Today's outline - February 04, 2020

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- Limiting cases of Fresnel equations

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- Parratt's exact recursive calculation

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Reading Assignment: Chapter 3.5–3.8

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Reading Assignment: Chapter 3.5–3.8

Homework Assignment #02:
Problems on Blackboard
due Tuesday, February 18, 2020

Fresnel equation review

The scattering vector (or momentum transfer) is given by

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$$r = \frac{q - q'}{q + q'}, \qquad t = \frac{2q}{q + q'}$$

Limiting cases - $q \gg 1$

Start by rearranging Snell's Law $q^2 = q'^2 + 1 - 2ib_\mu$

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Start by rearranging Snell's Law and since q is real by definition, when $q \gg 1$

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$$q^2 = q'^2 + 1 - 2ib_\mu$$
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$$q'^2 \approx q^2 + 2ib_\mu$$

Limiting cases - $q \gg 1$

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this implies $Re(q') \approx q$,

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$$q' = q + i\,Im(q')$$

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$$q'^2 = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2$$

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$$q'^2 = q^2 \left(1 + i\,\frac{Im(q')}{q}\right)^2$$
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reflected wave in phase with incident, almost total transmission

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When $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

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$$\Lambda \approx \frac{1}{Q_c}$$

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The reflected wave is out of phase with the incident wave, there is only small transmission in the form of an evanescent wave, and the penetration depth is very short.

Limiting cases - $q \sim 1$

If $q \sim 1$,

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$$q'^2 \approx 2ib_\mu$$

Limiting cases - $q \sim 1$

If $q \sim 1$, adding and subtracting b_μ,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1)$$

Limiting cases - $q \sim 1$

If $q \sim 1$, adding and subtracting b_μ, yields that q' is complex with real and imaginary parts of equal magnitude.

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1)$$

$$= b_\mu(1 + i)^2$$

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The reflected wave is in phase with the incident, there is significant (larger amplitude than the reflection) transmission with a large penetration depth.

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We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium.

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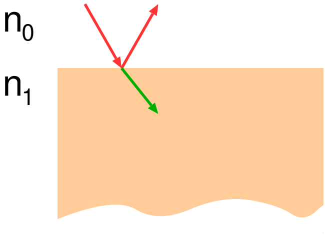

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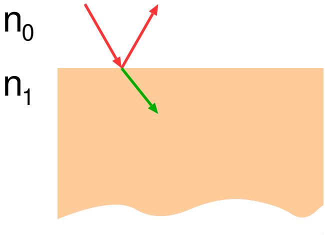

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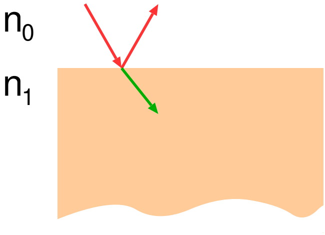

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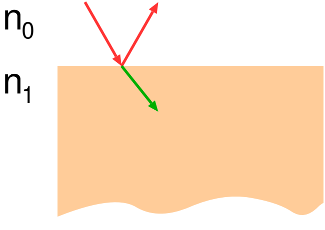

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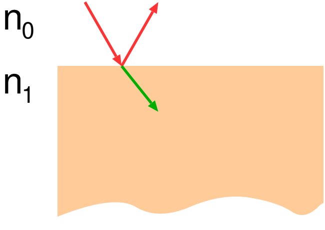

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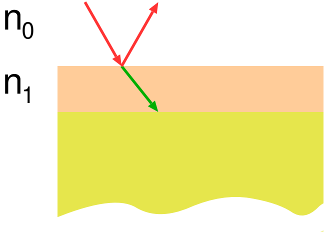

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We have assumed that the transmitted wave eventually attenuates to zero in all cases due to absorption. We now consider what happens if there is a second interface encountered by the transmitted wave before it dies away. That is, a thin slab of material on top of an infinite substrate

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:

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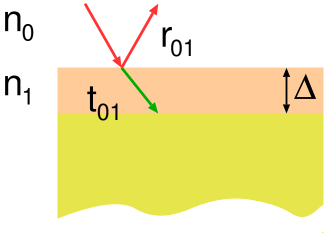

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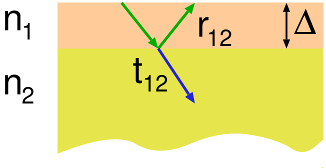

