Today's Outline - September 19, 2016

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- Reflection from a thin slab

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Homework Assignment #02:
Problems on Blackboard
due Monday, September 26, 2016

Fresnel equation review

The scattering vector (or momentum transfer) is given by

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$$q = \frac{Q}{Q_c}$$

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$$q^2 = q'^2 + 1 - 2ib_\mu, \quad b_\mu = \frac{2k}{Q_c^2}\mu$$

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Fresnel equations

$$r = \frac{q - q'}{q + q'},$$

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Fresnel equations

$$r = \frac{q - q'}{q + q'}, \qquad t = \frac{2q}{q + q'}$$

Limiting cases - $q \gg 1$

Start by rearranging Snell's Law $\qquad q^2 = q'^2 + 1 - 2ib_\mu$

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Limiting cases - $q \gg 1$

Start by rearranging Snell's Law and since q is real by definition, when $q \gg 1$

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Limiting cases - $q \gg 1$

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$$q^2 = q'^2 + 1 - 2ib_\mu$$
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$$q'^2 \approx q^2 + 2ib_\mu$$

Limiting cases - $q \gg 1$

Start by rearranging Snell's Law and since q is real by definition, when $q \gg 1$

this implies $Re(q') \approx q$,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

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Limiting cases - $q \gg 1$

Start by rearranging Snell's Law and since q is real by definition, when $q \gg 1$

this implies $Re(q') \approx q$, while the imaginary part can be computed by assuming

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reflected wave in phase with incident, almost total transmission

Limiting cases - $q \ll 1$

When $q \ll 1$

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Limiting cases - $q \ll 1$

When $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

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$$q'^2 \approx -1$$

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When $q \ll 1$, q' is mostly imaginary with magnitude 1 since b_μ is very small

Thus the reflection and transmission coefficients become

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$$r = \frac{(q - q')}{(q + q')}$$

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$$\Lambda \approx \frac{1}{Q_c}$$

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$$\Lambda \approx \frac{1}{Q_c}$$

The reflected wave is out of phase with the incident wave, there is only small transmission in the form of an evanescent wave, and the penetration depth is very short.

Limiting cases - $q \sim 1$

If $q \sim 1$, $q^2 = q'^2 + 1 - 2ib_\mu$

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If $q \sim 1$,

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$$q'^2 \approx 2ib_\mu$$

Limiting cases - $q \sim 1$

If $q \sim 1$, adding and subtracting b_μ,

$$q^2 = q'^2 + 1 - 2ib_\mu$$

$$q'^2 = q^2 - 1 + 2ib_\mu$$

$$q'^2 \approx 2ib_\mu = b_\mu(1 + 2i - 1)$$

Limiting cases - $q \sim 1$

If $q \sim 1$, adding and subtracting b_μ, yields that q' is complex with real and imaginary parts of equal magnitude.

$$q^2 = q'^2 + 1 - 2ib_\mu$$

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If $q \sim 1$, adding and subtracting b_μ, yields that q' is complex with real and imaginary parts of equal magnitude.

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The reflected wave is in phase with the incident, there is significant (larger amplitude than the reflection) transmission with a large penetration depth.

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We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium.

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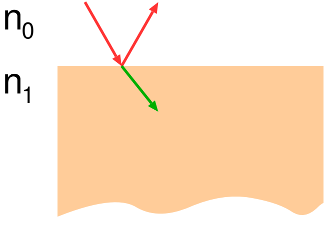

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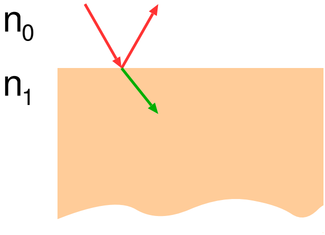

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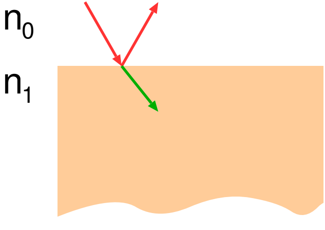

