Today's Outline - November 21, 2016

Today's Outline - November 21, 2016

- Imaging

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging

Today's Outline - November 21, 2016

- Imaging
    - Computed tomography
    - Microscopy
    - Phase contrast imaging
    - Grating interferometry

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging
  - Grating interferometry
  - Coherent diffraction imaging

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging
  - Grating interferometry
  - Coherent diffraction imaging
  - Holography

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging
  - Grating interferometry
  - Coherent diffraction imaging
  - Holography

Homework Assignment #7:
Chapter 7: 2,3,9,10,11
due Monday, November 28, 2016

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging
  - Grating interferometry
  - Coherent diffraction imaging
  - Holography

Homework Assignment #7:
Chapter 7: 2,3,9,10,11
due Monday, November 28, 2016

Final Exam, Wednesday, December 7, 2016, Stuart Building 213
2 sessions: 09:00-12:00; 13:00-17:00; (this may change)

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging
  - Grating interferometry
  - Coherent diffraction imaging
  - Holography

Homework Assignment #7:
Chapter 7: 2,3,9,10,11
due Monday, November 28, 2016

Final Exam, Wednesday, December 7, 2016, Stuart Building 213
2 sessions: 09:00-12:00; 13:00-17:00; (this may change)

Provide me with the paper you intend to present and a preferred session for the exam

Today's Outline - November 21, 2016

- Imaging
  - Computed tomography
  - Microscopy
  - Phase contrast imaging
  - Grating interferometry
  - Coherent diffraction imaging
  - Holography

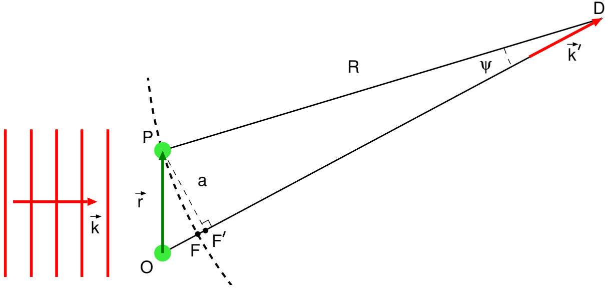
Homework Assignment #7:
Chapter 7: 2,3,9,10,11
due Monday, November 28, 2016

Final Exam, Wednesday, December 7, 2016, Stuart Building 213
2 sessions: 09:00-12:00; 13:00-17:00; (this may change)

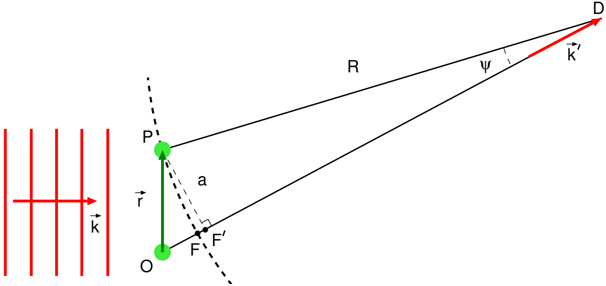
Provide me with the paper you intend to present and a preferred session for the exam

Send me your presentation in Powerpoint or PDF format before before your session

Phase difference in scattering

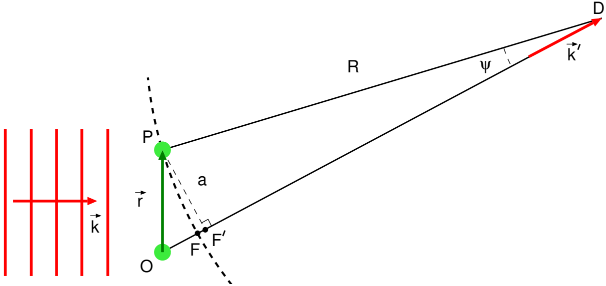


Phase difference in scattering



All imaging can be broken into a three step process

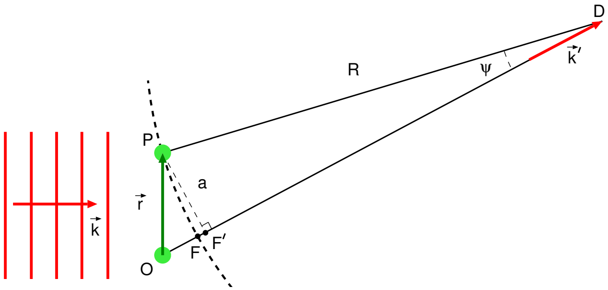
Phase difference in scattering



All imaging can be broken into a three step process

1. x-ray interaction with sample

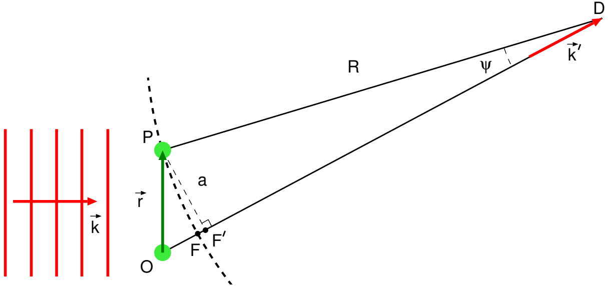
Phase difference in scattering



All imaging can be broken into a three step process

1. x-ray interaction with sample
2. scattered x-ray propagation

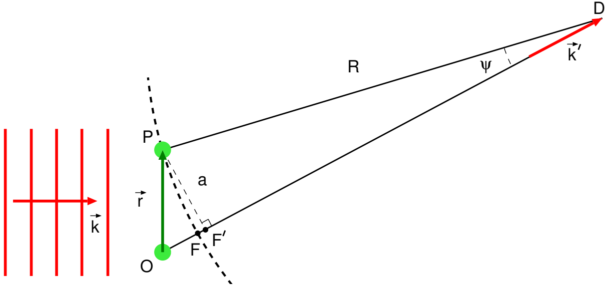
Phase difference in scattering



All imaging can be broken into a three step process

1. x-ray interaction with sample
2. scattered x-ray propagation
3. interaction with detector

Phase difference in scattering

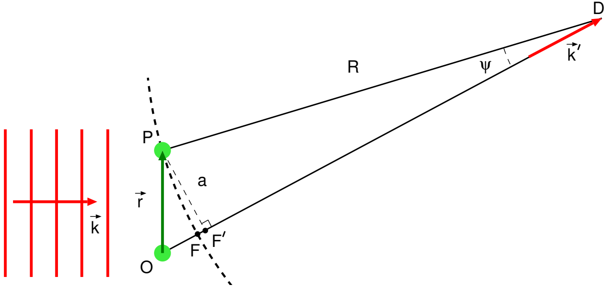


All imaging can be broken into a three step process

1. x-ray interaction with sample
2. scattered x-ray propagation
3. interaction with detector

The scattered waves from O and P will travel different distances

Phase difference in scattering



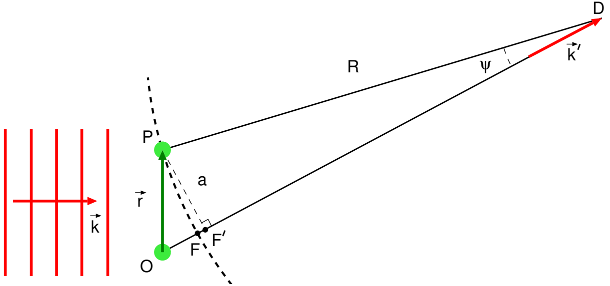
All imaging can be broken into a three step process

1. x-ray interaction with sample
2. scattered x-ray propagation
3. interaction with detector

The scattered waves from O and P will travel different distances

In the far field, the phase difference is $\phi \approx \vec{Q} \cdot \vec{r}$ with $\vec{Q} = \vec{k}' - \vec{k}$

Phase difference in scattering



All imaging can be broken into a three step process

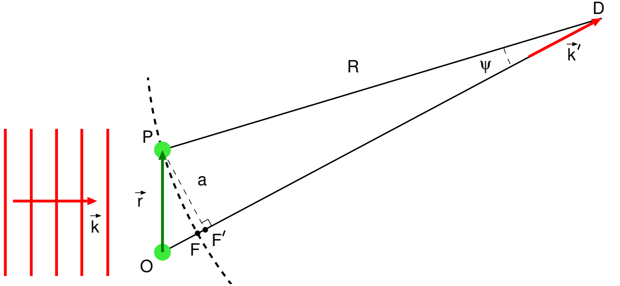
1. x-ray interaction with sample
2. scattered x-ray propagation
3. interaction with detector

The scattered waves from O and P will travel different distances

In the far field, the phase difference is $\phi \approx \vec{Q} \cdot \vec{r}$ with $\vec{Q} = \vec{k}' - \vec{k}$

Since $\vec{k} \perp \vec{r}$, $\phi \approx \vec{Q} \cdot \vec{r} = \vec{k'} \cdot \vec{r}$

Phase difference in scattering



All imaging can be broken into a three step process

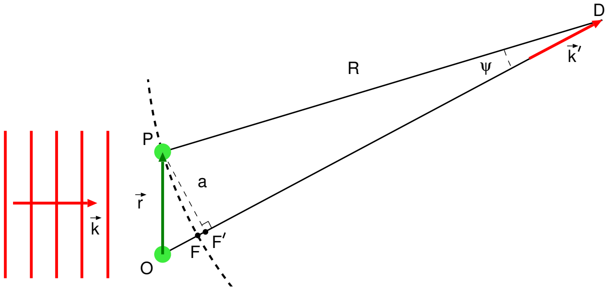
1. x-ray interaction with sample
2. scattered x-ray propagation
3. interaction with detector