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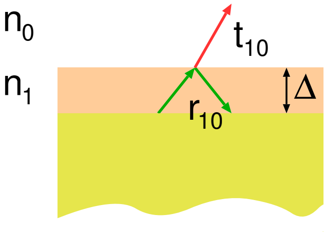

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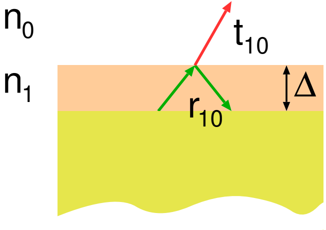

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Build the composite reflection coefficient from all possible events

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed

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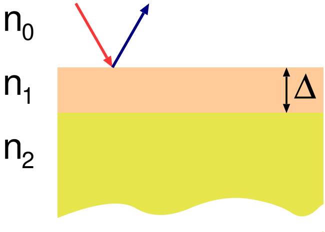

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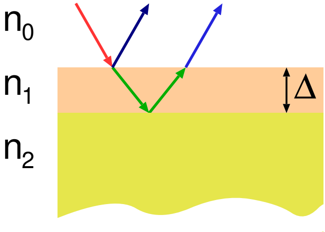

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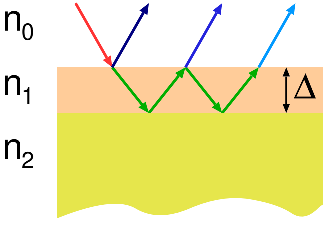

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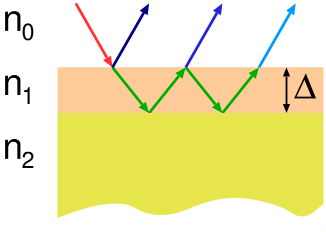

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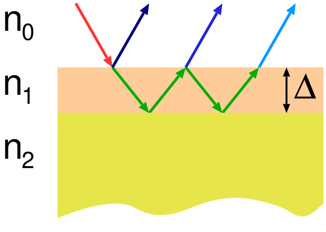

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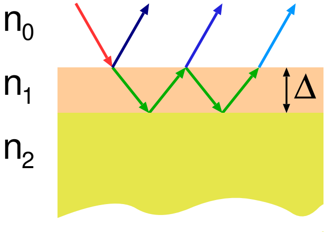

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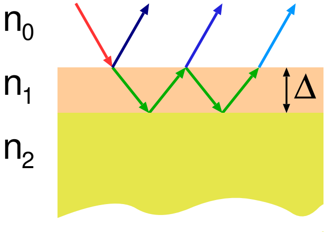

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$$= \frac{Q_0^2 - 2Q_0Q_1 + Q_1^2 + 4Q_0Q_1}{(Q_0 + Q_1)^2} = \frac{Q_0^2 + 2Q_0Q_1 + Q_1^2}{(Q_0 + Q_1)^2} = \frac{(Q_0 + Q_1)^2}{(Q_0 + Q_1)^2} = 1$$

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Starting with the reflection coefficient of the slab obtained earlier

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$$p^2 = e^{iQ_1\Delta}$$

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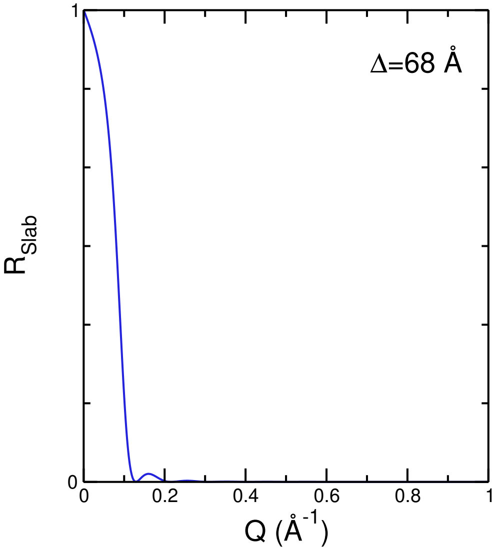