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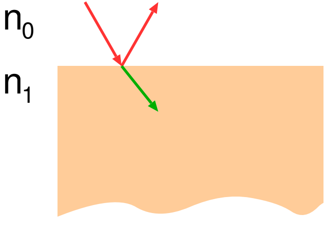

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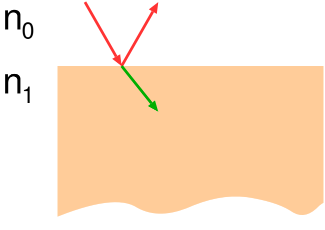

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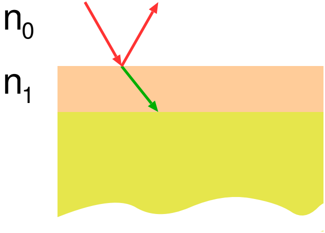

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We have assumed that the transmitted wave eventually attenuates to zero in all cases due to absorption. We now consider what happens if there is a second interface encountered by the transmitted wave before it dies away. That is, a thin slab of material on top of an infinite substrate

Reflection and transmission coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:

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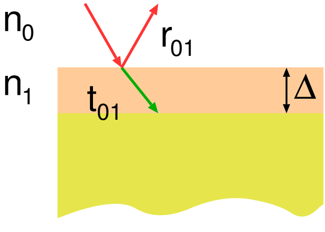

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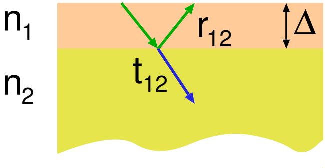

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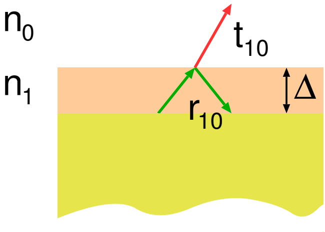

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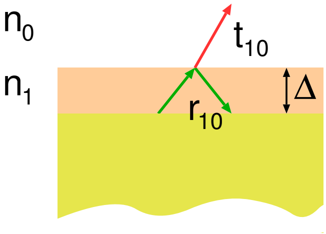

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Build the composite reflection coefficient from all possible events

Overall reflection from a slab

The composite reflection coefficient for each ray emerging from the top surface is computed

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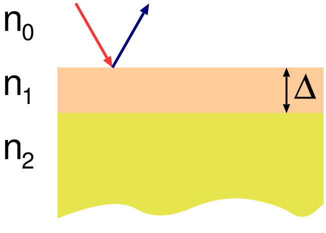

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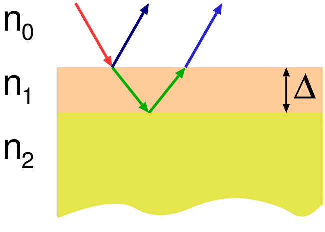

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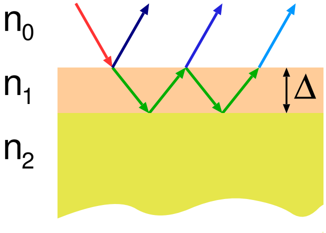

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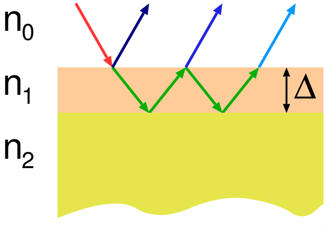

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Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

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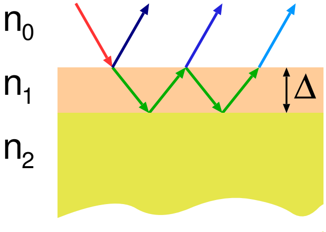

$$r_{01} + t_{01} r_{12} t_{10} + t_{01} r_{12} r_{10} r_{12} t_{10}$$

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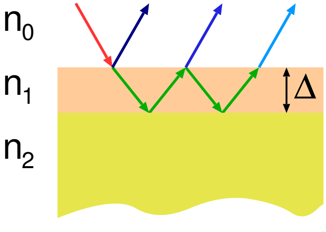

