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Homework Assignment #01:
Chapter 2: 2,3,5,6,8
due Thursday, January 30, 2020

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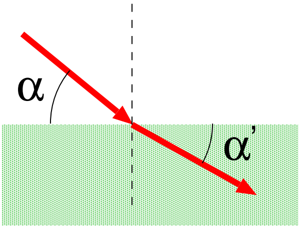

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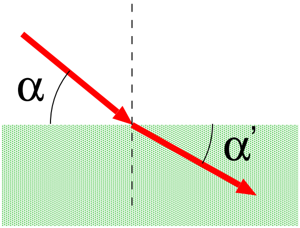

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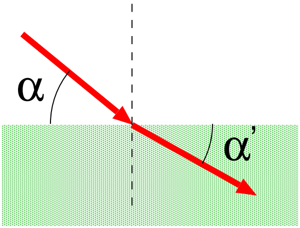

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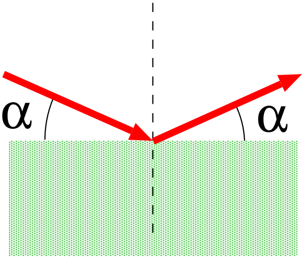

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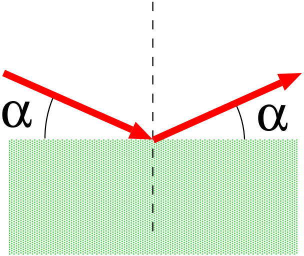

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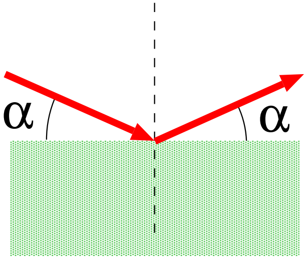

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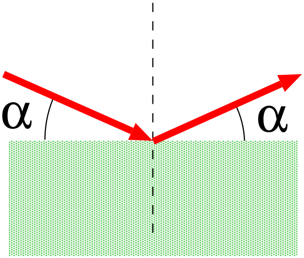

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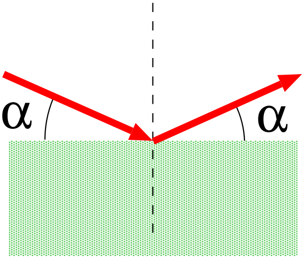

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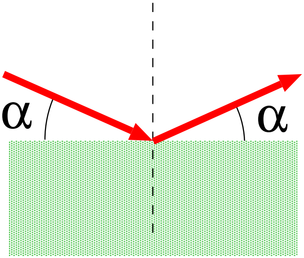

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$$\delta = \frac{\alpha_c^2}{2} \quad \longrightarrow \quad \alpha_c = \sqrt{2\delta}$$

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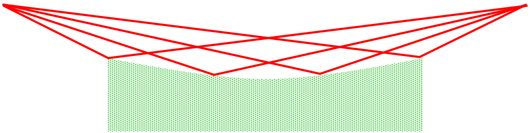

X-ray mirrors

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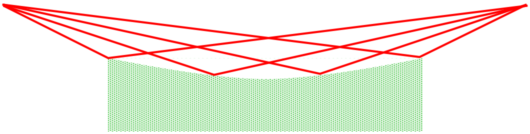

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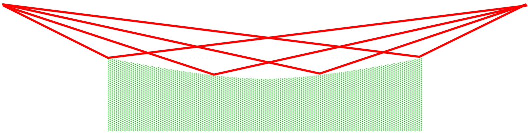

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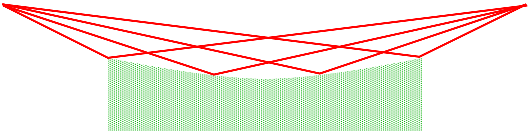

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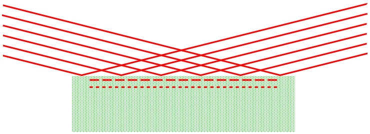

Evanscent wave experiments

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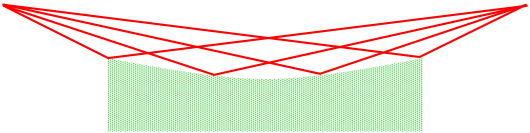

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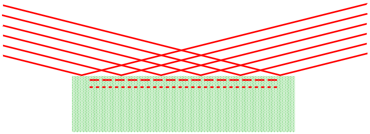

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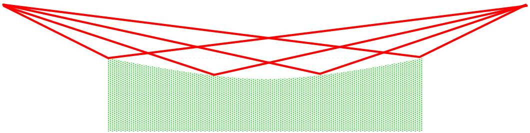

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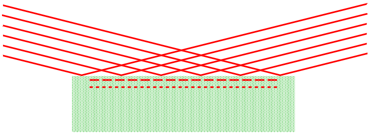

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Because of these imperfections the "coherence length" of an x-ray beam is finite and we can calculate it.

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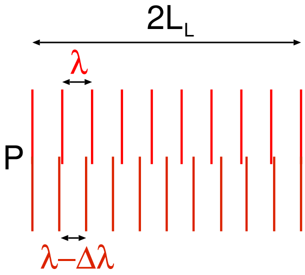

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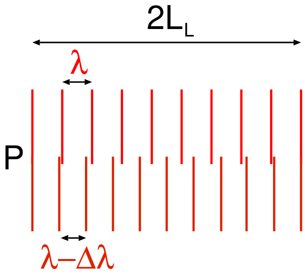

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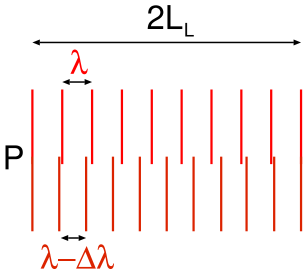

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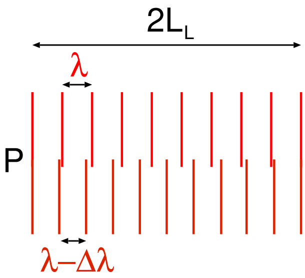

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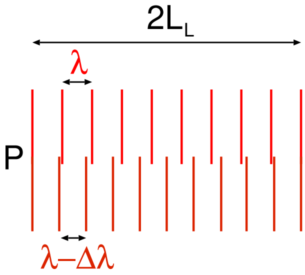

