Today's outline - January 30, 2020

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Reading Assignment: Chapter 3.4

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Reading Assignment: Chapter 3.4

Homework Assignment #02:
Problems to be provided
due Tuesday, February 18, 2020

HW #02

1. Knowing that the photoelectric absorption of an element scales as the inverse of the energy cubed, calculate:

(a) the absorption coefficient at 10keV for copper when the value at 5keV is 1698.3 cm^{-1};

(b) The actual absorption coefficient of copper at 10keV is 1942.1 cm^{-1}, why is this so different than your calculated value?

2. A 30 cm long, ionization chamber, filled with 80% helium and 20% nitrogen gases at 1 atmosphere, is being used to measure the photon rate (photons/sec) in a synchrotron beamline at 12 keV. If a current of 10 nA is measured, what is the photon flux entering the ionization chamber?

3. A 5 cm deep ionization chamber is used to measure the fluorescence from a sample containing arsenic (As). Using any noble gases or nitrogen, determine a gas fill (at 1 atmosphere) for this chamber which absorbs at least 60% of the incident photons. How does this change if you are measuring the fluorescence from ruthenium (Ru)?

HW #02

4. Calculate the critical angle of reflection of 10 keV and 30 keV x-rays for:

(a) A slab of glass (SiO_2);

(b) A thick chromium mirror;

(c) A thick platinum mirror.

(d) If the incident x-ray beam is 2 mm high, what length of mirror is required to reflect the entire beam for each material?

5. Calculate the fraction of silver (Ag) fluorescence x-rays which are absorbed in a 1 mm thick silicon (Si) detector and the charge pulse expected for each absorbed photon. Repeat the calculation for a 1 mm thick germanium (Ge) detector.

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The most advanced detectors can easily cost over a million dollars!

CCD detectors - direct

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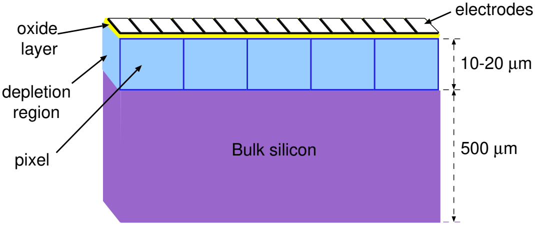

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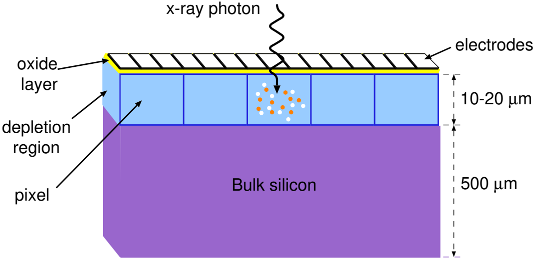

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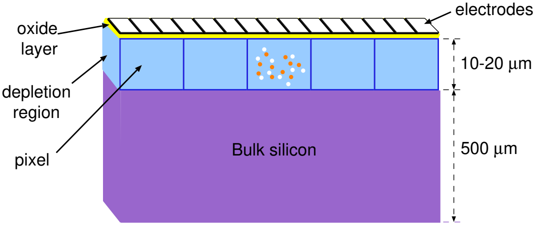

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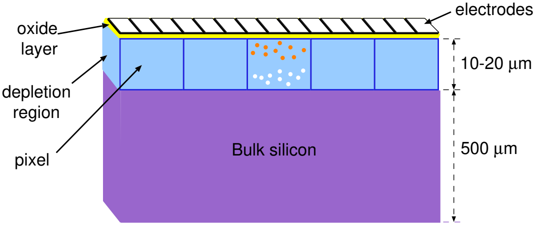

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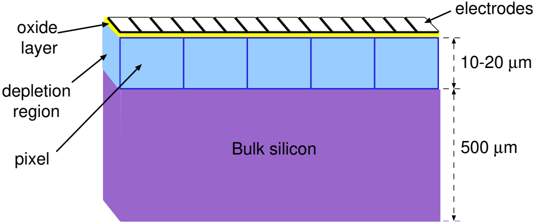

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expensive to make very large, limited sensitivity to high energies