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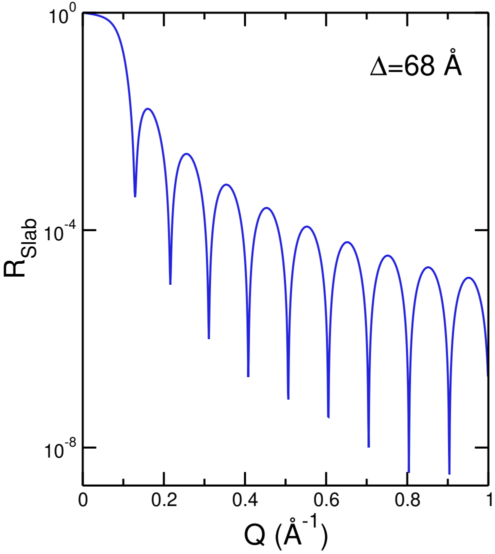
These are Kiessig fringes which arise from interference between reflections at the top and bottom of the slab.



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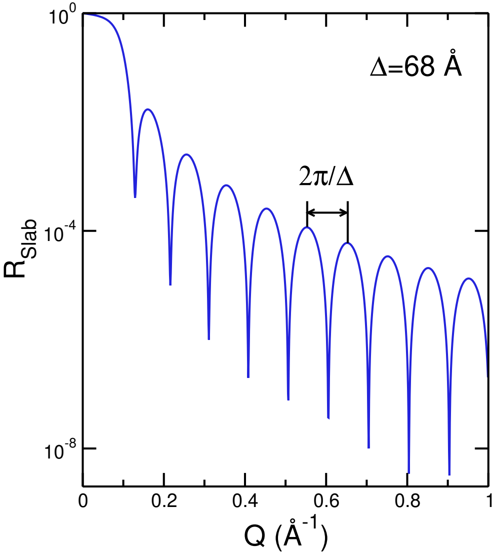
$$2\pi/\Delta = 0.092\text{Å}^{-1}$$



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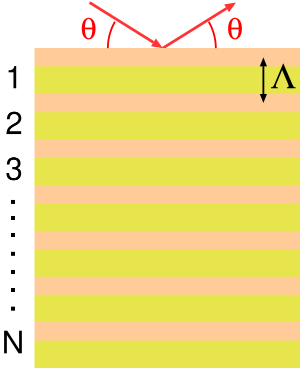
Multilayers in the kinematical regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

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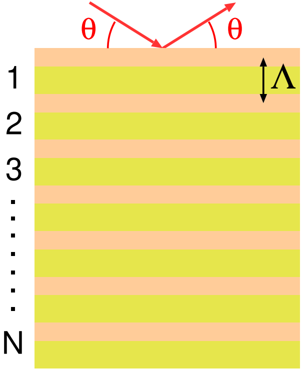

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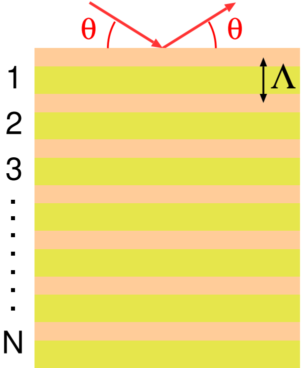

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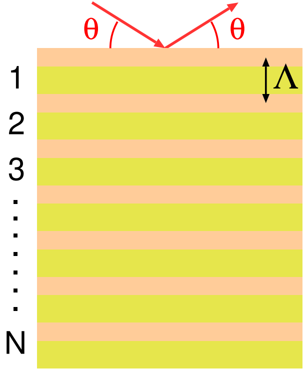

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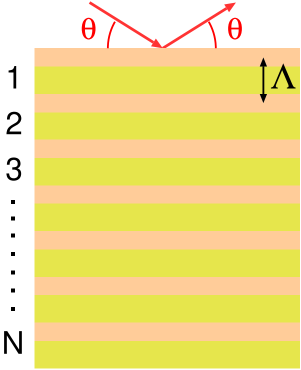

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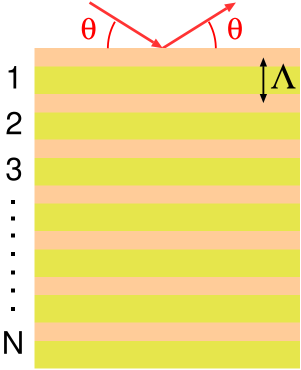

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Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components