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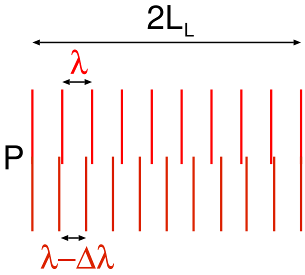

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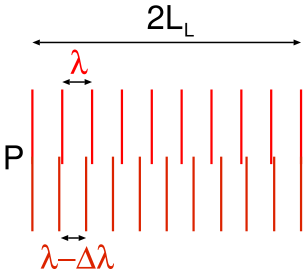

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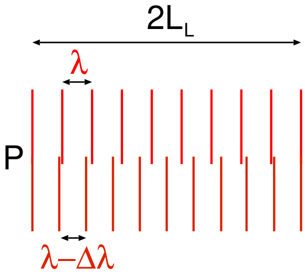

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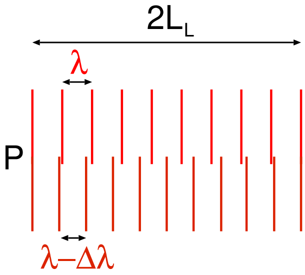

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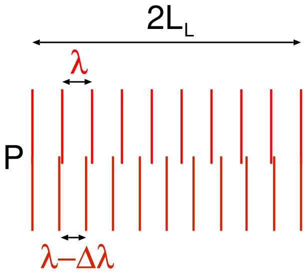

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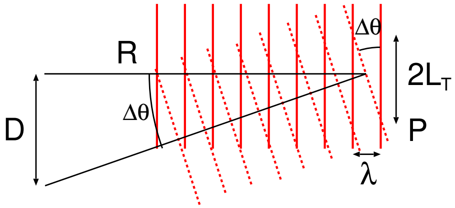

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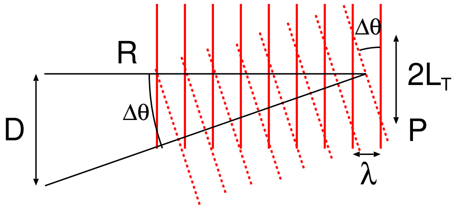

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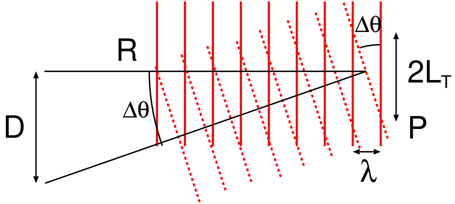

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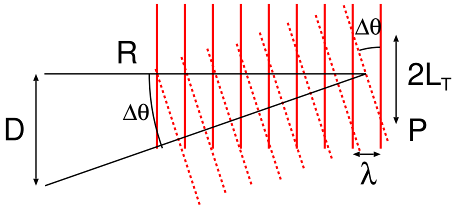

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$$\frac{\lambda}{2L_T} = \tan\Delta\theta \approx \Delta\theta \qquad \frac{D}{R} = \tan\Delta\theta \approx \Delta\theta$$

$$L_T = \frac{\lambda R}{2D}$$

Coherence lengths at the APS

For a typical 3^{rd} generation undulator source, such as at the Advanced Photon Source the vertical source size is $D_v = 10\mu$m and we are typically $R = 50$m away with our experiment. If we assume a typical wavelength of $\lambda = 1$Å, and a monochromator resolution of $\Delta\lambda/\lambda = 10^{-5}$ we have for the vertical direction:

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$$L_T = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (10 \times 10^{-6})}$$

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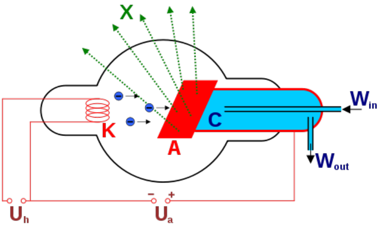
$$L_T = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (10 \times 10^{-6})} = 250\mu\text{m}$$

X-ray tube schematics

Fixed anode tube



- low power
- low maintenance

X-ray tube schematics

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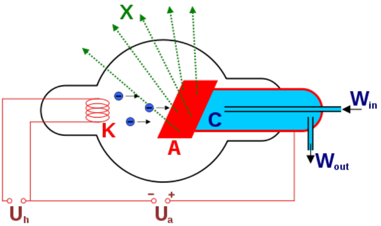

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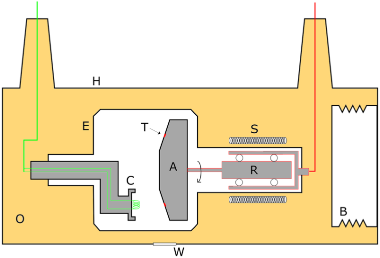
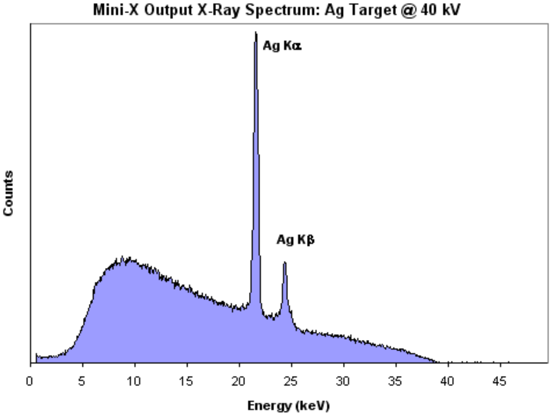
- low maintenance

Rotating anode tube



- high power
- high maintenance

X-ray tube spectrum



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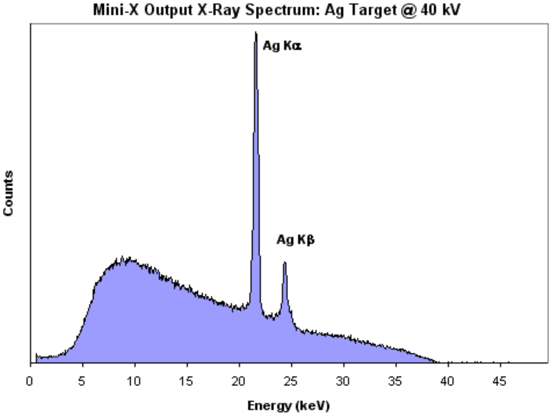
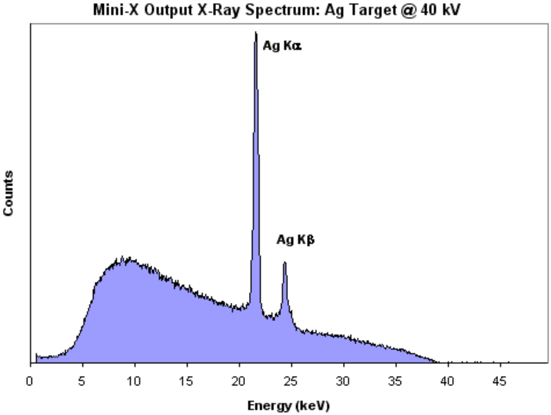