The scattered waves from O and P will travel different distances

In the far field, the phase difference is $\phi \approx \vec{Q} \cdot \vec{r}$ with $\vec{Q} = \vec{k}' - \vec{k}$

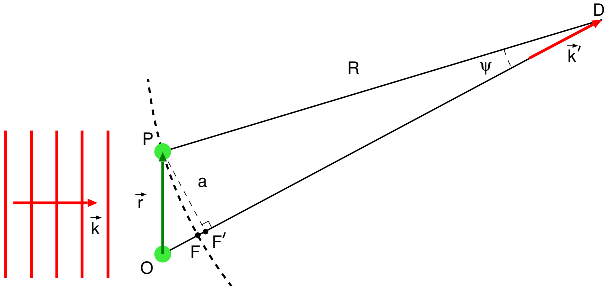
Since $\vec{k} \perp \vec{r}$, $\phi \approx \vec{Q} \cdot \vec{r} = \vec{k}' \cdot \vec{r}$

The path length difference corresponding to this phase shift is $\hat{k}' \cdot r = \overline{OF'}$



The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

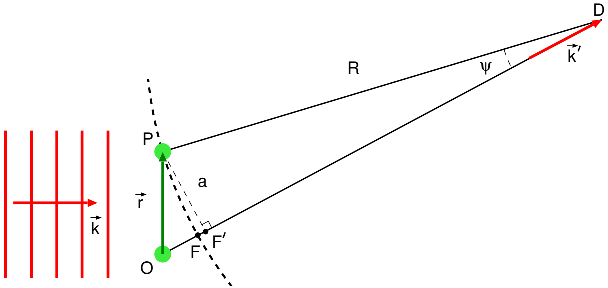
Franuhofer, Fresnel, and contact regimes



The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$\Delta = R - R\cos\psi$$

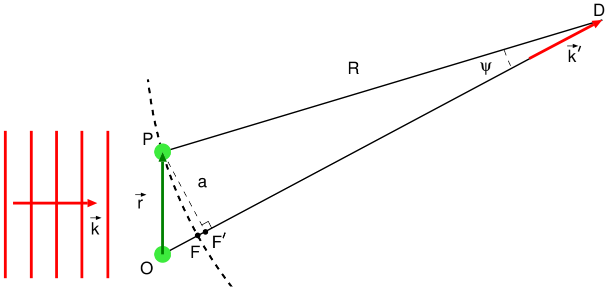
Franuhofer, Fresnel, and contact regimes



The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$\begin{aligned}\Delta &= R - R\cos\psi \\ &\approx R(1-(1-\psi^2/2))\end{aligned}$$

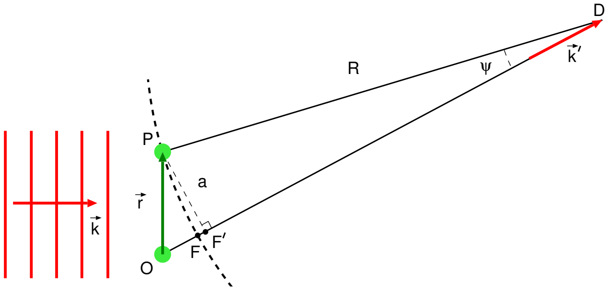
Franuhofer, Fresnel, and contact regimes



The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$\begin{aligned}\Delta &= R - R\cos\psi \\ &\approx R(1-(1-\psi^2/2)) \\ &= R\frac{a^2}{2R^2}\end{aligned}$$

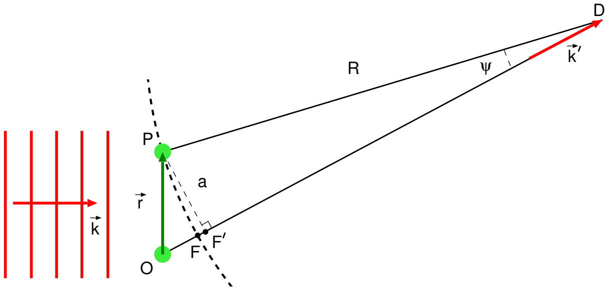
Franuhofer, Fresnel, and contact regimes



The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$
\begin{aligned}
\Delta &= R - R\cos\psi \\
&\approx R(1 - (1 - \psi^2/2)) \\
&= R\frac{a^2}{2R^2} = \frac{a^2}{2R}
\end{aligned}
$$

Franuhofer, Fresnel, and contact regimes

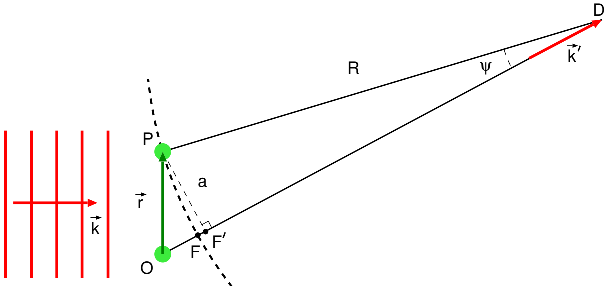


The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$\begin{aligned}\Delta &= R - R\cos\psi \\ &\approx R(1-(1-\psi^2/2)) \\ &= R\frac{a^2}{2R^2} = \frac{a^2}{2R}\end{aligned}$$

$$R \gg \frac{a^2}{\lambda}$$ Fraunhofer

Franuhofer, Fresnel, and contact regimes



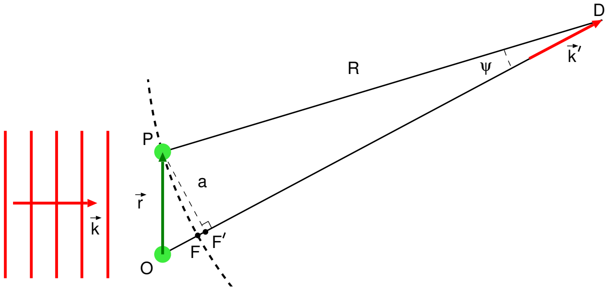
The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$\Delta = R - R\cos\psi$$
$$\approx R(1-(1-\psi^2/2))$$
$$= R\frac{a^2}{2R^2} = \frac{a^2}{2R}$$

$R \gg \frac{a^2}{\lambda}$ Fraunhofer

$R \approx \frac{a^2}{\lambda}$ Fresnel

Franuhofer, Fresnel, and contact regimes

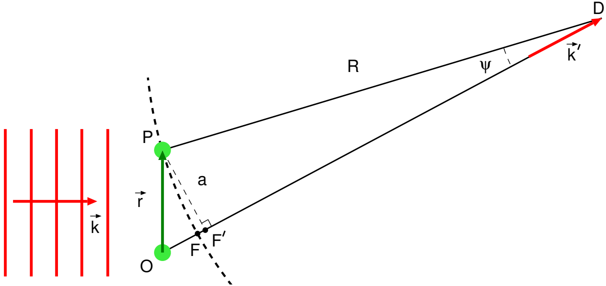


The path length difference computed with the far field approximation has a built in error of $\Delta = \overline{FF'}$ which sets a scale for different kinds of imaging

$$\begin{aligned}\Delta &= R - R\cos\psi \\ &\approx R(1-(1-\psi^2/2)) \\ &= R\frac{a^2}{2R^2} = \frac{a^2}{2R}\end{aligned}$$

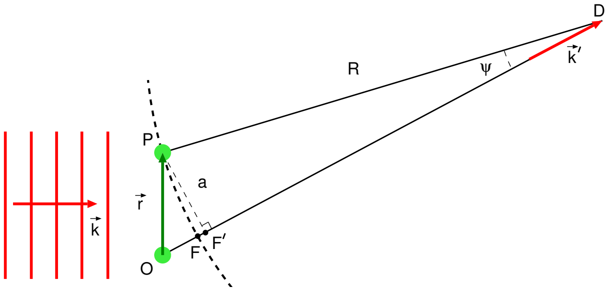
$R \gg \dfrac{a^2}{\lambda}$	Fraunhofer
$R \approx \dfrac{a^2}{\lambda}$	Fresnel
$R \ll \dfrac{a^2}{\lambda}$	Contact

Detector placement



If $\lambda = 1$ Å and the distance to be resolved is $a = 1$ Å, then $a^2/\lambda = 1$ Å and *any* detector placement is in the Fraunhofer (far field) regime

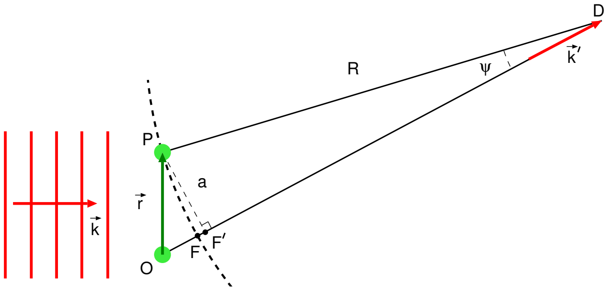
Detector placement



If $\lambda = 1$ Å and the distance to be resolved is $a = 1$ Å, then $a^2/\lambda = 1$ Å and *any* detector placement is in the Fraunhofer (far field) regime

if $a = \mu$m, then $a^2/\lambda = 10$ mm and the imaging regime can be selected by detector placement