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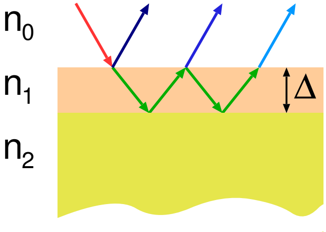

$$r_{01}$$
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$$+$$
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$$= \frac{Q_0^2 - 2Q_0Q_1 + Q_1^2 + 4Q_0Q_1}{(Q_0 + Q_1)^2} = \frac{Q_0^2 + 2Q_0Q_1 + Q_1^2}{(Q_0 + Q_1)^2} = \frac{(Q_0 + Q_1)^2}{(Q_0 + Q_1)^2} = 1$$

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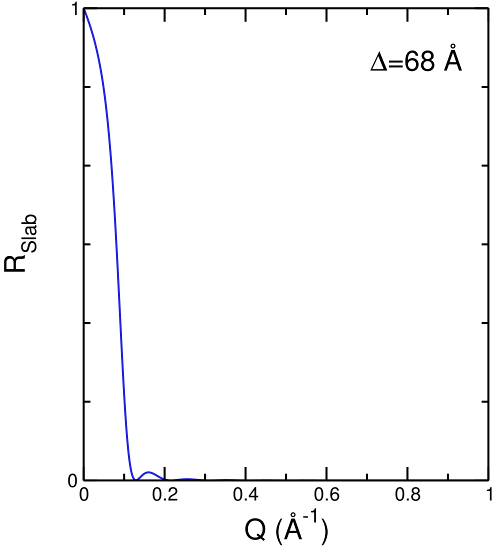

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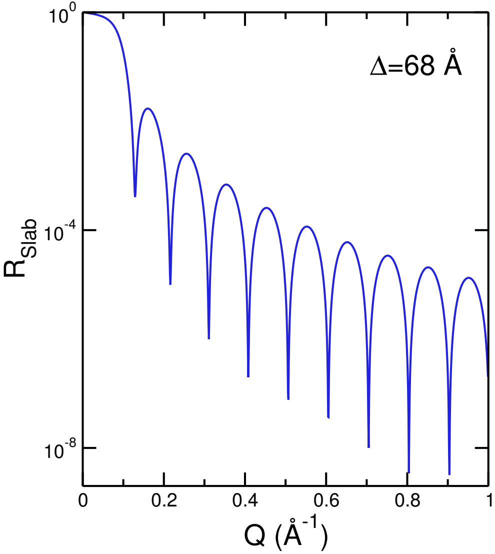

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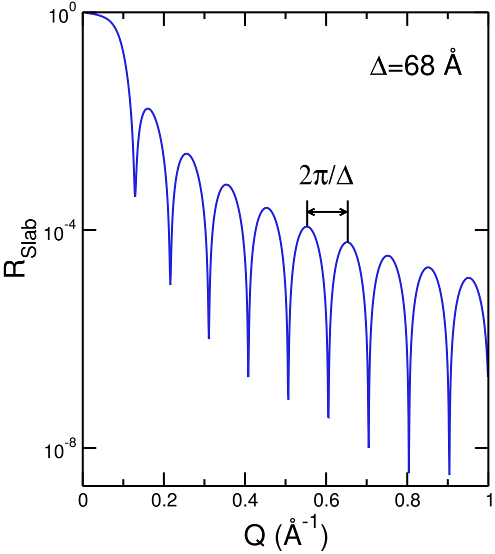
$$2\pi/\Delta = 0.092\text{Å}^{-1}$$



