Today's outline - January 23, 2020

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- The bending magnet source

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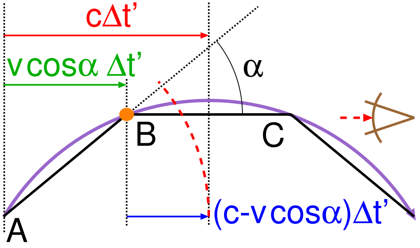
Homework Assignment #01:
Chapter Chapter 2: 2,3,5,6,8
due Thursday, January 30, 2020

Segmented arc review

The first approximation to a bending magnet source is the segmented arc



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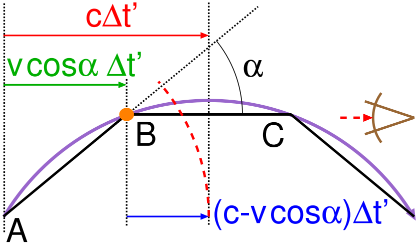

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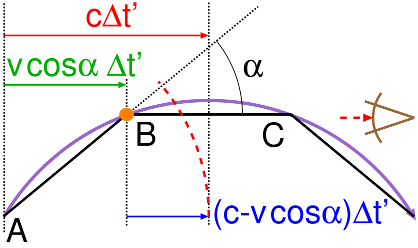

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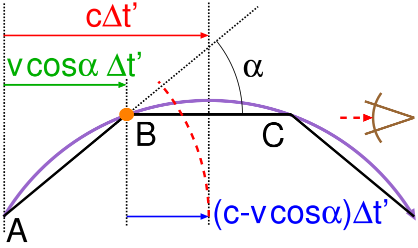

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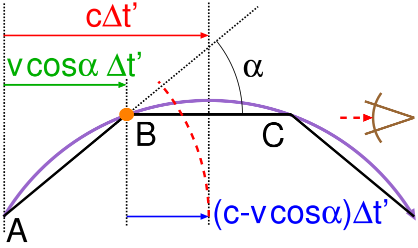

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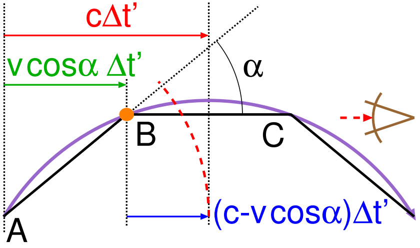

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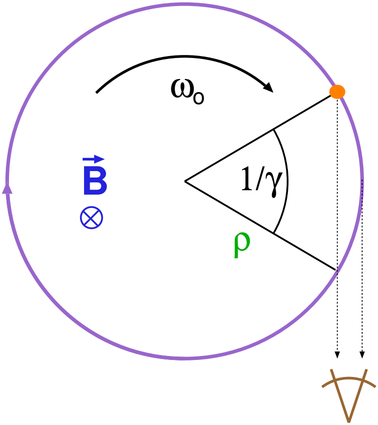
$$\frac{\Delta t}{\Delta t'} = (1 - \beta \cos \alpha)$$

Curved arc emission



But instantaneously, the compression ratio is:

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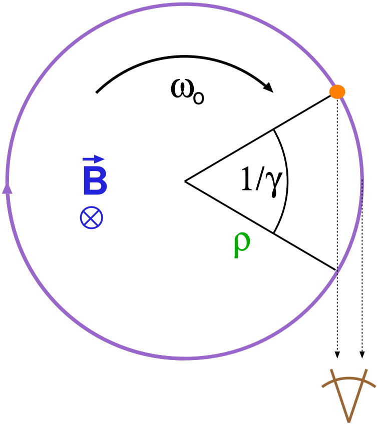

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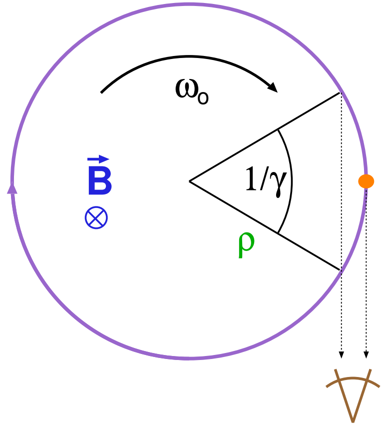

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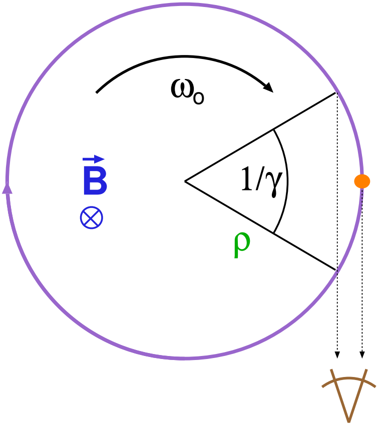

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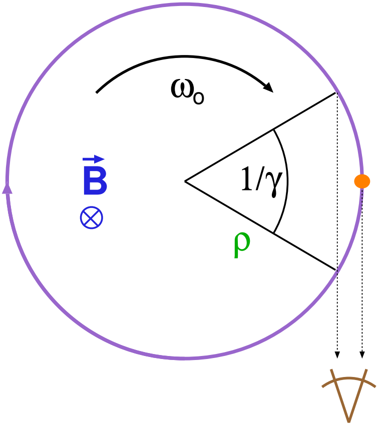

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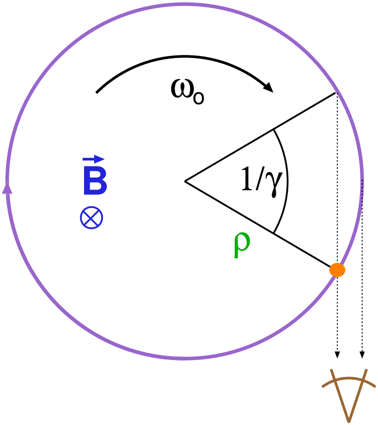

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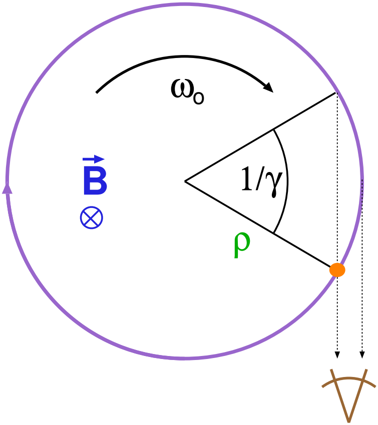

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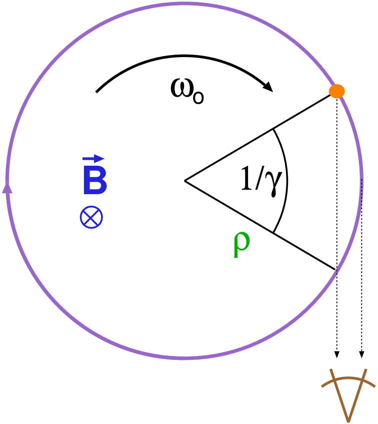
Electron bending radius



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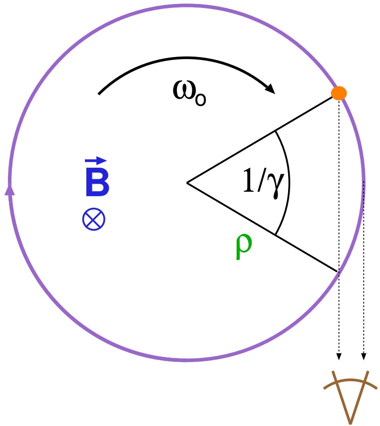

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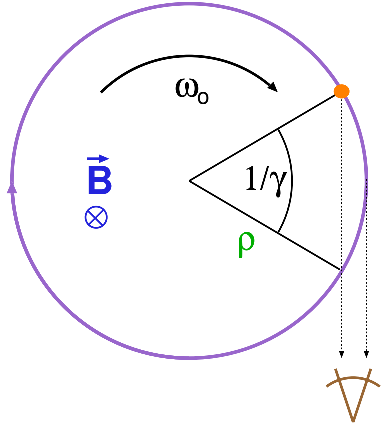

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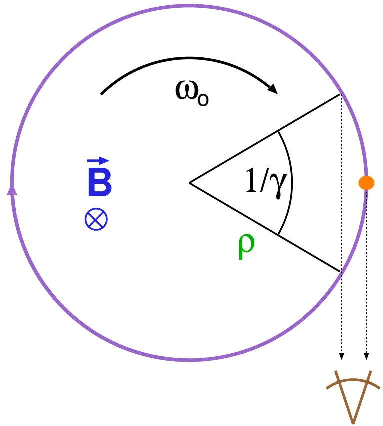