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Absorption coefficient of a bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_0\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin\left(\pi\Gamma\zeta\right)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

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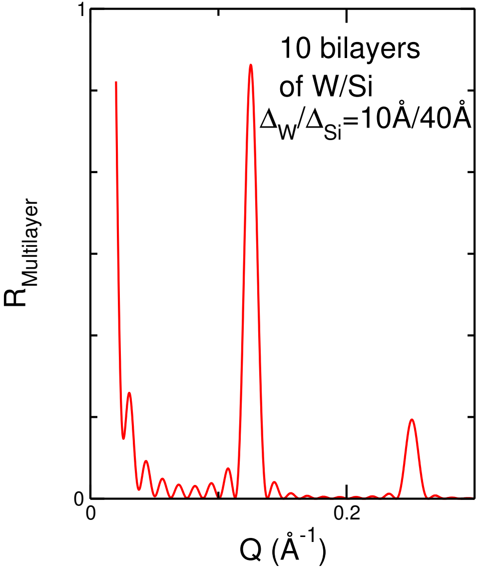
Reflectivity calculation



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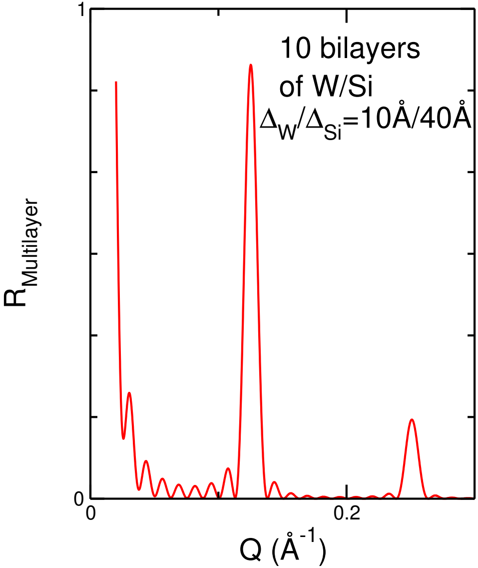

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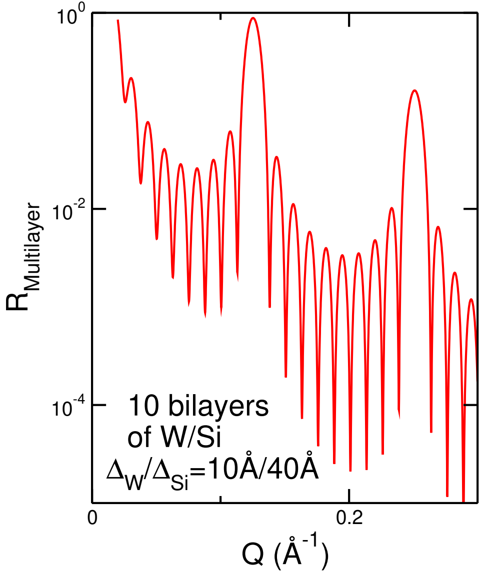

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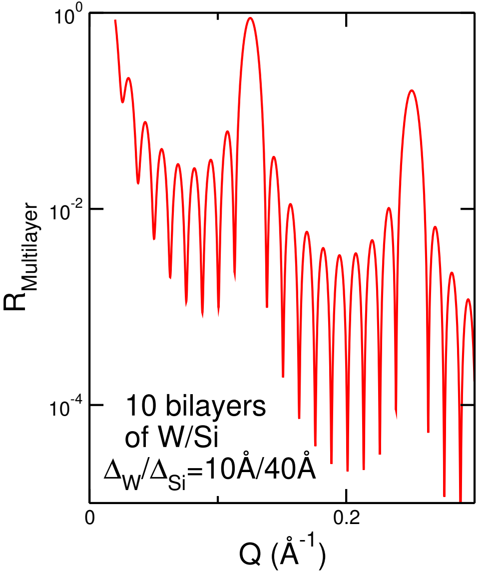

