Today's Outline - September 05, 2016

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- Liquid scattering

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Homework Assignment #04:
Chapter 4: 2, 4, 6, 7, 10
due Monday, October 24, 2016

The radial distribution function

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Ordered 2D crystal

Amorphous solid or liquid

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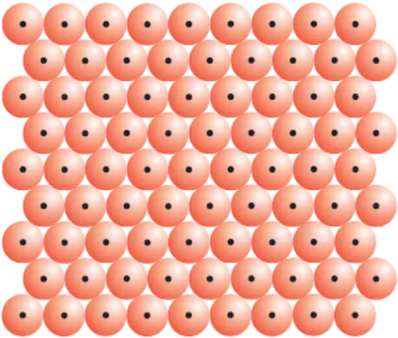

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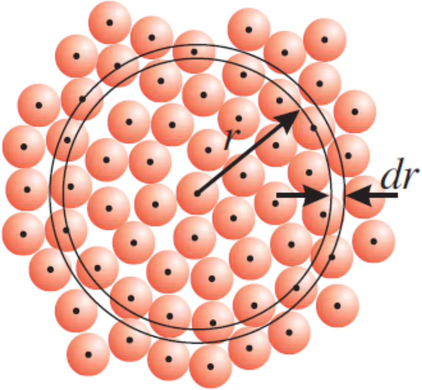

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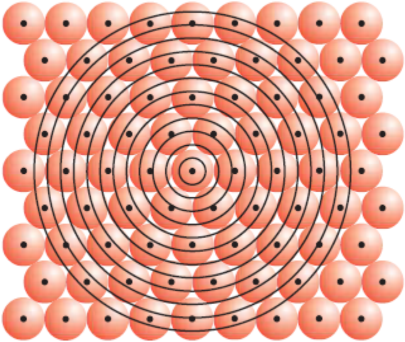

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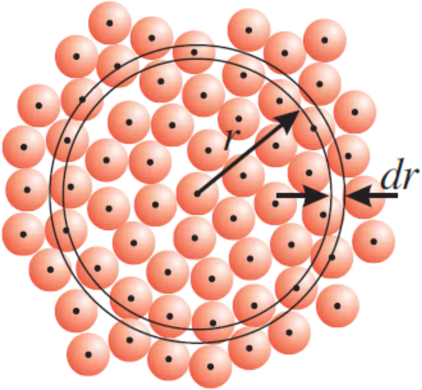

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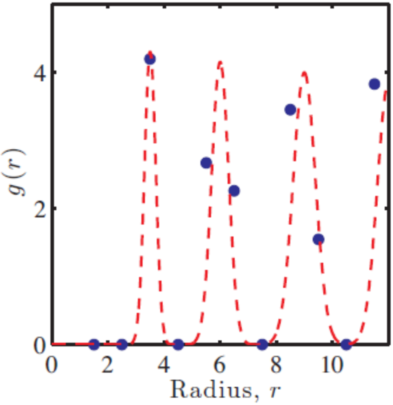

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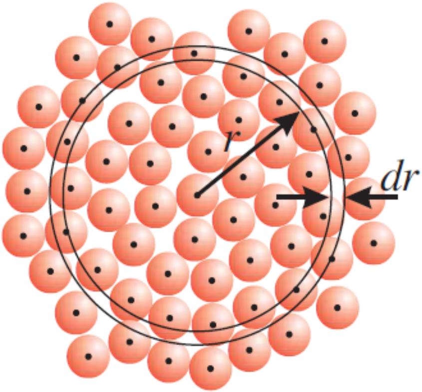

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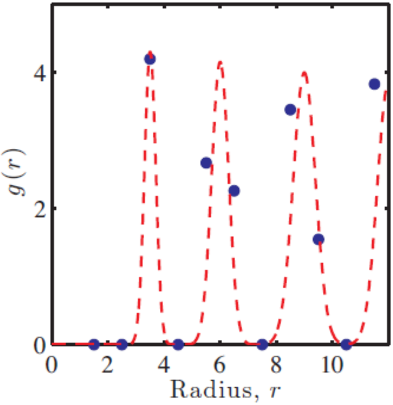

