Today's Outline - September 14, 2016

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- Refraction and reflection

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Reading Assignment: Chapter 3.5–3.8

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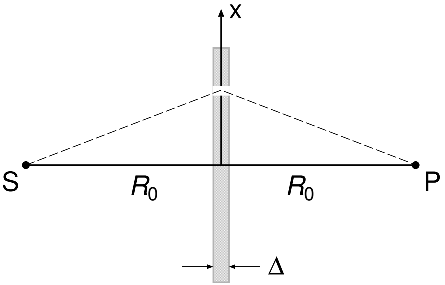
Reading Assignment: Chapter 3.5–3.8

Homework Assignment #02:
Problems on Blackboard
due Monday, September 26, 2016

Thin plate response - scattering approach

Consider a thin plate of thickness Δ onto which x-rays are incident from a point source S a perpendicular distance R_0 away.



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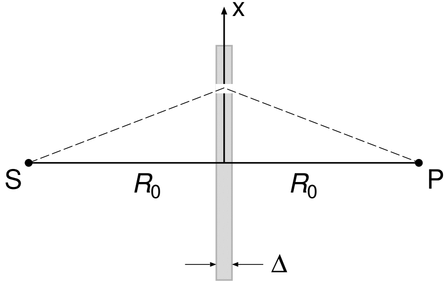

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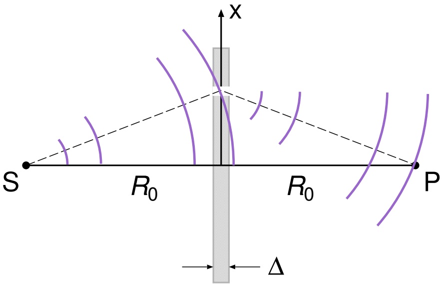

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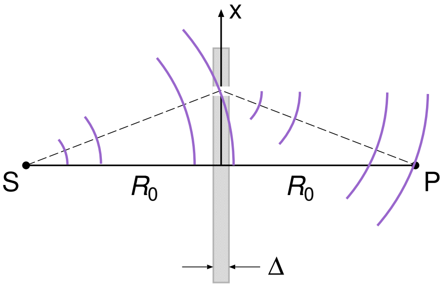

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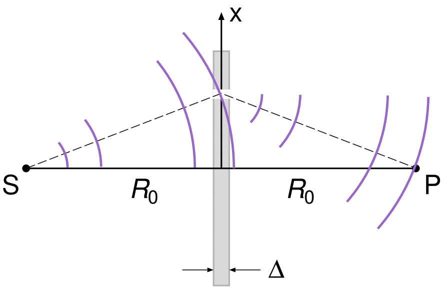

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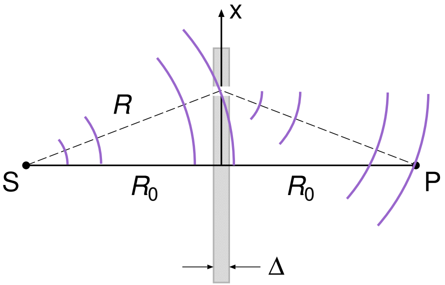

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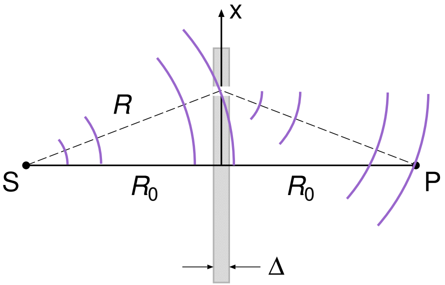

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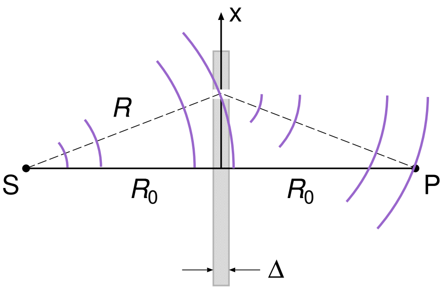

