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Homework Assignment #06:
Chapter 6: 1,6,7,8,9
due Monday, November 14, 2016

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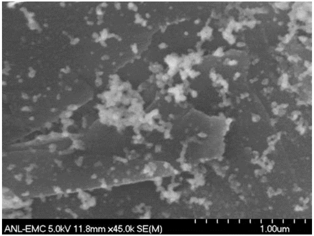
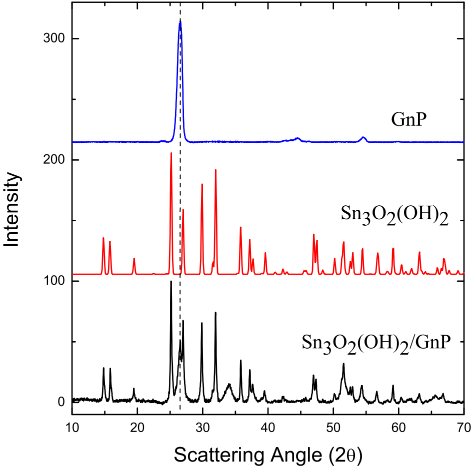
Homework Assignment #06:
Chapter 6: 1,6,7,8,9
due Monday, November 14, 2016

Homework Assignment #7:
Chapter 7: 2,3,9,10,11
due Monday, November 28, 2018

Synthesis of Sn-graphite nanocomposites

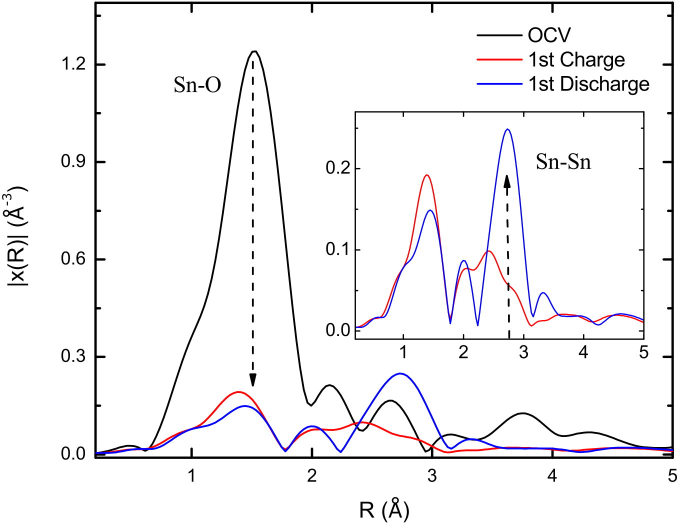




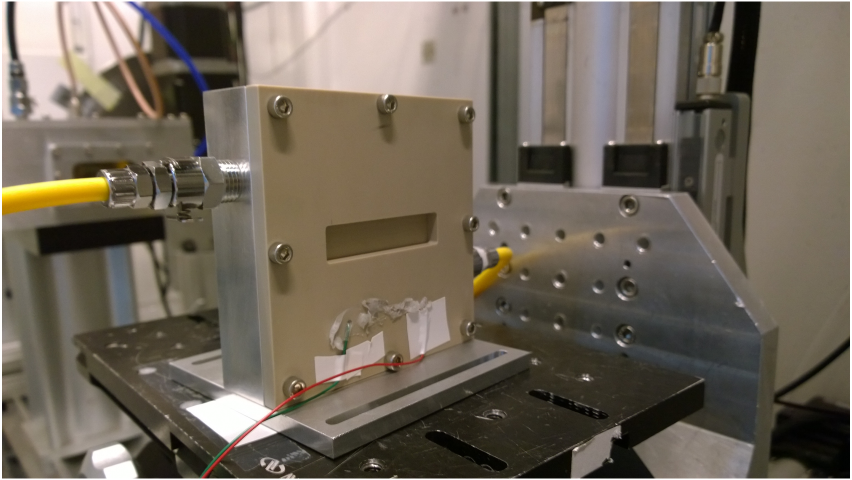
One-pot synthesis produces evenly distributed $Sn_3O_2(OH)_2$ nanoparticles on graphite nanoplatelets

