Today's Outline - October 03, 2016

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APS Visits:
10-ID: Friday, October 21, 2016
10-BM: Friday, October 28, 2016

Homework Assignment #03:
Chapter 3: 1, 3, 4, 6, 8
due Wednesday, September 05, 2016

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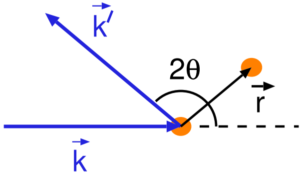

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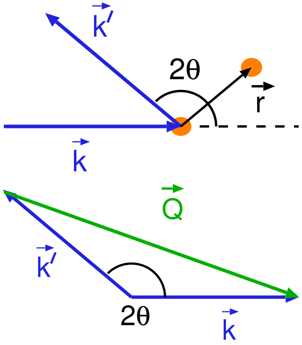

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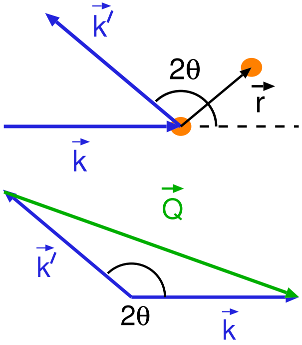

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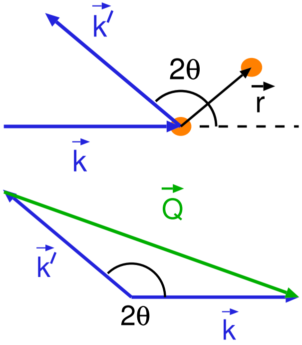

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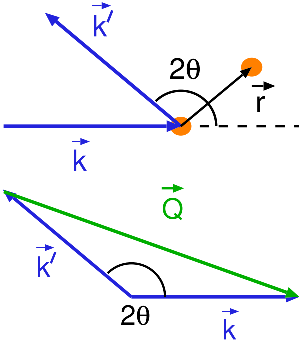

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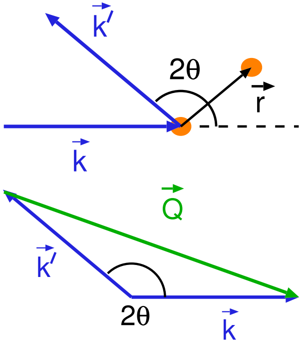

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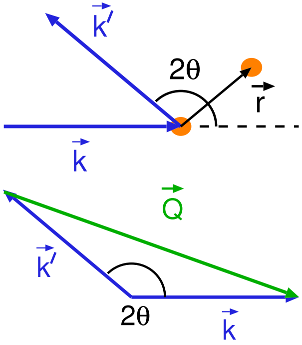

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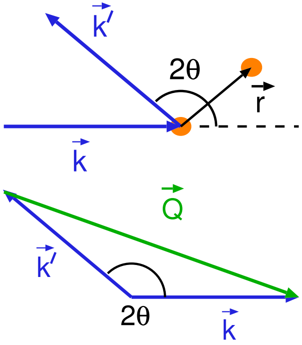

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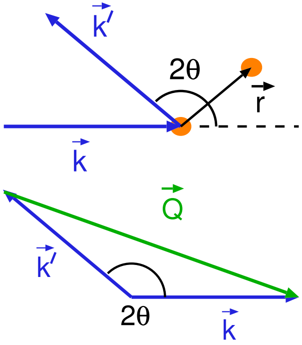

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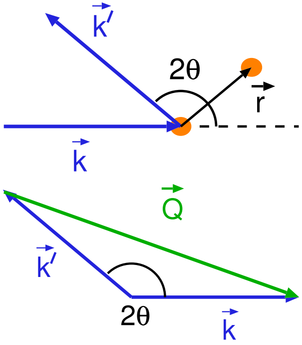

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We will now look at the consequences of this orientation and generalize to more than two electrons

Two electrons — fixed orientation

The expression

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assumes that the two electrons have a specific, fixed orientation. In this case the intensity as a function of Q is.

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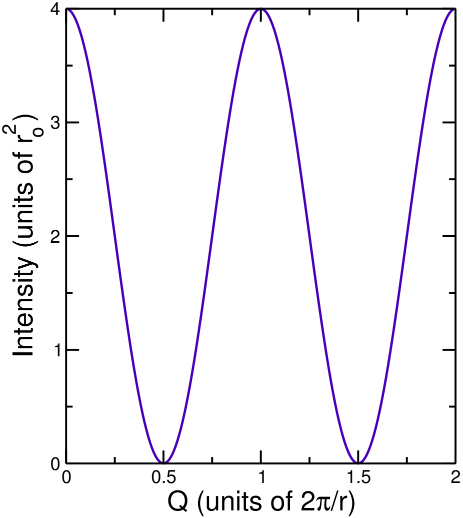

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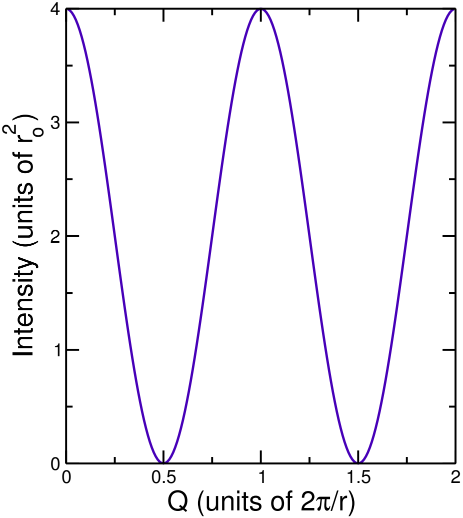
Fixed orientation is not the usual case, particularly for solution and small-angle scattering.