- Minimum wavelength (maximum energy) set by accelerating potential

X-ray tube spectrum



- Minimum wavelength (maximum energy) set by accelerating potential
- Bremßtrahlung radiation provides smooth background (charged particle deceleration)

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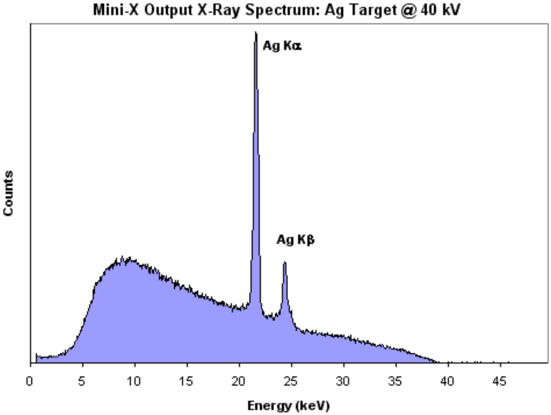

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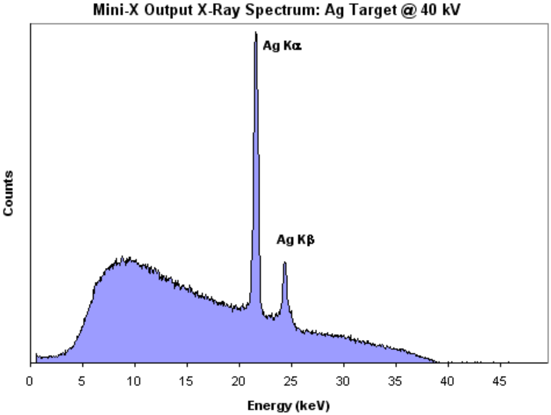
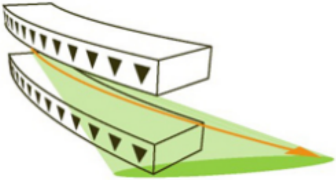


- Minimum wavelength (maximum energy) set by accelerating potential
- Bremßtrahlung radiation provides smooth background (charged particle deceleration)

- Highest intensity at the characteristic fluorescence emission energy of the anode material
- Unpolarized, incoherent x-rays emitted in all directions from anode surface, must be collimated with slits

Synchrotron sources

Bending magnet



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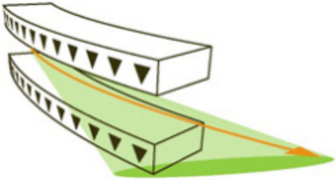

- Wide horizontal beam

Synchrotron sources

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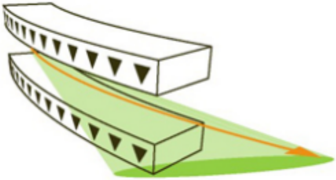

- Wide horizontal beam
- Broad spectrum to high energies

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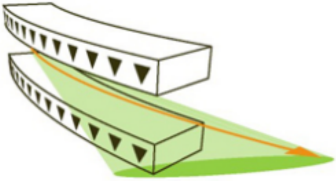
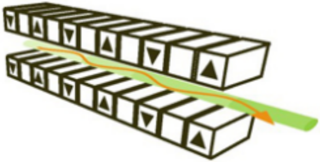


Undulator



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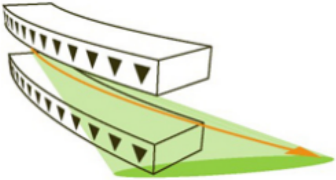

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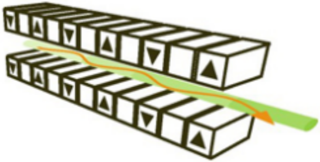

- Highly collimated beam

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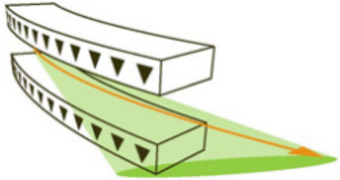

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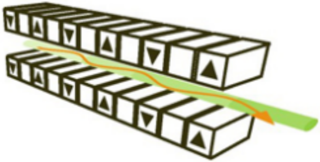
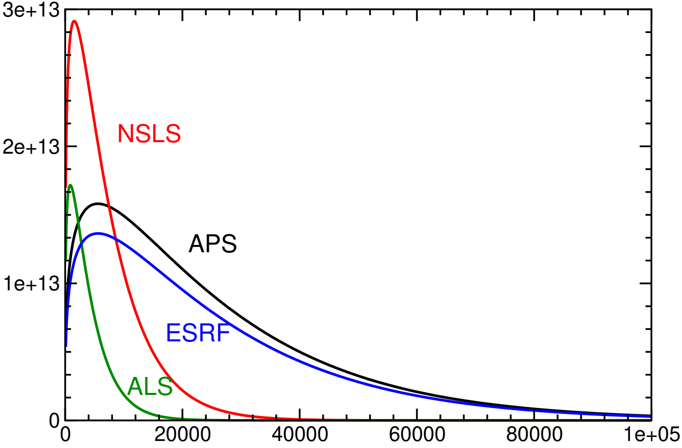
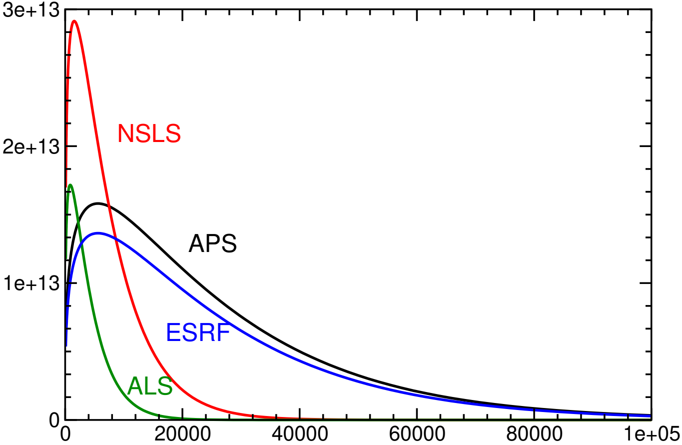