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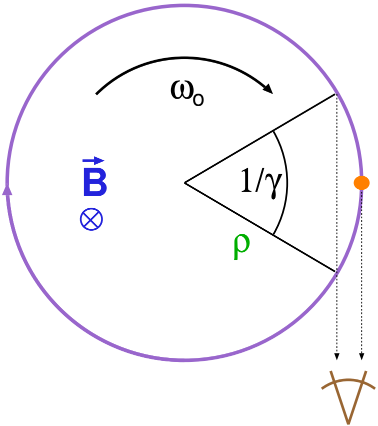

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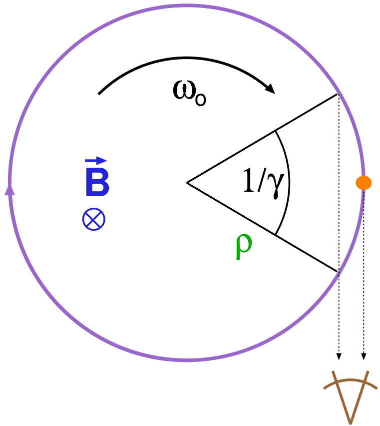

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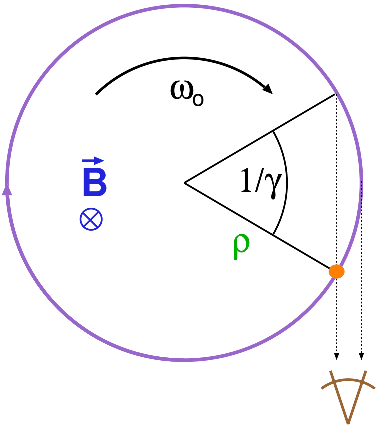

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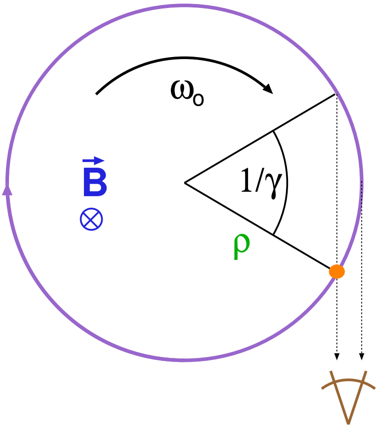

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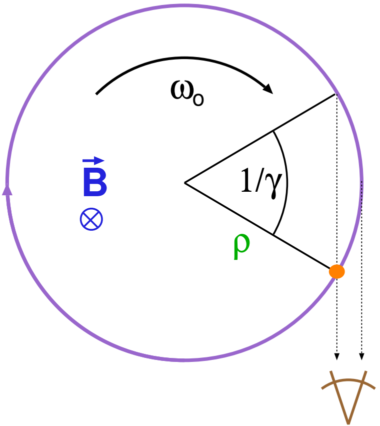

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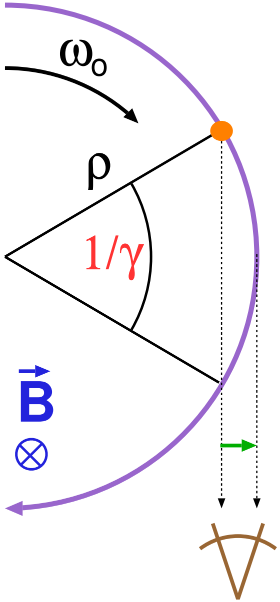
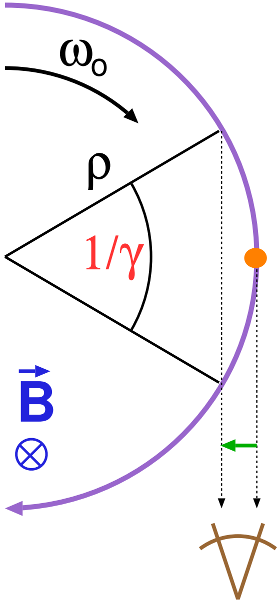


The observer, looking in the plane of the circular trajectory,

Curved arc emission



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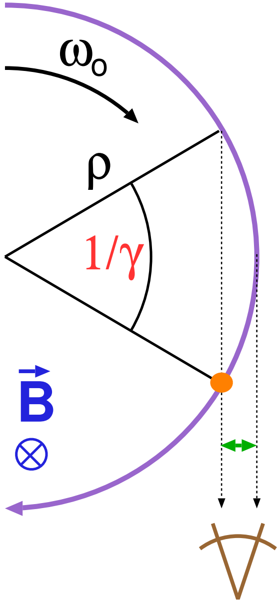

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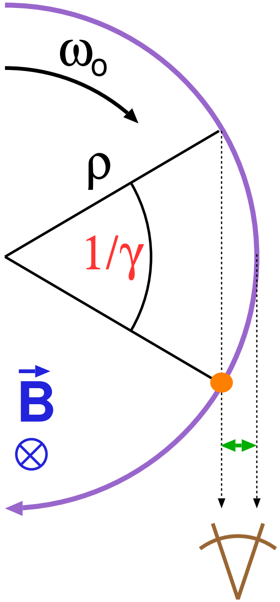

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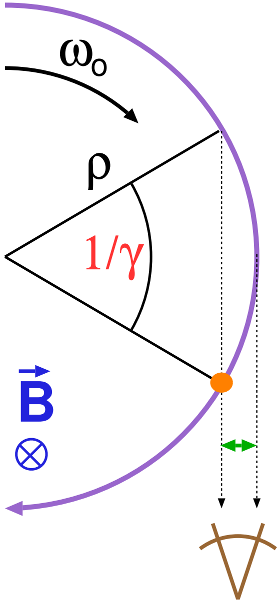

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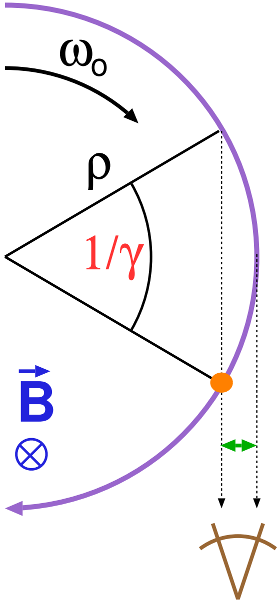

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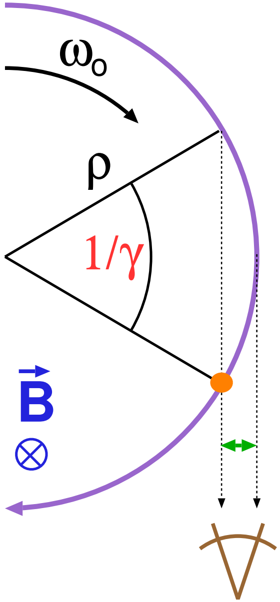

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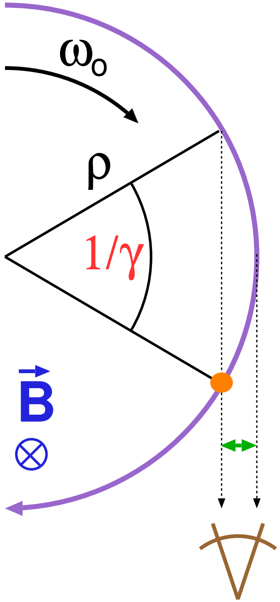

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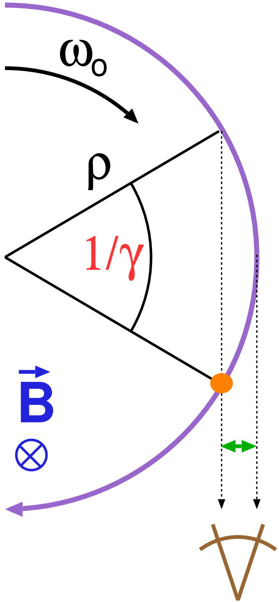

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The Fourier transform of this pulse is the spectrum of the radiation from the bending magnet.

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converting to storage ring units

Characteristic Energy of a Bending Magnet

The radiation from a bending magnet is defined by it's characteristic frequency, ω_c which, when the calculation is performed rigorously is:

$$\omega_c = \frac{3}{2}\gamma^3\omega_0$$

but since T is the period of the rotation through the full circle of radius ρ

$$\omega_0 = \frac{2\pi}{T} = 2\pi\frac{c}{2\pi\rho} = \frac{c}{\rho} = \frac{ceB}{\gamma mc}$$

we can therefore calculate the characteristic energy $\mathcal{E}_c$

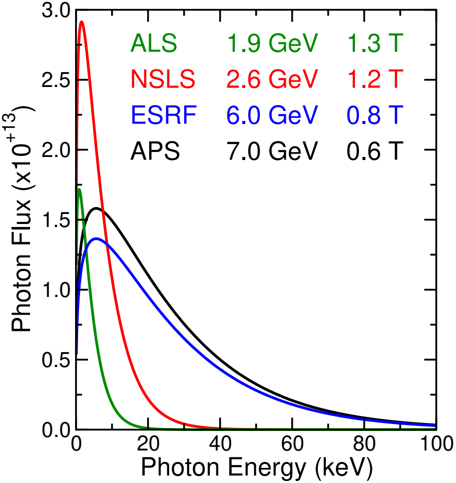
$$\mathcal{E}_c = \hbar\omega_c = \frac{3}{2}\hbar\gamma^3\frac{ceB}{\gamma mc} = \frac{3}{2}\hbar ceB\frac{\gamma^2}{mc} = \frac{3\hbar eB}{2m}\frac{\mathcal{E}^2}{(mc^2)^2}$$

converting to storage ring units

$$\mathcal{E}_c[\text{keV}] = 0.665\mathcal{E}^2[\text{GeV}]B[\text{T}]$$

Bending magnet spectrum

When the radiation pulse time is Fourier transformed, we obtain the spectrum of a bending magnet.