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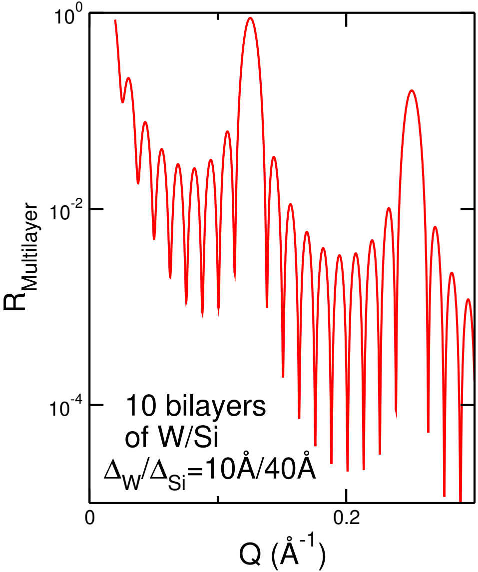

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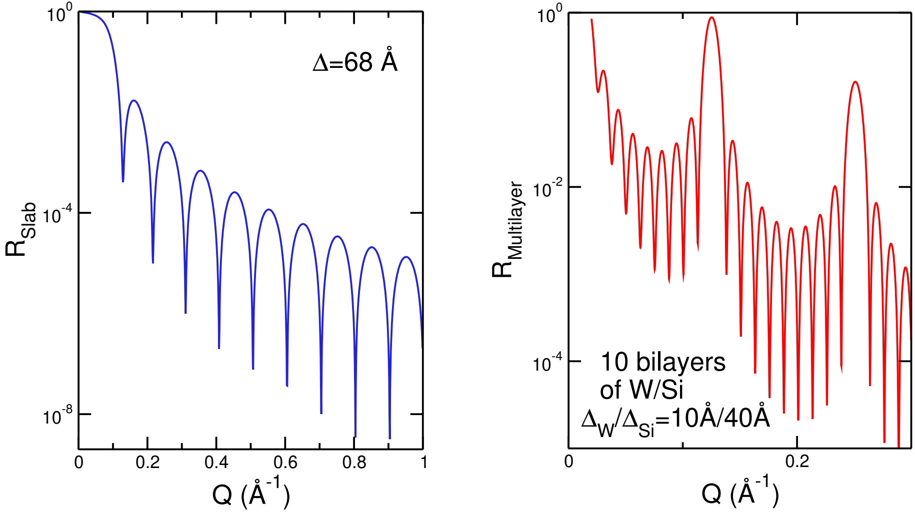
- Multilayers are used commonly on laboratory sources as well as at synchrotrons as mirrors

Slab - multilayer comparison



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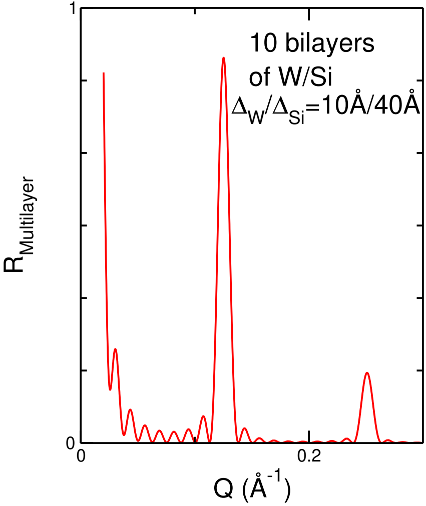

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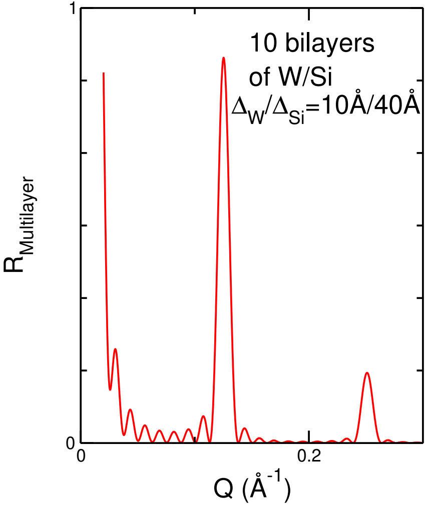

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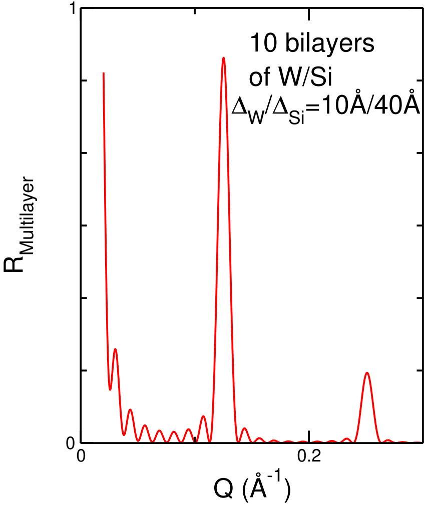