- Highly collimated beam
- Highly peaked spectrum with harmonics

Bending magnet spectra

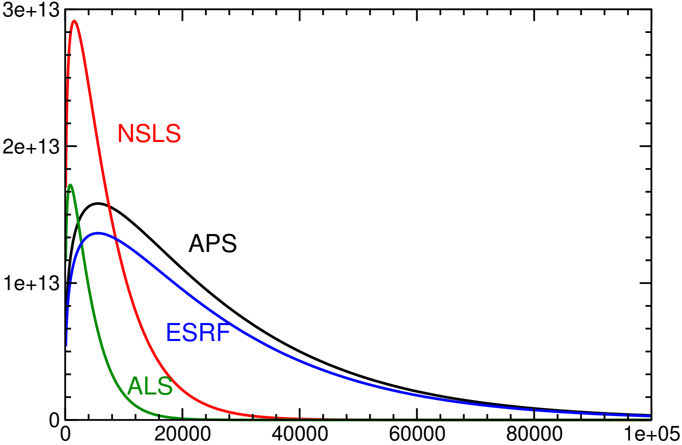


Bending magnet spectra



Lower energy sources, such as NSLS have lower peak energy and higher intensity at the peak.

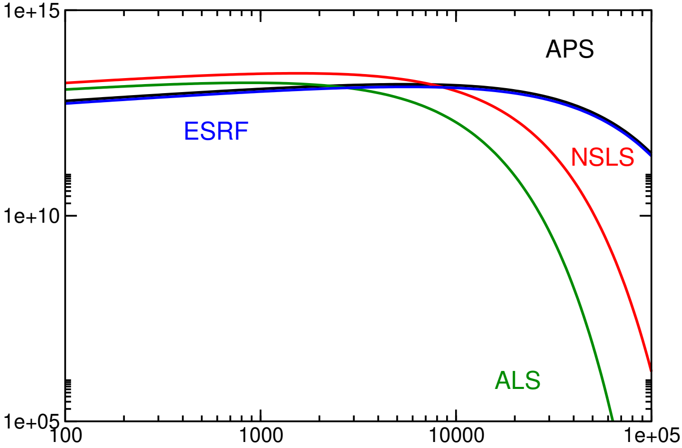
Bending magnet spectra



Lower energy sources, such as NSLS have lower peak energy and higher intensity at the peak.

Higher energy sources, such as APS have higher energy spectrum and are only off by a factor of 2 intensity at low energy.

Bending magnet spectra



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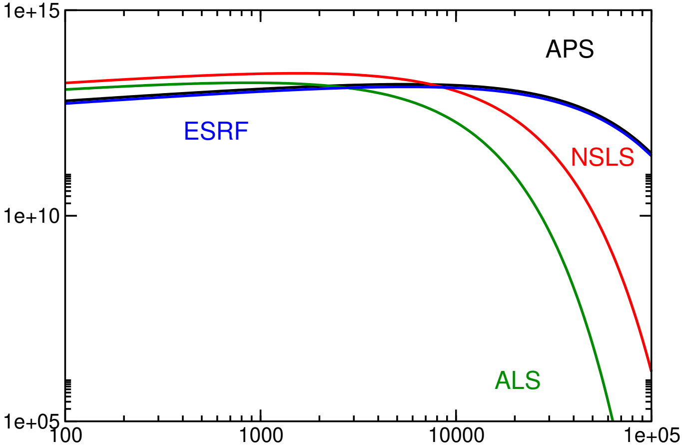
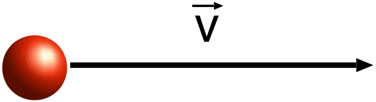
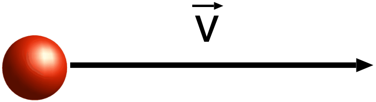


Logarithmic scale shows clearly how much more energetic and intense the bending magnet sources at the 6 GeV and 7 GeV sources are.

Review of special relativity

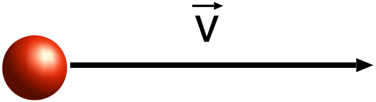


Review of special relativity



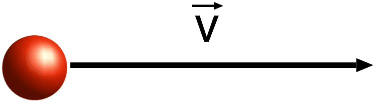
$$\beta = \frac{v}{c}$$

Review of special relativity



$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

Review of special relativity



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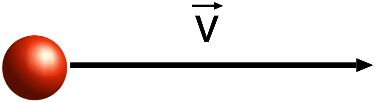

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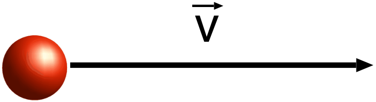
$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Review of special relativity



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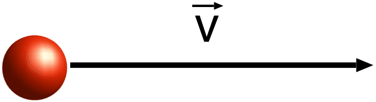
use binomial expansion since $1/\gamma^2 << 1$

Review of special relativity



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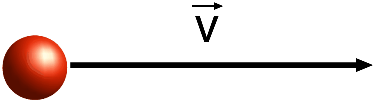
Let's calculate these quantities for an electron at NSLS and APS



Let's calculate these quantities for an electron at NSLS and APS

$$m_e = 0.511 \text{ MeV}/c^2$$

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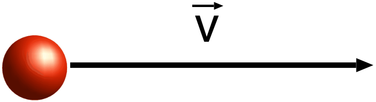

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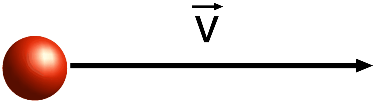
$$m_e = 0.511 \text{ MeV/c}^2$$

