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10-ID: Friday, October 21, 2016
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Homework Assignment #03:
Chapter 3: 1, 3, 4, 6, 8
due Wednesday, October 05, 2016

Layering in liquid films

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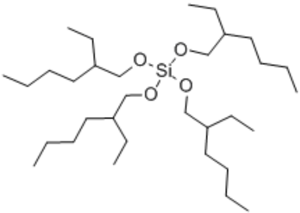

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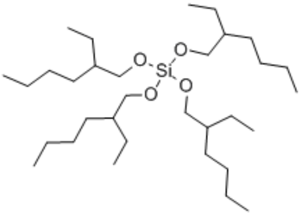

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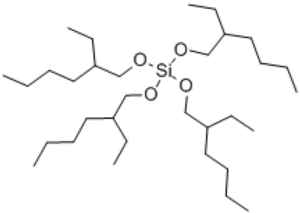

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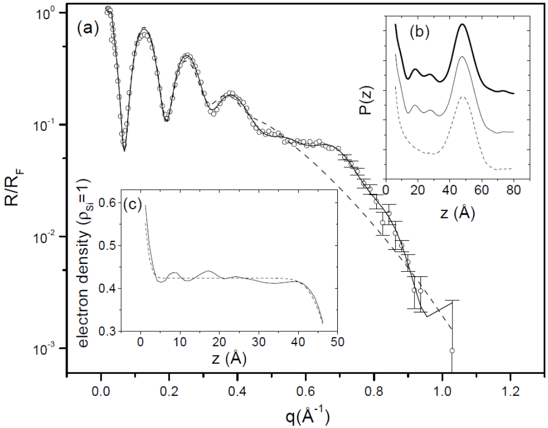

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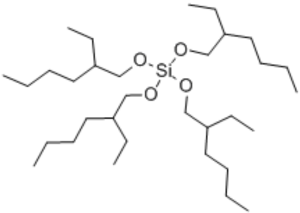

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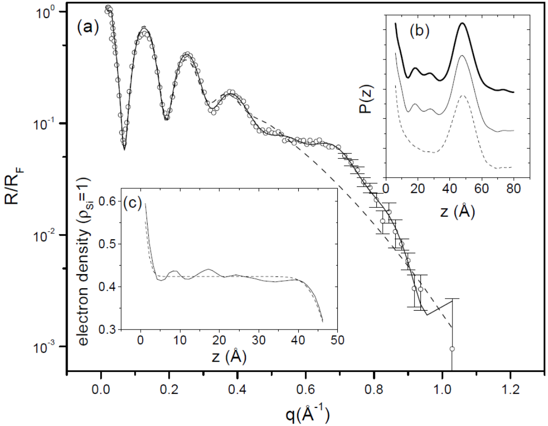

Deviations from uniform density are used to fit experimental reflectivity

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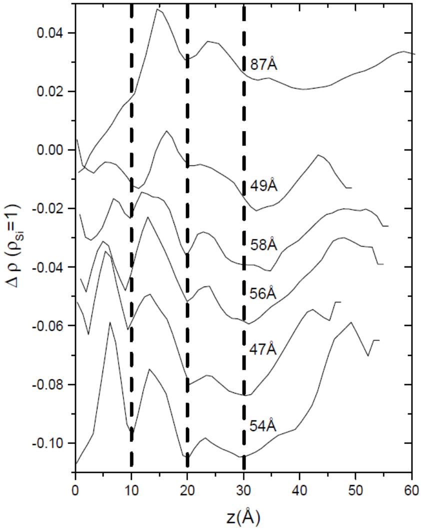

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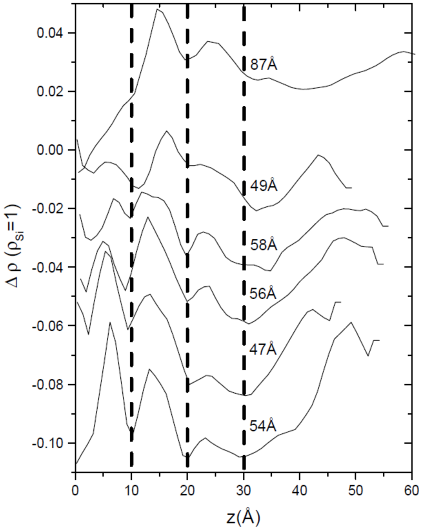

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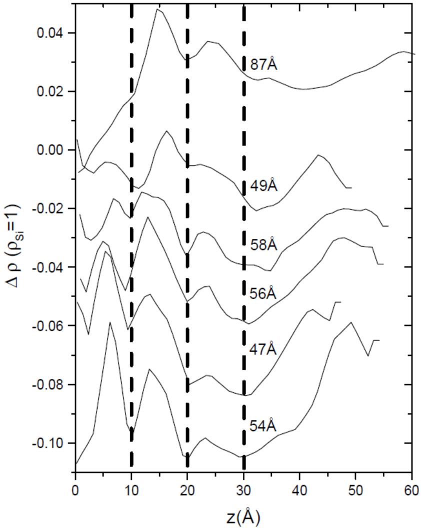

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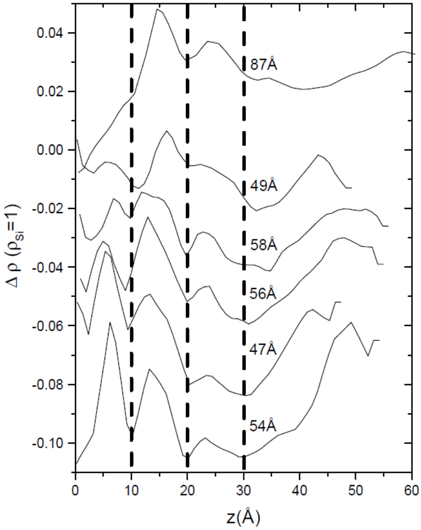

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A broad peak appears at free surface indicating that ordering requires a hard smooth surface.