Orientation averaging

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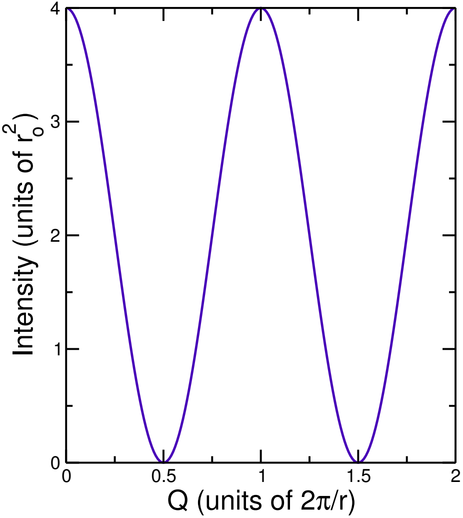

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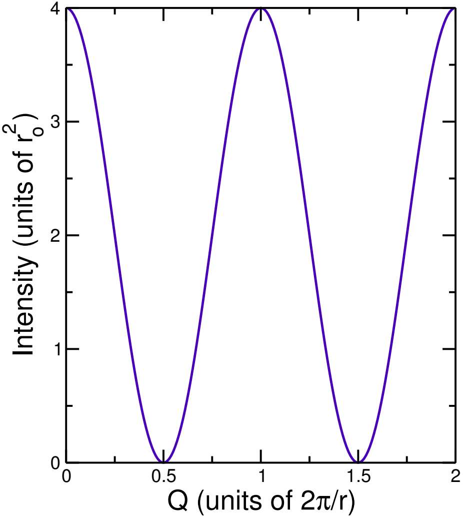

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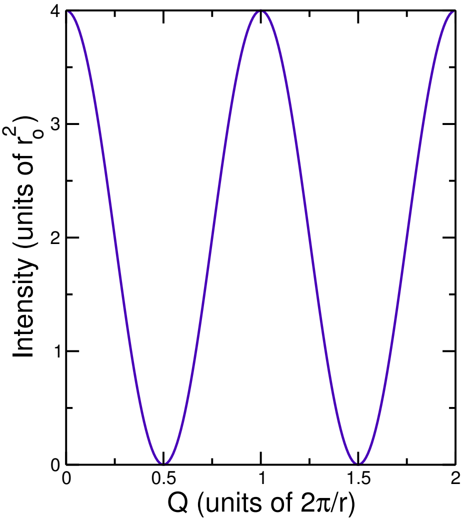

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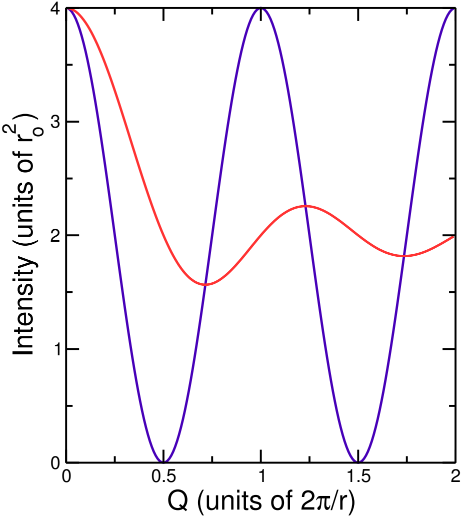

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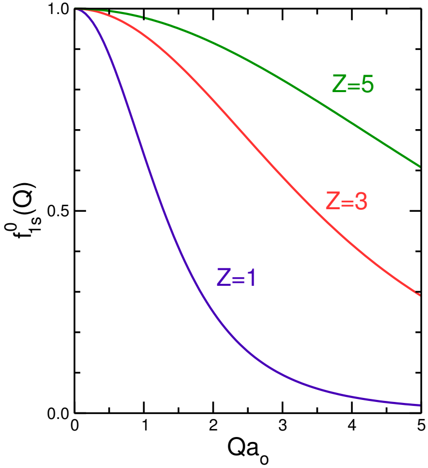

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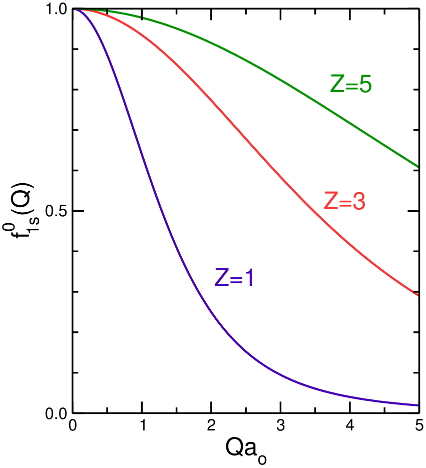