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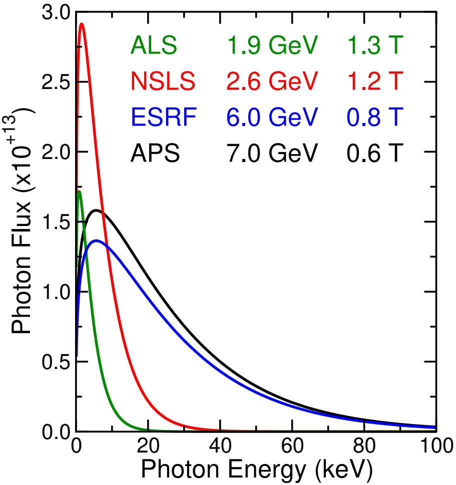

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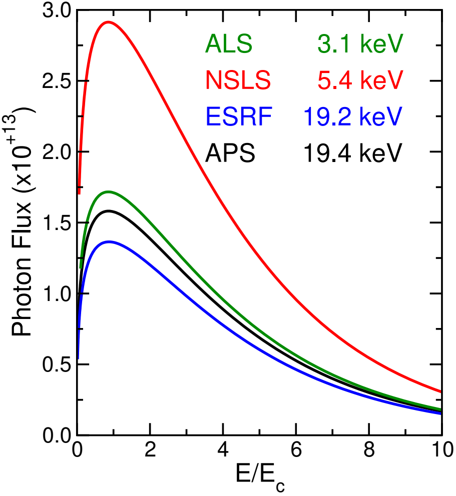

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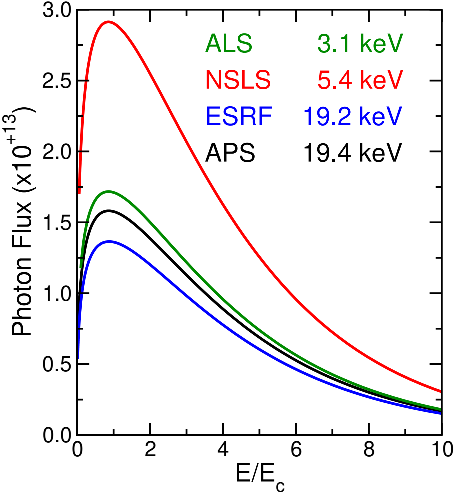
$$1.33\times10^{13}\mathcal{E}^2\, I \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\right)$$

where $K_{2/3}$ is a modified Bessel function of the second kind.



Power from a bending magnet

The radiated power is given in storage ring units by:

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We can calculate this for the ESRF where $\mathcal{E} = 6$ GeV, $B = 0.8$ T, $\mathcal{E}_c = 19.2$ keV and the bending radius $\rho = 24.8$ m. Assuming that the aperture is 1 mm^2 at a distance of 20 m, the angular aperture is $1/20 = 0.05$ mrad and the flux at the characteristic energy is given by:

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$$\text{Flux} = (1.95\times10^{13})(0.05^2\text{mrad}^2)(6^2\text{GeV}^2)(0.2\text{A}) = 3.5\times10^{11}\text{ph/s/0.1\%BW}$$

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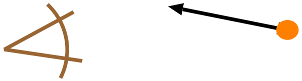
$$P = 1.266(6\text{GeV})^2(0.8\text{T})^2(1.24\times10^{-3}\text{m})(0.2\text{A}) = 7.3\text{W}$$

Polarization

A bending magnet also produces circularly polarized radiation



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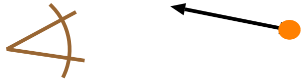

- If the observer is in the plane of the electron orbit, the electron motion looks like a half period of linear sinusoidal motion

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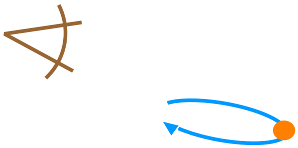

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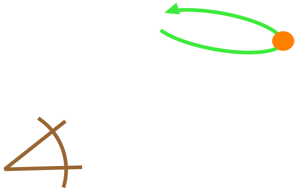

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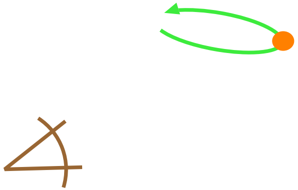

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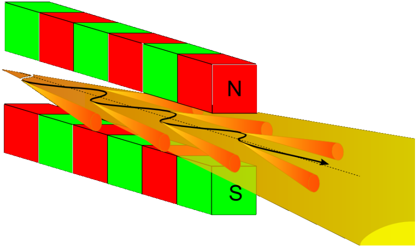
The result is circularly polarized radiation above and below the on-axis radiation.

Wigglers and undulators

Wiggler



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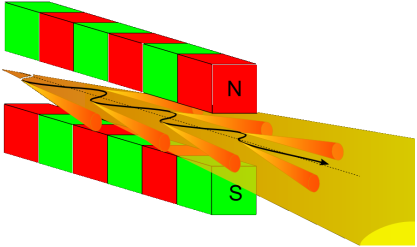

Like bending magnet except:

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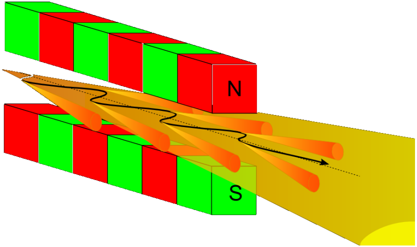

Like bending magnet except:

- larger $\vec{B} \rightarrow$ higher E_c

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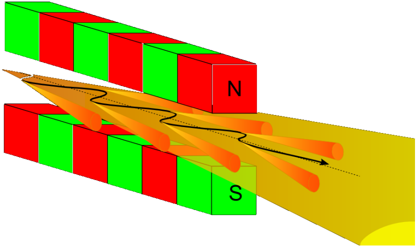

Like bending magnet except:

- larger $\vec{B} \rightarrow$ higher E_c
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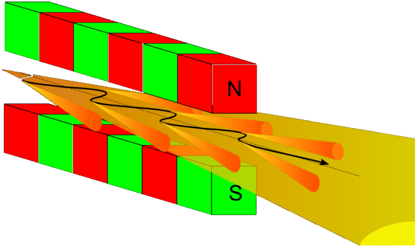
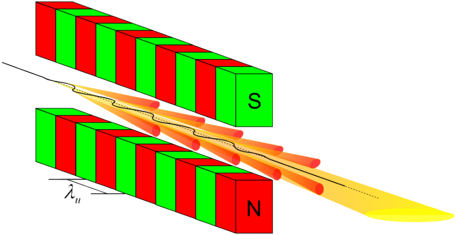


Undulator



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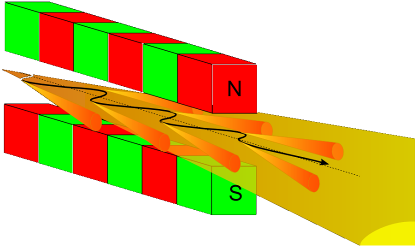

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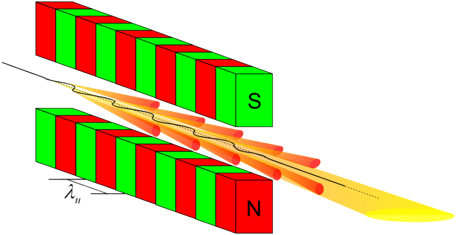

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Different from bending magnet:

Wigglers and undulators

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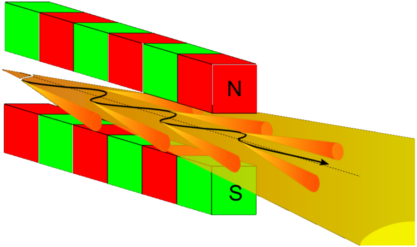