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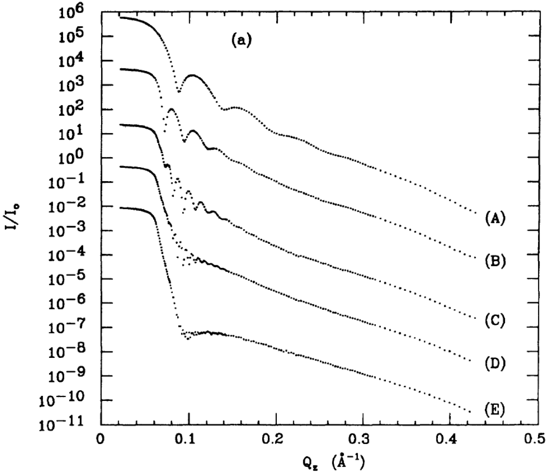

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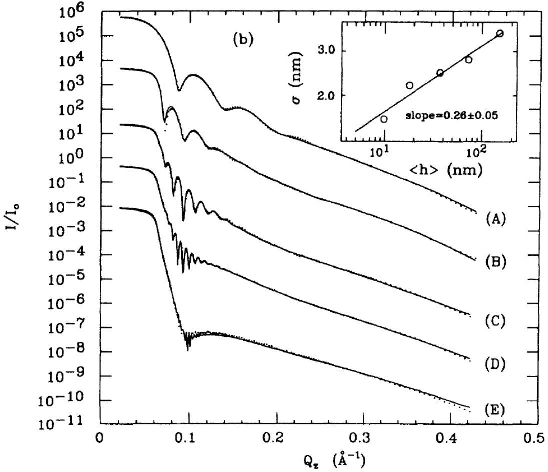

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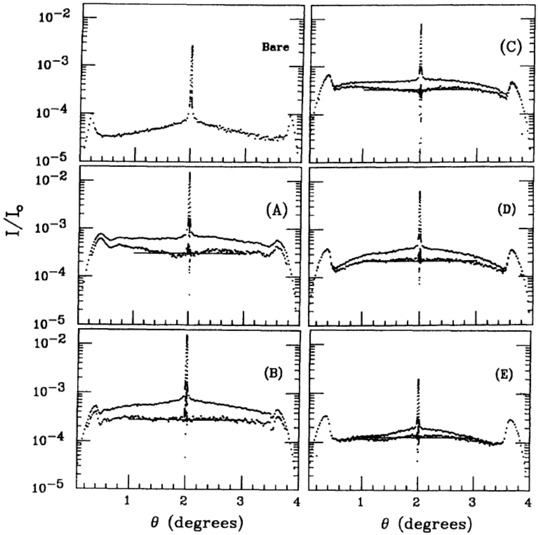

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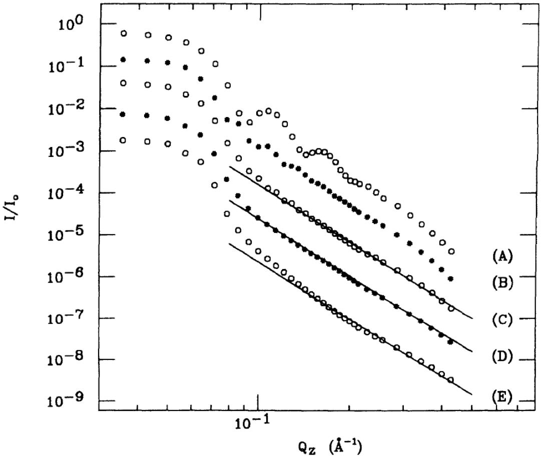

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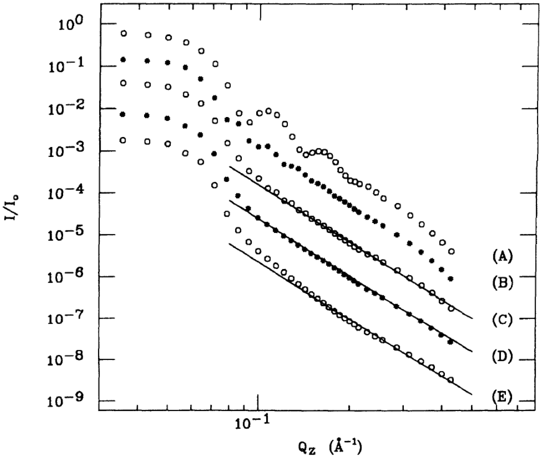

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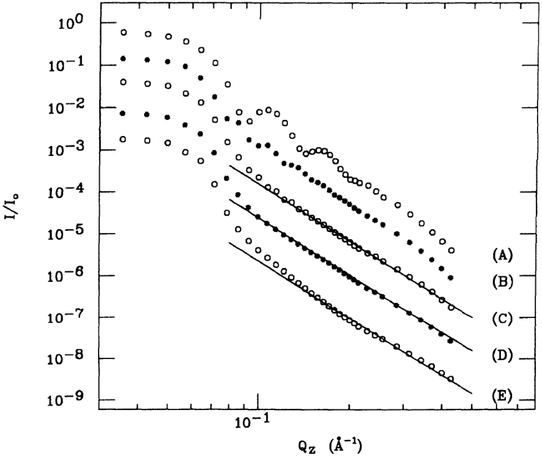

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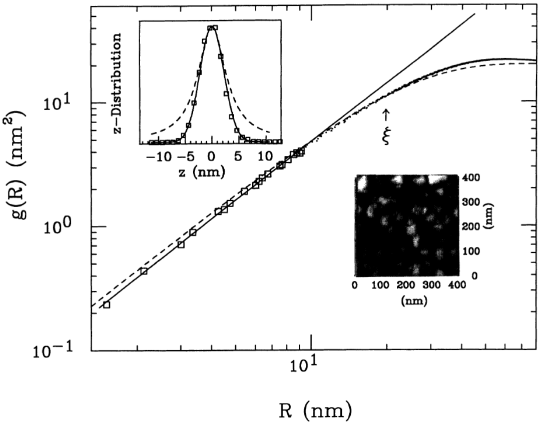

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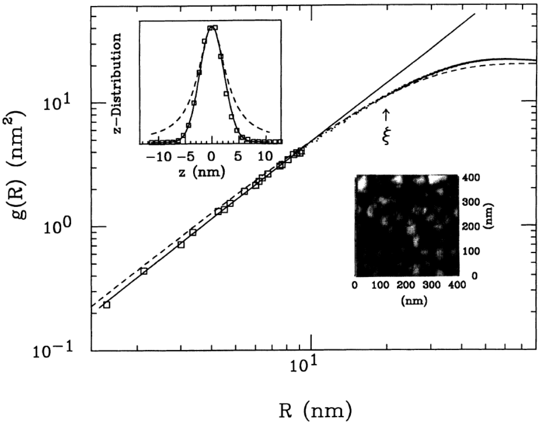

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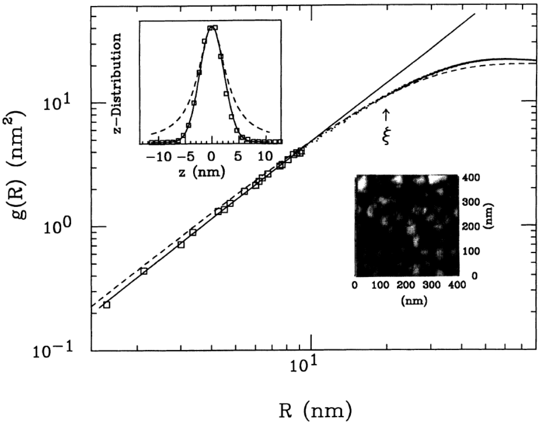
Thus $z_s = h/\beta = 2.7$