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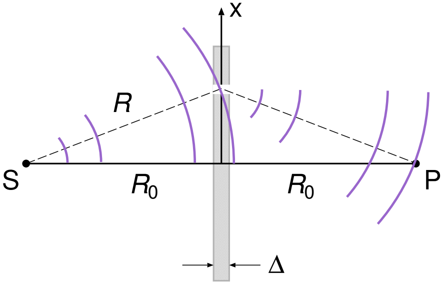
$$R = R_0\sqrt{1 + \frac{x^2 + y^2}{R_0^2}} \approx R_0\left[1 + \frac{x^2 + y^2}{2R_0^2}\right]$$

Thin plate response - scattering approach

R is also the distance between the scattering volume and P so, a wave (x-ray) which travels from $S \to P$ through the scattering volume will have an extra phase shift



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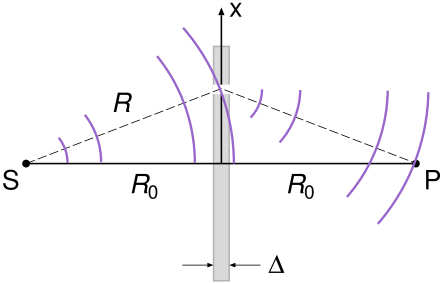

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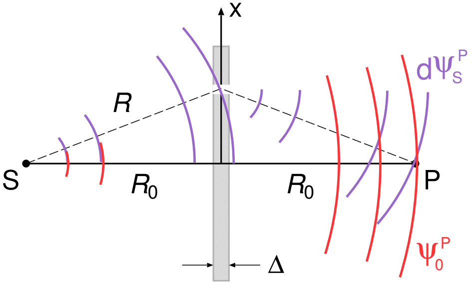

$$\phi(x, y) = 2k\frac{x^2+y^2}{2R_0^2} = \frac{x^2+y^2}{R_0^2}k$$

compared to a wave which travels directly along the z-axis.

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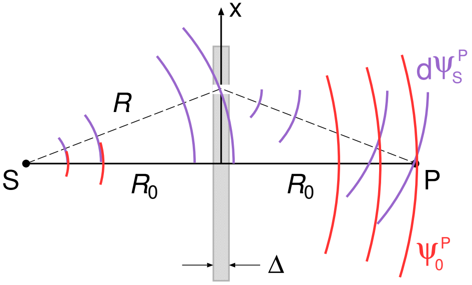

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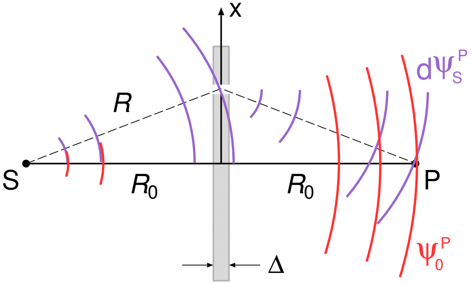

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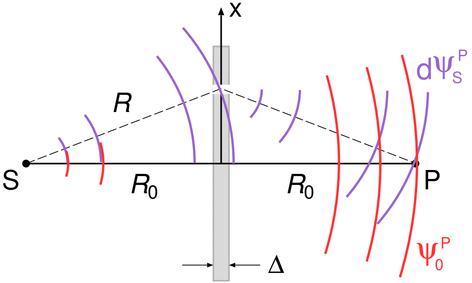

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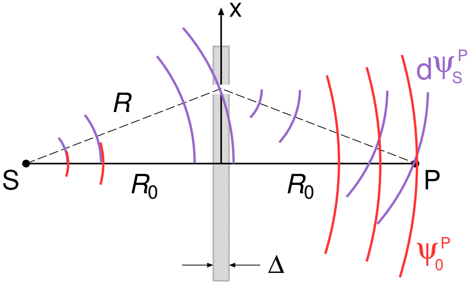

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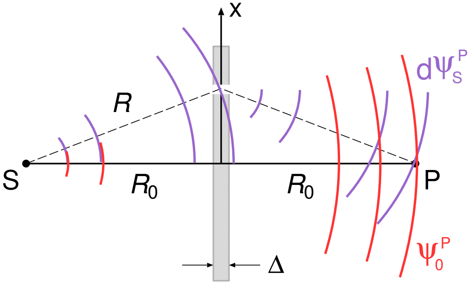

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Integrate the scattered wave over the entire plate.