XRD shows a small amount of Sn metal in addition to $Sn_3O_2(OH)_2$

In situ XAS studies of lithiation

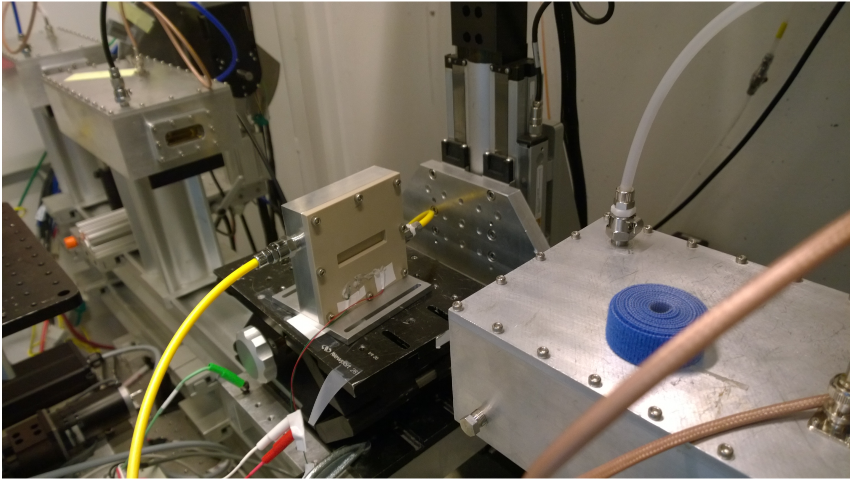


In situ battery box



Pouch cell clamped against front window in helium environment

In situ battery box



Suitable for both transmission and fluorescence measurements

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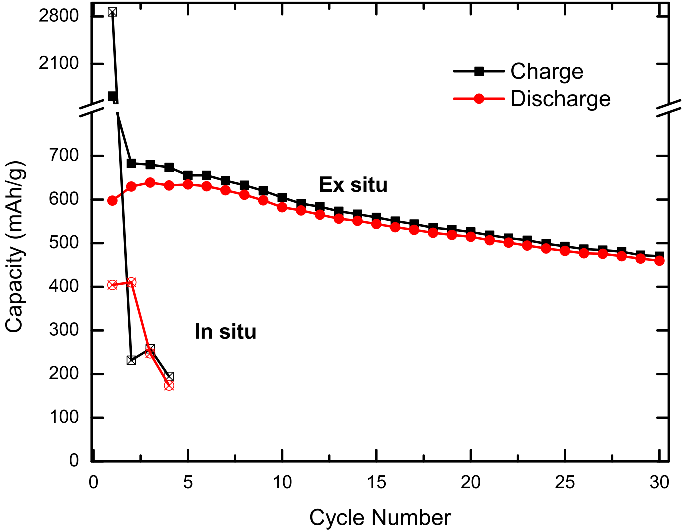

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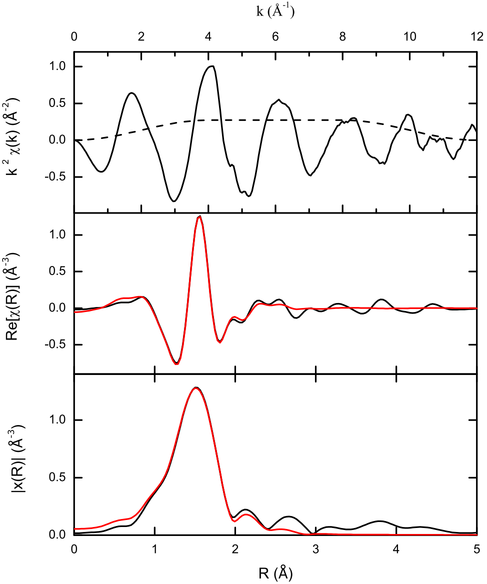