Detector placement

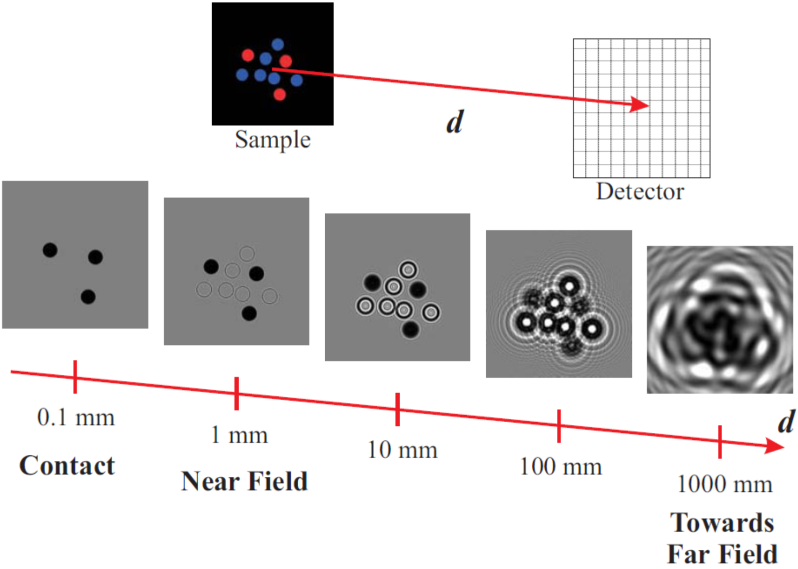


If $\lambda = 1$ Å and the distance to be resolved is $a = 1$ Å, then $a^2/\lambda = 1$ Å and *any* detector placement is in the Fraunhofer (far field) regime

if $a = \mu$m, then $a^2/\lambda = 10$ mm and the imaging regime can be selected by detector placement

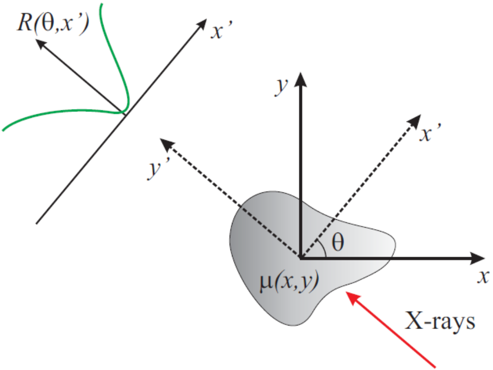
if $a = 1$ mm, then $a^2/\lambda = 10$ km and the detector will always be in the contact regime

Contact to far-field imaging



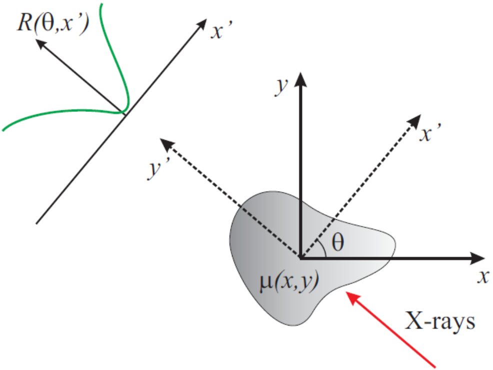
Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895.



Radiography to tomography

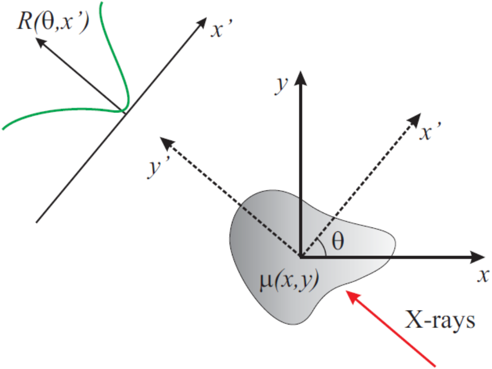
Radiography started immediately after the discovery of x-rays in 1895. In 1970 Computer Tomography revolutionized x-ray imaging



Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895. In 1970 Computer Tomography revolutionized x-ray imaging

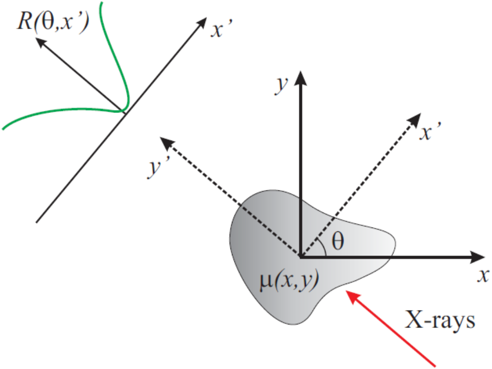
Assume the object to be imaged has a non uniform absorption coefficient $\mu(x, y)$



Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895. In 1970 Computer Tomography revolutionized x-ray imaging

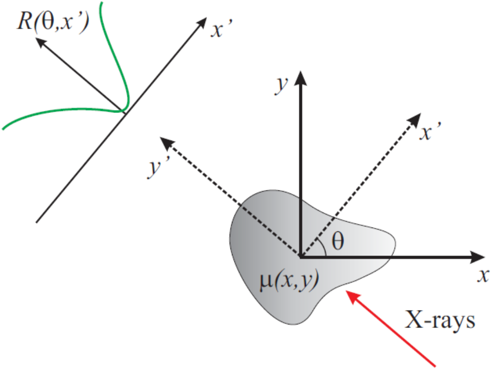
Assume the object to be imaged has a non uniform absorption coefficient $\mu(x, y)$ The line integral of the absorption coefficient at a particular value of x' is measured as the ratio of the transmitted to the incident beam



Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895. In 1970 Computer Tomography revolutionized x-ray imaging

Assume the object to be imaged has a non uniform absorption coefficient $\mu(x, y)$ The line integral of the absorption coefficient at a particular value of x' is measured as the ratio of the transmitted to the incident beam

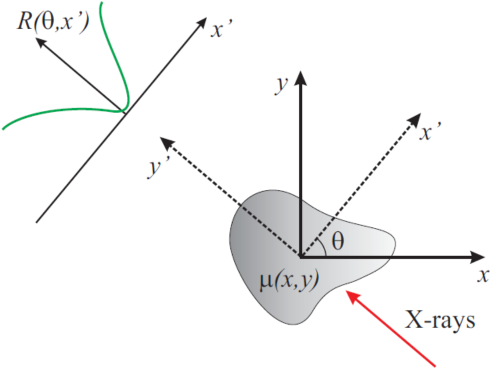


$$I = I_0 e^{-\int \mu(x,y) dy'}$$

Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895. In 1970 Computer Tomography revolutionized x-ray imaging

Assume the object to be imaged has a non uniform absorption coefficient $\mu(x,y)$ The line integral of the absorption coefficient at a particular value of x' is measured as the ratio of the transmitted to the incident beam



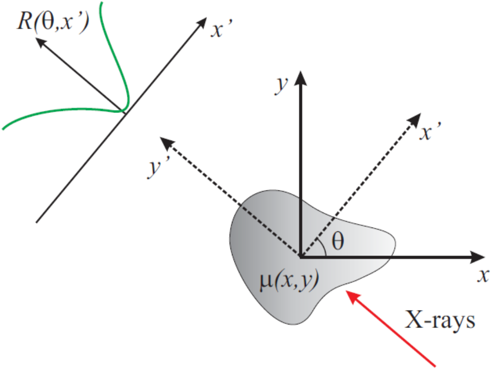
$$I = I_0 e^{-\int \mu(x,y)dy'}$$

$$\ln\left(\frac{I_0}{I}\right) = \int \mu(x,y)dy'$$

Radiography to tomography

Radiography started immediately after the discovery of x-rays in 1895. In 1970 Computer Tomography revolutionized x-ray imaging

Assume the object to be imaged has a non uniform absorption coefficient $\mu(x,y)$ The line integral of the absorption coefficient at a particular value of x' is measured as the ratio of the transmitted to the incident beam



$$I = I_0 e^{-\int \mu(x,y)dy'}$$

$$\ln\left(\frac{I_0}{I}\right) = \int \mu(x,y)dy'$$

The radon transform $R(\theta, x')$ is used to reconstruct the 3D absorption image of the object numerically.

Fourier slice theorem

Start with a general function $f(x,y)$ which is projected onto the x-axis

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x) e^{iq_x x} dx$$

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x) e^{iq_x x} dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x, y)$?