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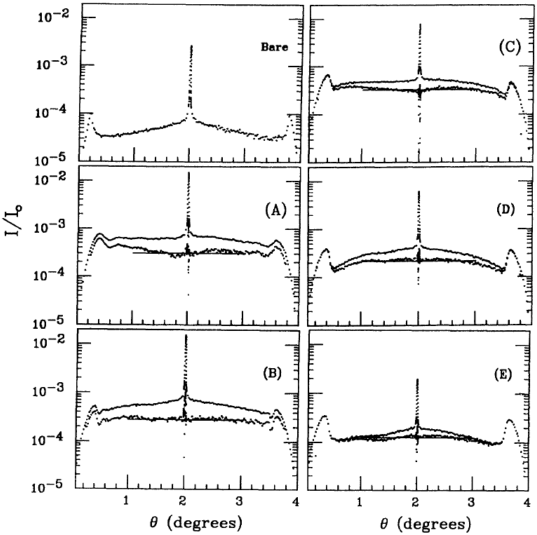

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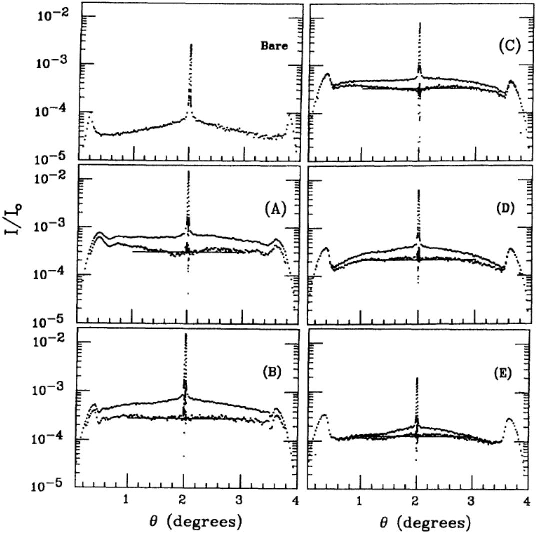

Thus $z_s = h/\beta = 2.7$ and diffraction data confirm $\xi = 19.9\langle h\rangle^{1/2.7}$ Å

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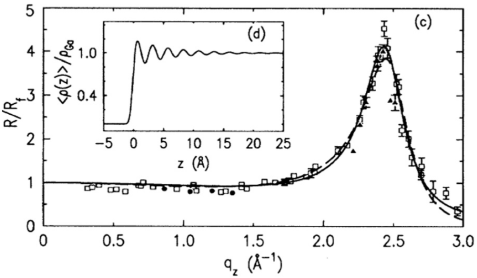

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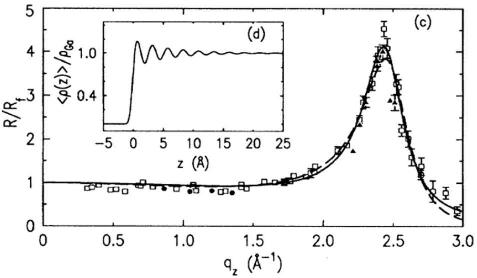

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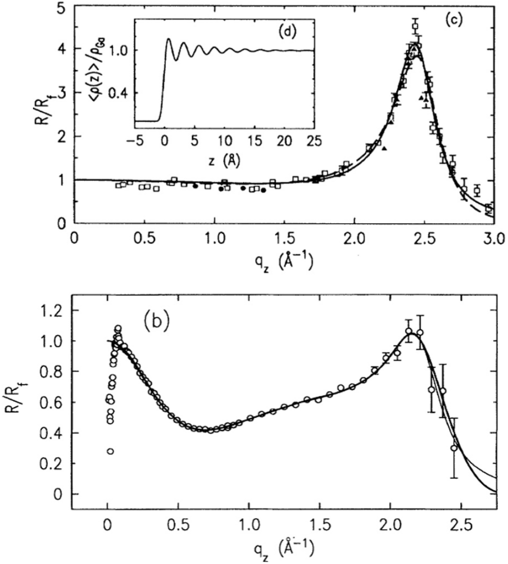

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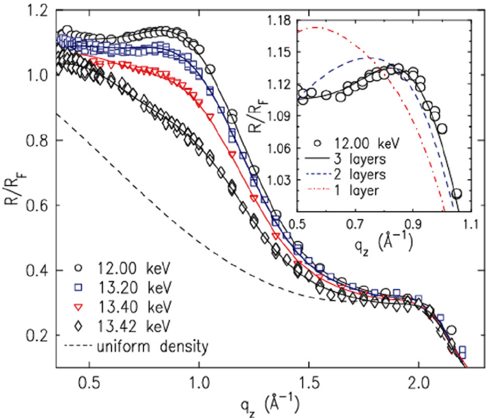

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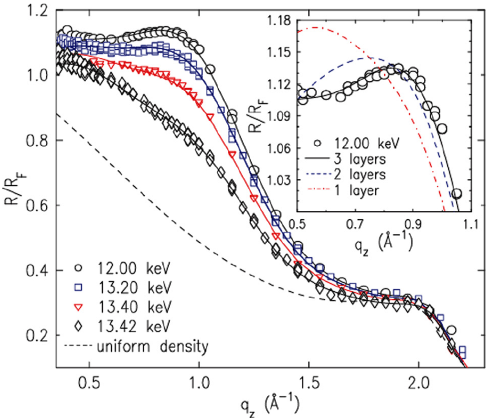
surface layer is rich in Bi (95%), second layer is deficient (25%), and third layer is rich in Bi (53%) once again



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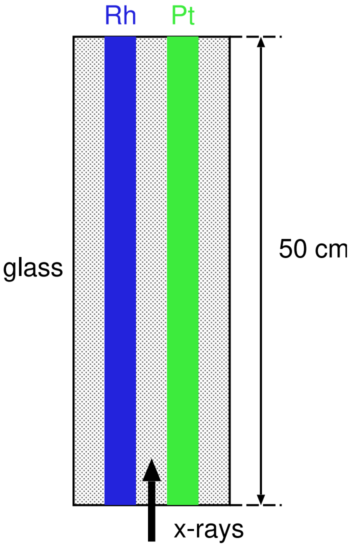
The MRCAT mirror