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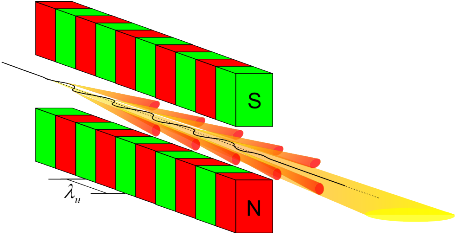

Different from bending magnet:

- shallow bends $\rightarrow$ smaller source

Wigglers and undulators

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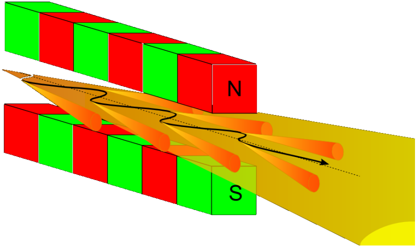

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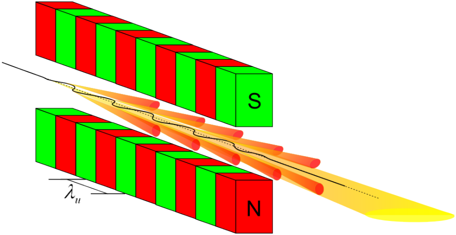

Like bending magnet except:

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Different from bending magnet:

- shallow bends $\rightarrow$ smaller source
- interference $\rightarrow$ peaked spectrum

Wiggler radiation

- The electron's trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet

$$Power[\mathrm{kW}] = 1.266\mathcal{E}_e^2[\mathrm{GeV}]B^2[\mathrm{T}]L[\mathrm{m}]I[\mathrm{A}]$$

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- This results in a significantly higher power load on all downstream components

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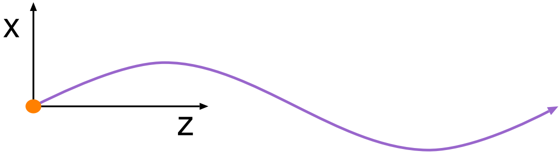
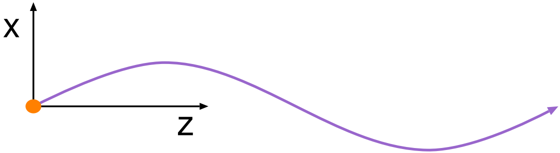
Undulator characterization



Undulator radiation is characterized by three parameters:

Undulator characterization



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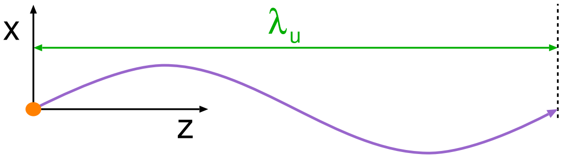

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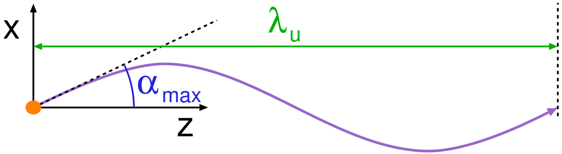

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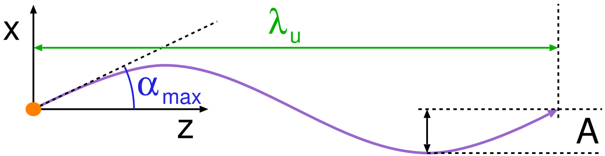
Undulator characterization



Undulator radiation is characterized by three parameters:

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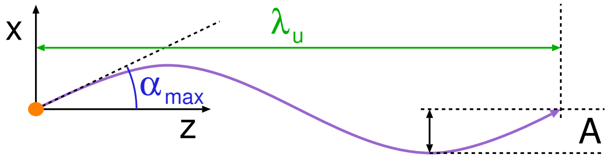

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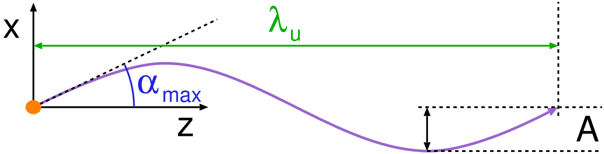

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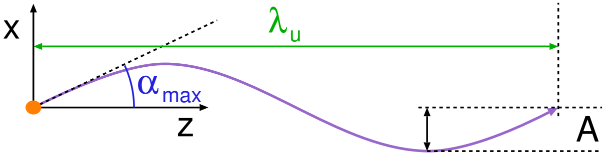

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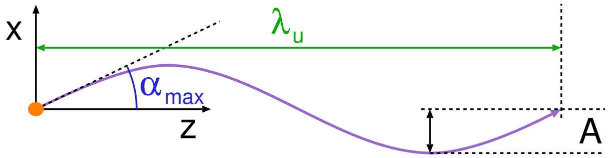

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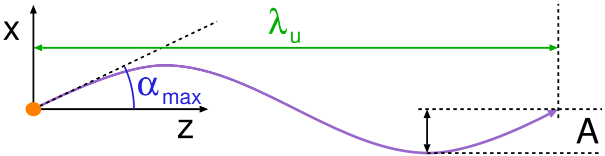

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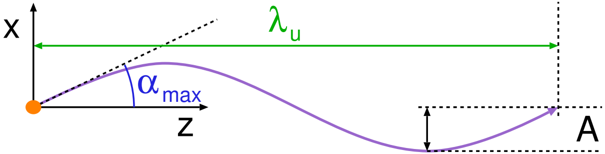

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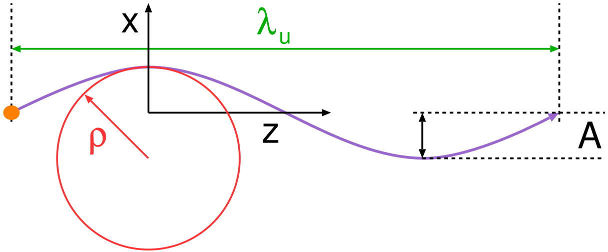
Circular path approximation



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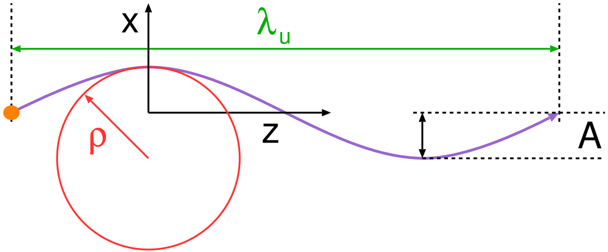

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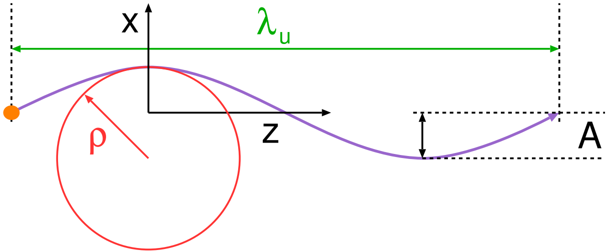

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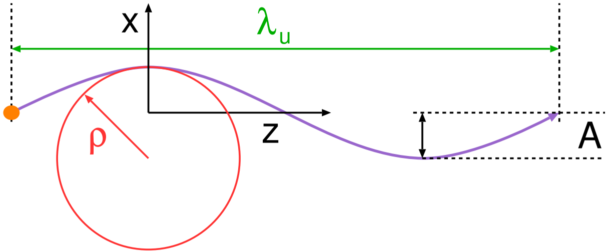

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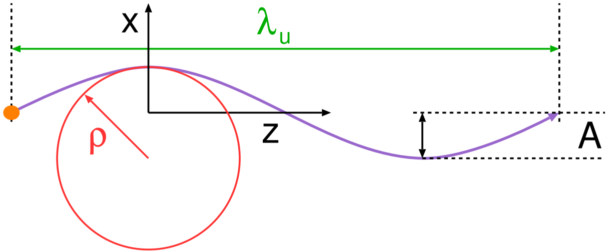

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From the equation for a circle:

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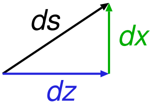
Electron path length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



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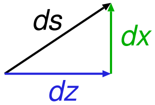

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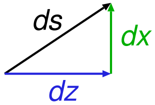

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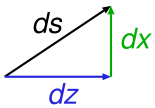

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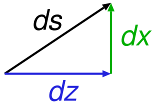

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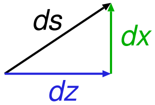

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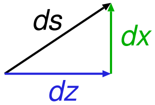

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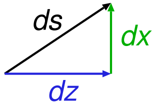

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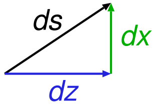

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$$\rho = \frac{1}{A k_u^2} \quad \longrightarrow \quad \rho = \frac{\gamma}{K k_u}$$

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$$\rho = \frac{1}{Ak_u^2} \quad \longrightarrow \quad \rho = \frac{\gamma}{Kk_u}$$