Thin plate response - scattering approach

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Thus the total wave (electric field) at P can be written

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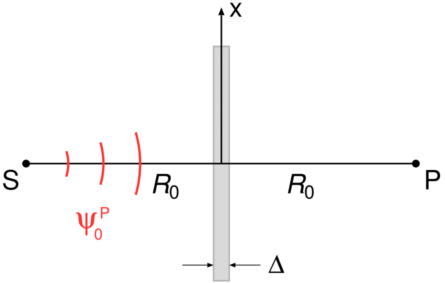
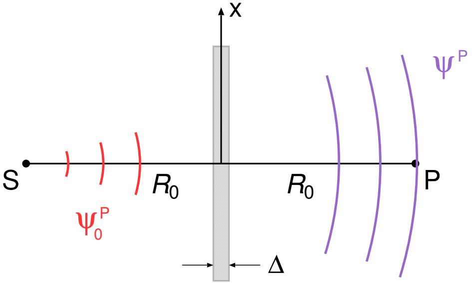
Thin plate response - refraction approach

Now let's look at this phenomenon from a different point of view, that of refraction.



Thin plate response - refraction approach

Now let's look at this phenomenon from a different point of view, that of refraction. Assume that the wave passing through the plate simply gains a phase shift because it passes through a medium compared to a wave which does not have the plate present.



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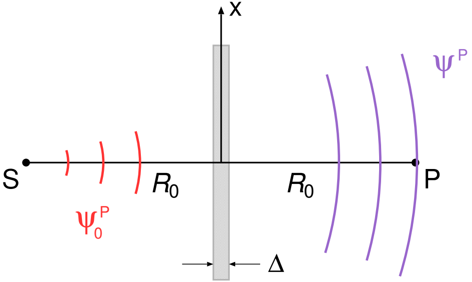

The phase shift depends on the thickness and the difference between the index of refraction of the medium and that of vacuum

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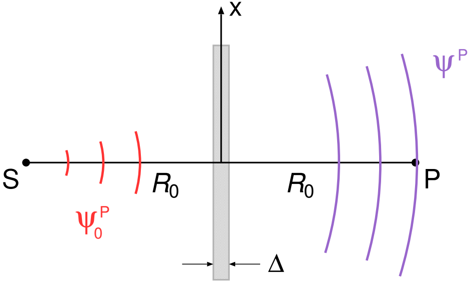

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$$\phi = 2\pi\left(\frac{\Delta}{\lambda'} - 2\pi\frac{\Delta}{\lambda}\right)$$

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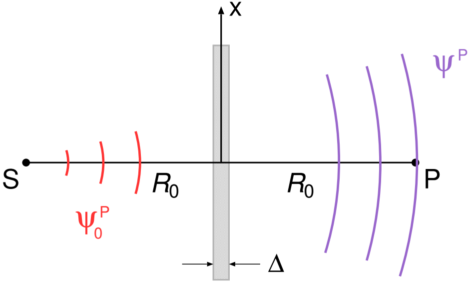

The phase shift depends on the thickness and the difference between the index of refraction of the medium and that of vacuum

$$\phi = 2\pi\left(\frac{n\Delta}{\lambda} - 2\pi\frac{\Delta}{\lambda}\right)$$

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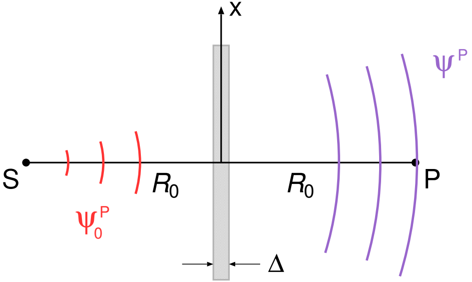