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y)dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x)e^{iq_x x}dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x, y)$? The Fourier transform of $f(x, y)$ is $F(q_x, q_y)$

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x) e^{iq_x x} dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x, y)$? The Fourier transform of $f(x, y)$ is $F(q_x, q_y)$

$$F(q_x, q_y) = \int \int f(x, y) e^{iq_x x + q_y y} dxdy$$

Fourier slice theorem

Start with a general function $f(x,y)$ which is projected onto the x-axis

$$p(x) = \int f(x,y)dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x)e^{iq_x x}dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x,y)$? The Fourier transform of $f(x,y)$ is $F(q_x, q_y)$ and by choosing $q_y \equiv 0$, we get a slice

$$F(q_x, q_y) = \int\int f(x,y)e^{iq_x x + q_y y}dxdy$$

Fourier slice theorem

Start with a general function $f(x,y)$ which is projected onto the x-axis

$$p(x) = \int f(x,y)dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x)e^{iq_x x}dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x,y)$? The Fourier transform of $f(x,y)$ is $F(q_x,q_y)$ and by choosing $q_y \equiv 0$, we get a slice

$$F(q_x,q_y) = \int\int f(x,y)e^{iq_x x + q_y y}dxdy$$

$$F(q_x,q_y=0) = \int\left[\int f(x,y)dy\right]e^{iq_x x}dx$$

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x) e^{iq_x x} dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x, y)$? The Fourier transform of $f(x, y)$ is $F(q_x, q_y)$ and by choosing $q_y \equiv 0$, we get a slice

$$F(q_x, q_y) = \int \int f(x, y) e^{iq_x x + q_y y} dx dy$$

$$F(q_x, q_y = 0) = \int \left[\int f(x, y) dy \right] e^{iq_x x} dx = \int p(x) e^{iq_x x} dx$$

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x) e^{iq_x x} dx$$

What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x, y)$? The Fourier transform of $f(x, y)$ is $F(q_x, q_y)$ and by choosing $q_y \equiv 0$, we get a slice

$$F(q_x, q_y) = \int \int f(x, y) e^{iq_x x + q_y y} dx dy$$

$$F(q_x, q_y = 0) = \int \left[\int f(x, y) dy \right] e^{iq_x x} dx = \int p(x) e^{iq_x x} dx = P(q_x)$$

Fourier slice theorem

Start with a general function $f(x, y)$ which is projected onto the x-axis

$$p(x) = \int f(x, y) dy$$

the Fourier transform of the projection is

$$P(q_x) = \int p(x) e^{iq_x x} dx$$

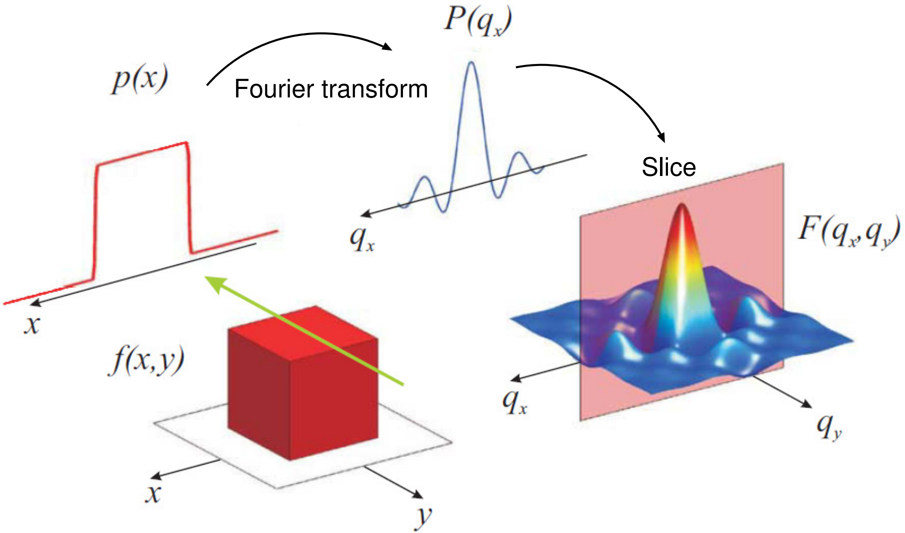
What is the relationship of the Fourier transform, $P(q_x)$, to the original function, $f(x, y)$? The Fourier transform of $f(x, y)$ is $F(q_x, q_y)$ and by choosing $q_y \equiv 0$, we get a slice

$$F(q_x, q_y) = \int \int f(x, y) e^{iq_x x + q_y y} dx dy$$

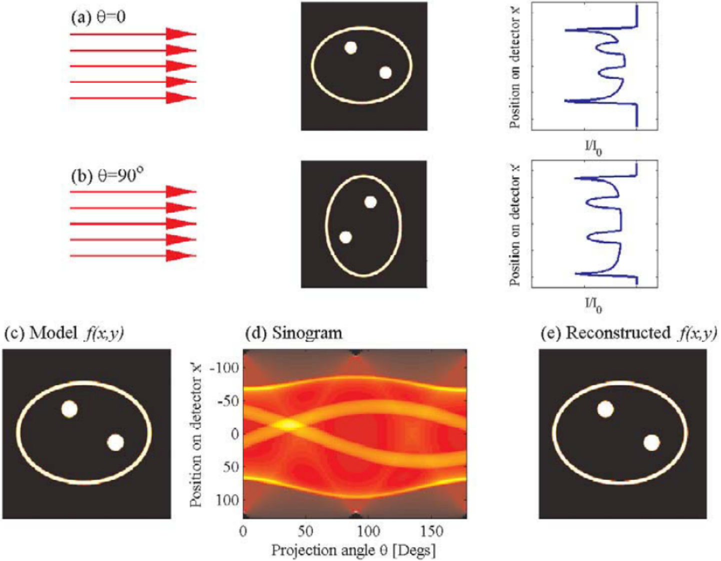
$$F(q_x, q_y = 0) = \int \left[\int f(x, y) dy \right] e^{iq_x x} dx = \int p(x) e^{iq_x x} dx = P(q_x)$$

The Fourier transform of the projection is equal to a slice through the Fourier transform of the object at the origin in the direction of propagation

Fourier transform reconstruction

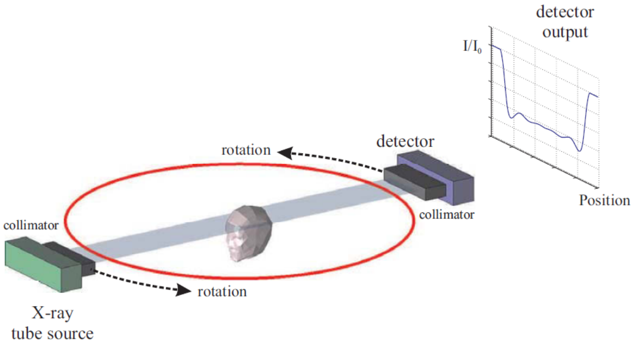


Sinograms



(a) θ=0
Position on detector x'
I/I0
(b) θ=90°
Position on detector x'
I/I0
(c) Model f(x,y)
(d) Sinogram
Position on detector x'
-100
-50
0
50
100
0
50
100
150
Projection angle θ [Degs]
(e) Reconstructed f(x,y)

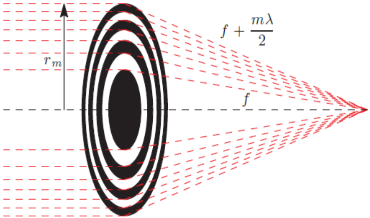
Medical tomography



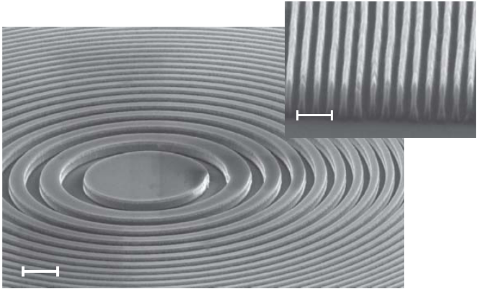
Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

(a)

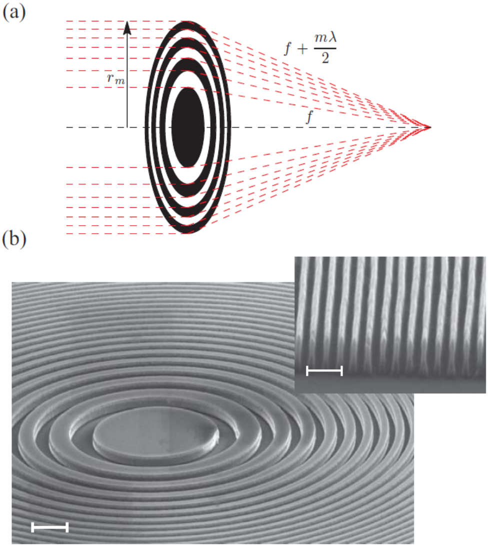


(b)