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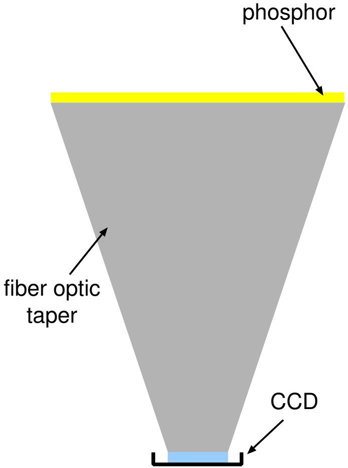

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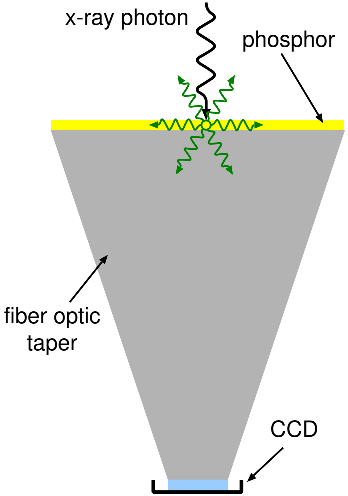

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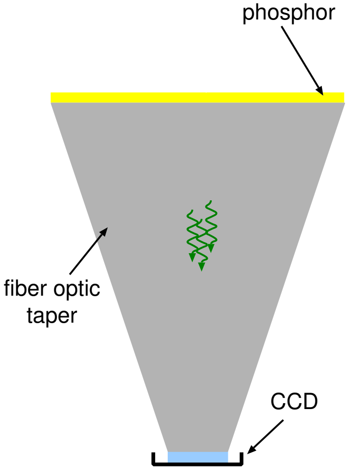

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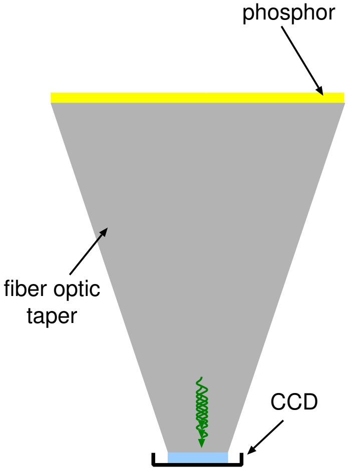

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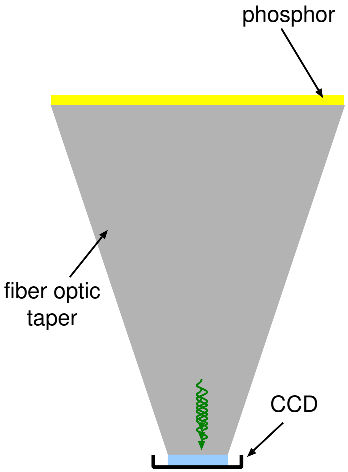
Pixel sizes are usually rather large (50 μm $\times$ 50 μm)



Pixel array detectors - schematic

The Pixel Array Detector combines area detection with on-board electronics for fast signal processing

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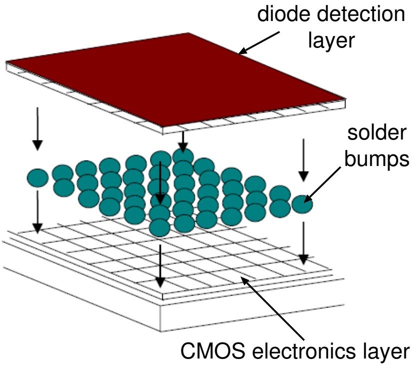

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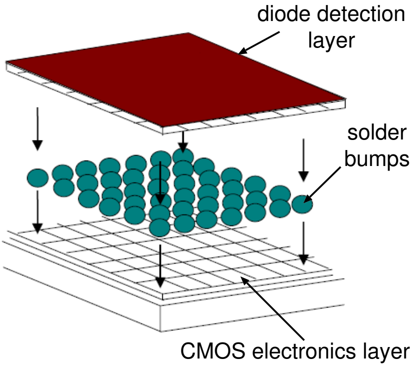

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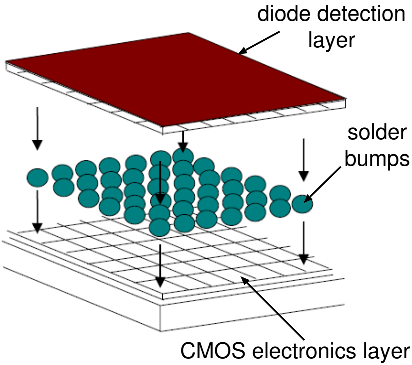

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This permits fast processing and possibly energy discrimination on a per-pixel level

Pixel array detectors - Pilatus

Pixel array detector with 1,000,000 pixels.