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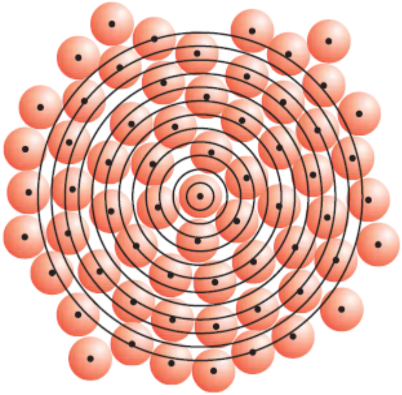

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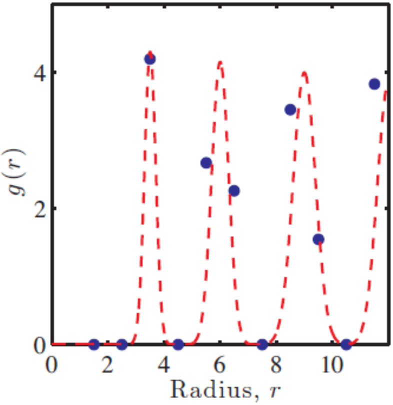

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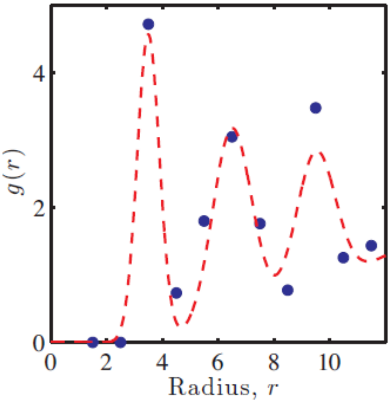

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Which is the sine Fourier Transform of the deviation of the atomic density from its average, $\mathcal{H}(r) = 4\pi r\left[g(r) - 1\right]$

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The relation between radial distribution function and structure factor can be extended to multi-component systems where $g(r) \rightarrow g_{ij}(r)$ and $S(Q) \rightarrow S_{ij}(Q)$.

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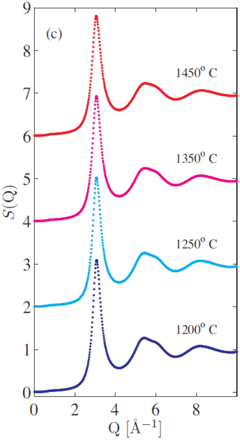

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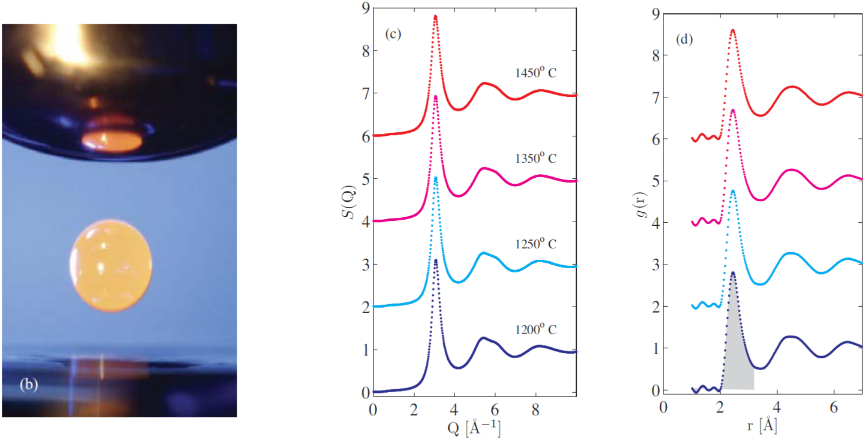

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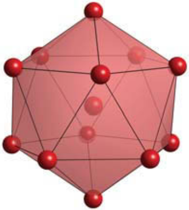
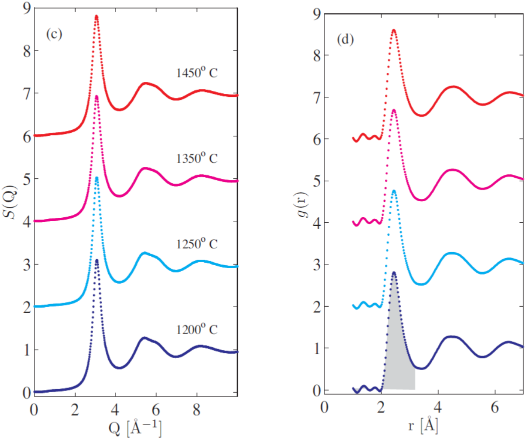
This indicates the presence of icosahedral clusters which inhibit crystallization.