Recalling that the radius of curvature is related to the electron momentum by the Lorentz force, we have

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$$K = \frac{e B_0}{m c k_u} = \frac{e}{2\pi m c} \lambda_u B_0$$

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$$\rho = \frac{1}{Ak_u^2} \quad \longrightarrow \quad \rho = \frac{\gamma}{Kk_u}$$

Recalling that the radius of curvature is related to the electron momentum by the Lorentz force, we have

$$p = \gamma mv \approx \gamma mc = \rho eB_0 \quad \longrightarrow \quad \gamma mc \approx \frac{\gamma}{Kk_u} eB_0$$

Combining the above expressions yields

$$K = \frac{eB_0}{mck_u} = \frac{e}{2\pi mc}\lambda_u B_0 = 0.934\lambda_u[\text{cm}]B_0[\text{T}]$$

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For APS Undulator A, $\lambda_u = 3.3\text{cm}$ and $B_0 = 0.6\text{T}$ at closed gap, so

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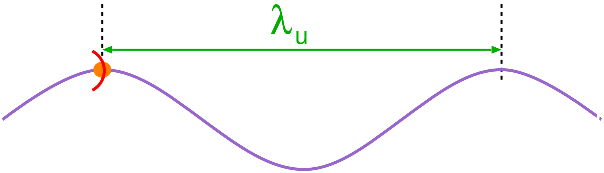
For APS Undulator A, $\lambda_u = 3.3\mathrm{cm}$ and $B_0 = 0.6\mathrm{T}$ at closed gap, so

$$K = 0.934 \cdot 3.3[\mathrm{cm}] \cdot 0.6[\mathrm{T}] = 1.85$$

Undulator wavelength

Consider an electron traveling through the undulator and emitting radiation at the first maximum excursion from the center.



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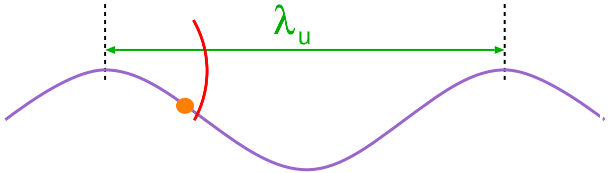

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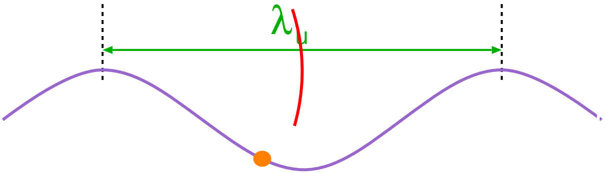

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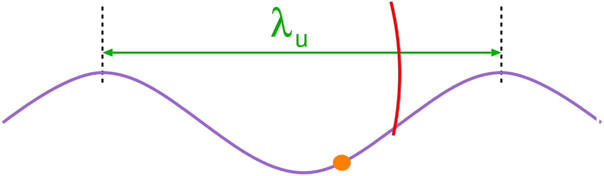

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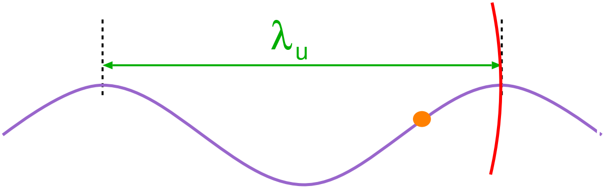

The emitted wave travels slightly faster than the electron.

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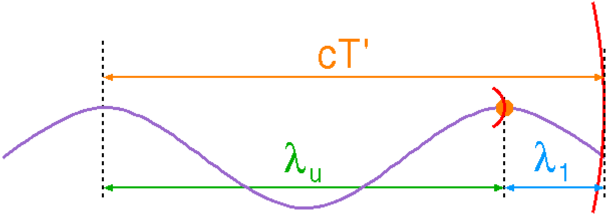

The emitted wave travels slightly faster than the electron.
It moves cT' in the time the electron travels a distance λ_u along the undulator.

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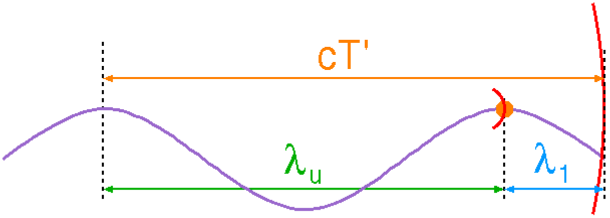

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The observer sees radiation with a compressed wavelength,

λ_1

Undulator wavelength

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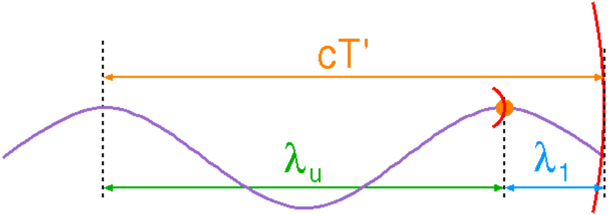

The emitted wave travels slightly faster than the electron.
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The observer sees radiation with a compressed wavelength,

$$\lambda_1 = cT' - \lambda_u$$

Undulator wavelength

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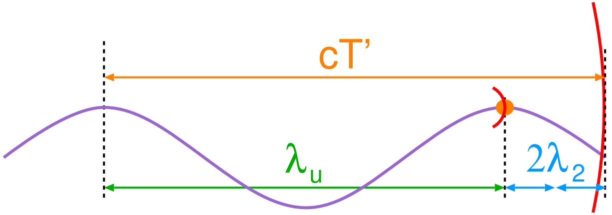

The emitted wave travels slightly faster than the electron.
It moves cT' in the time the electron travels a distance λ_u along the undulator.

The observer sees radiation with a compressed wavelength, along with harmonics which satisfy the same condition.

$$n\lambda_n = cT' - \lambda_u$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

Over the time T' the electron actually travels a distance $S\lambda_u$, so that

$$T' = \frac{S\lambda_u}{v}$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

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The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = c\,T' - \lambda_u \cos\theta$$

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Over the time T' the electron actually travels a distance $S\lambda_u$, so that

$$T' = \frac{S\lambda_u}{v}$$

$$S \approx 1 + \frac{K^2}{4\gamma^2}$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = c\,T' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

$$= \lambda_u \left(\left[1 + \frac{K^2}{4\gamma^2} \right] \frac{1}{\beta} - \cos\theta \right)$$

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Since γ is large, the maximum observation angle θ is small so

$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right)$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

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$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right) = \frac{\lambda_u}{2\gamma^2} \left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2 \right)$$

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2\right)$$

The fundamental wavelength

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regrouping terms

The fundamental wavelength

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regrouping terms

$$\approx \frac{\lambda_u}{2\gamma^2}\left(2\gamma^2\left[\frac{1}{\beta} - 1\right] + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

The fundamental wavelength

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$$\gamma = \sqrt{\frac{1}{1-\beta^2}}$$

The fundamental wavelength

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$$1 - \beta^2 = (1+\beta)(1-\beta)$$

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2\right)$$

regrouping terms

$$\approx \frac{\lambda_u}{2\gamma^2}\left(2\gamma^2\left[\frac{1}{\beta} - 1\right] + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

$$\gamma = \sqrt{\frac{1}{1-\beta^2}}$$

$$\approx \frac{\lambda_u}{2\gamma^2}\left(2\frac{1}{1-\beta^2}\left[\frac{1-\beta}{\beta}\right] + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

$$1-\beta^2 = (1+\beta)(1-\beta)$$

$$\approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - [\gamma\theta]^2\right)$$

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

The fundamental wavelength

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and directly on axis

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

The fundamental wavelength

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$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

for a typical undulator $\gamma \sim 10^4$, $K \sim 1$, and $\lambda_u \sim 2\text{cm}$ so we estimate

$$\lambda_1 \approx \frac{2 \times 10^{-2}}{2\,(10^4)^2}\left(1 + \frac{(1)^2}{2}\right)$$

The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

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$$\lambda_1 \approx \frac{2 \times 10^{-2}}{2\,(10^4)^2}\left(1 + \frac{(1)^2}{2}\right) = 1.5 \times 10^{-10}\text{m} = 1.5\text{Å}$$

The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

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This corresponds to an energy $\mathcal{E}_1 \approx 8.2\text{keV}$ but as the undulator gap is widened

The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

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The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

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This corresponds to an energy $\mathcal{E}_1 \approx 8.2\text{keV}$ but as the undulator gap is widened, B_0 decreases, K decreases

The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

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The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - (\gamma\theta)^2\right) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

and directly on axis

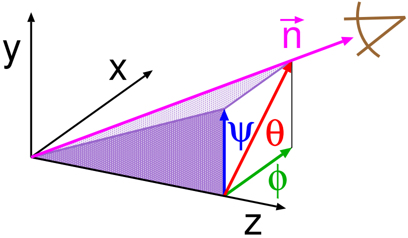
$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

for a typical undulator $\gamma \sim 10^4$, $K \sim 1$, and $\lambda_u \sim 2\text{cm}$ so we estimate

$$\lambda_1 \approx \frac{2 \times 10^{-2}}{2\,(10^4)^2}\left(1 + \frac{(1)^2}{2}\right) = 1.5 \times 10^{-10}\text{m} = 1.5\text{Å}$$

This corresponds to an energy $\mathcal{E}_1 \approx 8.2\text{keV}$ but as the undulator gap is widened, B_0 decreases, K decreases, λ_1 decreases, and $\mathcal{E}_1$ increases.