Each pixel has energy resolving capabilities & high speed readout.

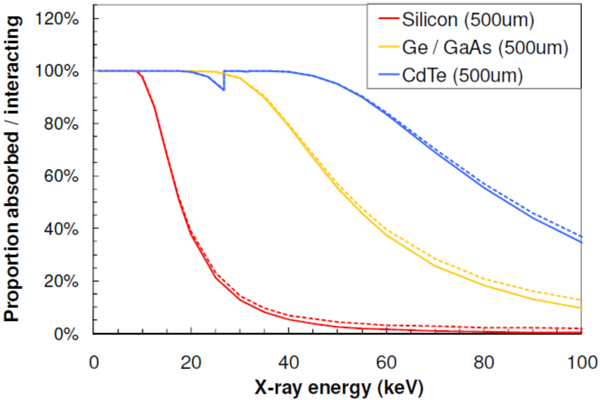
Silicon sensor limits energy range of operation.

from Swiss Light Source

High energy solutions

One of the major problems with pixel array detectors and SDDs is the low absorption cross section at high energies

High energy solutions



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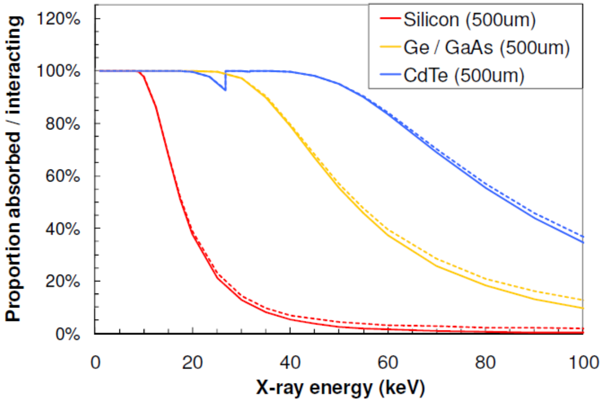

One of the major problems with pixel array detectors and SDDs is the low absorption cross section at high energies

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The absorption can be significantly enhanced with these higher Z elements while maintaining good energy discrimination capabilities.

Refractive index in the x-ray region

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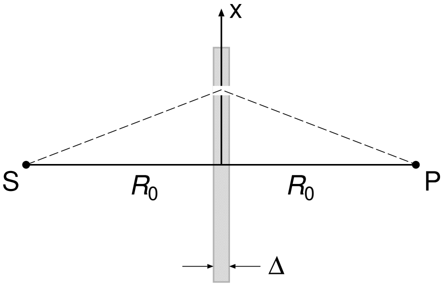
Initially assume that all interfaces are perfectly flat and ignore all absorption processes.

Thin plate response - scattering approach

Consider a thin plate of thickness Δ onto which x-rays are incident from a point source S a perpendicular distance R_0 away.



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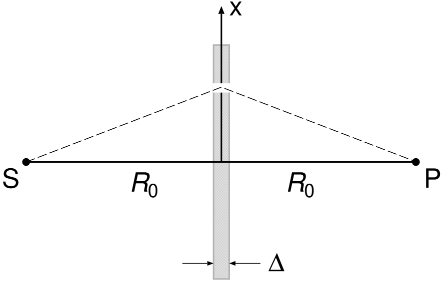

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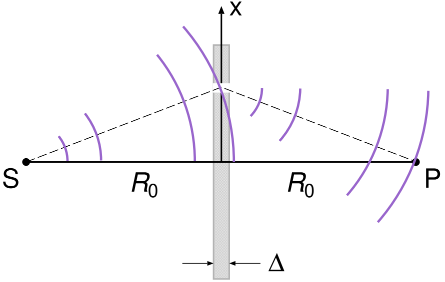