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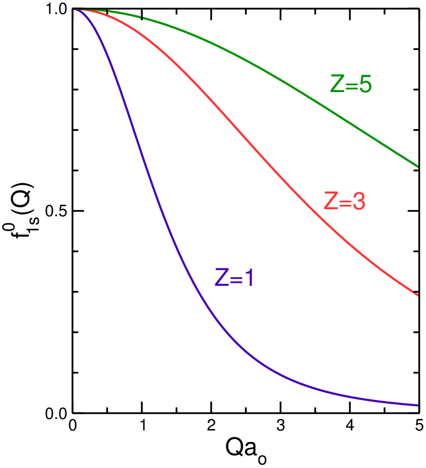

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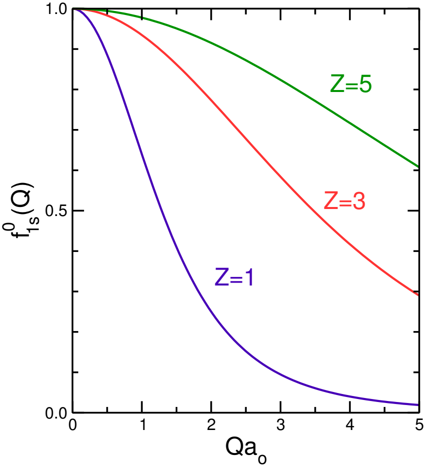

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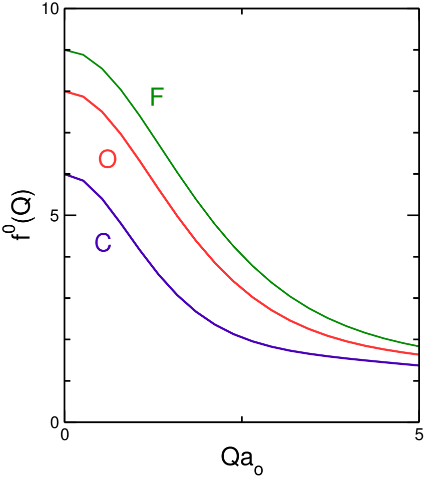

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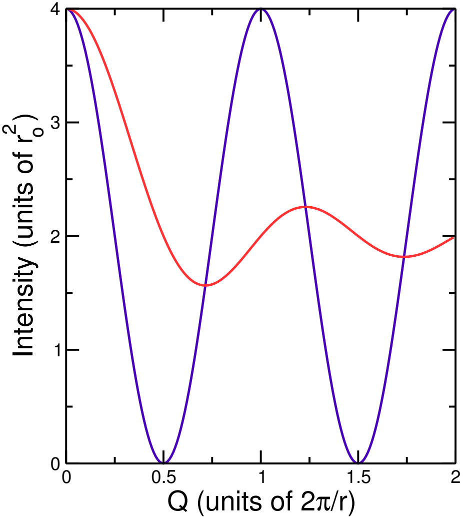
Previously we derived the scattering intensity from two localized electrons both fixed and randomly oriented to the x-rays



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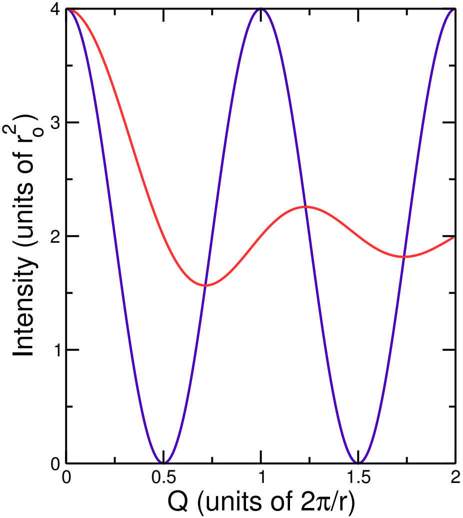

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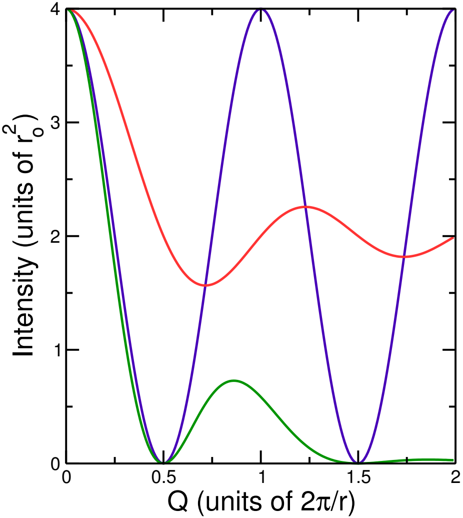

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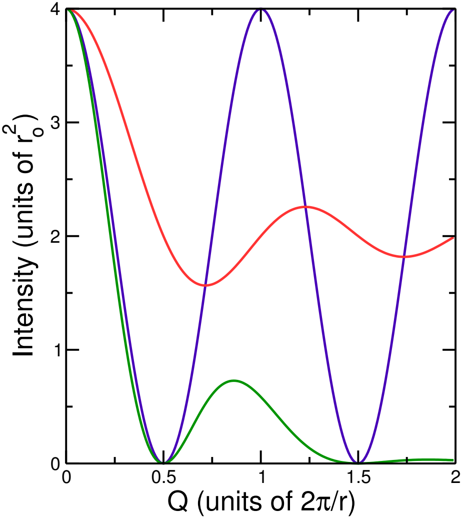