Higher harmonics



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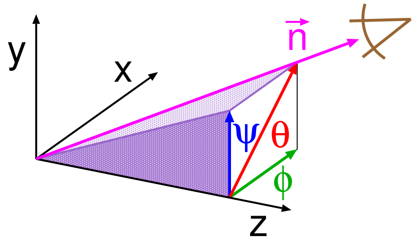

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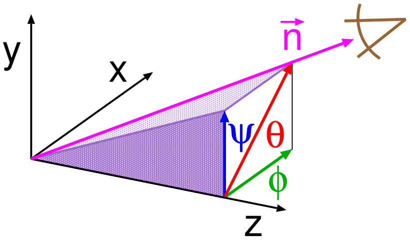

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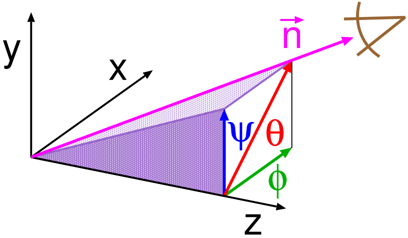

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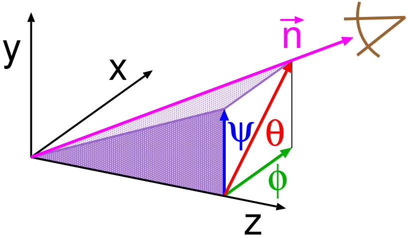

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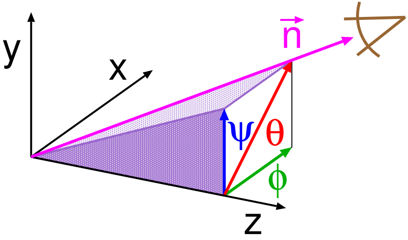

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The motion of the electron, $\sin \omega_u t'$, is always sinusoidal, but because of the additional terms, the motion as seen by the observer, $\sin \omega_1 t$, is not.

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin\left(2\omega_u t'\right)$$

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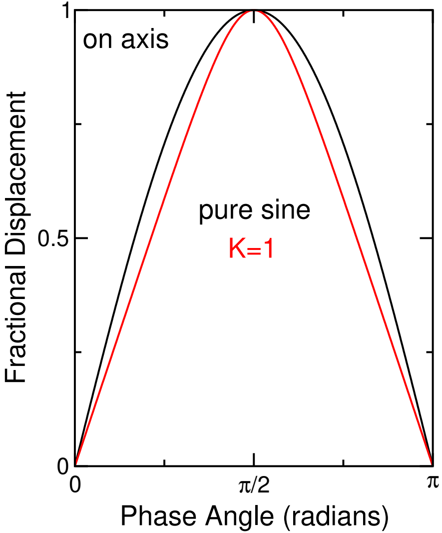

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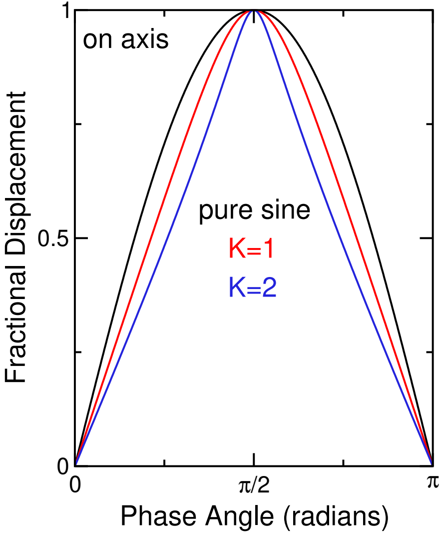

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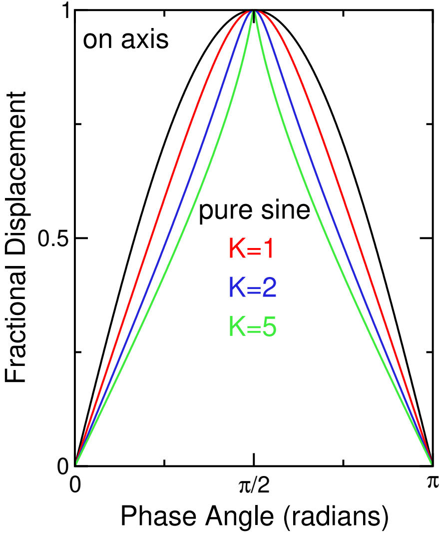

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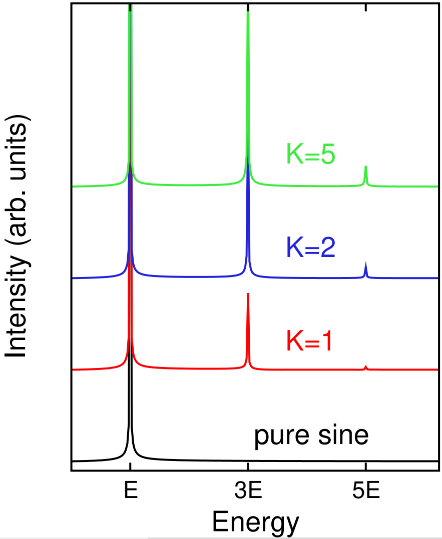

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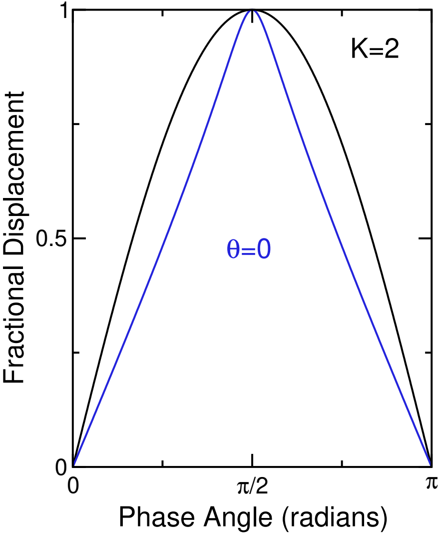

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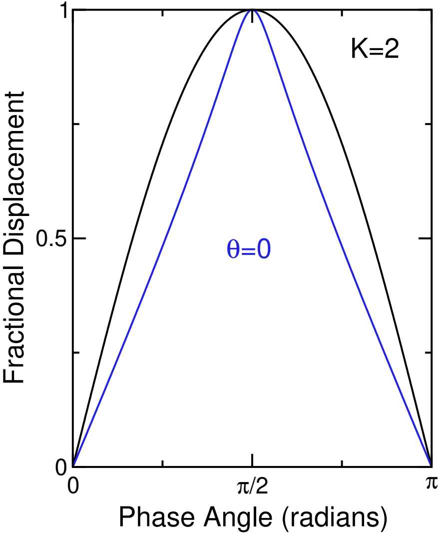

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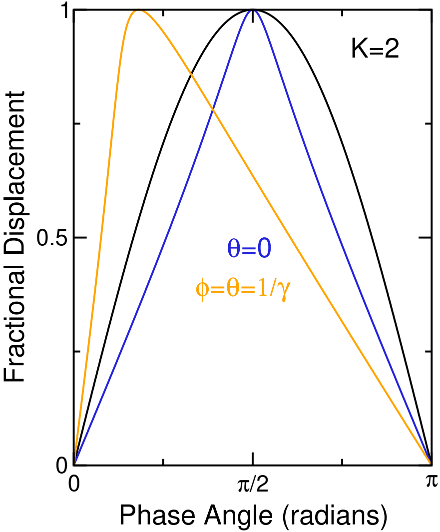

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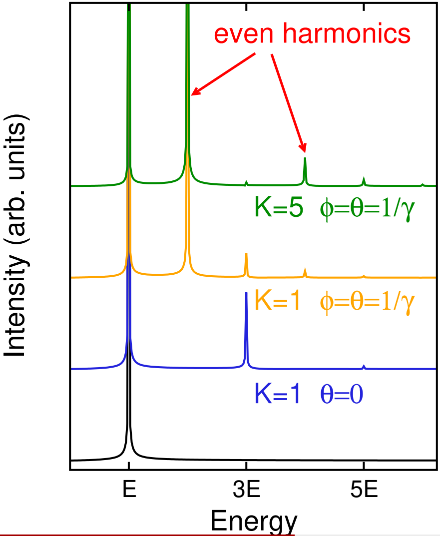

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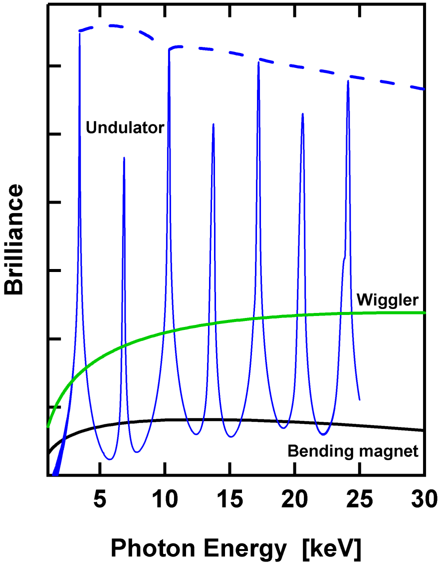
The last term introduces an antisymmetric term which skews the function and leads to the presence of forbidden harmonics (2^{nd}, 4^{th}, etc) in the radiation from the undulator compared to the on-axis radiation.