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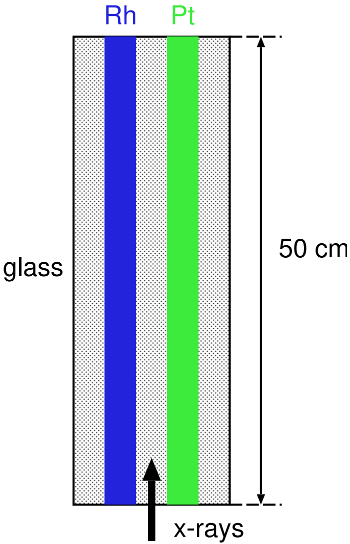

Ultra low expansion glass polished to a few Å roughness

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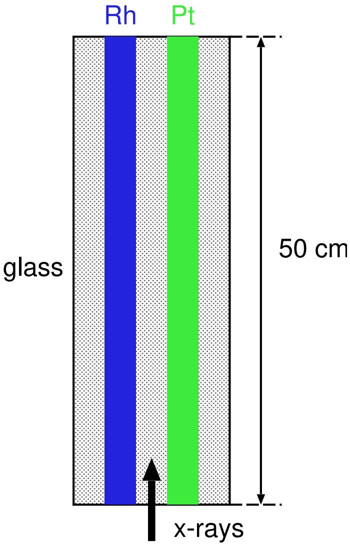

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One platinum stripe and one rhodium stripe deposited along the length of the mirror on top of a chromium buffer layer

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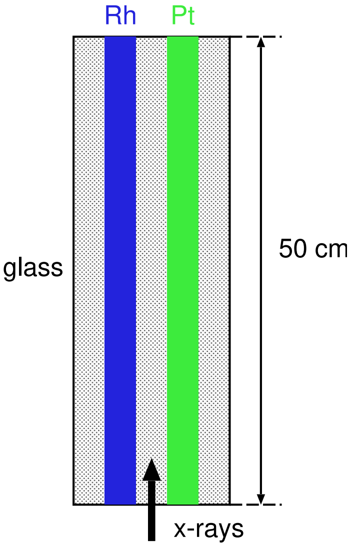

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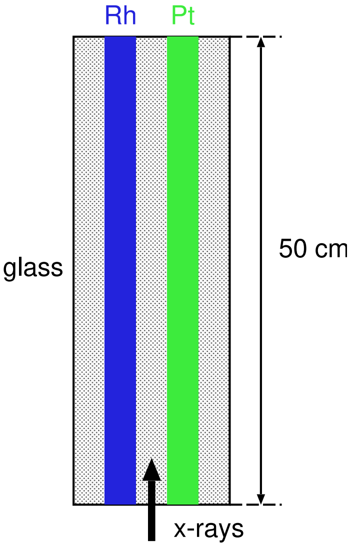

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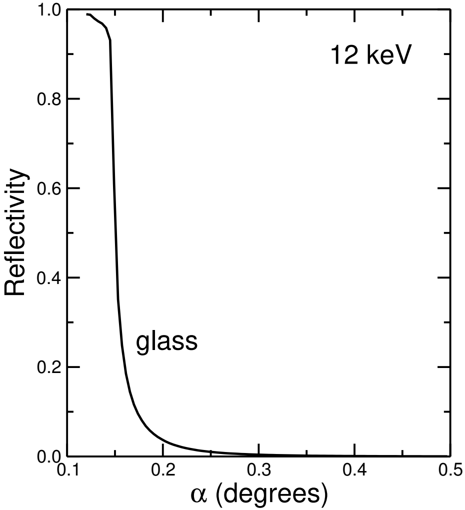
A bending mechanism to permit vertical focusing of the beam to $\sim 60\ \mu\mathrm{m}$

Mirror performance

When illuminated with 12 keV x-rays on the glass “stripe”, the reflectivity is measured as:



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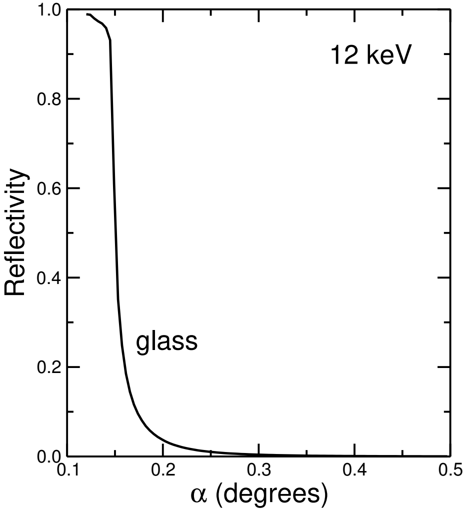

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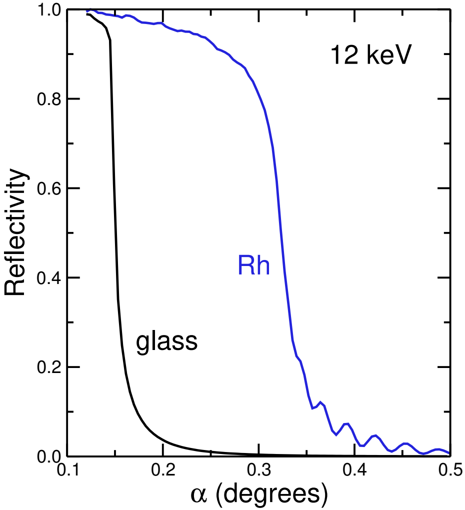

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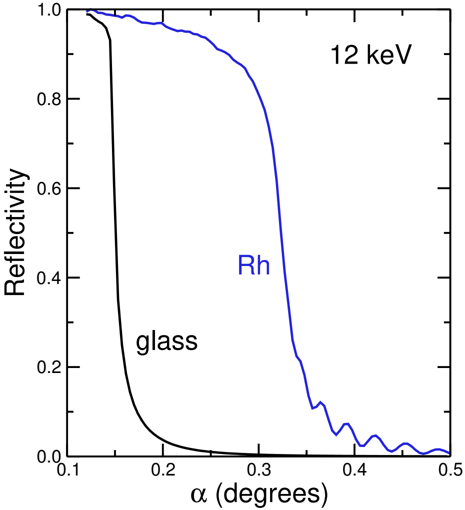
The Pt stripe gives a higher critical angle still but a lower reflectivity and it looks like an infinite slab.