"Difference in Icosahedral Short-Range Order in Early and Late Transition Metal Liquids", G.W. Lee et al. *Phys. Rev. Lett* **93**, 037802 (2004).

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Small angle x-ray scattering

Recall that there was an additional term in the scattering intensity which becomes important at small Q.

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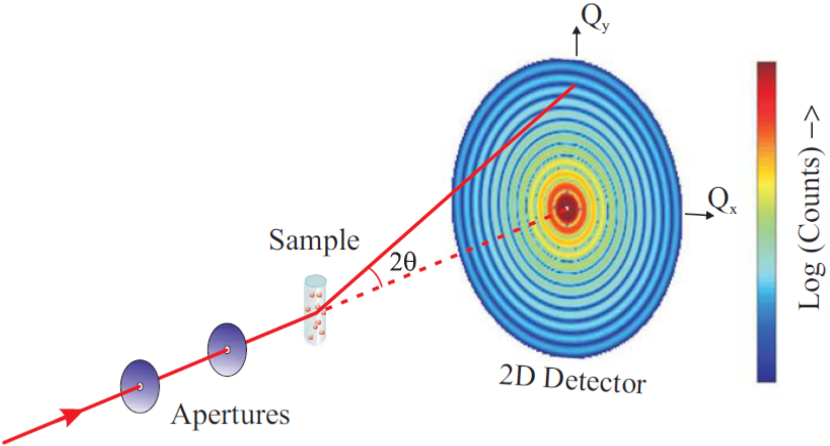
Where we have assumed sufficient averaging and introduced $\rho_{sl} = f\rho_{at}$. This final expression looks just like an atomic form factor but the charge density that we consider here is on a much longer length scale than an atom.

The SAXS experiment



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The simplest case is for a dilute solution of non-interacting molecules.

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Where $\Delta\rho = (\rho_{sl,p} - \rho_{sl,0})$, and the form factor depends on the morphology of the particle (size and shape).

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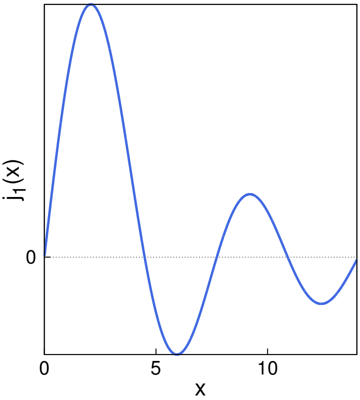

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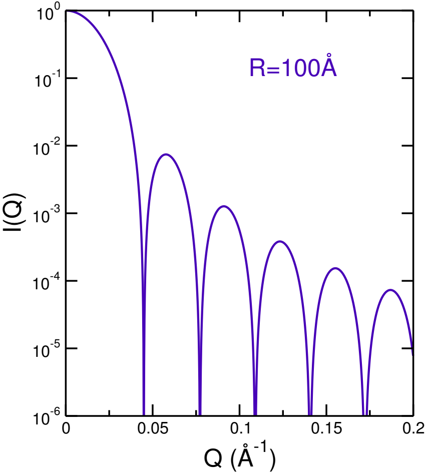

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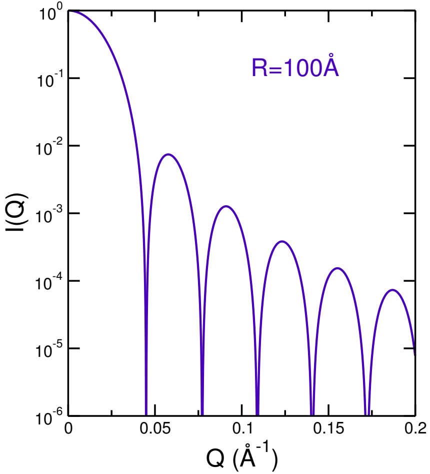