The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

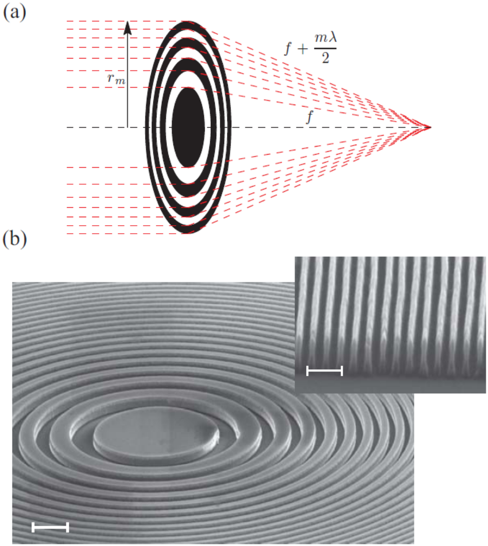


Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1})$$

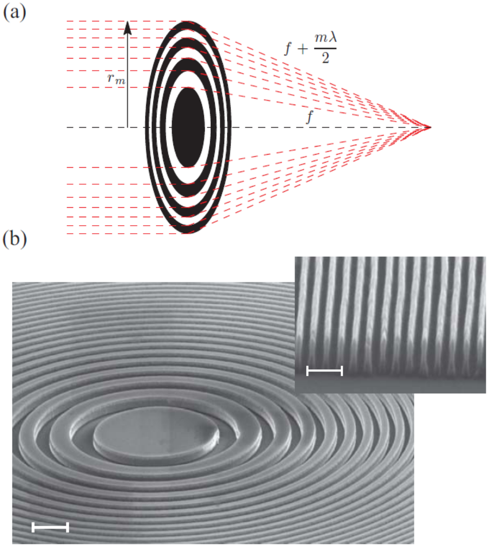


Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1}) \approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$

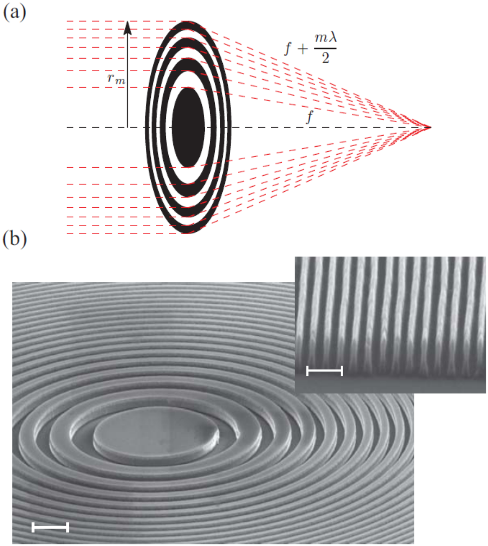


Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1})$$
$$\approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$
$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$



Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

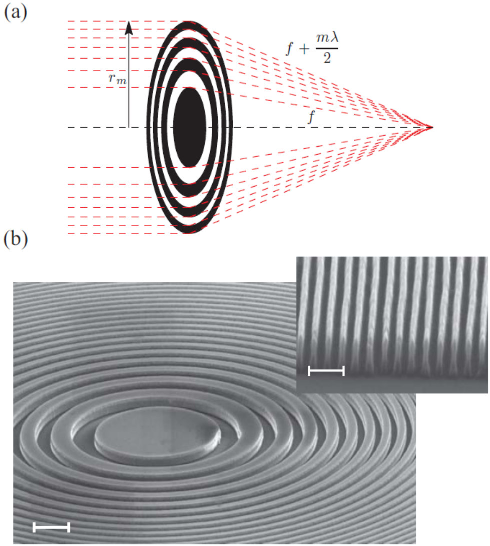
the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1})$$

$$\approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$

$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$

$$D = 2r_M$$



Microscopy

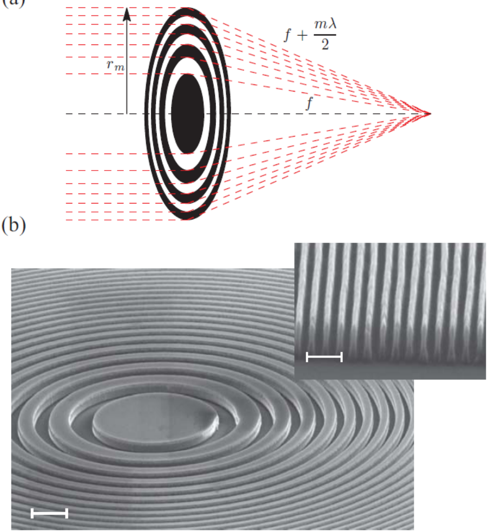
The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1})$$
$$\approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$
$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$
$$D = 2r_M = 2\sqrt{M\lambda f}$$

(a)

(b)

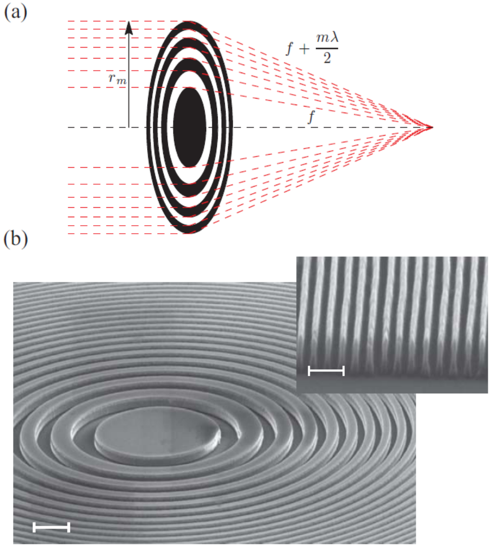


Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1})$$
$$\approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$
$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$
$$D = 2r_M = 2\sqrt{M\lambda f}$$
$$= 2\sqrt{M}\sqrt{\lambda f}$$



The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

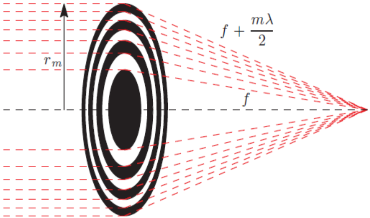
the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1}) \approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$

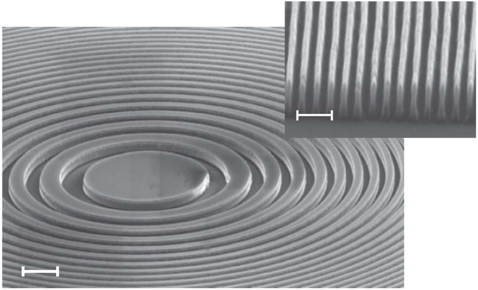
$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$

$$D = 2r_M = 2\sqrt{M\lambda f} = 2\sqrt{M}\sqrt{\lambda f} = 4M\Delta r_M$$

(a)



(b)



Microscopy

The radius of the m^{th} zone is $r_m \approx \sqrt{m\lambda f}$

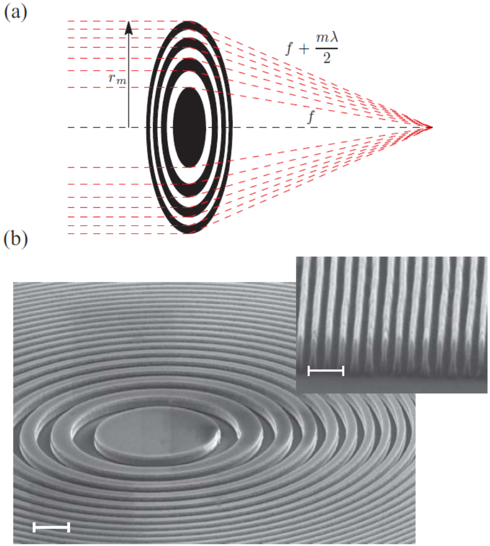
the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1}) \approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$

$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$

$$D = 2r_M = 2\sqrt{M\lambda f} = 2\sqrt{M}\sqrt{\lambda f} = 4M\Delta r_M$$

$$\Delta x = 1.22\frac{\lambda f}{D}$$



Microscopy

The radius of the m^{th} zone is
$r_m \approx \sqrt{m\lambda f}$

the width of the outermost zone is

$$\Delta r_M = \sqrt{\lambda f}(\sqrt{M} - \sqrt{M-1}) \approx \frac{\sqrt{\lambda f}}{2\sqrt{M}}$$

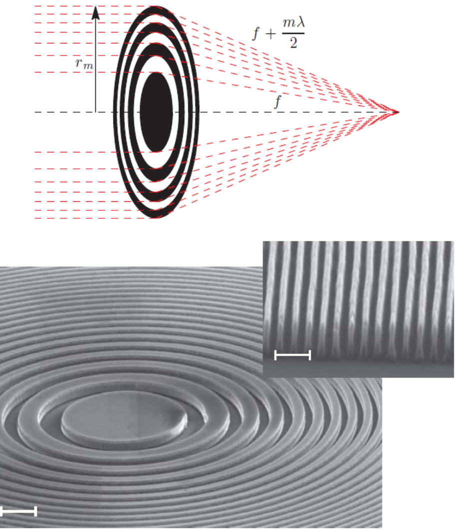
$$f = 4M\frac{(\Delta r_M)^2}{\lambda}$$

$$D = 2r_M = 2\sqrt{M\lambda f} = 2\sqrt{M}\sqrt{\lambda f} = 4M\Delta r_M$$

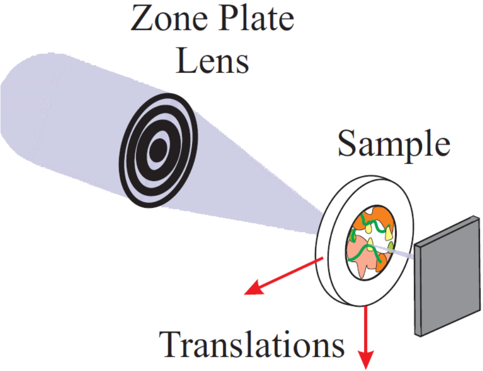
$$\Delta x = 1.22\frac{\lambda f}{D} = 1.22\Delta r_M$$