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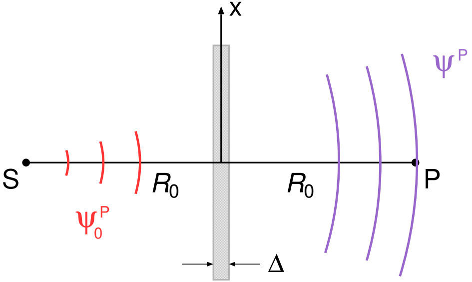

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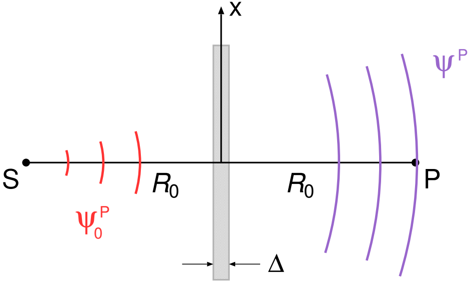

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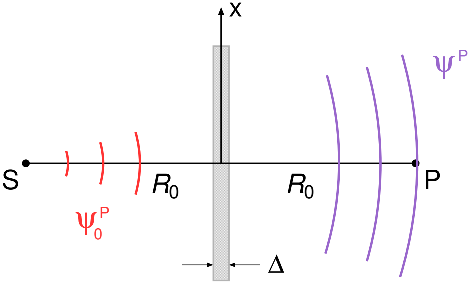

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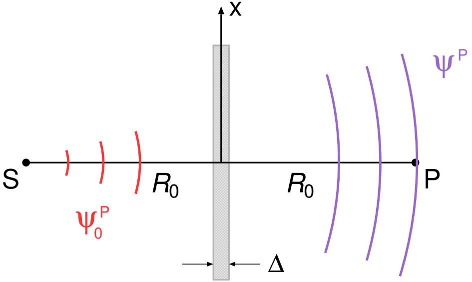

The phase shift depends on the thickness and the difference between the index of refraction of the medium and that of vacuum

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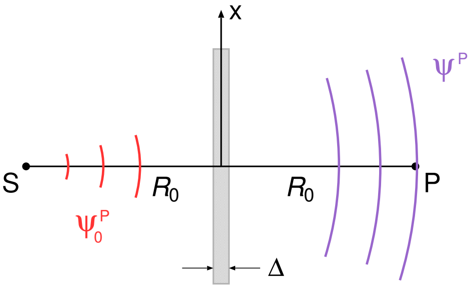

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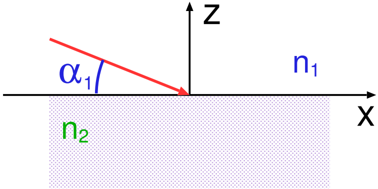

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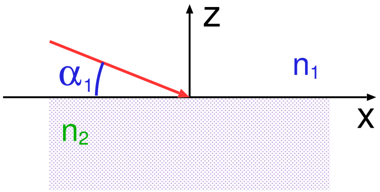

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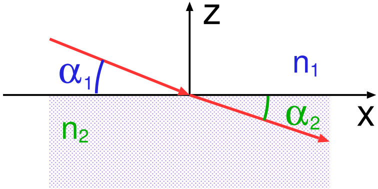

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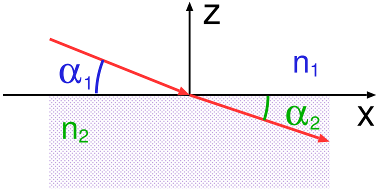

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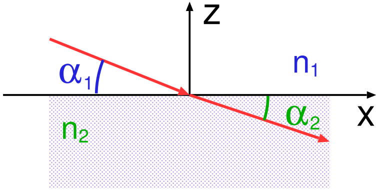

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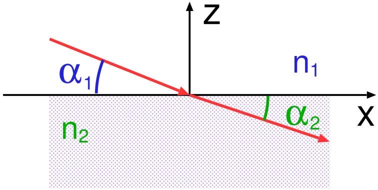

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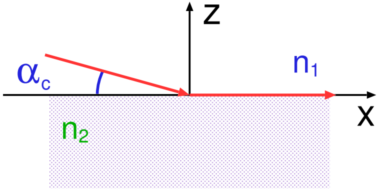

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When the incident angle becomes small enough, there will be total external reflection

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Connection to atomic scattering

So far, we have made the assumption that the charge distribution is uniform. We know that this is not correct, and that usually electron charge distributions are those of the atoms making up the solid.