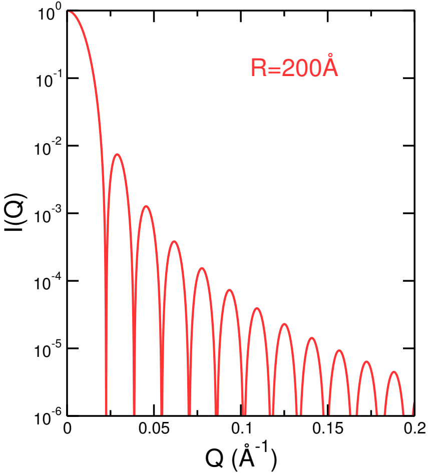

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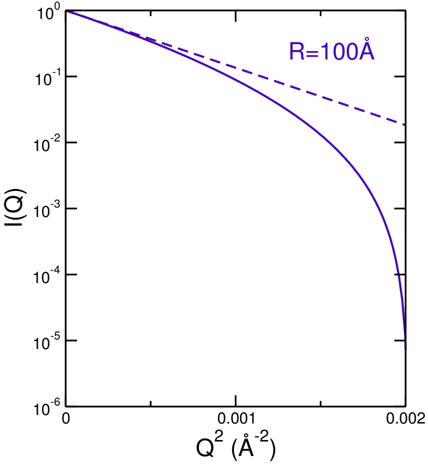

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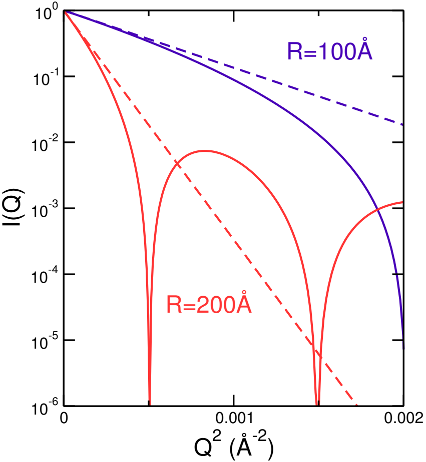

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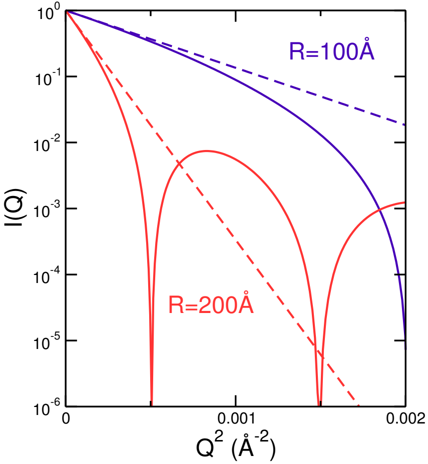

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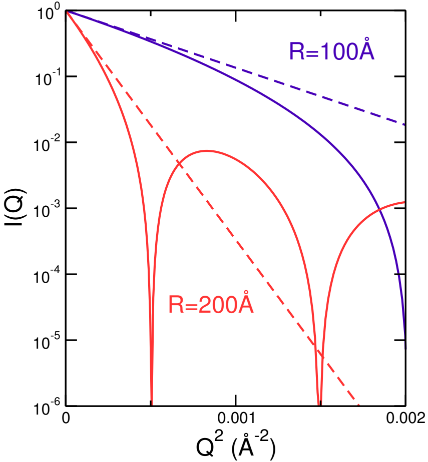

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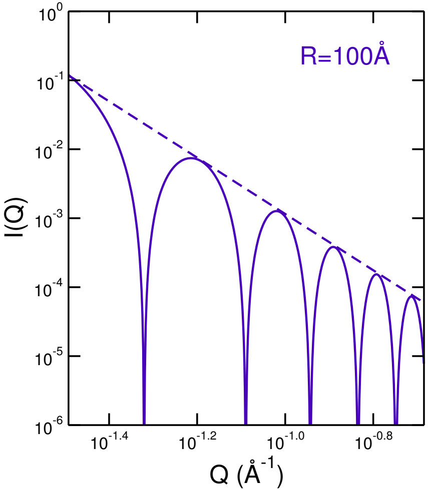