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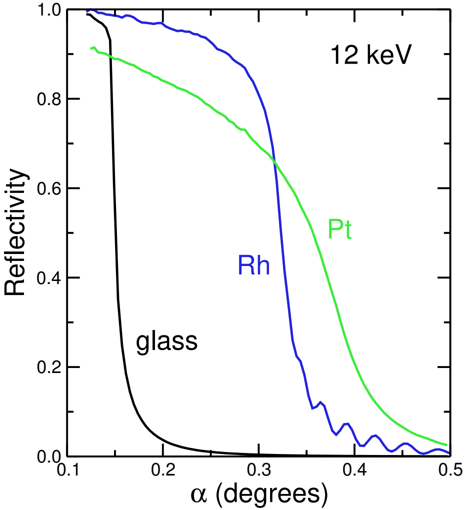

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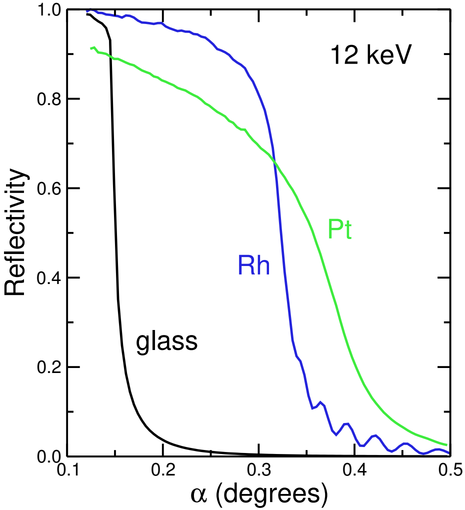

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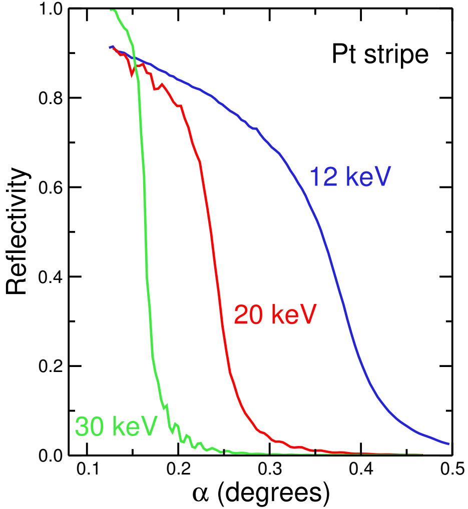

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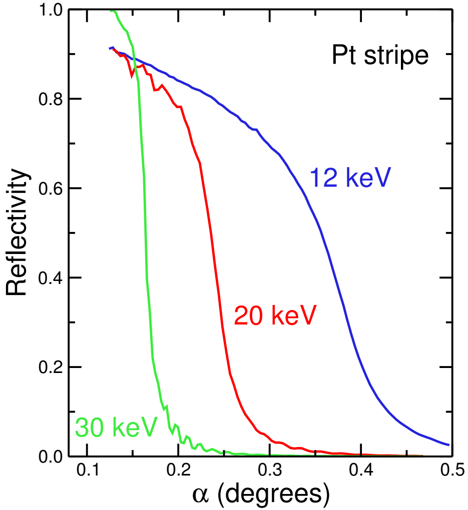

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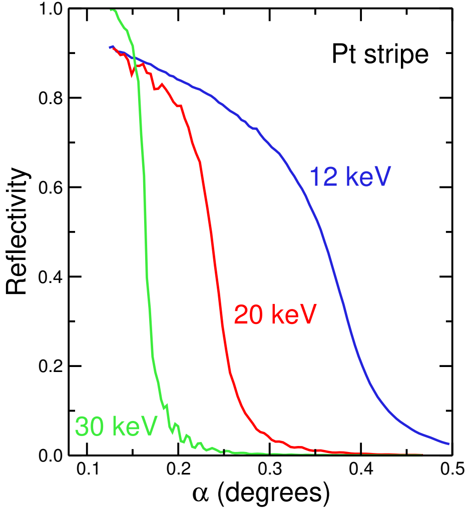

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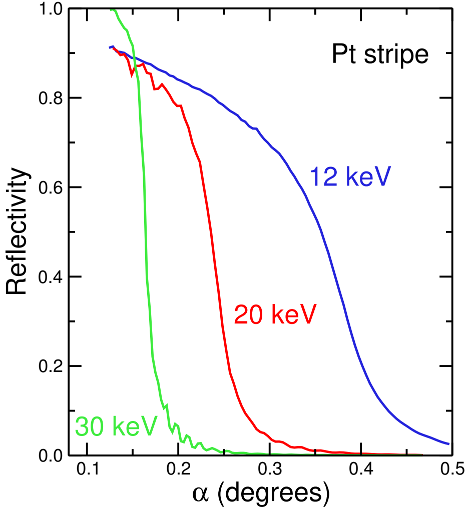

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As energy rises, the Pt layer begins to show the reflectivity of a thin slab.

Tangential focusing mirror

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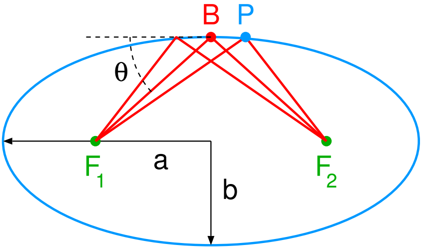

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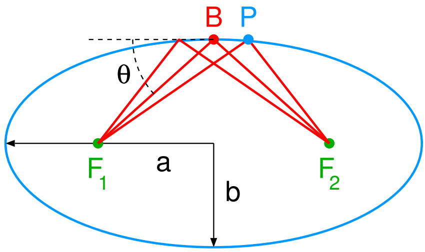

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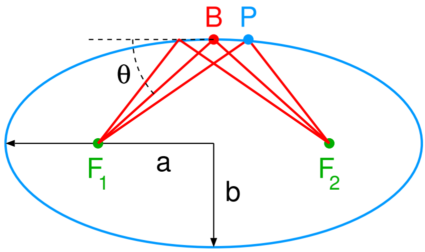

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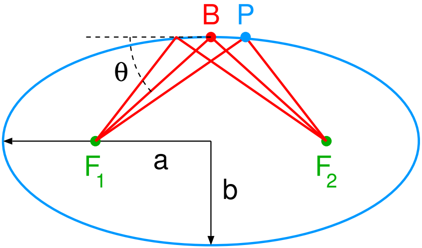

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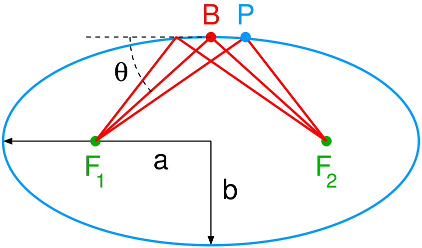

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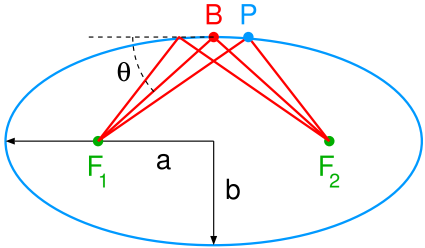

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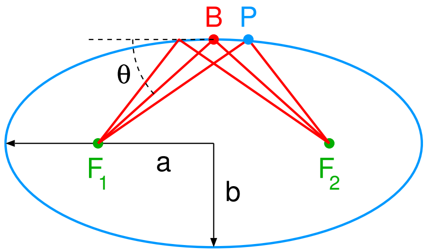

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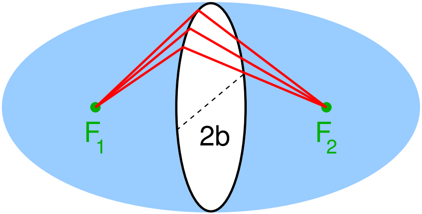
Saggital focusing mirror

Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of saggital focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.