(a)

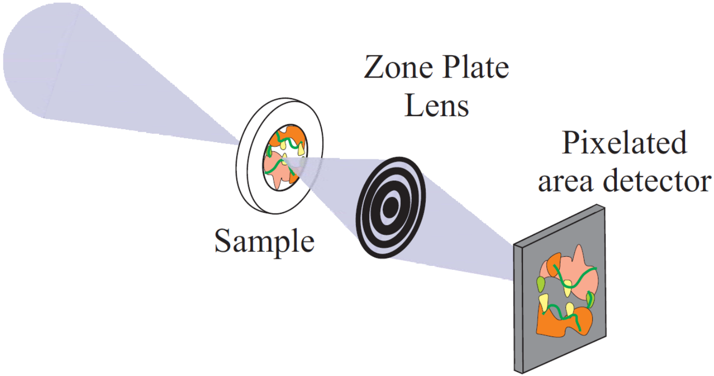
(b)



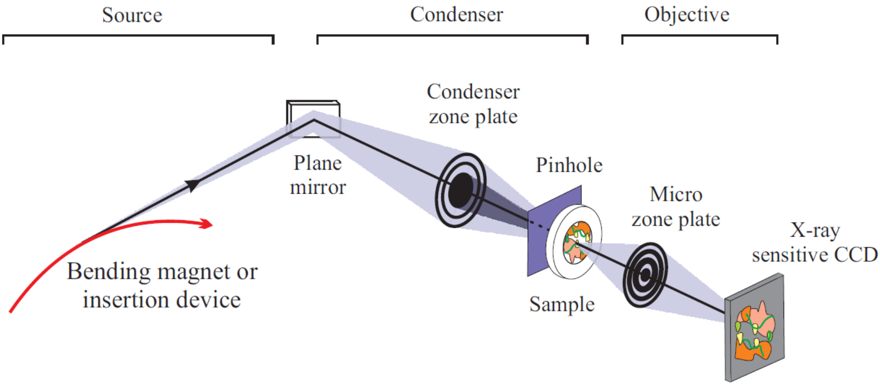
Scanning transmission x-ray microscope



Full field microscope



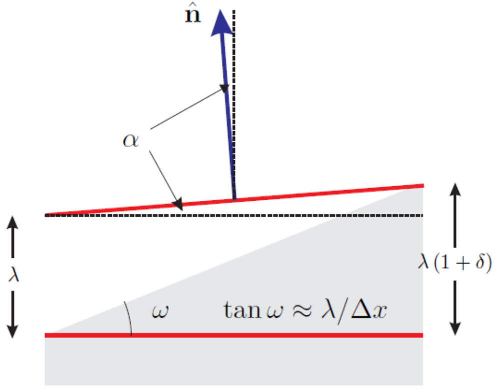
Transmission x-ray microscope



Angular deviation from refraction

When x-rays cross an interface that is not normal to their direction, there is refraction

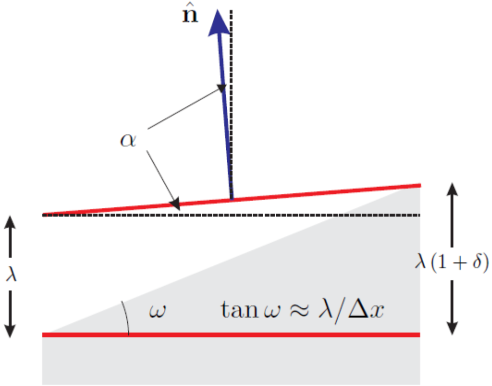
Angular deviation from refraction



When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

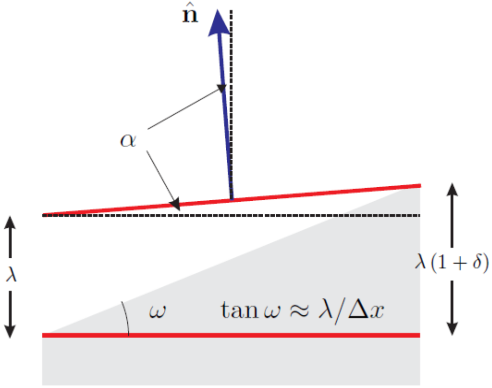
Angular deviation from refraction



When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

Angular deviation from refraction

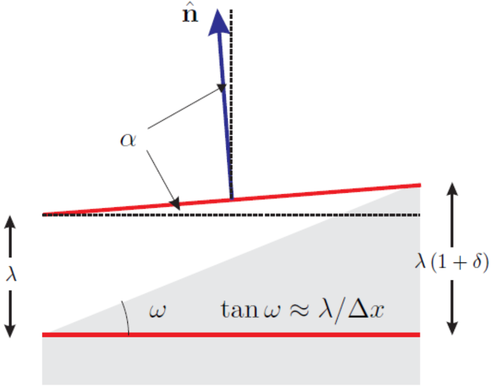


When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

$$\lambda_n = \frac{\lambda}{n}$$

Angular deviation from refraction

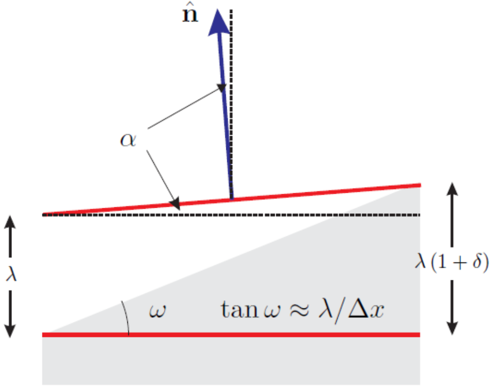


When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

$$\lambda_n = \frac{\lambda}{n} = \frac{\lambda}{1-\delta}$$

Angular deviation from refraction

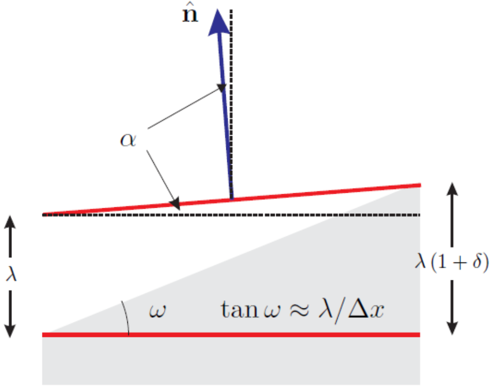


When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

$$\lambda_n = \frac{\lambda}{n} = \frac{\lambda}{1-\delta}$$
$$\approx \lambda(1+\delta)$$

Angular deviation from refraction

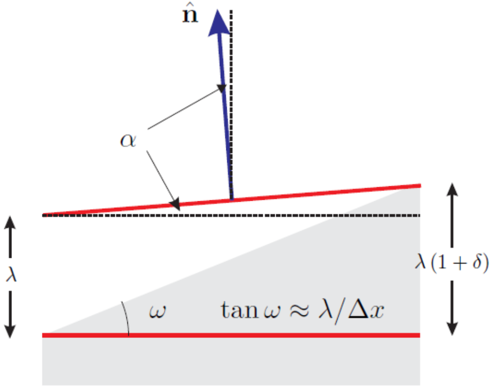


When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

$$\lambda_n = \frac{\lambda}{n} = \frac{\lambda}{1-\delta}$$
$$\approx \lambda(1+\delta)$$
$$\alpha = \frac{\lambda(1+\delta)-\lambda}{\Delta x}$$

Angular deviation from refraction

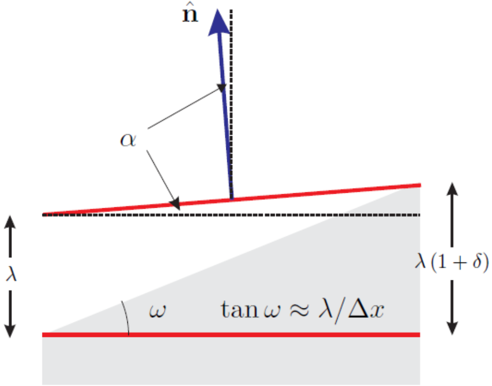


When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

$$
\begin{aligned}
\lambda_n &= \frac{\lambda}{n} = \frac{\lambda}{1-\delta} \\
&\approx \lambda(1+\delta) \\
\alpha &= \frac{\lambda(1+\delta)-\lambda}{\Delta x} \\
&= \delta\frac{\lambda}{\Delta x}
\end{aligned}
$$

Angular deviation from refraction



When x-rays cross an interface that is not normal to their direction, there is refraction

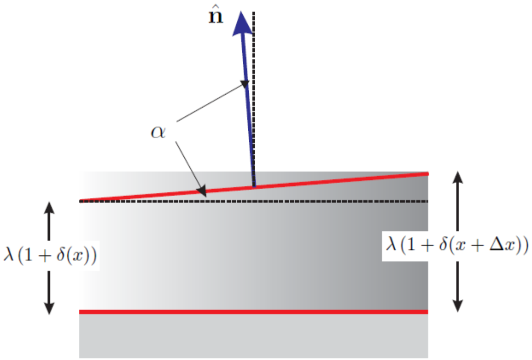
The angle of refraction α can be calculated

$$\lambda_n = \frac{\lambda}{n} = \frac{\lambda}{1-\delta}$$
$$\approx \lambda(1+\delta)$$
$$\alpha = \frac{\lambda(1+\delta)-\lambda}{\Delta x}$$
$$= \delta\frac{\lambda}{\Delta x} \approx \delta \tan\omega$$