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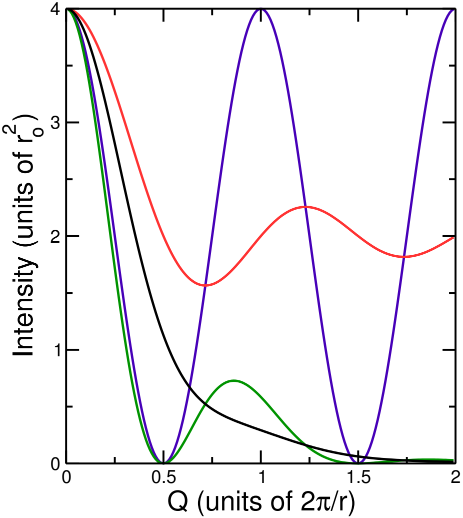

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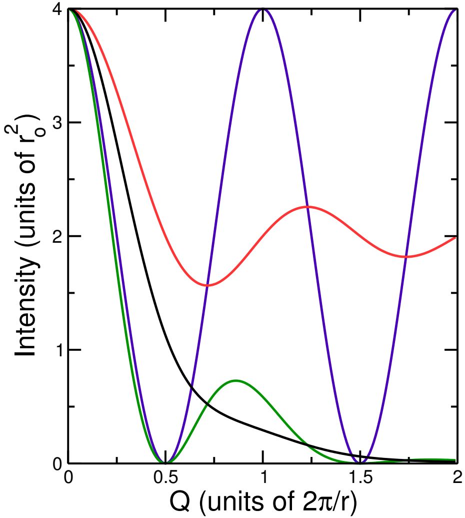

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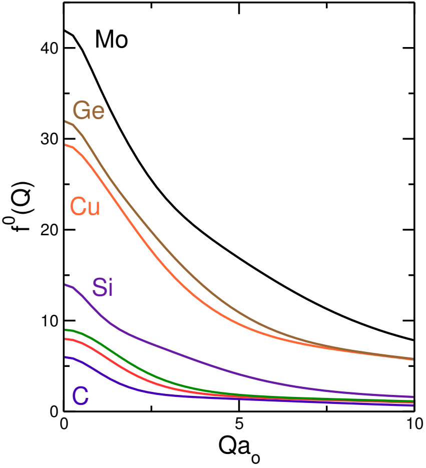

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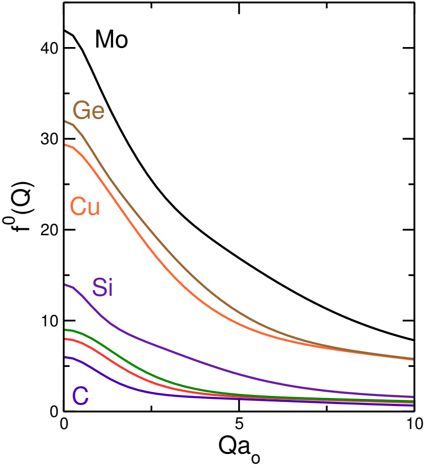

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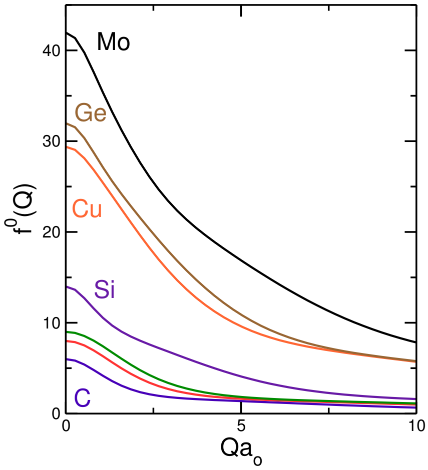

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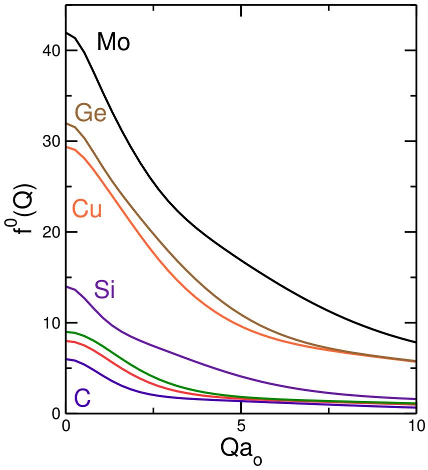

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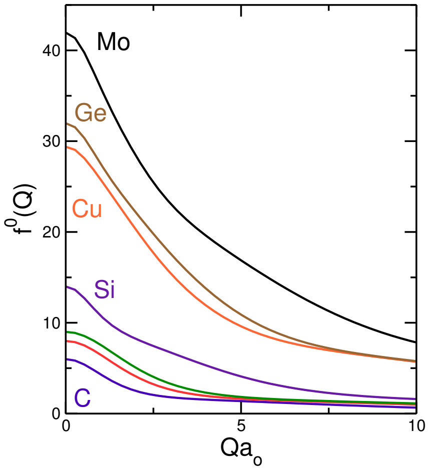

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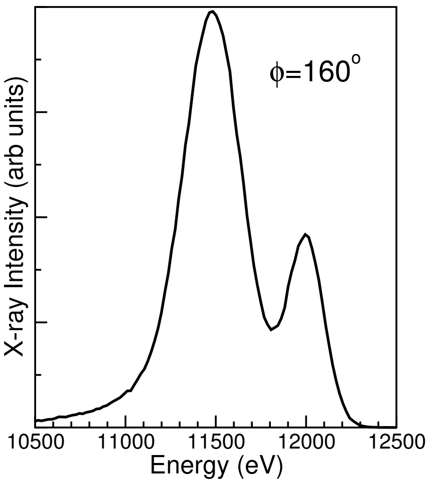