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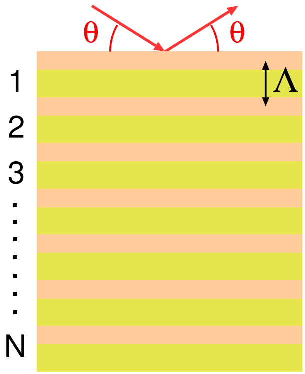
Multilayers in the kinematical regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

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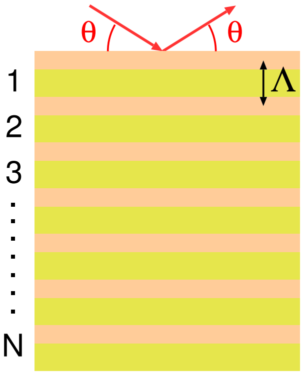

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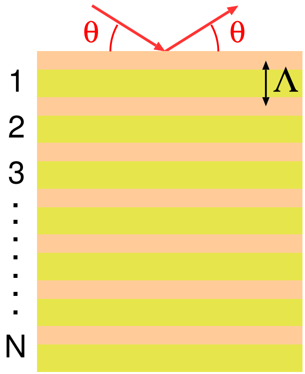

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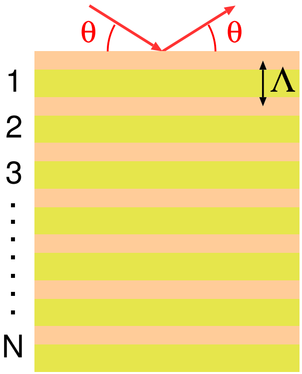

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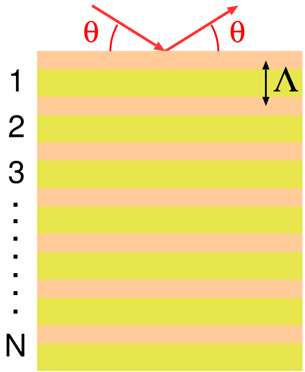

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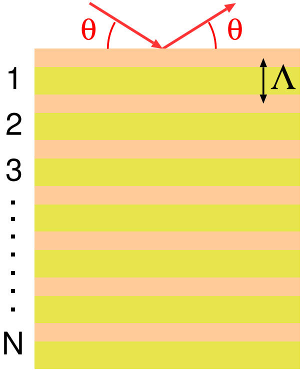
$$r_N(\zeta) = \sum_{\nu=0}^{N-1} r_1(\zeta) e^{i2\pi\zeta\nu} e^{-\beta\nu}$$

Multilayers in the kinematical regime



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Reflectivity of a bilayer

The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components

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Absorption coefficient of a bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_o\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin\left(\pi\Gamma\zeta\right)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

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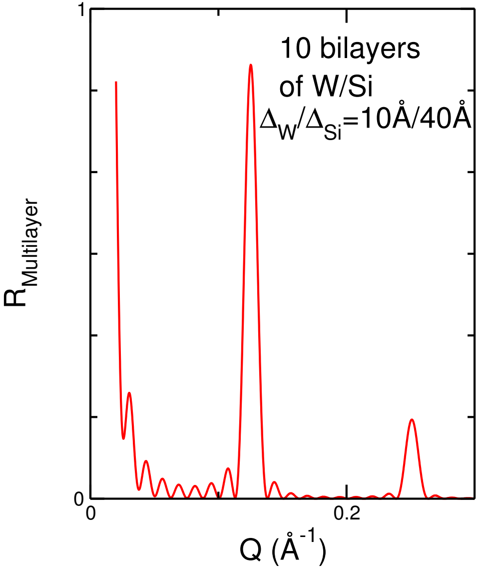
$$\beta = 2\left[\frac{\mu_A}{2}\frac{\Gamma\Lambda}{\sin\theta} + \frac{\mu_B}{2}\frac{(1-\Gamma)\Lambda}{\sin\theta}\right] = \frac{\Lambda}{\sin\theta}\left[\mu_A\Gamma + \mu_B(1-\Gamma)\right]$$