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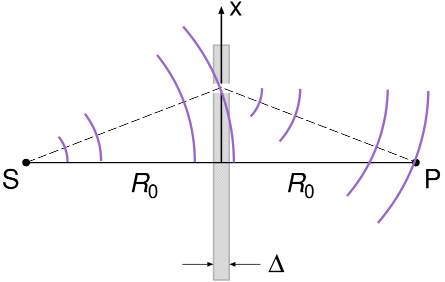

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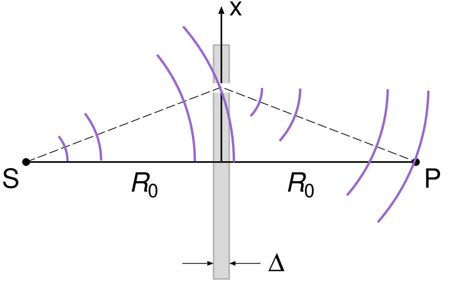

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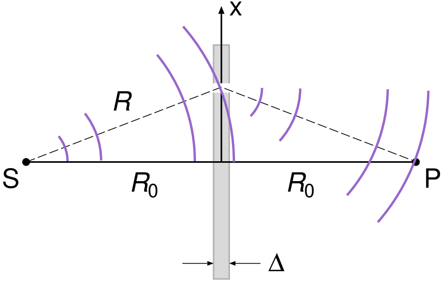

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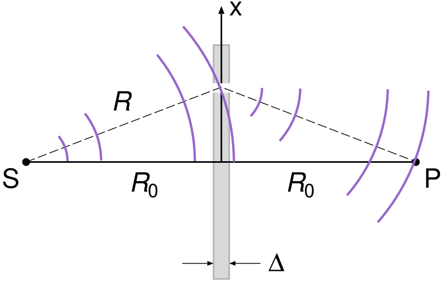

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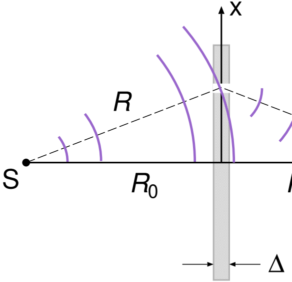

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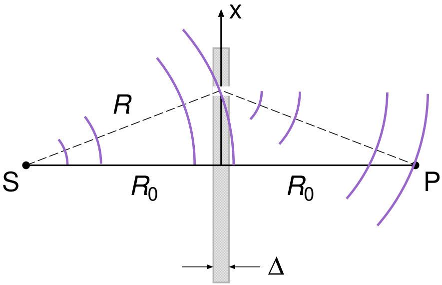
R is also the distance between the scattering volume and P so, a wave (x-ray) which travels from $S \to P$ through the scattering volume will have an extra phase shift



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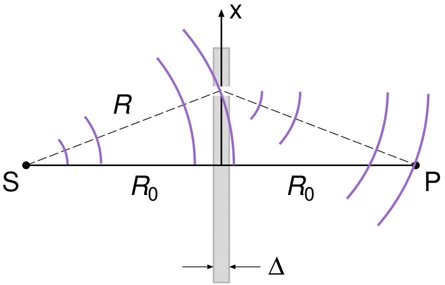

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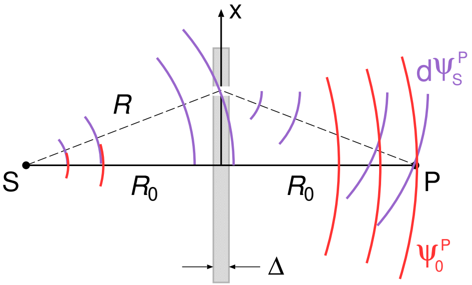

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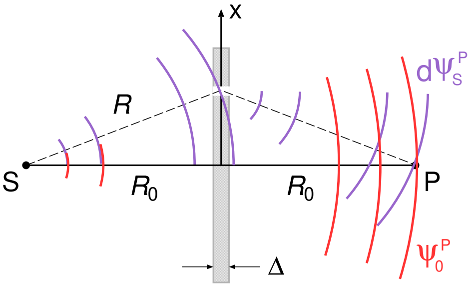

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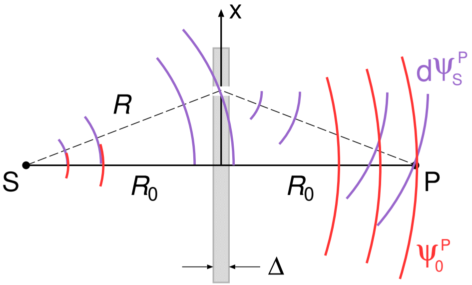