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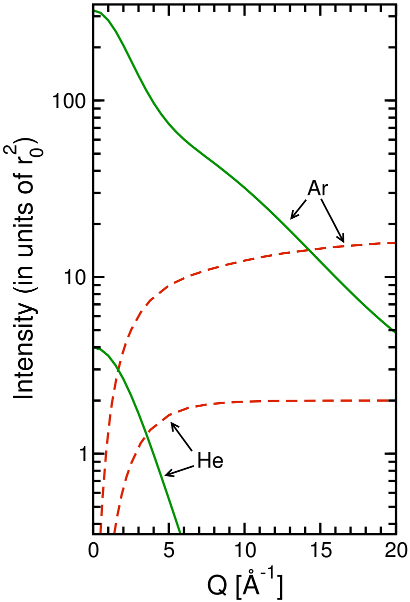

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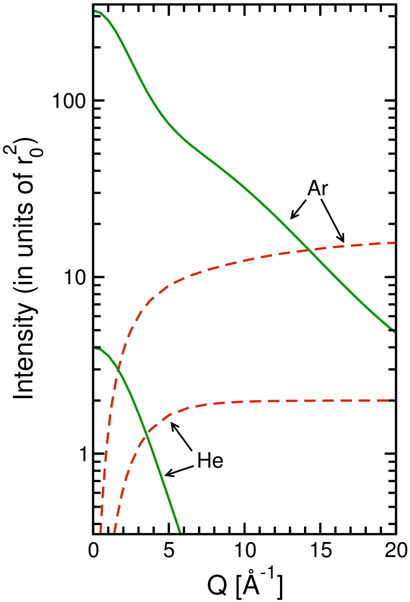
$Z_{He} = 2$ $\qquad Z_{Ar} = 18$



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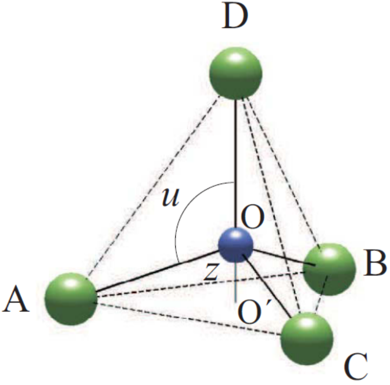

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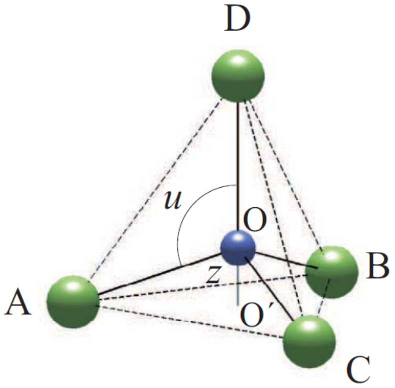

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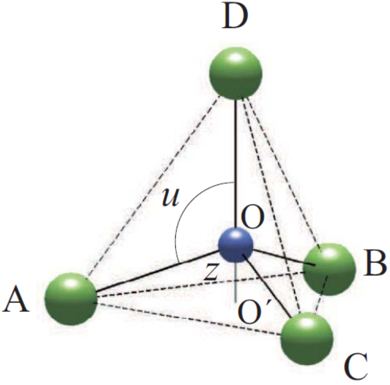

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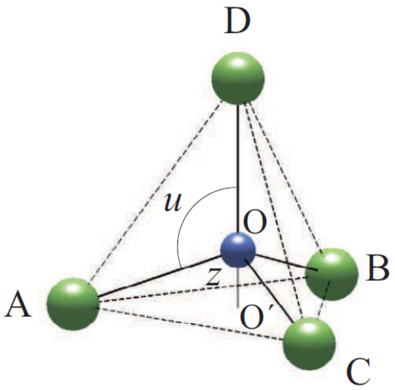

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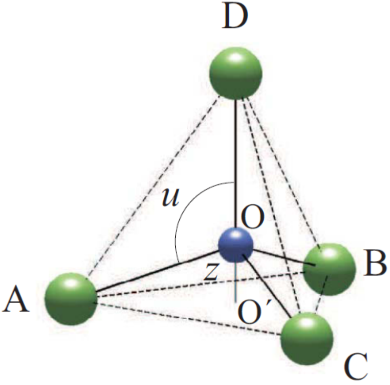

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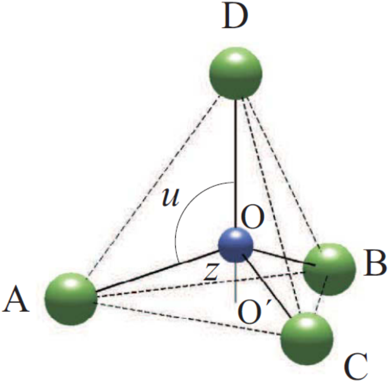

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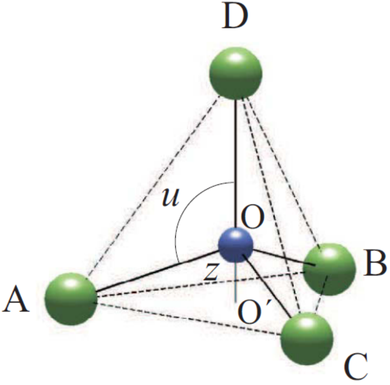