Reflectivity calculation



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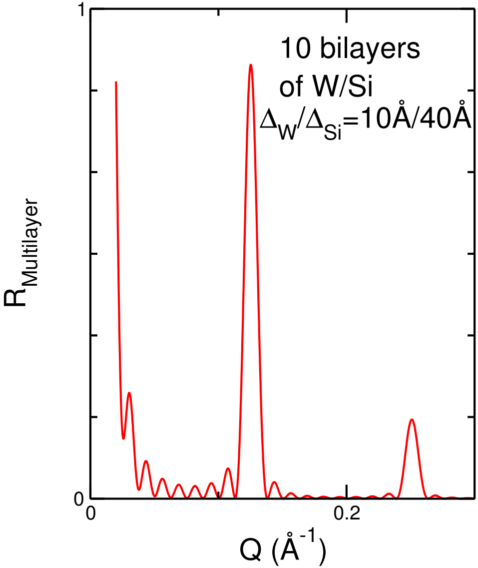

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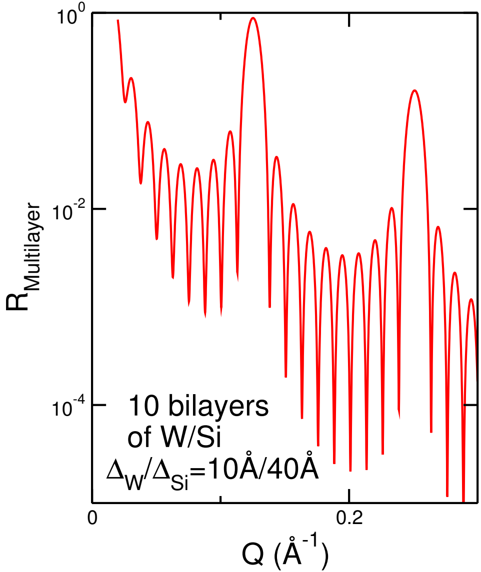

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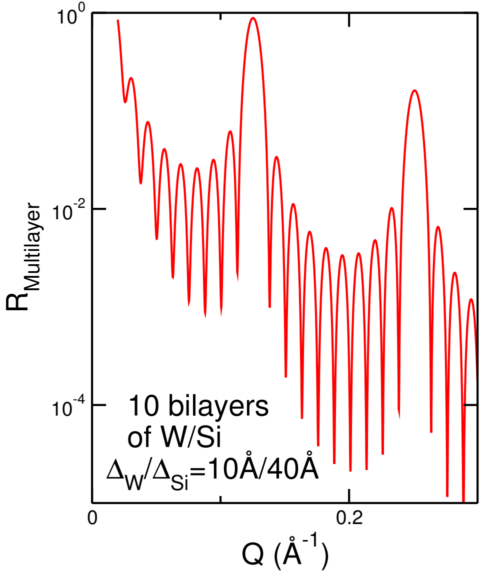

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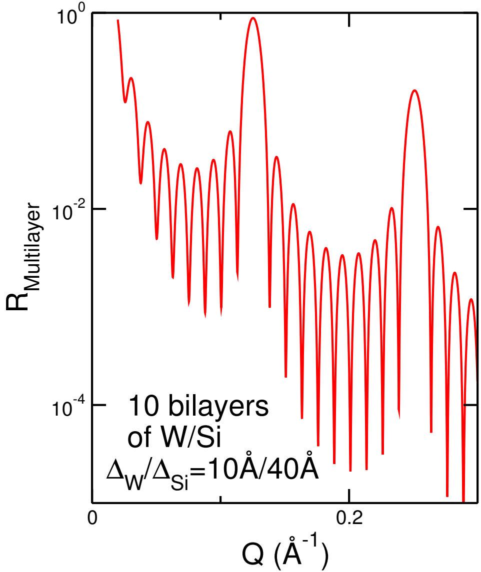

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- Multilayers are used commonly on laboratory sources as well as at synchrotrons as mirrors

Slab - multilayer comparison