NSLS: $E = 1.5$ GeV

$$\gamma = \frac{1.5 \times 10^9}{0.511 \times 10^6} = 2935$$

Review of special relativity



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Let's calculate these quantities for an electron at NSLS and APS

$$m_e = 0.511 \text{ MeV/c}^2$$

NSLS: $E = 1.5$ GeV

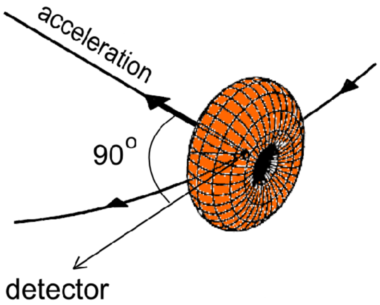
$$\gamma = \frac{1.5 \times 10^9}{0.511 \times 10^6} = 2935$$

APS: $E = 7.0$ GeV

$$\gamma = \frac{7.0 \times 10^9}{0.511 \times 10^6} = 13700$$

“Headlight” effect

In electron rest frame:



emission is symmetric about the axis of the acceleration vector

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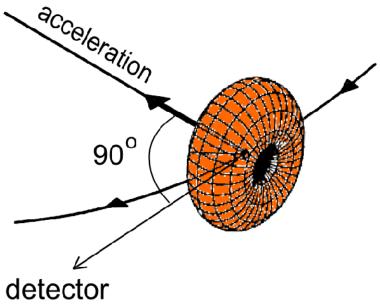

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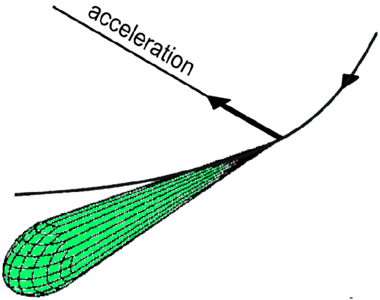
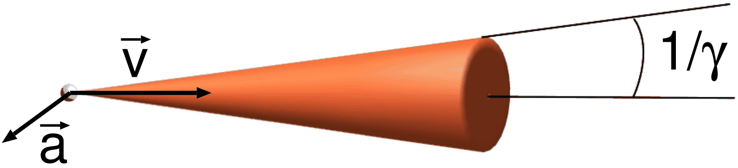
In lab frame:



emission is pushed into the direction of motion of the electron

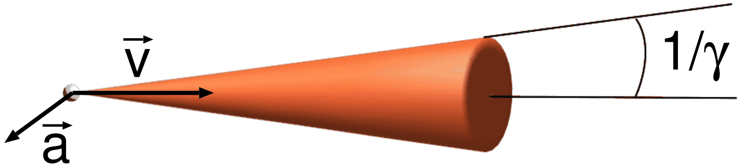
Relativistic emission



the electron is in constant transverse acceleration due to the Lorentz force from the magnetic field of the bending magnet

$$\vec{F} = e\vec{v} \times \vec{B} = m_e\vec{a}$$

Relativistic emission



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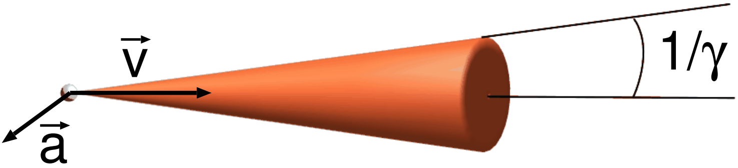
the aperture angle of the radiation cone is $1/\gamma$

Relativistic emission



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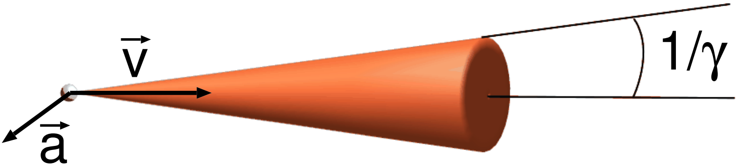
Relativistic emission



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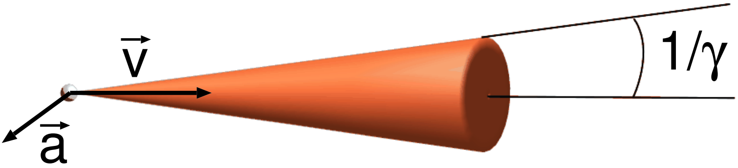
the aperture angle of the radiation cone is $1/\gamma$

the angular frequency of the electron in the ring is $\omega_0 \approx 10^6$ and the cutoff energy for emission is

$$E_{max} \approx \gamma^3\omega_0$$

Relativistic emission



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the aperture angle of the radiation cone is $1/\gamma$

the angular frequency of the electron in the ring is $\omega_0 \approx 10^6$ and the cutoff energy for emission is

$$E_{max} \approx \gamma^3\omega_0$$

for the APS, with $\gamma \approx 10^4$ we have

$$E_{max} \approx (10^4)^3 \cdot 10^6 = 10^{18}$$

Flux and brilliance

There are a number of important quantities which are relevant to the quality of an x-ray source:

Flux and brilliance

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photon flux

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photon flux
photon density

Flux and brilliance

There are a number of important quantities which are relevant to the quality of an x-ray source:

photon flux
photon density
beam divergence

Flux and brilliance

There are a number of important quantities which are relevant to the quality of an x-ray source:

photon flux
photon density
beam divergence
energy resolution

Flux and brilliance

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photon flux	source type
photon density	source type
beam divergence	source type
energy resolution	

Flux and brilliance

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photon flux	source type	optics
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beam divergence	source type	optics
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Flux and brilliance

There are a number of important quantities which are relevant to the quality of an x-ray source:

photon flux	source type	optics
photon density	source type	optics
beam divergence	source type	optics
energy resolution		optics

All these quantities are conveniently taken into account in a measure called brilliance

Flux and brilliance

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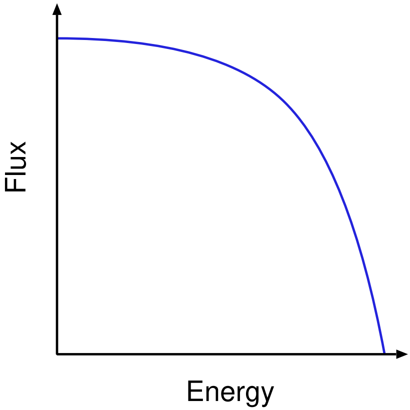