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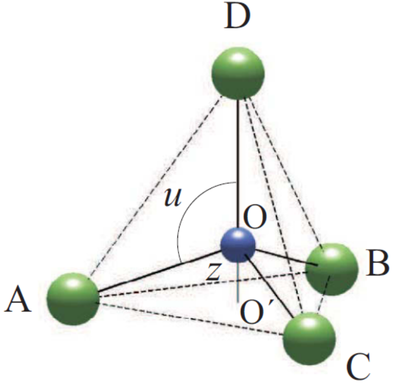

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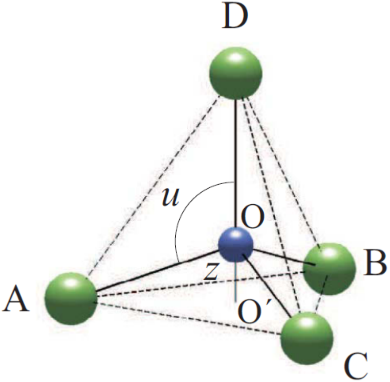

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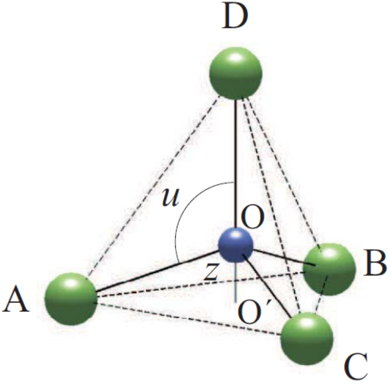

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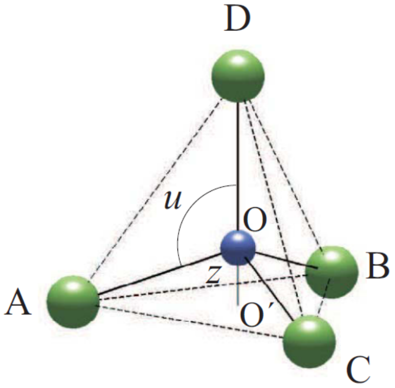

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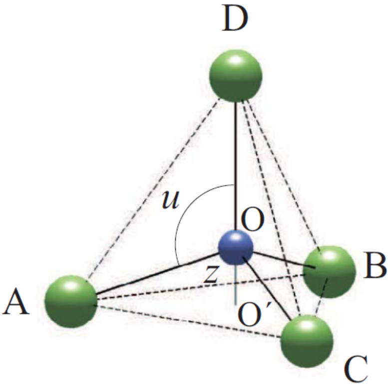

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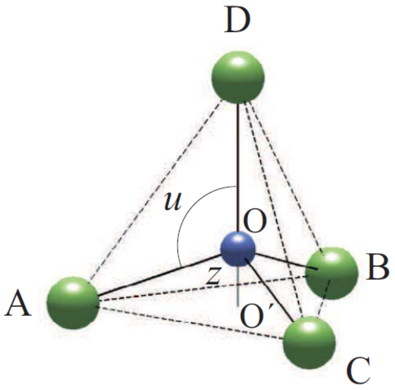

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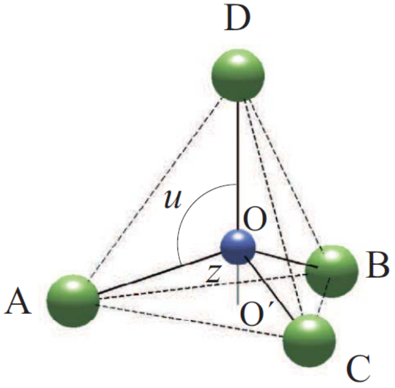

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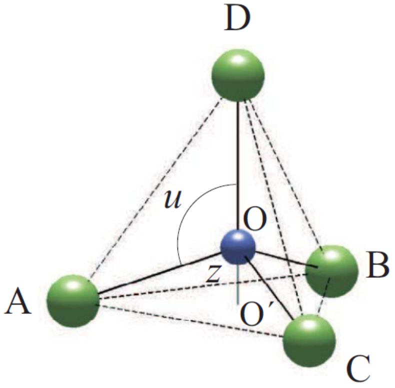

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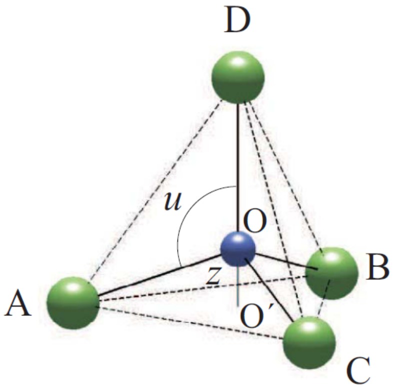

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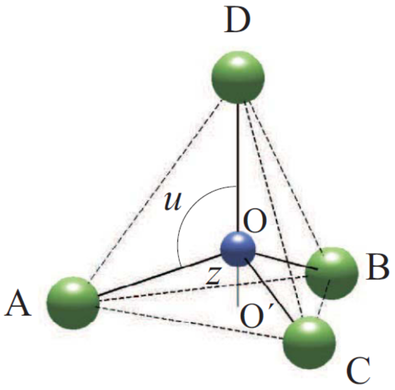