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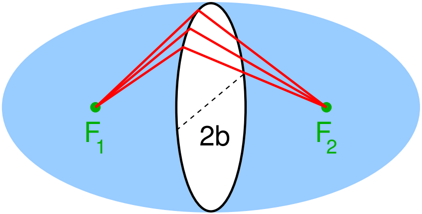

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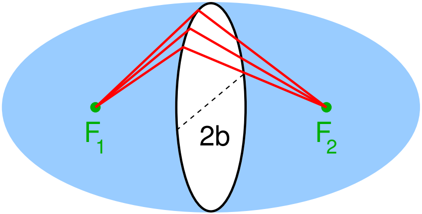

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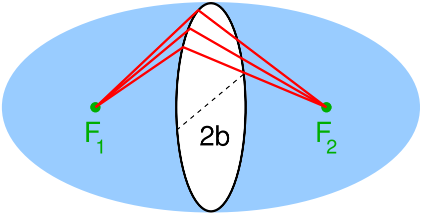

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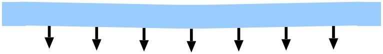

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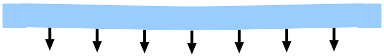

A cost effective way to focus in both directions is a toroidal mirror which has a fixed bend in the transverse direction but which can be bent longitudinally to change the vertical focus.

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- K-B mirrors & zone plates — in combination with an initial focusing element, gives smallest focal spot, but hard to vary energy

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visible light:

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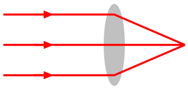

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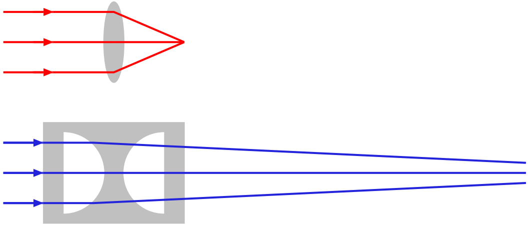

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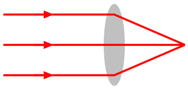

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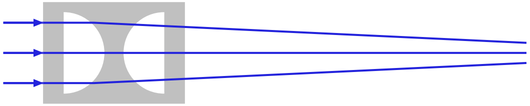

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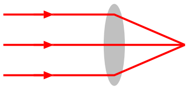

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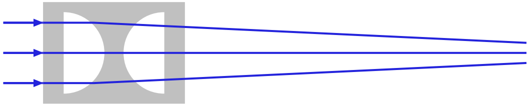

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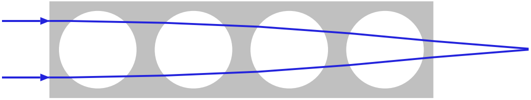
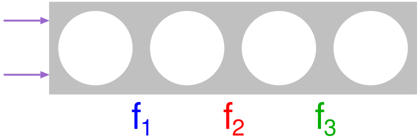
getting manageable focal distances requires making compound lenses



Focal length of a compound lens



Start with a 3-element compound lens, calculate effective focal length

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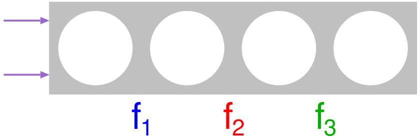

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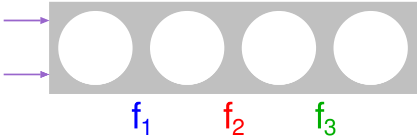
Focal length of a compound lens



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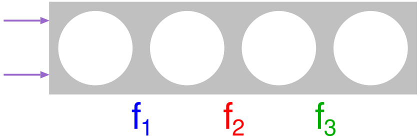

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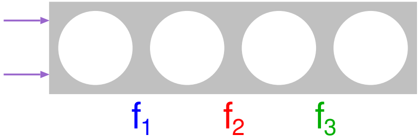

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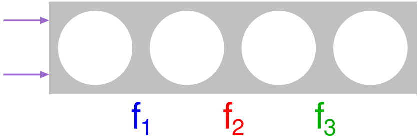

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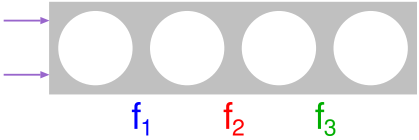

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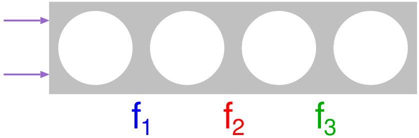

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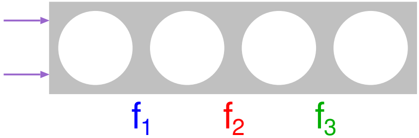

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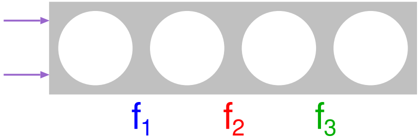

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similarly for the third lens, $o_3 = -i_2$

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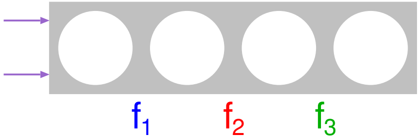

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$$\frac{1}{i_2}=\frac{1}{f_2}-\frac{1}{o_2}\rightarrow\frac{1}{i_2}=\frac{1}{f}+\frac{1}{f}\rightarrow i_2=\frac{f}{2}$$

similarly for the third lens, $o_3 = -i_2$

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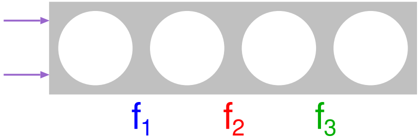
Focal length of a compound lens



Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$$\frac{1}{i}+\frac{1}{o}=\frac{1}{f}\rightarrow\frac{1}{i}=\frac{1}{f}-\frac{1}{o}$$

$f_1 = f,\ o_1 = \infty$

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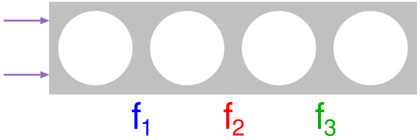

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$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$f_1 = f$, $o_1 = \infty$

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

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so for N lenses $f_{eff} = f/N$

Rephasing distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.