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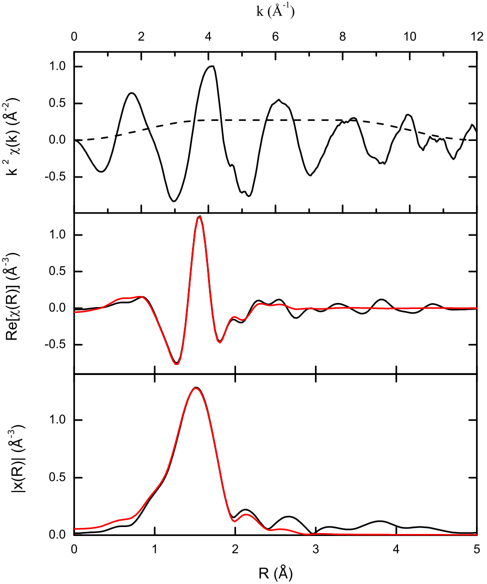

Fresh electrode can be fit with $Sn_3O_2(OH)_2$ structure which is dominated by the near neighbor Sn-O distances

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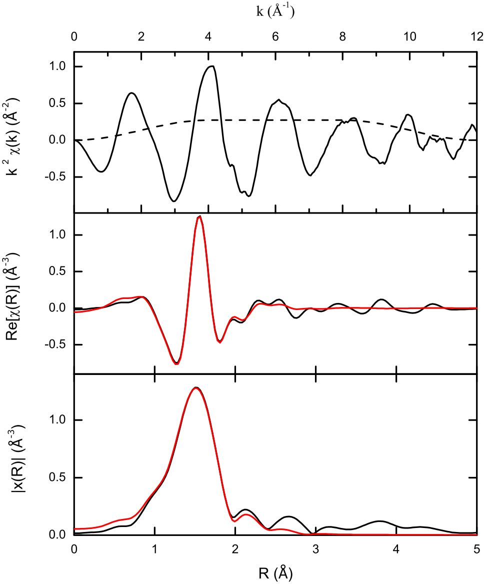

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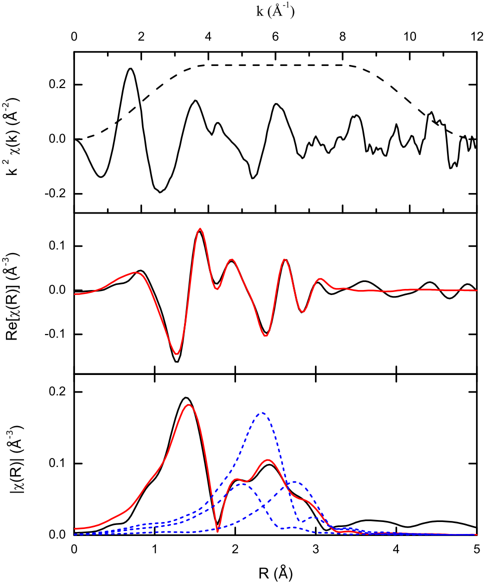
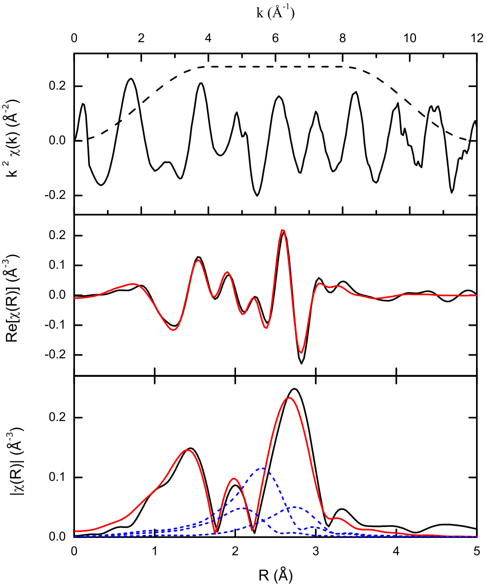
Only a small amount of metallic Sn-Sn distances can be seen



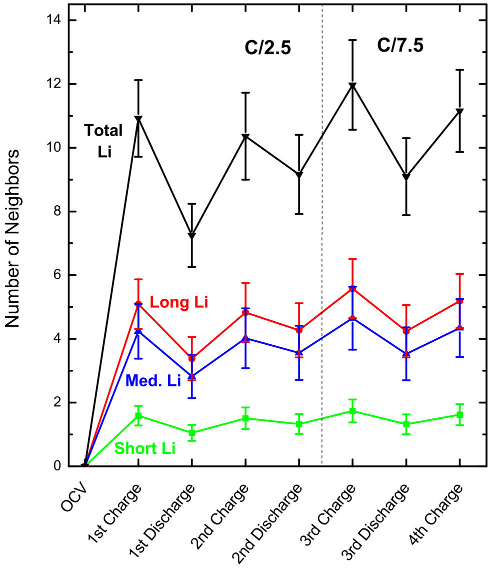
Reduction of number of Sn-O near neighbors and 3 Sn-Li paths characteristic of the $Li_{22}Sn_5$ structure