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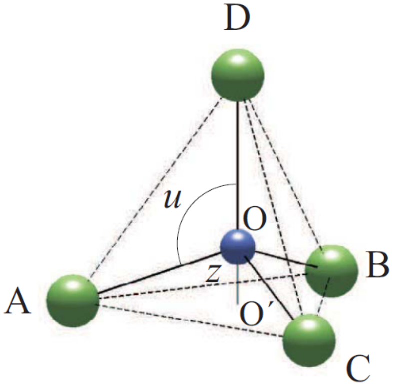

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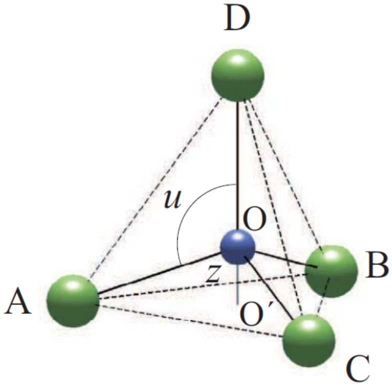

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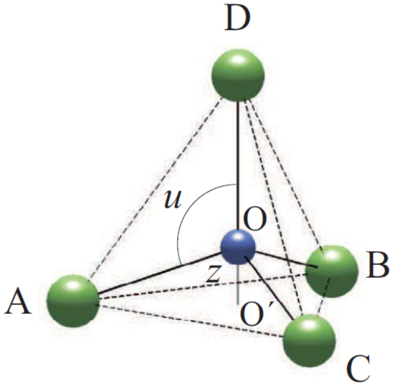

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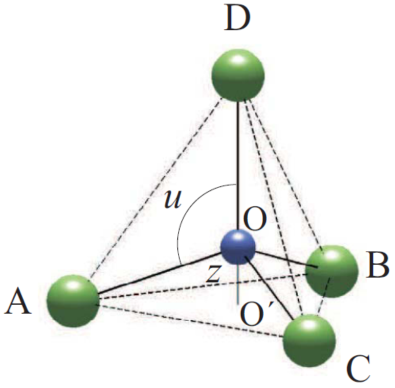

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\end{aligned}
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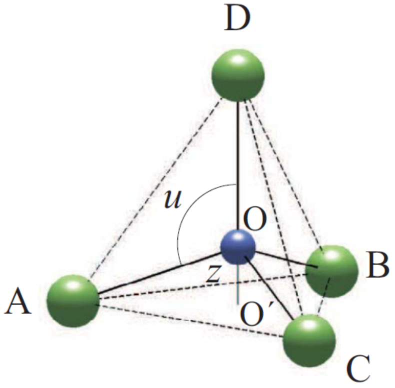

but from the triangle $OO'A$

$$(O'A)^2 = 1 - z^2$$

Scattering from molecules

$$
\begin{aligned}
-z &= (\overline{OO'} + \overline{O'A}) \cdot (\overline{OO'} + \overline{O'B}) \\
&= z^2 + 0 + 0 + \overline{O'A} \cdot \overline{O'B} \\
&= z^2 + (O'A)^2 \cos(120^\circ) \\
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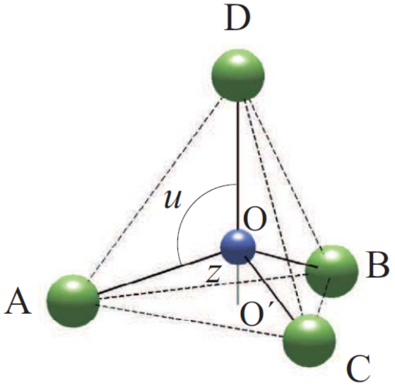

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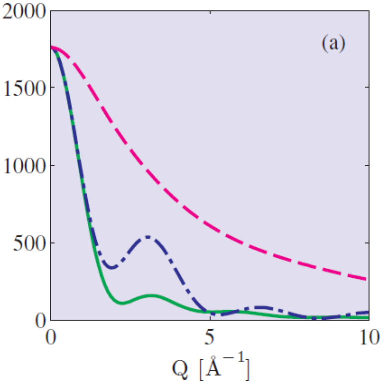

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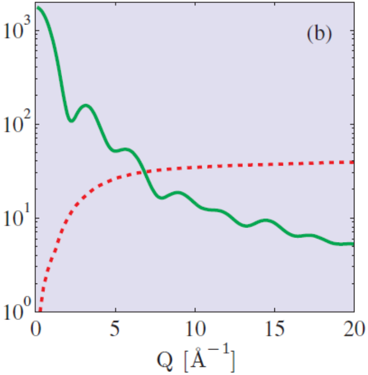

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$$|F^{mol}|^2 = |f^C|^2 + 4|f^F|^2 + 8f^C f^F \frac{\sin(QR)}{QR} + 12|f^F|^2 \frac{\sin(Q\sqrt{8/3}R)}{Q\sqrt{8/3}R}$$