Spectral comparison



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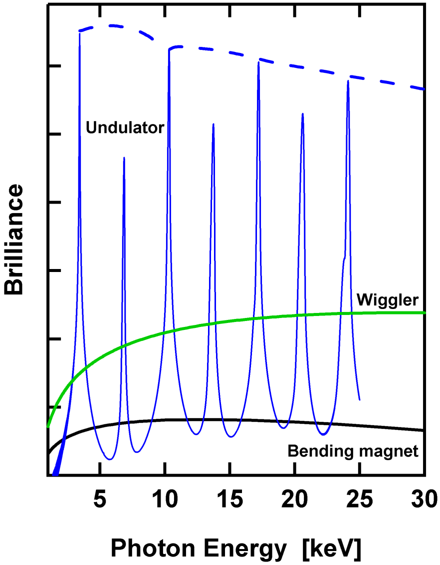

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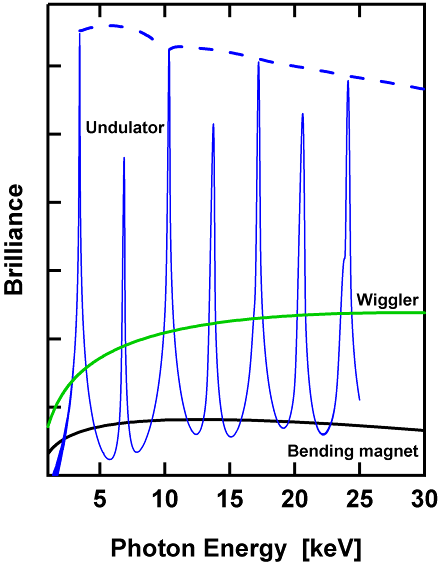

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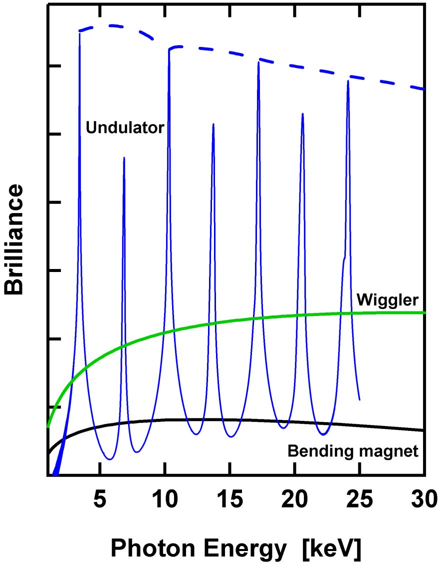

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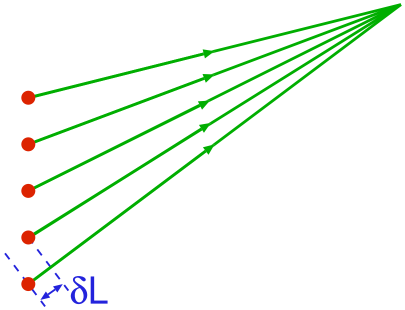

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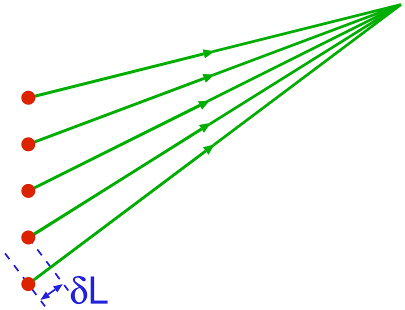

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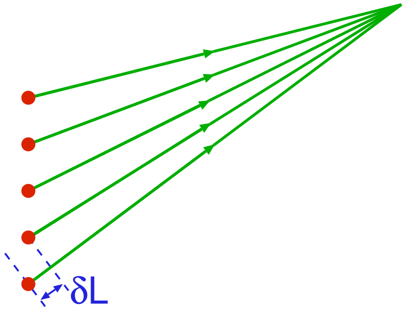

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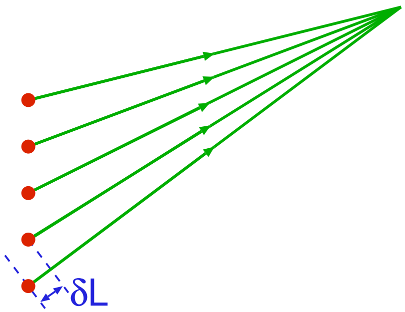

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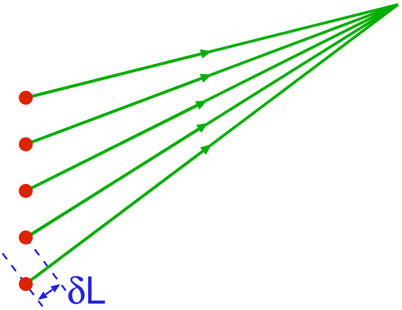

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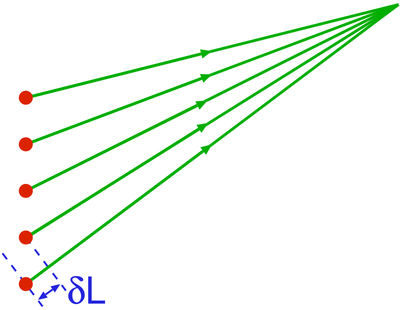

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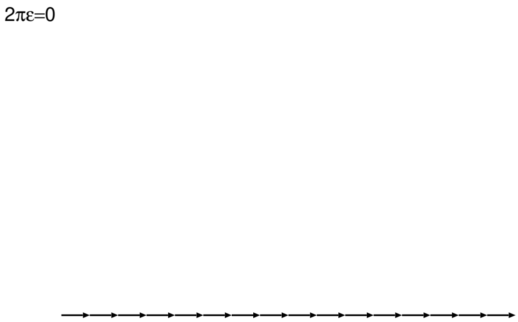

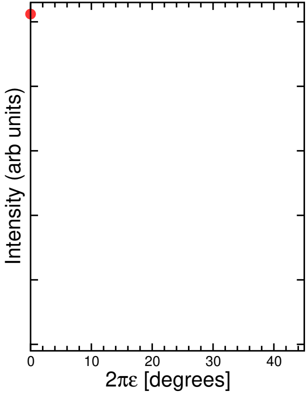

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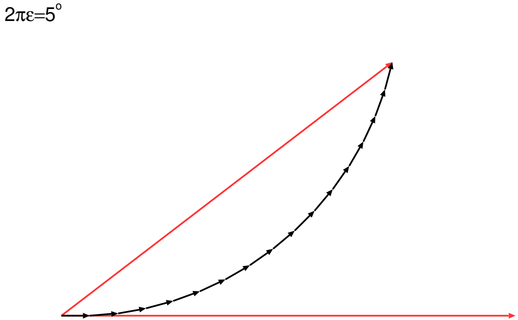

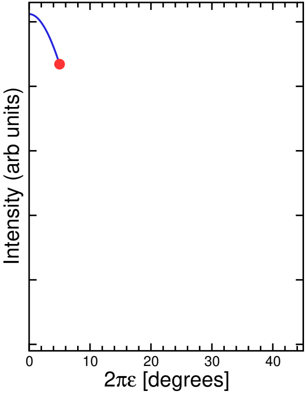

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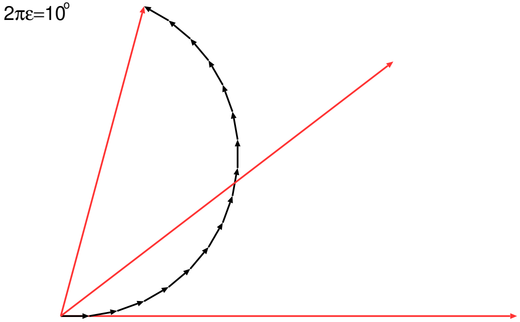