In situ XAS studies of lithiation



Metallic Sn-Sn distances appear but Sn-Li paths are still present, further reduction in Sn-O near neighbors.

In situ XAS studies of lithiation



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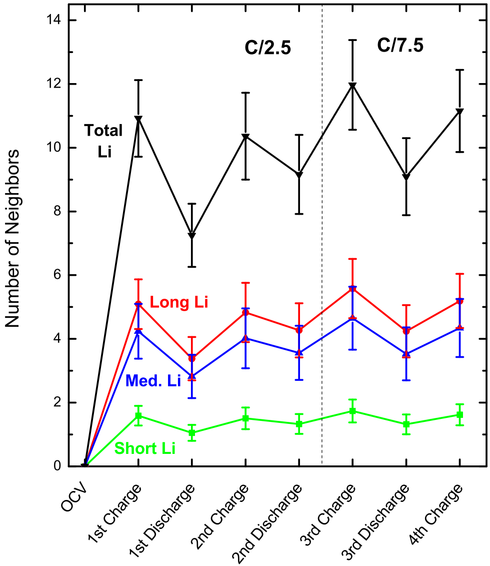

Number of Li near neighbors oscillates with the charge/discharge cycles but never returns to zero

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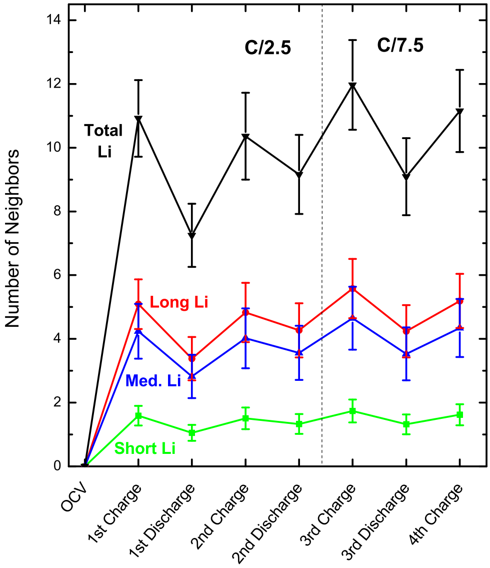

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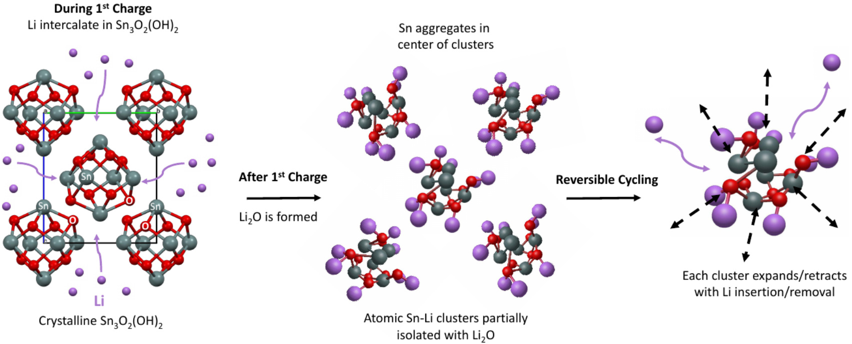
In situ cell promotes accelerated aging because of Sn swelling and the reduced pressure of the thin PEEK pouch cell assembly