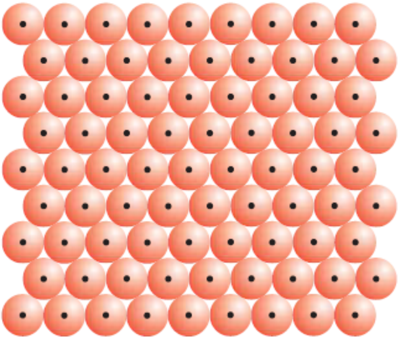
The Radial Distribution Function

The Radial Distribution Function

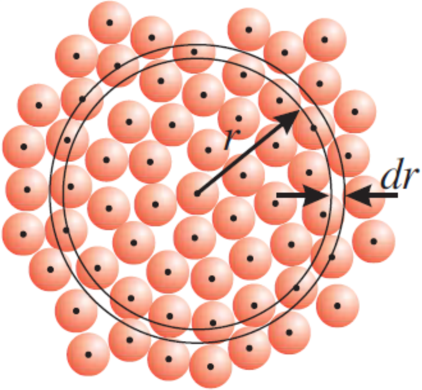
Ordered 2D crystal

Amorphous solid or liquid

The Radial Distribution Function



Ordered 2D crystal



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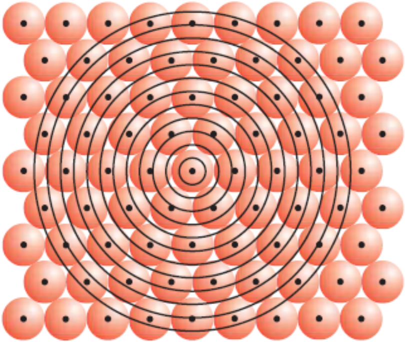

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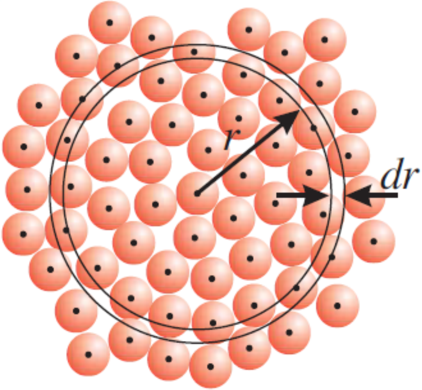

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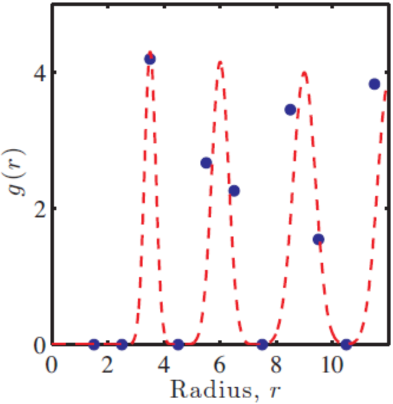

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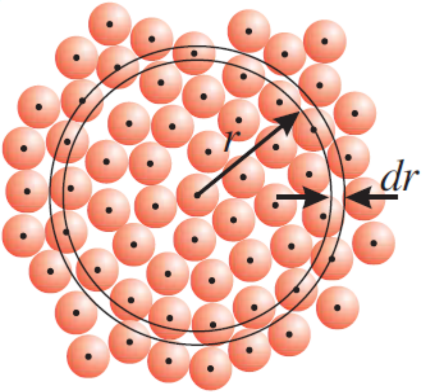

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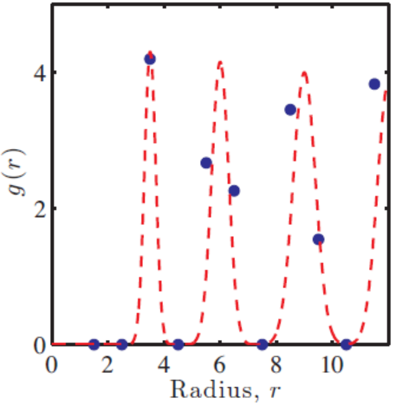

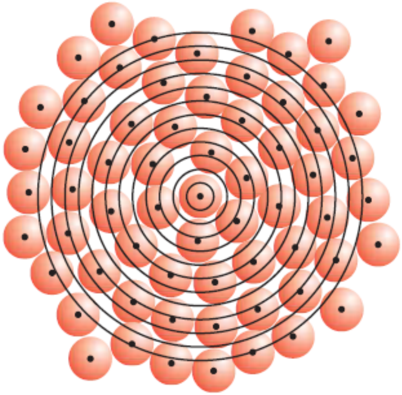

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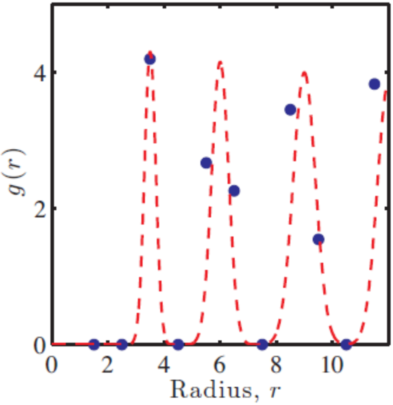

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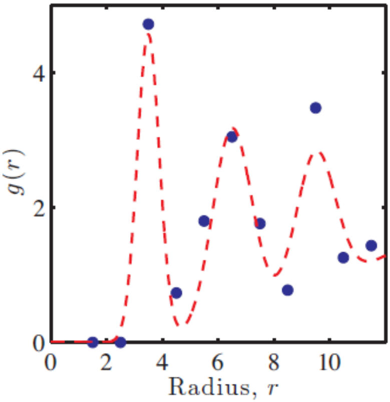

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