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So far, we have made the assumption that the charge distribution is uniform. We know that this is not correct, and that usually electron charge distributions are those of the atoms making up the solid. Therefore, it is useful to replace the uniform charge distribution, ρ, with a more realistic one, including the atom distribution ρ_a:

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In terms of the absorption coefficient, μ

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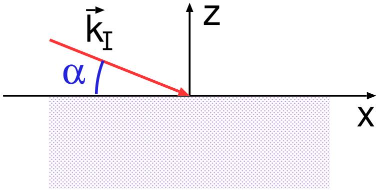
$$\begin{aligned} f'' &= -\frac{k^2}{2\pi\rho_a r_0}\frac{\mu}{2k} \\ &= -\frac{k}{4\pi r_0}\sigma_a \end{aligned}$$

Electromagnetic Boundary Conditions

Maxwell's equations require that an electromagnetic wave and its derivative be continuous in all directions at any interface. This condition places restrictions on the waves which exist at any interface:



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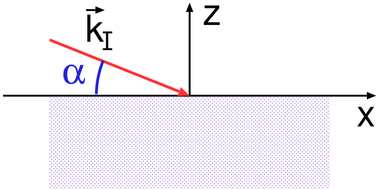

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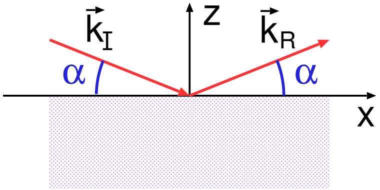

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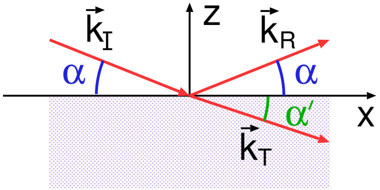

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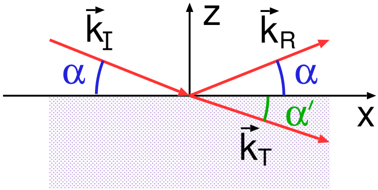

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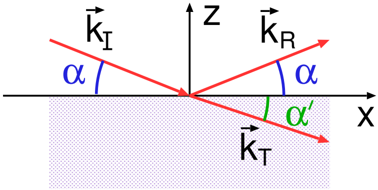

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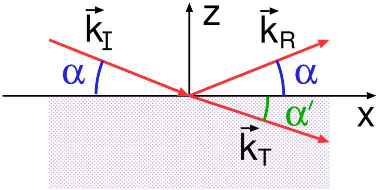

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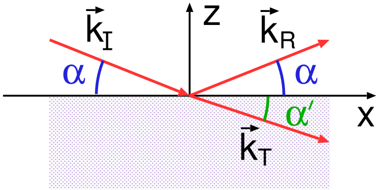

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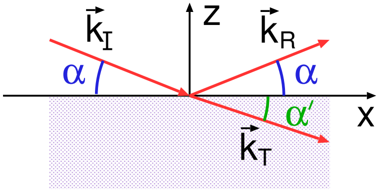

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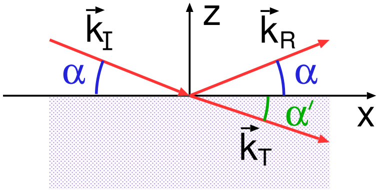

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Taking the perpendicular projection, substituting for the wave vectors

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Reflectivity and Transmittivity

r and t are called the reflection and transmission coefficients, respectively.

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q is a convenient parameter to use because it is a combination of two parameters which are often varied in experiments, the angle of incidence α and the wavenumber (energy) of the x-ray, k.

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reflected wave in phase with incident, almost total transmission