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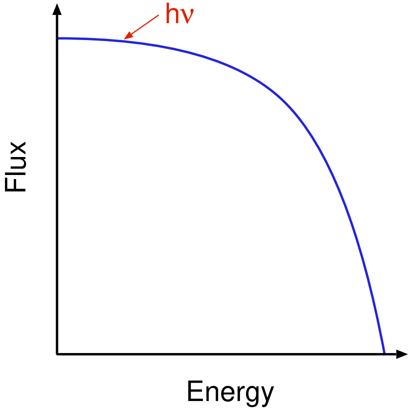

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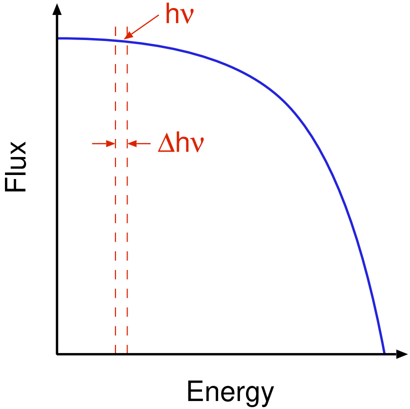

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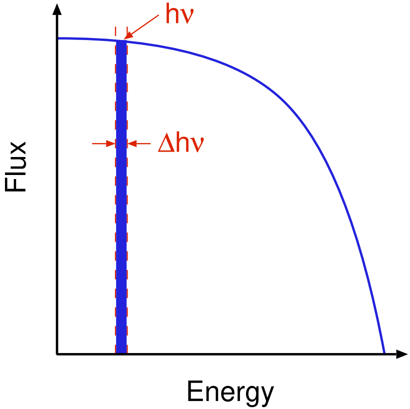

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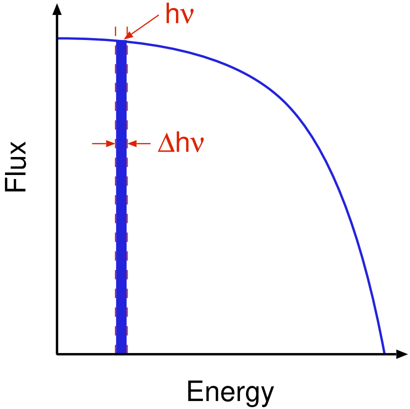

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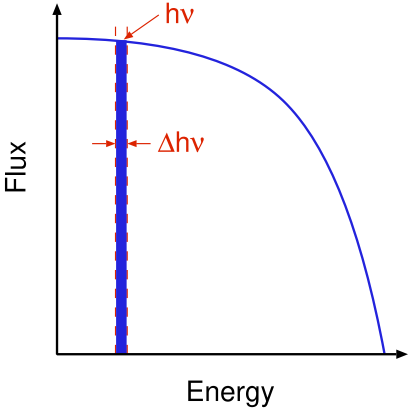

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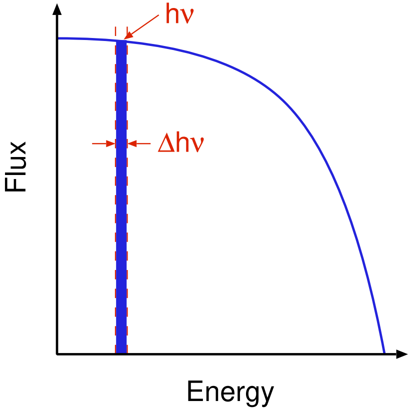

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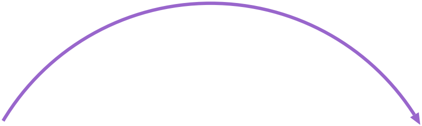
$$\alpha \approx x/z \qquad \beta \approx y/z,$$

where z is the distance from the source over which there is a lateral spread x and y in each direction

Segmented arc approximation



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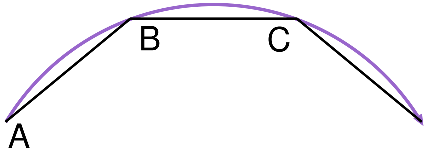

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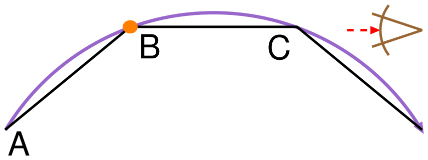

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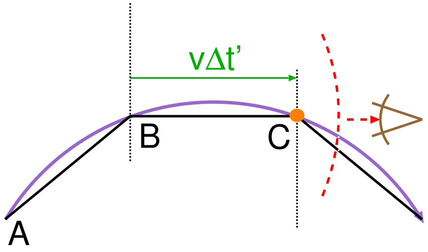

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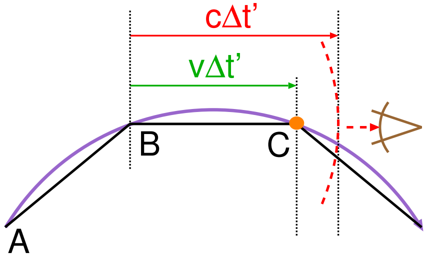

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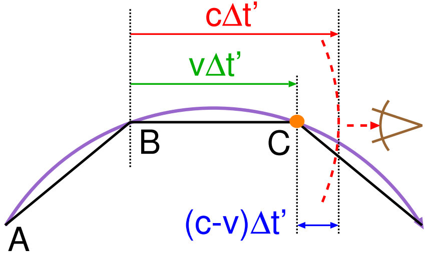

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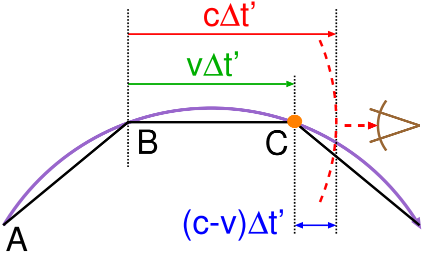

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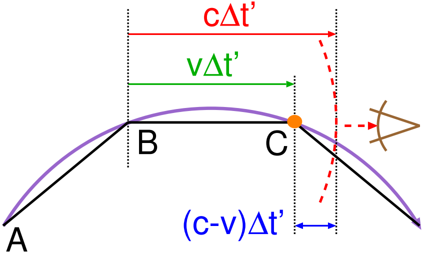

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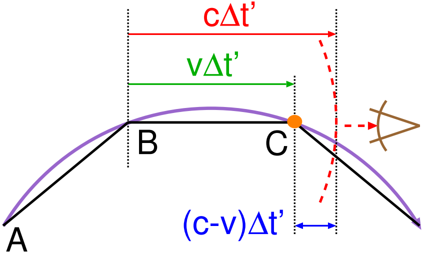