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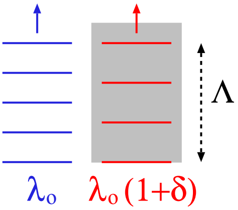

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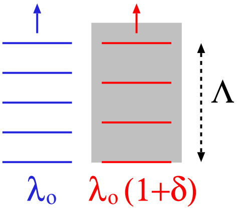

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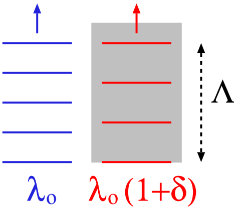

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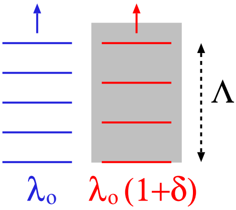

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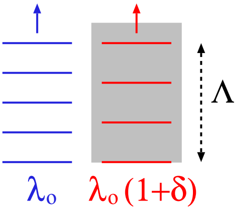

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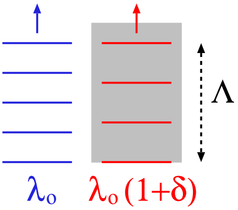

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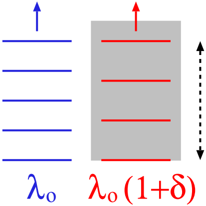

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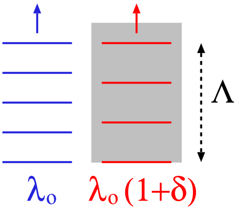

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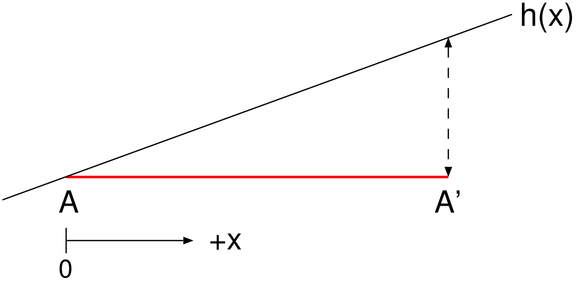
Ideal interface profile



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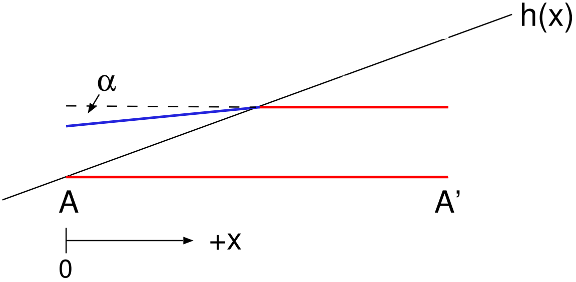
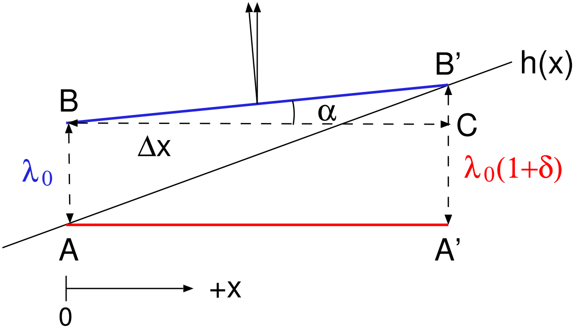


The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α.

Ideal interface profile



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Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

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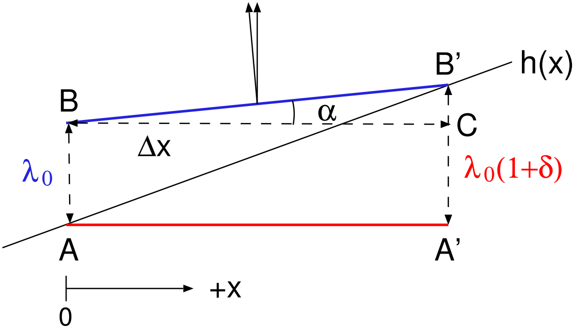

from the $AA'B'$ triangle

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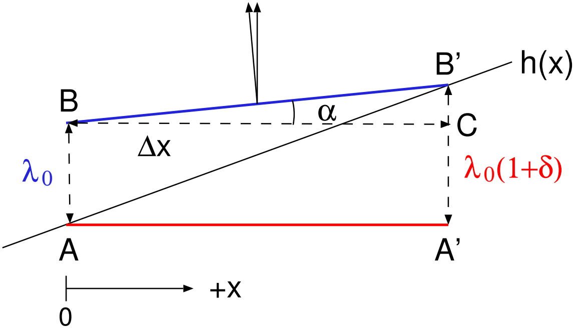

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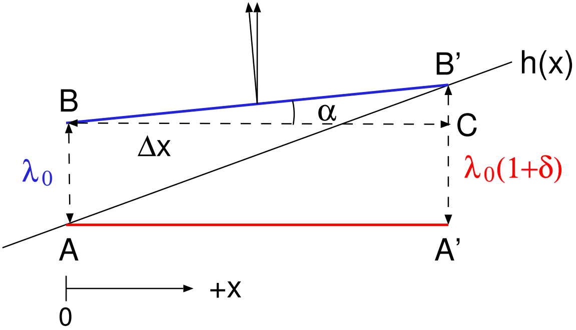

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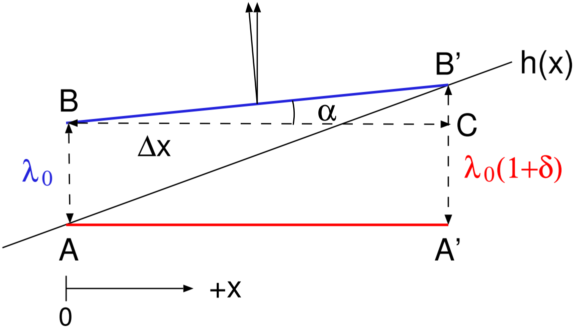

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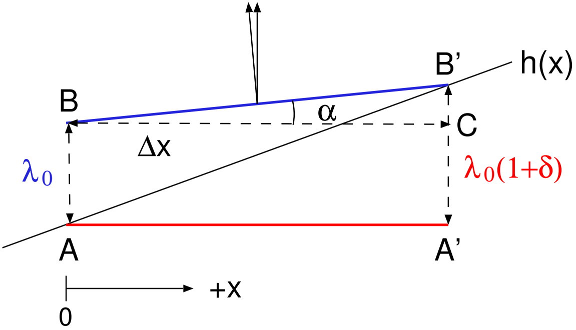