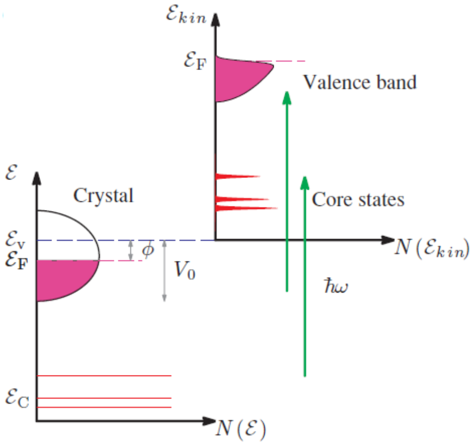
In situ XAS studies of lithiation



C. Pelliccione, E.V. Timofeeva, and C.U. Segre, "In situ XAS study of the capacity fading mechanism in hybrid $Sn_3O_2(OH)_2$/graphite battery anode nanomaterials" *Chem. Mater.* **27**, 574-580 (2015).

The photoemission process

Photoemission is the complement to XAFS. It probes the filled states below the Fermi level



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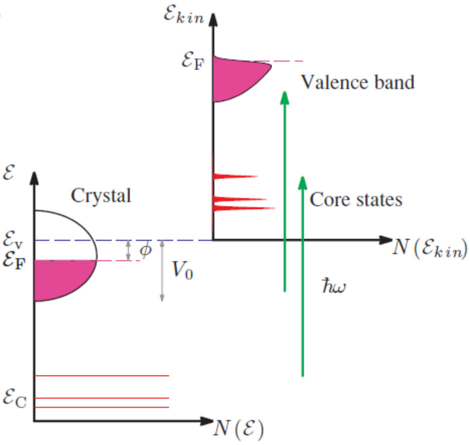

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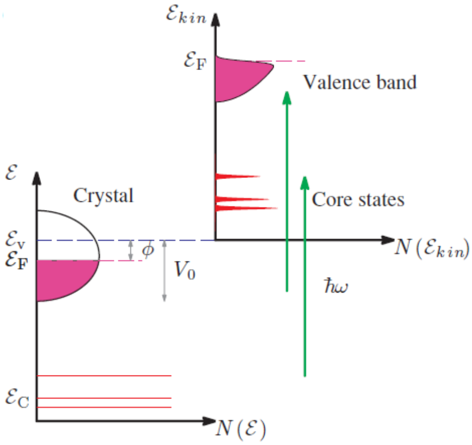

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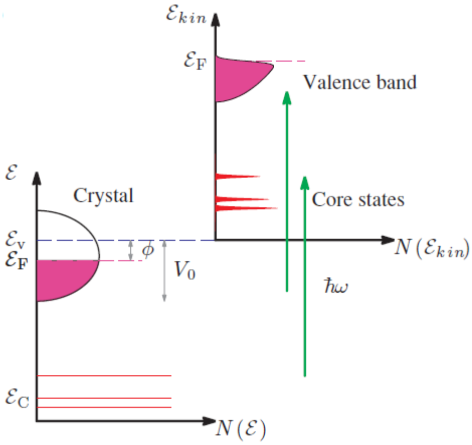
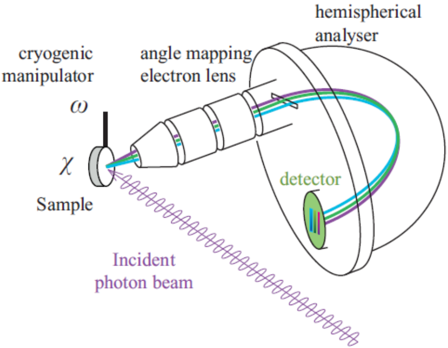
$$\mathcal{E}_{kin} = \frac{\hbar^2 q_v^2}{2m} = \hbar\omega - \phi - \mathcal{E}_B$$

$$\mathcal{E}_B = \mathcal{E}_F - \mathcal{E}_i$$



Hemispherical mirror analyzer

The electric field between the two hemispheres has a R^2 dependence from the center of the hemispheres



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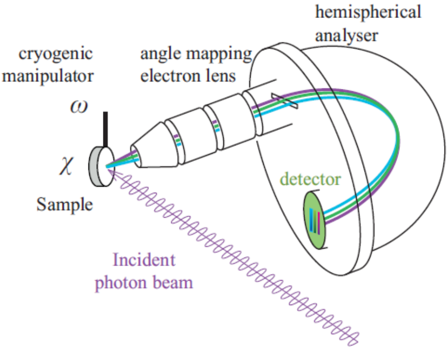
Electrons with $\mathcal{E}_0$, called the "pass energy", will follow a circular path of radius $R_0 = (R_1 + R_2)/2$