Angular deviation from graded density

In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

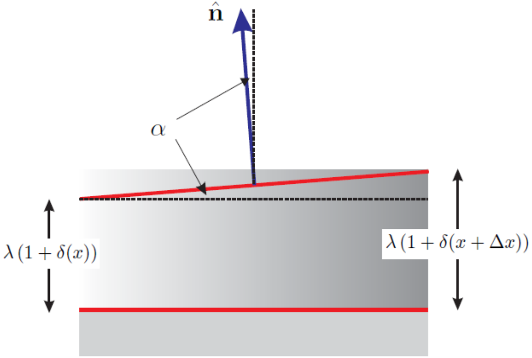
Angular deviation from graded density



In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

The angle of refraction α can be calculated

Angular deviation from graded density

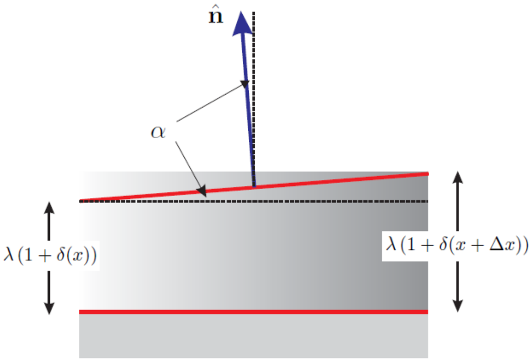


In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

The angle of refraction α can be calculated

$$\alpha = \frac{\lambda(1+\delta(x+\Delta x)) - \lambda(1+\delta(x))}{\Delta x}$$

Angular deviation from graded density



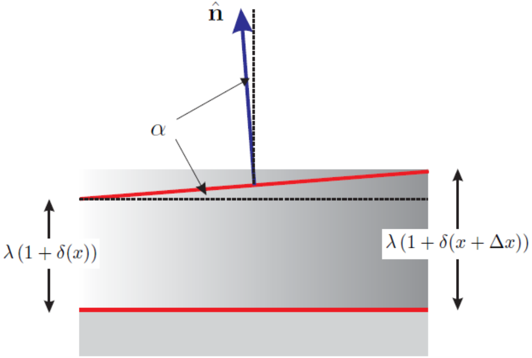
In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

The angle of refraction α can be calculated

$$\alpha = \frac{\lambda(1+\delta(x+\Delta x)) - \lambda(1+\delta(x))}{\Delta x}$$

$$\delta(x+\Delta x) \approx \delta(x) + \Delta x \frac{\partial\delta(x)}{\partial x}$$

Angular deviation from graded density



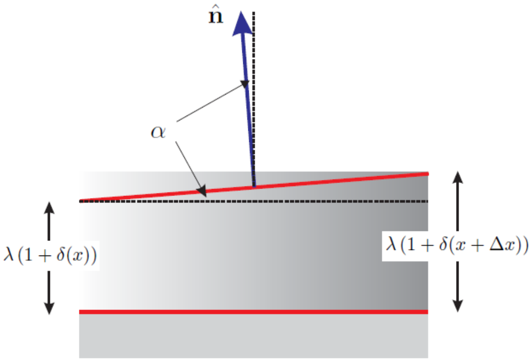
In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

The angle of refraction α can be calculated

$$\alpha = \frac{\lambda(1+\delta(x+\Delta x)) - \lambda(1+\delta(x))}{\Delta x} = \frac{\lambda \Delta x \frac{\partial \delta(x)}{\partial x}}{\Delta x}$$

$$\delta(x+\Delta x) \approx \delta(x) + \Delta x \frac{\partial \delta(x)}{\partial x}$$

Angular deviation from graded density



In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

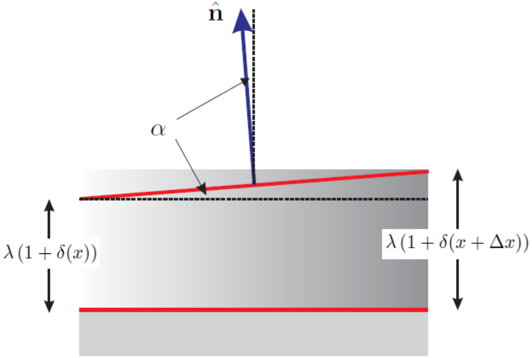
The angle of refraction α can be calculated

$$\alpha = \frac{\lambda(1+\delta(x+\Delta x)) - \lambda(1+\delta(x))}{\Delta x} = \frac{\lambda \Delta x \frac{\partial \delta(x)}{\partial x}}{\Delta x}$$

$$\delta(x+\Delta x) \approx \delta(x) + \Delta x \frac{\partial \delta(x)}{\partial x}$$

$$\alpha_{gradient} = \lambda \frac{\partial \delta(x)}{\partial x}$$

Angular deviation from graded density



In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

The angle of refraction α can be calculated

$$\alpha = \frac{\lambda(1+\delta(x+\Delta x)) - \lambda(1+\delta(x))}{\Delta x} = \frac{\lambda \Delta x \frac{\partial \delta(x)}{\partial x}}{\Delta x}$$

$$\delta(x+\Delta x) \approx \delta(x) + \Delta x \frac{\partial \delta(x)}{\partial x}$$

$$\alpha_{gradient} = \lambda \frac{\partial \delta(x)}{\partial x} \qquad \text{compare to} \qquad \alpha_{\text{refrac}} = \lambda \frac{\delta}{\Delta x}$$

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

$$\hat{n} = \frac{\vec{k}'}{k'}$$

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

$$\hat{n} = \frac{\vec{k}'}{k'} = \frac{\lambda}{2\pi} \nabla \phi(\vec{r})$$

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

$$\hat{n} = \frac{\vec{k}'}{k'} = \frac{\lambda}{2\pi} \nabla \phi(\vec{r})$$

Thus the angular deviation, in each of the x and y directions in the plane perpendicular to the original propagation direction becomes

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

$$\hat{n} = \frac{\vec{k}'}{k'} = \frac{\lambda}{2\pi}\nabla\phi(\vec{r})$$

Thus the angular deviation, in each of the x and y directions in the plane perpendicular to the original propagation direction becomes

$$\alpha_x = \frac{\lambda}{2\pi}\frac{\partial\phi(x,y)}{\partial x}$$

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

$$\hat{n} = \frac{\vec{k}'}{k'} = \frac{\lambda}{2\pi}\nabla\phi(\vec{r})$$

Thus the angular deviation, in each of the x and y directions in the plane perpendicular to the original propagation direction becomes

$$\alpha_x = \frac{\lambda}{2\pi}\frac{\partial\phi(x,y)}{\partial x}$$

$$\alpha_y = \frac{\lambda}{2\pi}\frac{\partial\phi(x,y)}{\partial y}$$

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

$$\hat{n} = \frac{\vec{k}'}{k'} = \frac{\lambda}{2\pi}\nabla\phi(\vec{r})$$

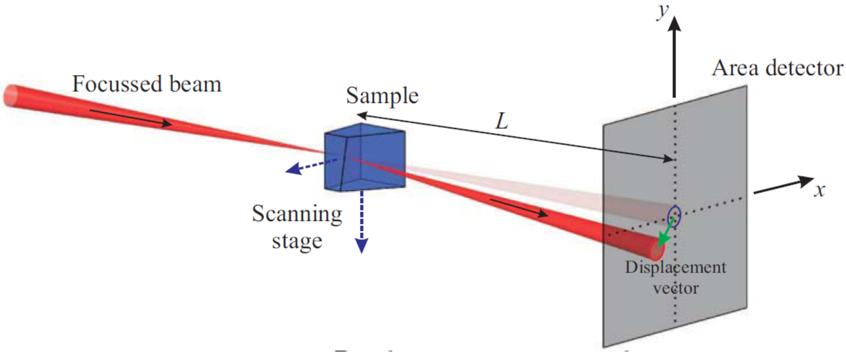
$$\alpha_x = \frac{\lambda}{2\pi}\frac{\partial\phi(x,y)}{\partial x}$$

$$\alpha_y = \frac{\lambda}{2\pi}\frac{\partial\phi(x,y)}{\partial y}$$

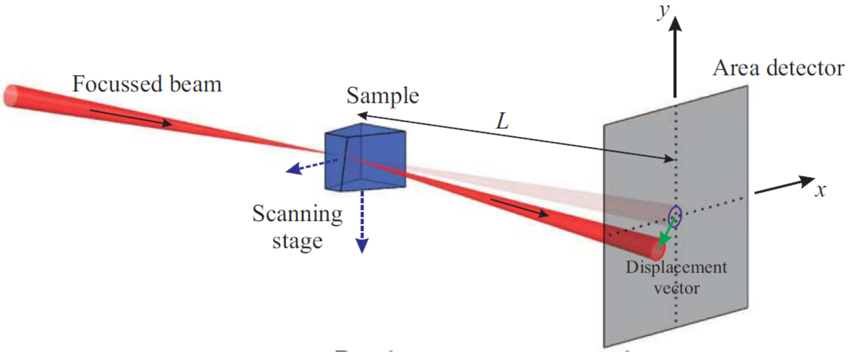
Thus the angular deviation, in each of the x and y directions in the plane perpendicular to the original propagation direction becomes

By measuring the angular deviation as a function of position in a sample, one can reconstruct the phase shift $\phi(x, y)$ due to the sample by integration.