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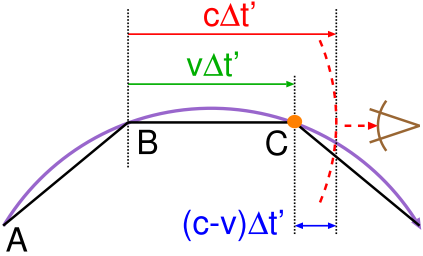

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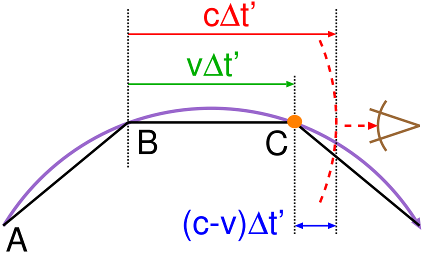

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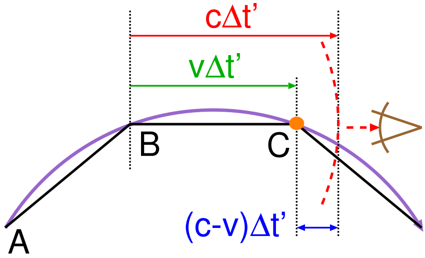

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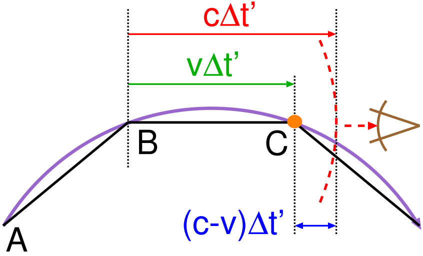

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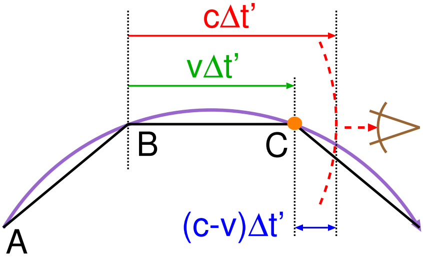

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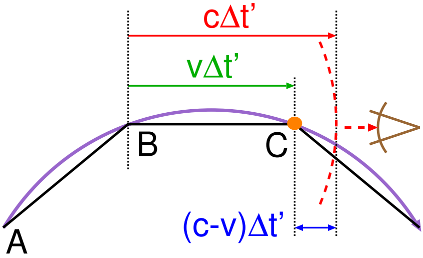

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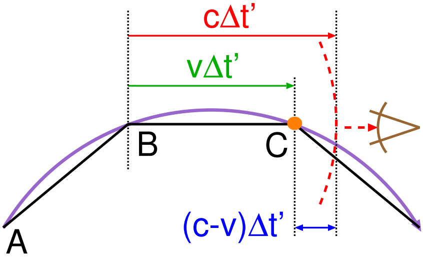

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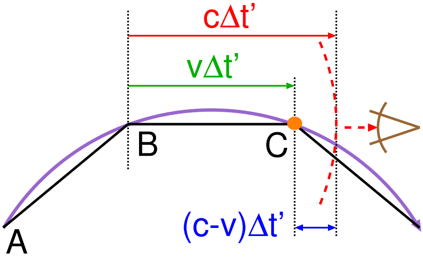

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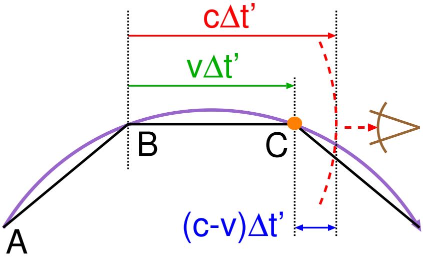

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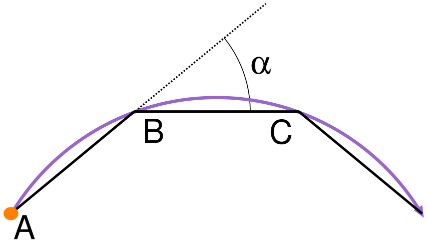
Off-axis emission



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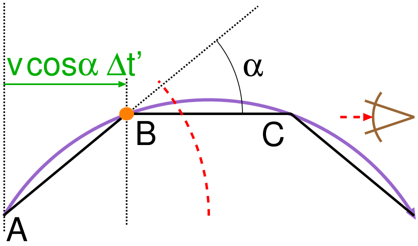

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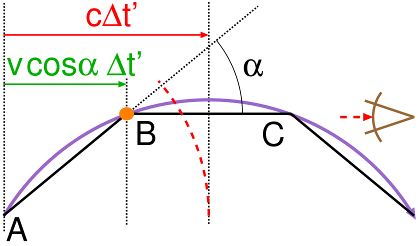

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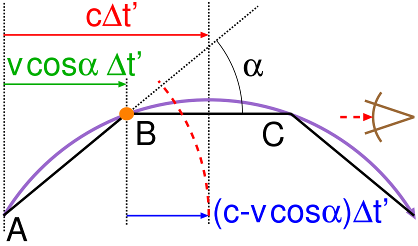

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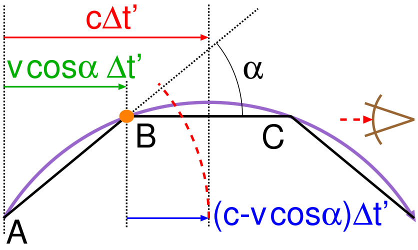

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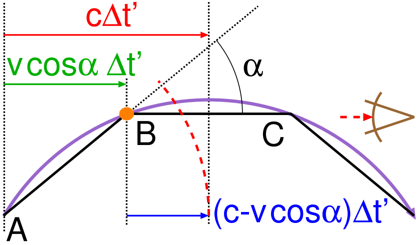

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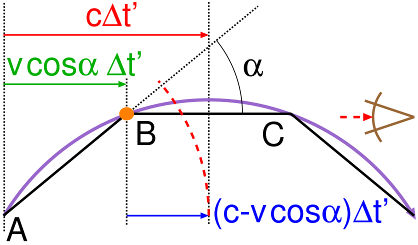