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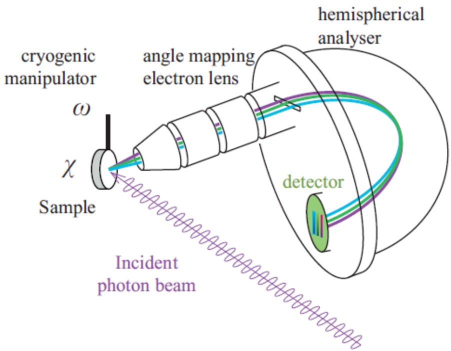

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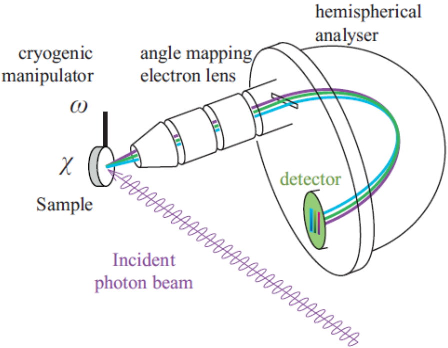
Electrons with lower energy will fall inside this circular path while those with higher enegy will fall outside



A better scattering model

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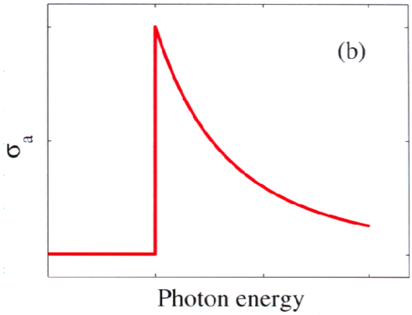

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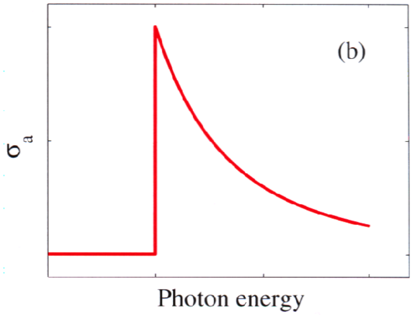

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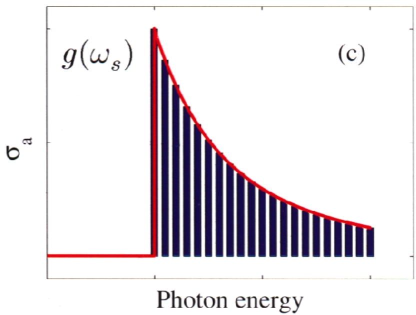

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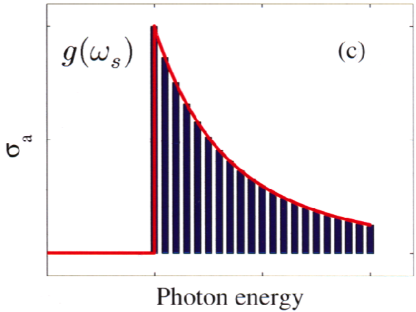

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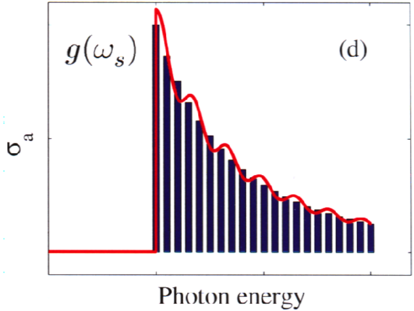

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The amplitude of the response has a resonance and dissipation

Radiated field

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which is an outgoing spherical wave
with scattering amplitude