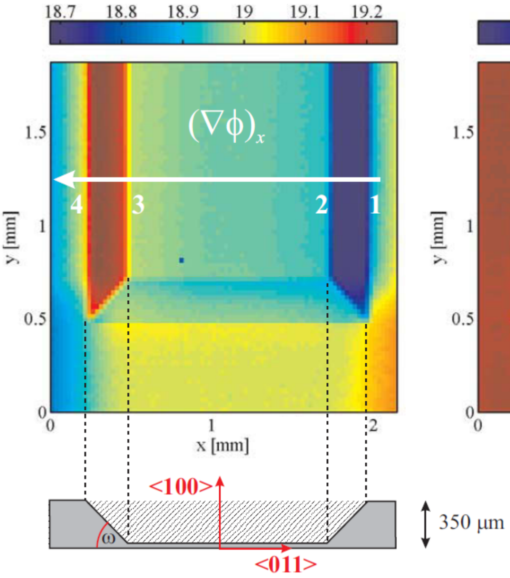
Phase contrast experiment



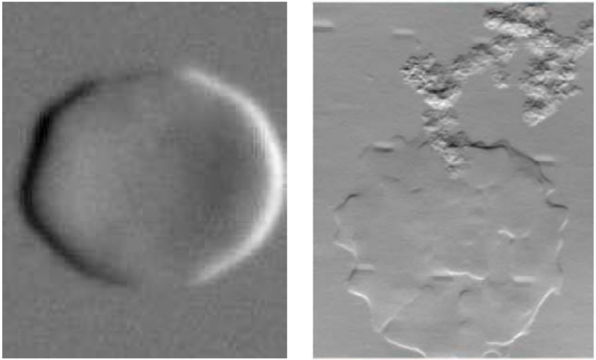
Phase contrast experiment



Imaging a silicon trough



Imaging blood cells



Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

start with a monochromatic plane wave propagating in the z direction, described by the function e^{ikz} with wavelength $\lambda = 2\pi/k$

Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

start with a monochromatic plane wave propagating in the z direction, described by the function e^{ikz} with wavelength $\lambda = 2\pi/k$

when the wavefront passes through an optical element or a sample, it is distorted such that the wavefield at $z = 0$ is now a function of the transverse dimensions, x and y: $\psi_0(x, y)$

Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

start with a monochromatic plane wave propagating in the z direction, described by the function e^{ikz} with wavelength $\lambda = 2\pi/k$

when the wavefront passes through an optical element or a sample, it is distorted such that the wavefield at $z = 0$ is now a function of the transverse dimensions, x and y: $\psi_0(x, y)$

our goal is to determine the wavefield at a distance z downstream and this is done by constructing a propagation operator $\hat{D}_z$ such that

Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

start with a monochromatic plane wave propagating in the z direction, described by the function e^{ikz} with wavelength $\lambda = 2\pi/k$

when the wavefront passes through an optical element or a sample, it is distorted such that the wavefield at $z = 0$ is now a function of the transverse dimensions, x and y: $\psi_0(x, y)$

our goal is to determine the wavefield at a distance z downstream and this is done by constructing a propagation operator $\hat{D}_z$ such that

$$\psi_z = \hat{D}_z \psi_0(x, y)$$

Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

start with a monochromatic plane wave propagating in the z direction, described by the function e^{ikz} with wavelength $\lambda = 2\pi/k$

when the wavefront passes through an optical element or a sample, it is distorted such that the wavefield at $z = 0$ is now a function of the transverse dimensions, x and y: $\psi_0(x, y)$

our goal is to determine the wavefield at a distance z downstream and this is done by constructing a propagation operator $\hat{D}_z$ such that

$$\psi_z = \hat{D}_z \psi_0(x, y)$$

The Fourier transform is used to generate this propagation operator in the following way (showing only the x dependence for simplicity)

Wavefield propagation

In order to understand many of these imaging techniques, it is important to have a good model for wave propagation in an optical system

start with a monochromatic plane wave propagating in the z direction, described by the function e^{ikz} with wavelength $\lambda = 2\pi/k$

when the wavefront passes through an optical element or a sample, it is distorted such that the wavefield at $z = 0$ is now a function of the transverse dimensions, x and y: $\psi_0(x, y)$

our goal is to determine the wavefield at a distance z downstream and this is done by constructing a propagation operator $\hat{D}_z$ such that

$$\psi_z = \hat{D}_z \psi_0(x, y)$$

The Fourier transform is used to generate this propagation operator in the following way (showing only the x dependence for simplicity)

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x\hat{x} + k_z\hat{z}$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows

$$k_z = \sqrt{k^2 - k_x^2}$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2}$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

each of the plane waves $\tilde{\psi}_0 e^{-ik_x x}$ propagates to z by multiplication with a phase factor

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

each of the plane waves $\tilde{\psi}_0 e^{-ik_x x}$ propagates to z by multiplication with a phase factor

$$\tilde{\psi}_0 e^{-ik_x x} \rightarrow \tilde{\psi}_0 e^{-ik_x x} e^{ik_z z}$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

each of the plane waves $\tilde{\psi}_0 e^{-ik_x x}$ propagates to z by multiplication with a phase factor

$$\tilde{\psi}_0 e^{-ik_x x} \rightarrow \tilde{\psi}_0 e^{-ik_x x} e^{ik_z z} = \tilde{\psi}_0 e^{-ik_x x} e^{ikz} e^{-ik_x^2 z/2k} = \tilde{\psi}_z(k_x)$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi} \int \tilde{\psi}_0(k_x) e^{-ik_x x} dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x \hat{x} + k_z \hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

each of the plane waves $\tilde{\psi}_0 e^{-ik_x x}$ propagates to z by multiplication with a phase factor

$$\tilde{\psi}_0 e^{-ik_x x} \rightarrow \tilde{\psi}_0 e^{-ik_x x} e^{ik_z z} = \tilde{\psi}_0 e^{-ik_x x} e^{ikz} e^{-ik_x^2 z/2k} = \tilde{\psi}_z(k_x)$$

$$\psi_z(x) = \mathcal{FT}^{-1}[\tilde{\psi}_z(k_x)]$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi}\int \tilde{\psi}_0(k_x)e^{-ik_x x}dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x\hat{x} + k_z\hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

each of the plane waves $\tilde{\psi}_0 e^{-ik_x x}$ propagates to z by multiplication with a phase factor

$$\tilde{\psi}_0 e^{-ik_x x} \rightarrow \tilde{\psi}_0 e^{-ik_x x}e^{ik_z z} = \tilde{\psi}_0 e^{-ik_x x}e^{ikz}e^{-ik_x^2 z/2k} = \tilde{\psi}_z(k_x)$$

$$\psi_z(x) = \mathcal{FT}^{-1}[\tilde{\psi}_z(k_x)] = \frac{1}{2\pi}\int \tilde{\psi}_z(k_x)e^{-ik_x x}dk_x$$

Wavefield propagation

$$\psi_0(x) = \frac{1}{2\pi}\int \tilde{\psi}_0(k_x)e^{-ik_x x}dk_x$$

in words, this says that at $z = 0$ the wavefield is a superposition of plane waves each of which is a function of the total wavevector $\vec{k} = k_x\hat{x} + k_z\hat{z}$

We can rewrite the relationships as follows but if $k_x \ll k$ we have

$$k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$$

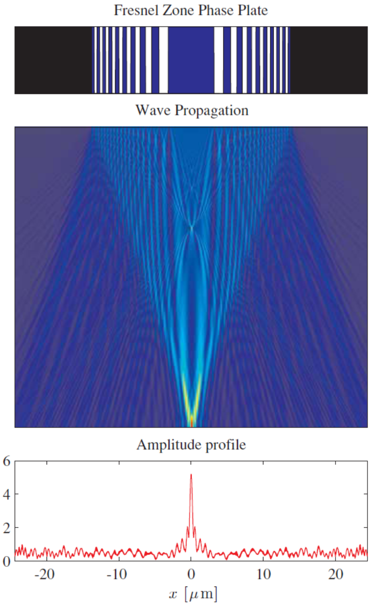
each of the plane waves $\tilde{\psi}_0 e^{-ik_x x}$ propagates to z by multiplication with a phase factor

$$\tilde{\psi}_0 e^{-ik_x x} \rightarrow \tilde{\psi}_0 e^{-ik_x x} e^{ik_z z} = \tilde{\psi}_0 e^{-ik_x x} e^{ikz} e^{-ik_x^2 z/2k} = \tilde{\psi}_z(k_x)$$

$$\psi_z(x) = \mathcal{FT}^{-1}[\tilde{\psi}_z(k_x)] = \frac{1}{2\pi}\int \tilde{\psi}_z(k_x)e^{-ik_x x}dk_x$$

$$\psi_z(x,y) = \hat{D}_z\psi_0(x,y) = e^{ikz}\,\mathcal{FT}^{-1}\left[e^{-iz(k_x^2+k_y^2)/2k}\,\mathcal{FT}[\psi_0(x,y)]\right]$$

Fresnel zone plates



Fresnel zone plates