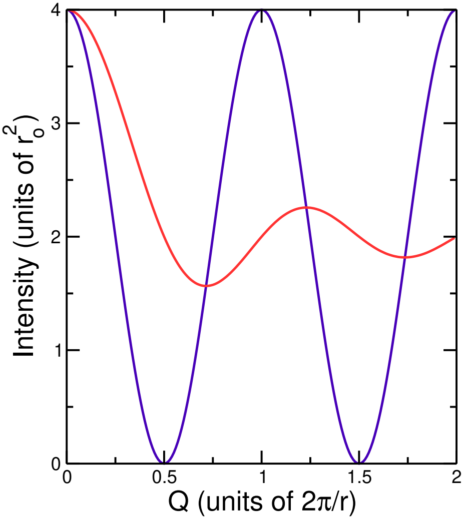
Previously we derived the scattering intensity from two localized electrons both fixed and randomly oriented to the x-rays



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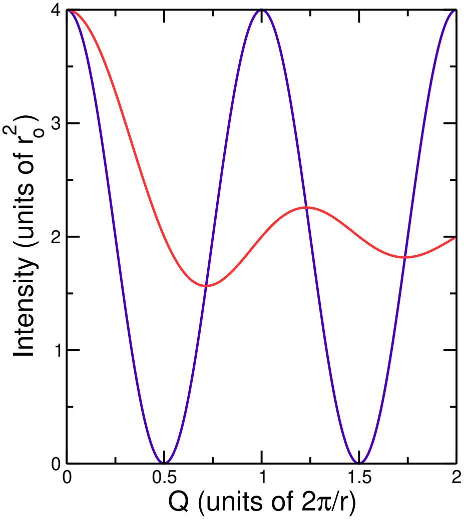

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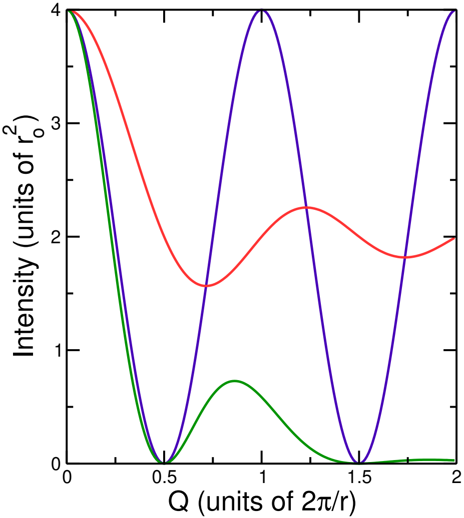

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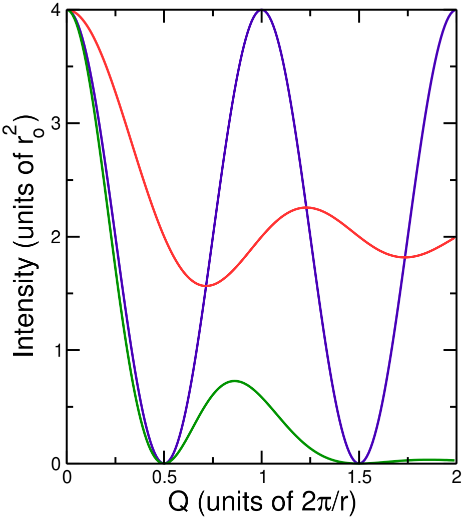

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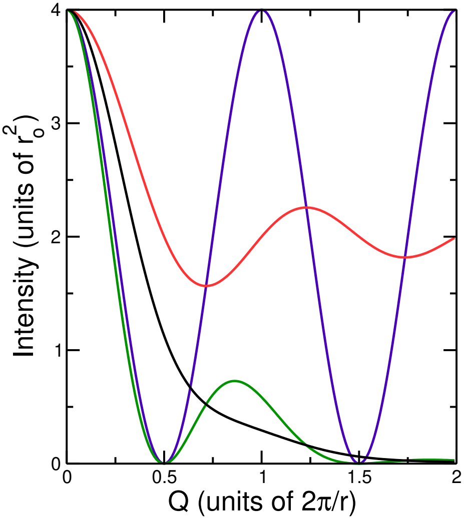

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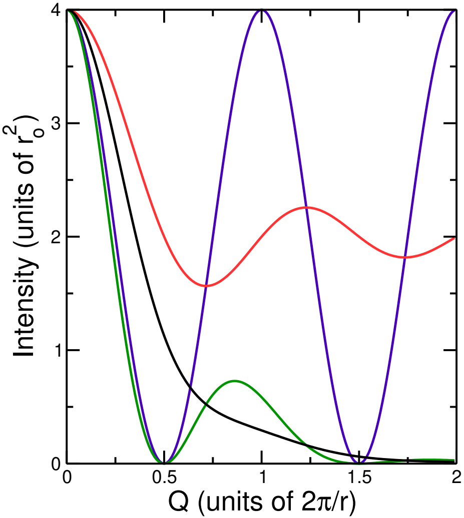