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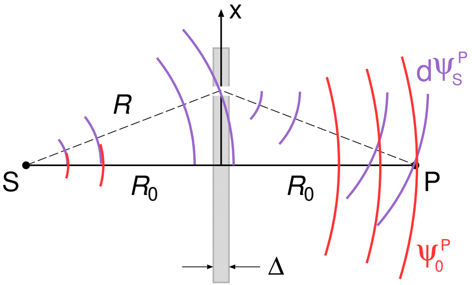

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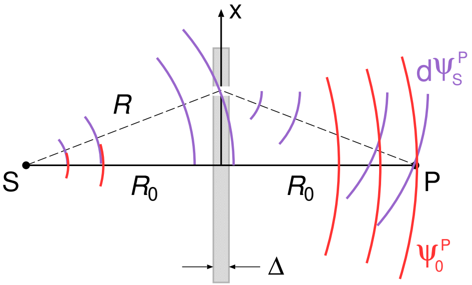

$$\phi(x,y) = 2k\frac{x^2+y^2}{2R_0^2} = \frac{x^2+y^2}{R_0^2}k$$

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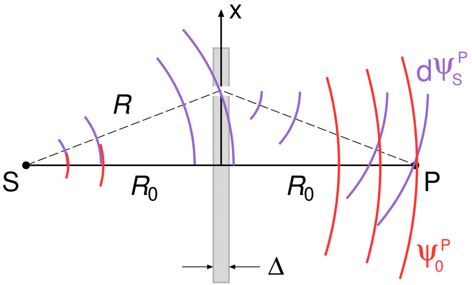

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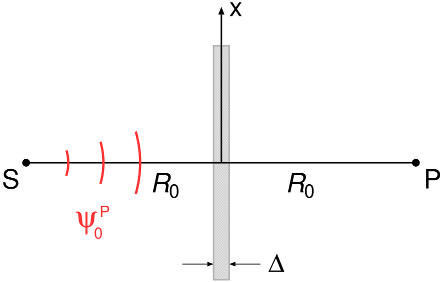

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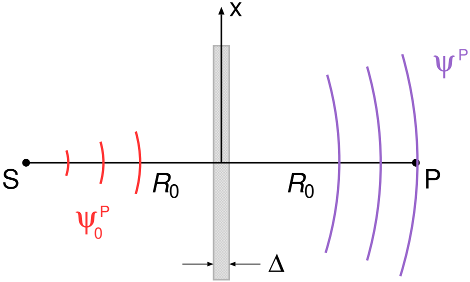

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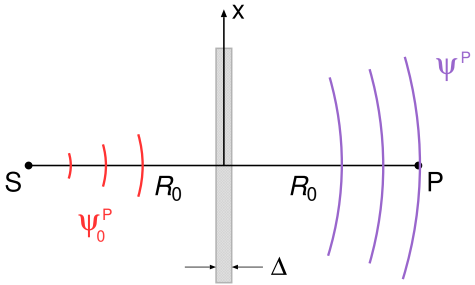

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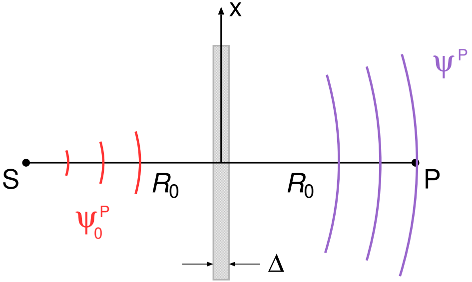

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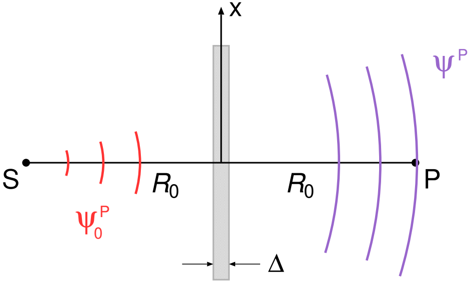