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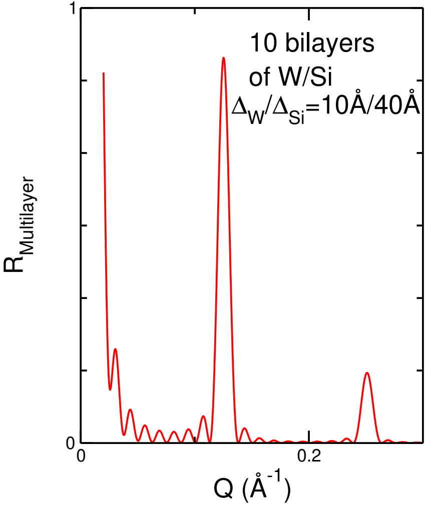

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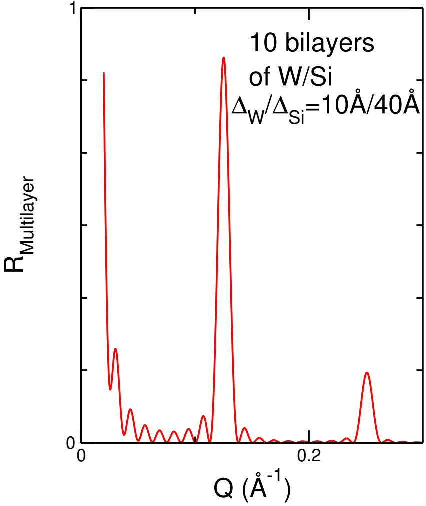

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This is Parratt's recursive approach and needs to be computed numerically

Parratt's recursive method

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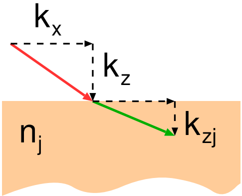
Because of continuity, $k_{xj} = k_x$ and therefore, we can compute the z-component of $\vec{k}_j$



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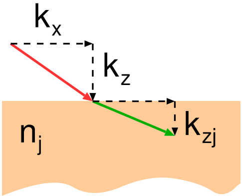

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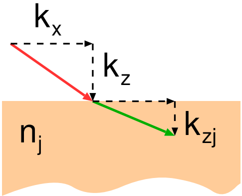

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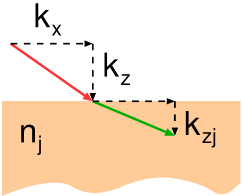

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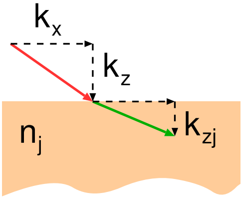

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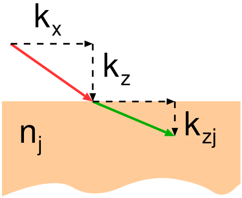

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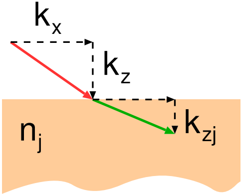

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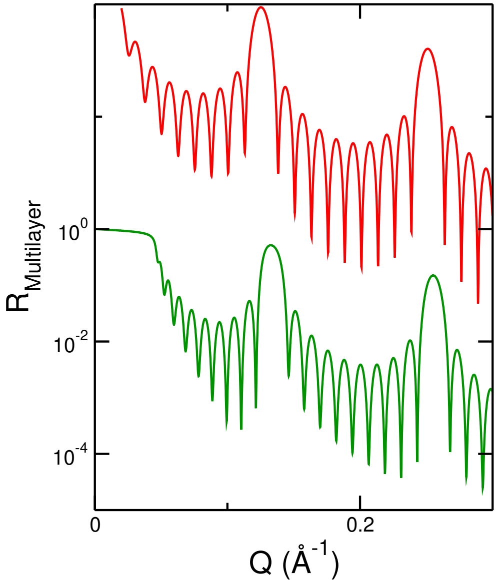
The recursive relation can be seen from the calculation of reflectivity of the next layer up

$$r_{N-2,N-1} = \frac{r'_{N-2,N-1} + r_{N-1,N} p_{N-1}^2}{1 + r'_{N-2,N-1} r_{N-1,N} p_{N-1}^2}$$