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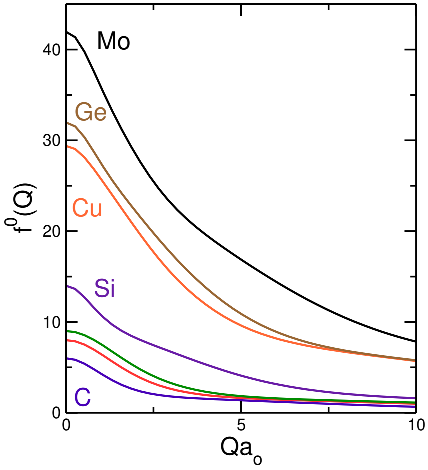

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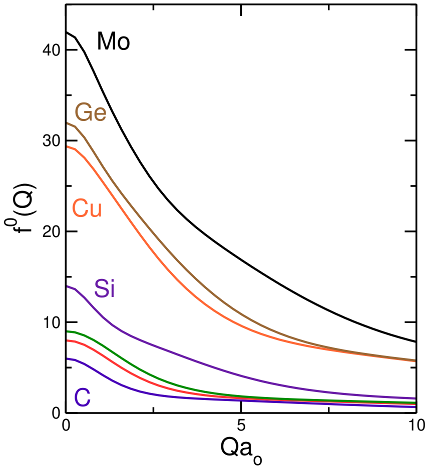

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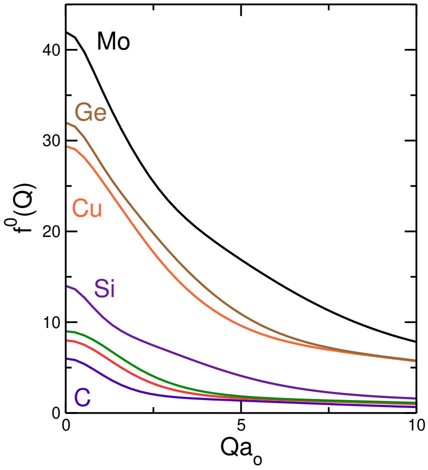

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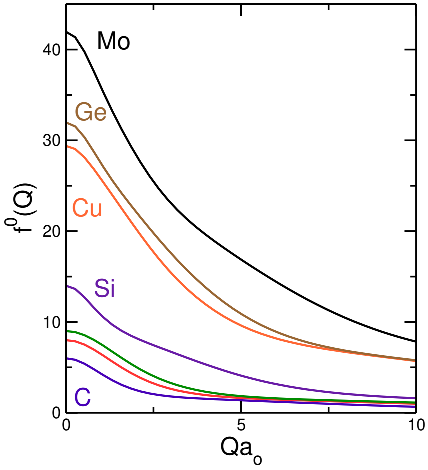

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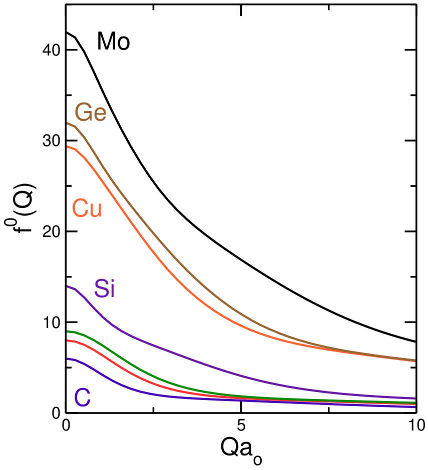

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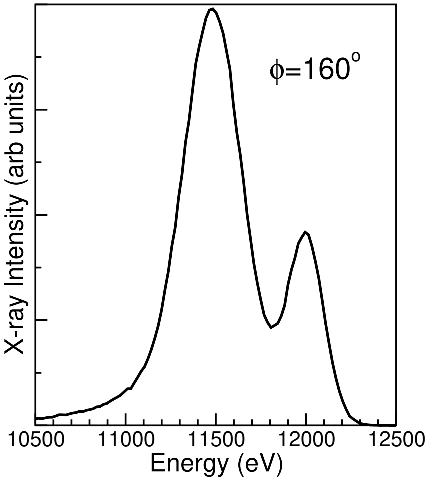

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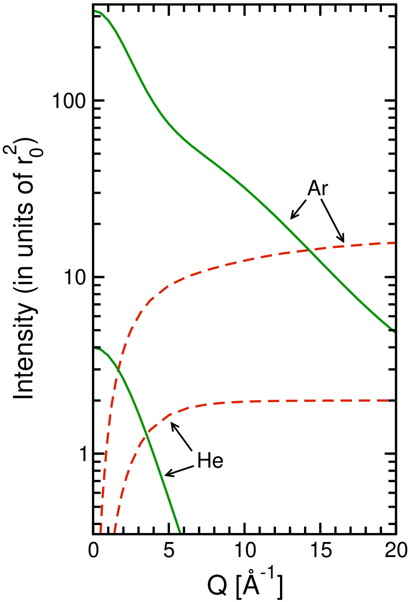

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