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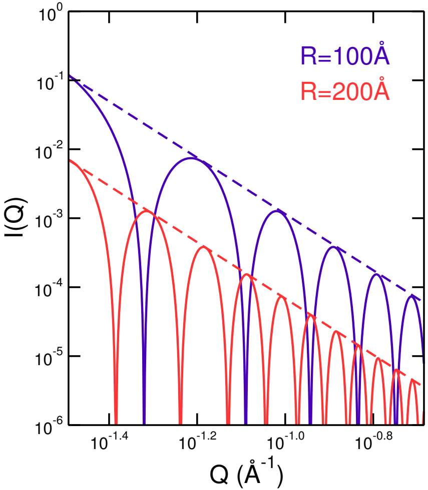
$$= 9\Delta\rho^2V_p^2\frac{\langle\cos^2(QR)\rangle}{Q^4R^4}$$

$$= \frac{9\Delta\rho^2V_p^2}{Q^4R^4}\left(\frac{1}{2}\right)$$

$$I(Q) = \frac{2\pi\Delta\rho^2}{Q^4}S_p$$



power law drop as Q^{-4} for spheres

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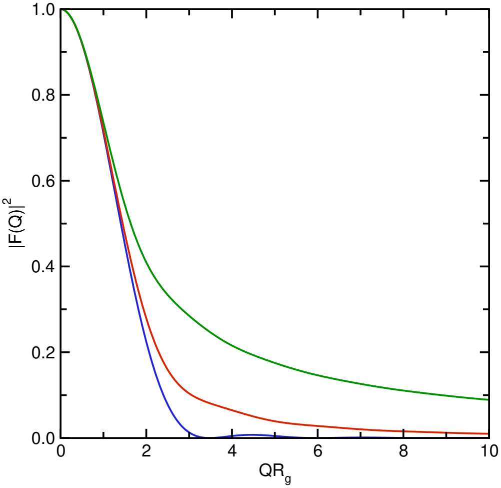

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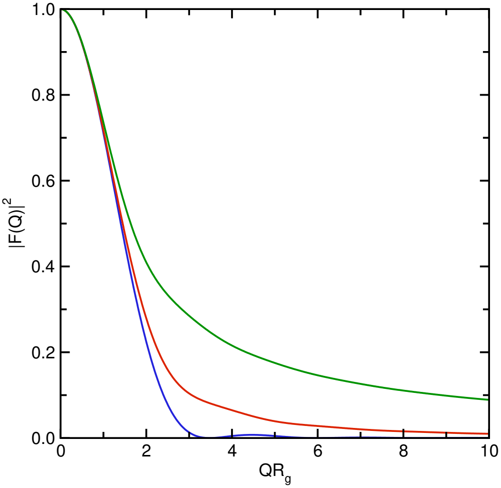
$dV_p = 4\pi r^2 dr$	sphere	-4
$dA_p = 2\pi r dr$	disk	
$dL_p = dr$	rod	



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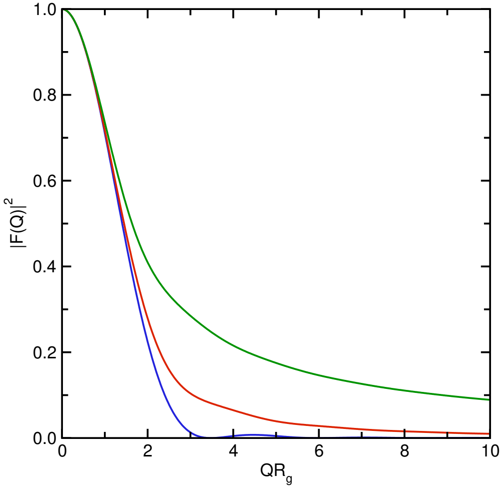
$dV_p = 4\pi r^2 dr$	sphere	-4
$dA_p = 2\pi r dr$	disk	-2
$dL_p = dr$	rod	



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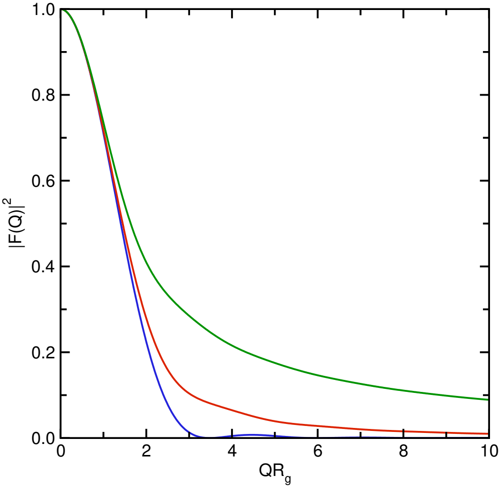
$dA_p = 2\pi r dr$	disk	-2
$dL_p = dr$	rod	-1



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