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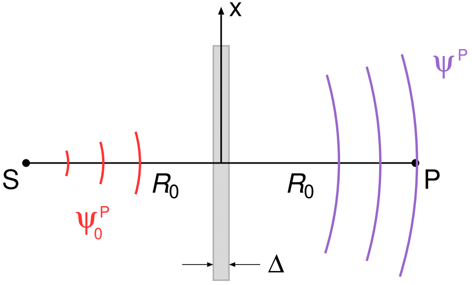

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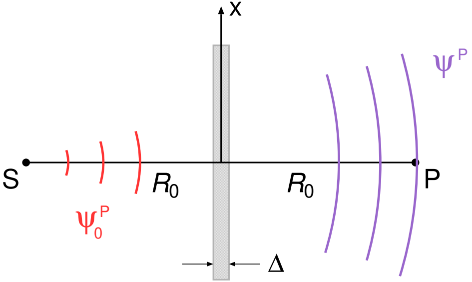

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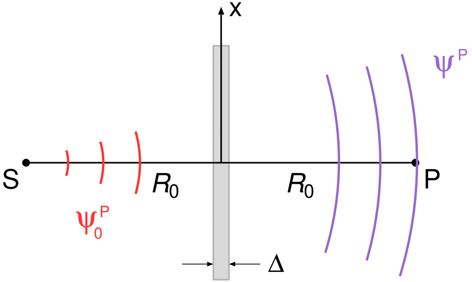

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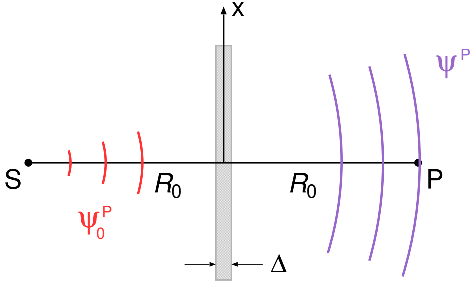

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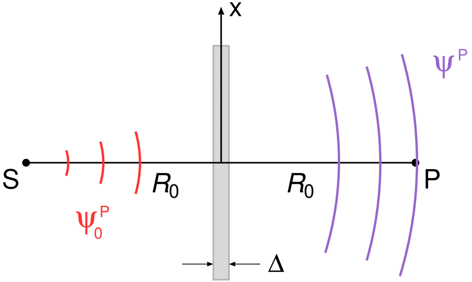

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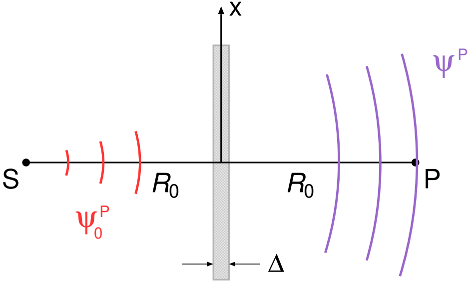

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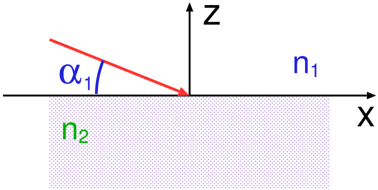

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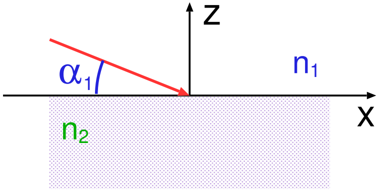

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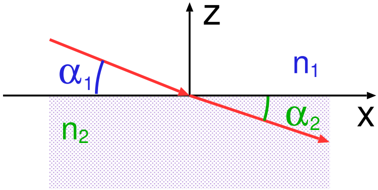

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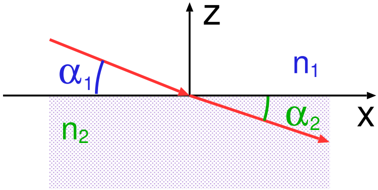

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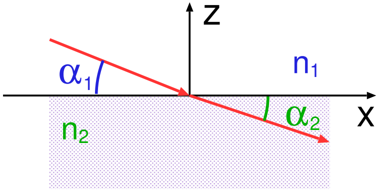

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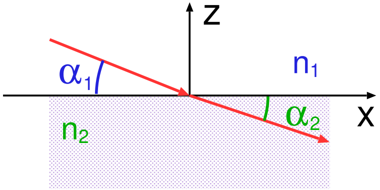

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If we now apply the known form of the index of refraction for the medium ($n_2 = 1 - \delta$).