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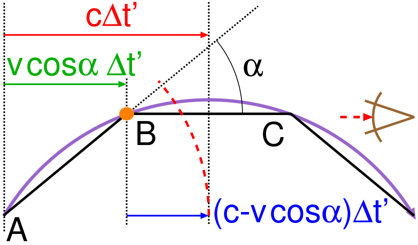

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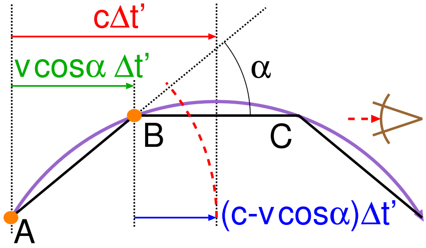
Corrected Doppler shift



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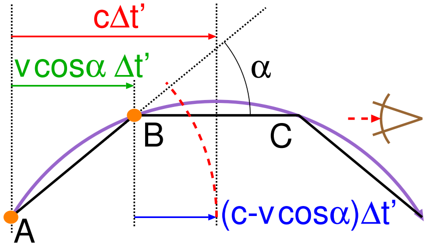

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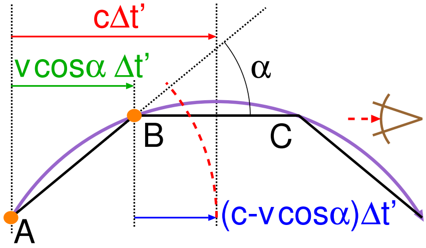

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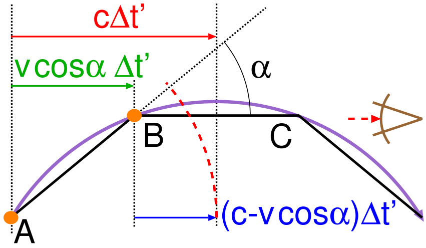

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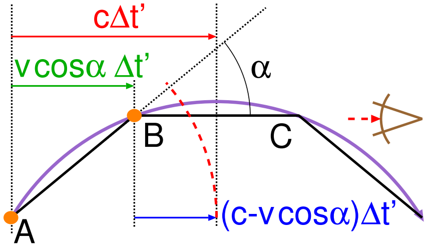
and γ is very large, we have

$$\frac{\Delta t}{\Delta t'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{\alpha^2}{2}\right)$$

Corrected Doppler shift



$$\Delta t = (1 - \beta \cos\alpha)\Delta t'$$

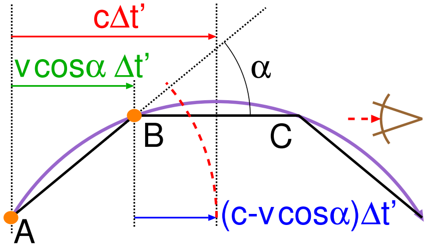
Since α is very small:

$$\cos\alpha \approx 1 - \frac{\alpha^2}{2}$$

and γ is very large, we have

$$\frac{\Delta t}{\Delta t'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{\alpha^2}{2}\right) = 1 - 1 + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} - \frac{\alpha^2}{2\gamma^2}$$

Corrected Doppler shift



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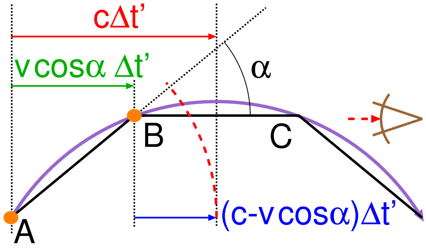

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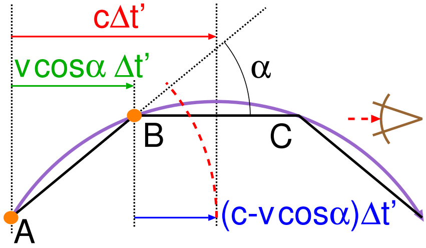

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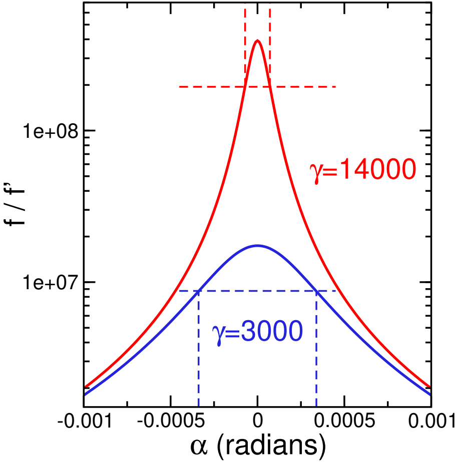
called the time compression ratio.

Radiation opening angle

The Doppler shift is defined in terms of the time compression ratio

$$\frac{f}{f'} = \frac{\Delta t'}{\Delta t} = \frac{2\gamma^2}{1+\alpha^2\gamma^2}$$



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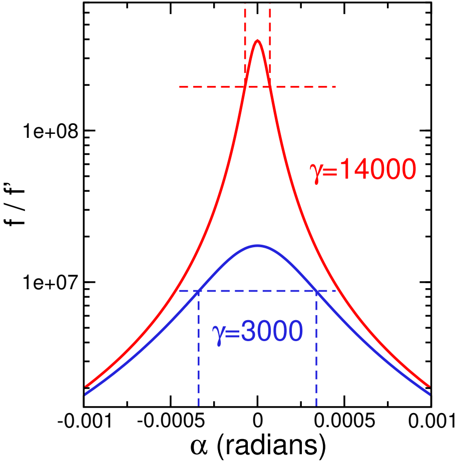

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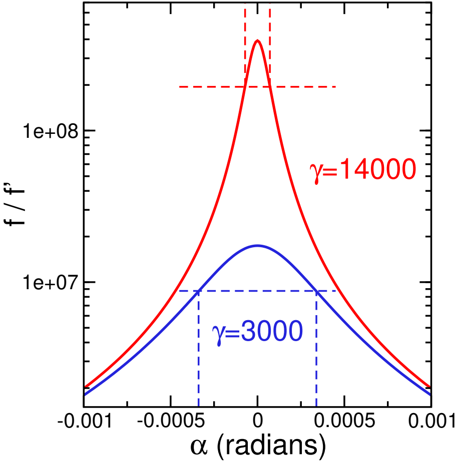

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- The highest energy emitted radiation appears within a cone of half angle $1/\gamma$