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$$\alpha(x) \approx \frac{\lambda_0 \delta}{\Delta x}$$

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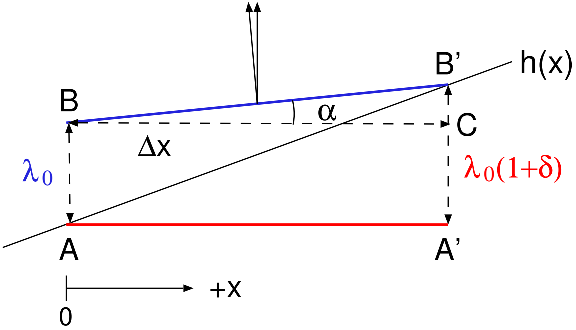

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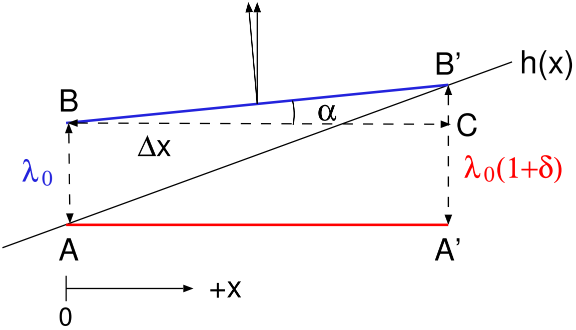

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using $\Lambda = \lambda_0/\delta$

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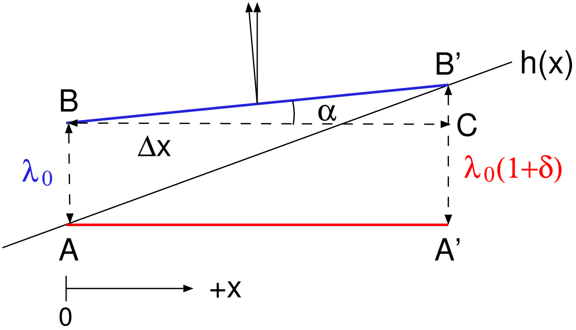

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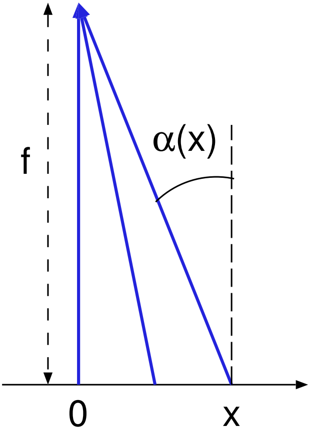
using $\Lambda = \lambda_0/\delta$

Ideal interface profile

If the desired focal length of this lens is f, the wave must be redirected at an angle which depends on the distance from the optical axis



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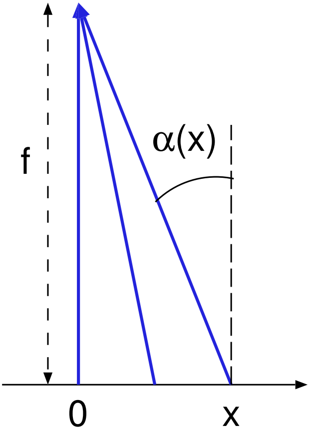
$$\alpha(x) = \frac{x}{f}$$



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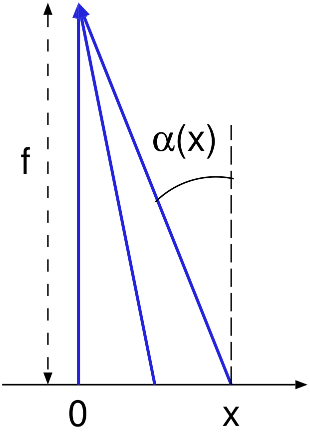
combining, we have

$$\frac{\lambda_0 h'(x)}{\Lambda} = \frac{x}{f}$$



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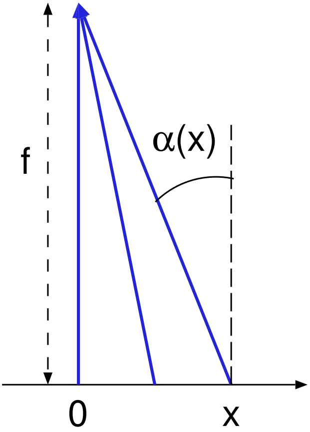

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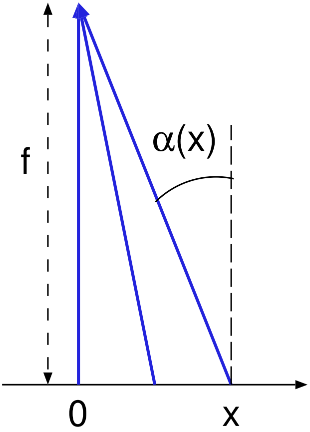
this can be directly integrated



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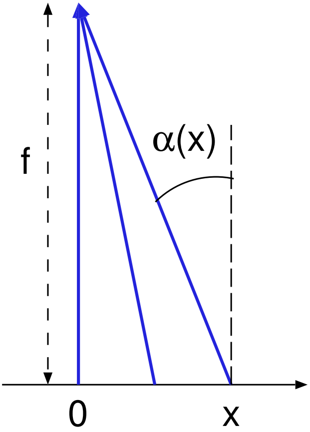
$$\frac{h(x)}{\Lambda} = \frac{x^2}{2f\lambda_0}$$



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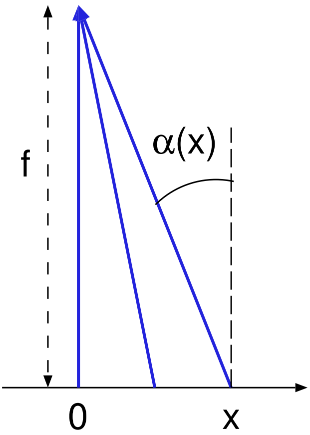
this can be directly integrated

$$\frac{h(x)}{\Lambda} = \frac{x^2}{2f\lambda_0} = \left[\frac{x}{\sqrt{2f\lambda_0}}\right]^2$$



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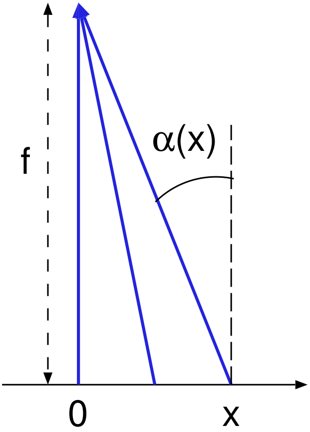

a parabola is the ideal surface for focusing be refraction