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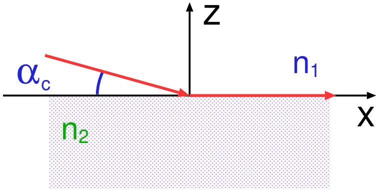

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When the incident angle becomes small enough, there will be total external reflection

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$$\delta \approx \frac{{\alpha_c}^2}{2}$$

$$\alpha_c = \sqrt{2\delta}$$

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Estimation of critical angle

For small angles, the cosine function can expanded to give a simple relation for the critical angle

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$$
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&= 0.26^\circ
\end{aligned}
$$

Connection to atomic scattering

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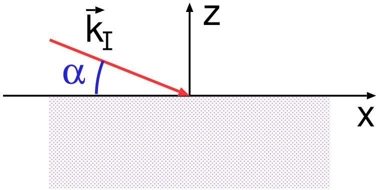
$$
\begin{aligned}
f'' &= -\frac{k^2}{2\pi\rho_a r_0}\frac{\mu}{2k} \\
&= -\frac{k}{4\pi r_0}\sigma_a
\end{aligned}
$$

Electromagnetic boundary conditions

Maxwell's equations require that an electromagnetic wave and its derivative be continuous in all directions at any interface. This condition places restrictions on the waves which exist at any interface:



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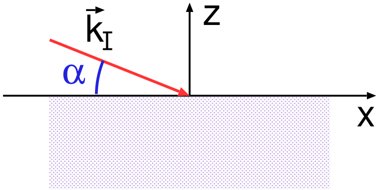

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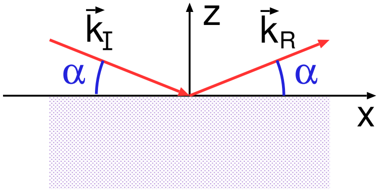

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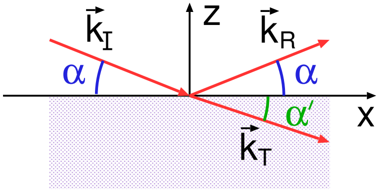

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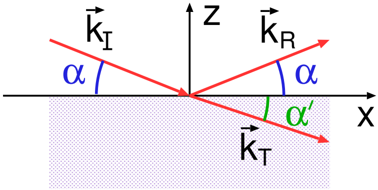

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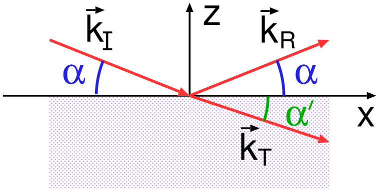

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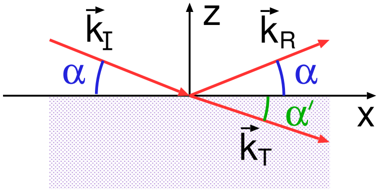

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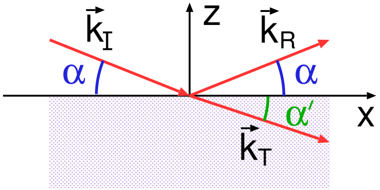

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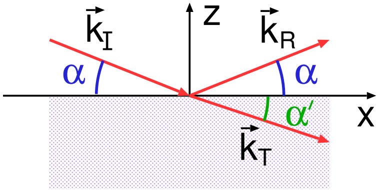

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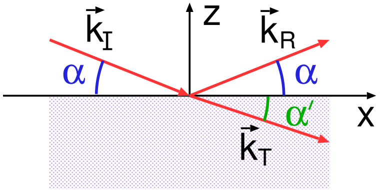

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Parallel projection & Snell's Law

Starting with the equation for the parallel projection of the field on the surface and noting that

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q is a convenient parameter to use because it is a combination of two parameters which are often varied in experiments, the angle of incidence α and the wavenumber (energy) of the x-ray, k.

Defining equations in q

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$$q^2 = q'^2 + 1 - 2ib_\mu$$

Similarly, we convert the reflection and transmission coefficients.

$$r = \frac{q - q'}{q + q'} \qquad t = \frac{2q}{q + q'}$$