Kinematical - Parratt comparison



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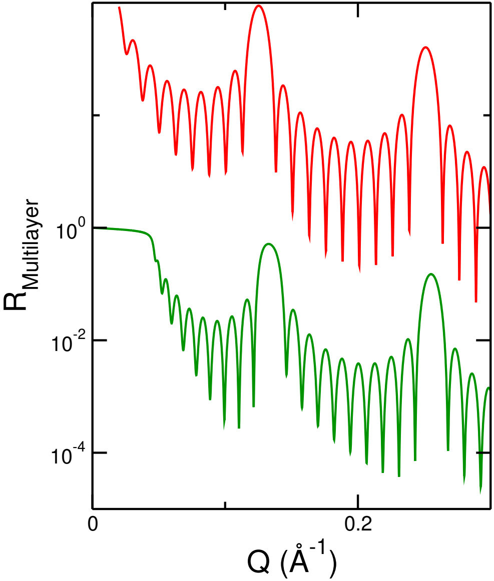

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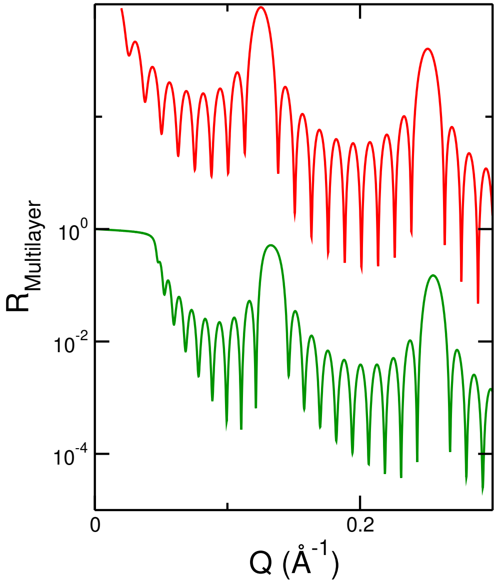

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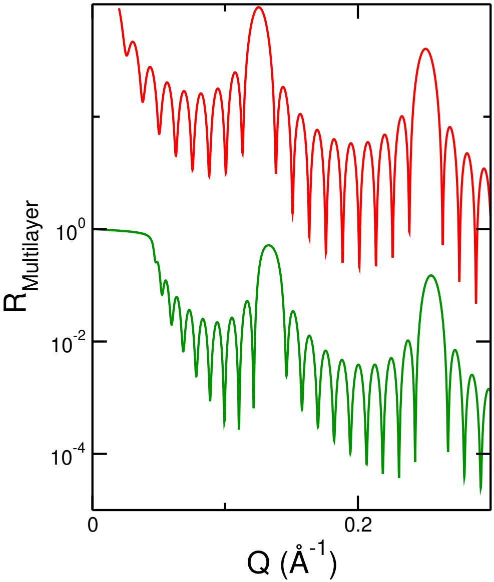

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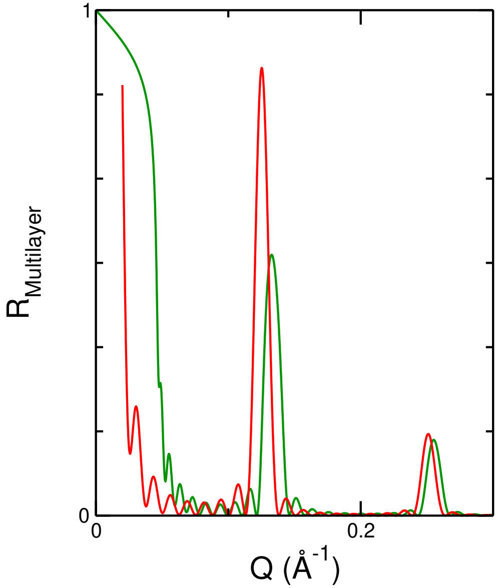

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Peaks in kinematical calculation are somewhat higher reflectivity than true value.