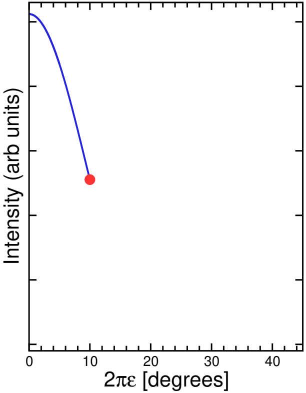

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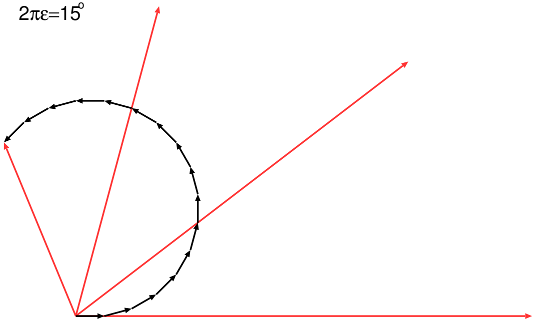

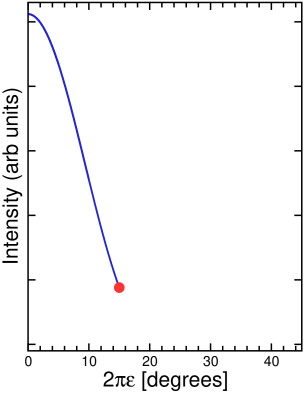

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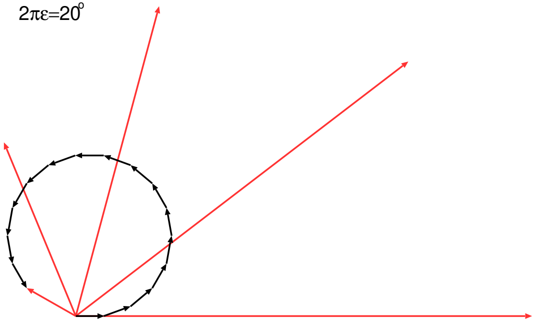

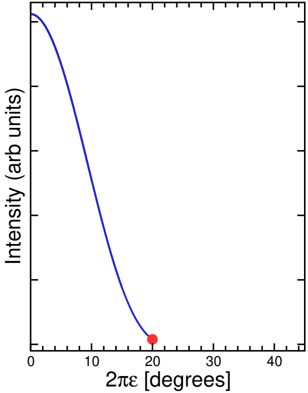

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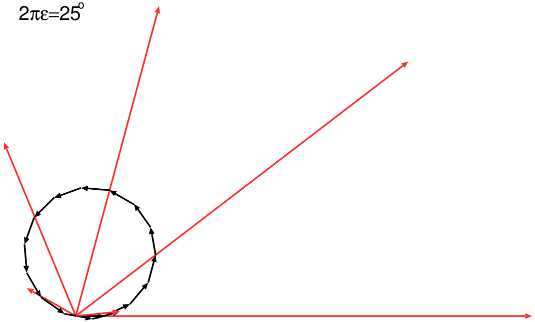

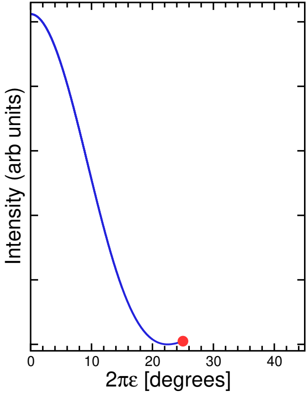

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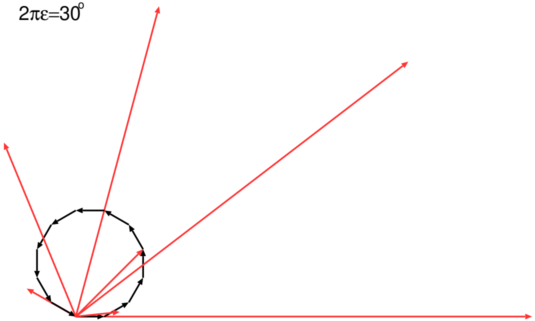

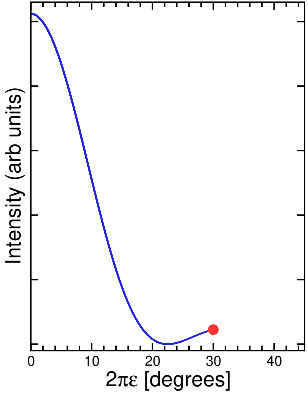

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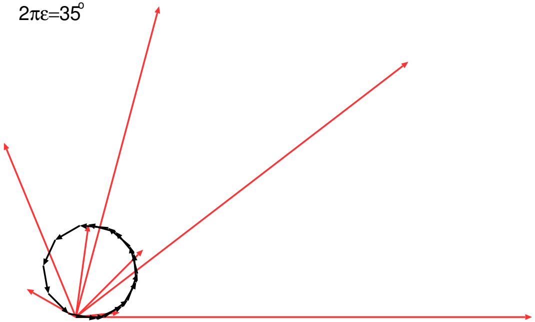

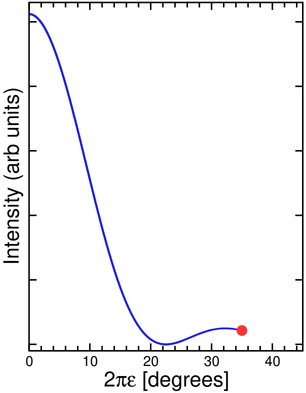

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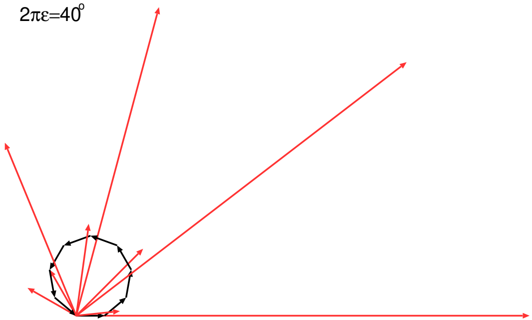

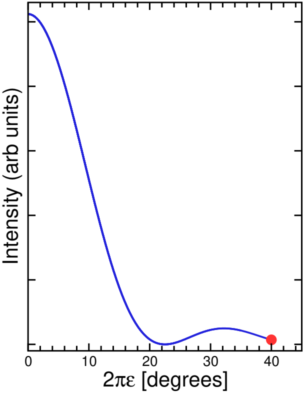

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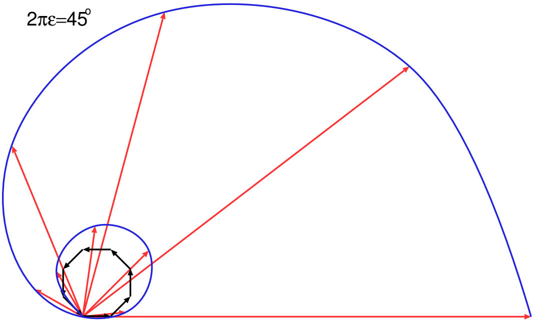

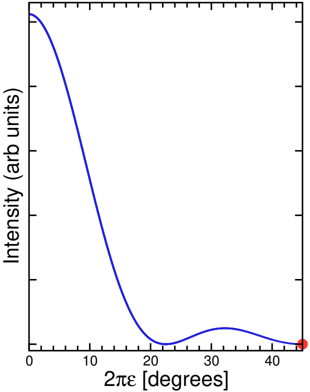

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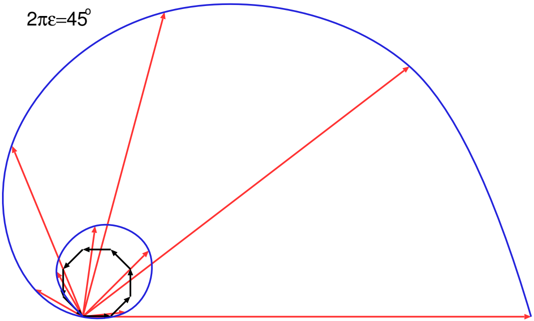

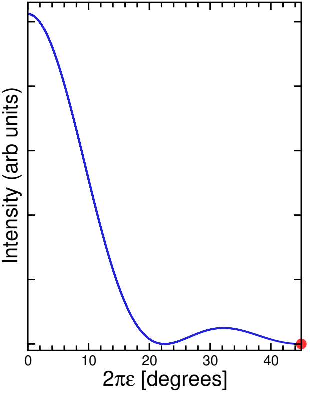

With the height and width of the peak dependent on the number of poles.