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$$
\begin{aligned}
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\end{aligned}
$$

which is an outgoing spherical wave with scattering amplitude

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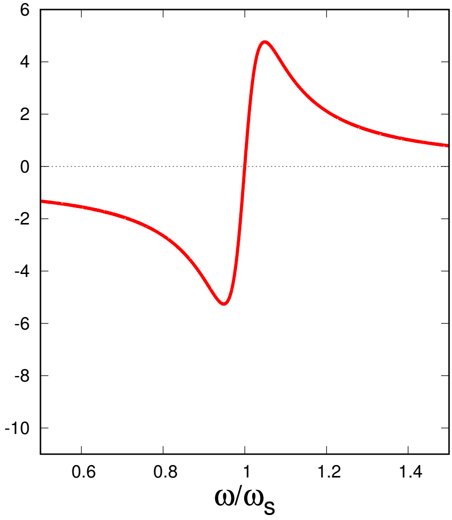

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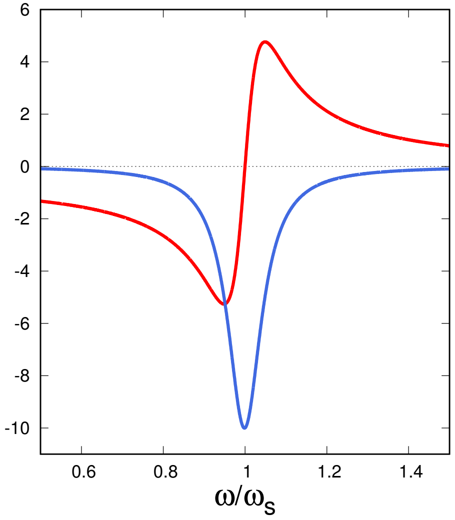

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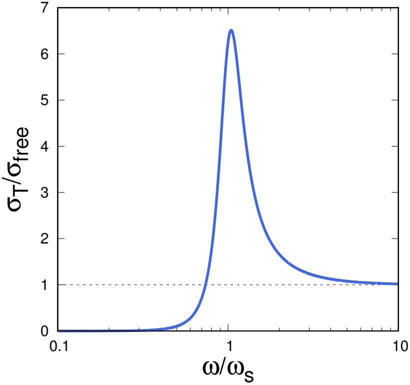

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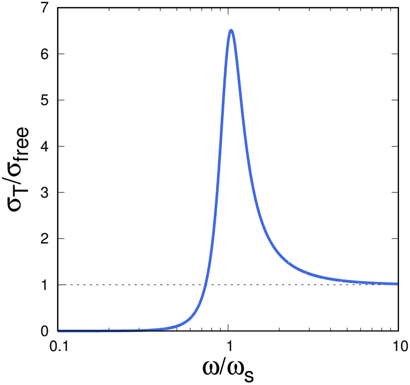

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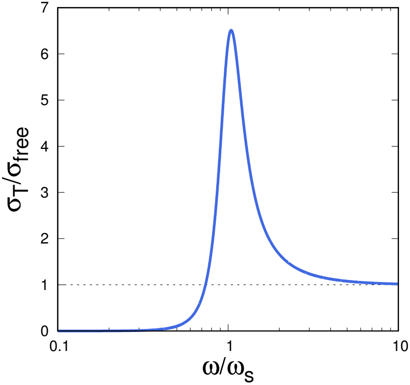

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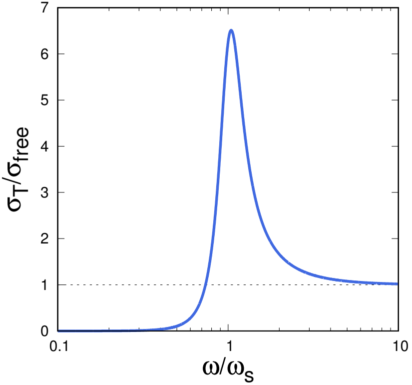

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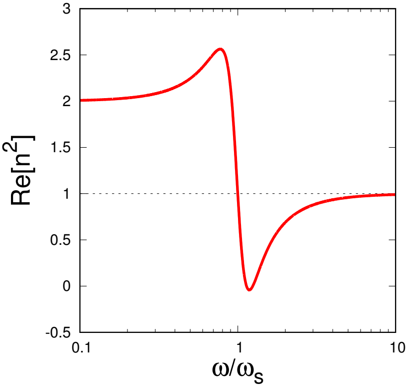

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