Today's Outline - September 07, 2016

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- Undulator harmonics

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Reading Assignment: Chapter 3.1–3.3

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Homework Assignment #01:
Chapter Chapter 2: 2,3,5,6,8
due Monday, September 12, 2016

Undulator review

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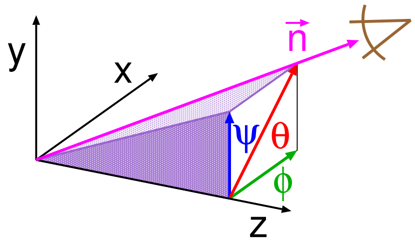
Now let us look at the higher harmonics and the coherence of the undulator radiation

Higher harmonics



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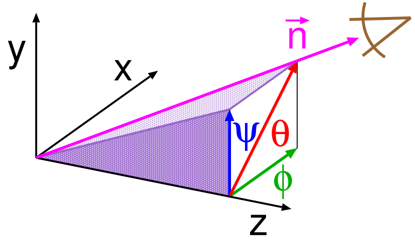

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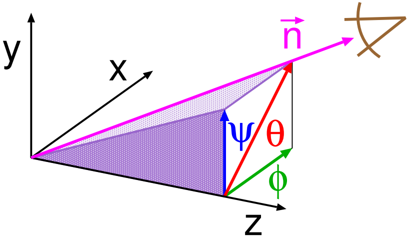

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This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

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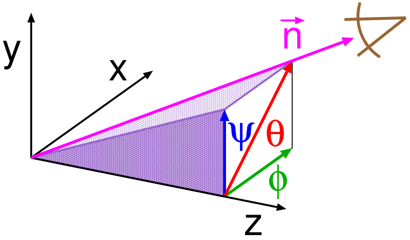

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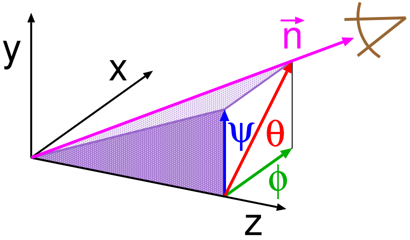

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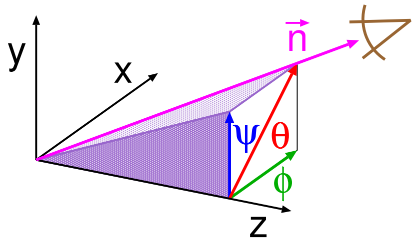

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The motion of the electron, $\sin\omega_u t'$, is always sinusoidal, but because of the additional terms, the motion as seen by the observer, $\sin\omega_1 t$, is not.

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t')$$

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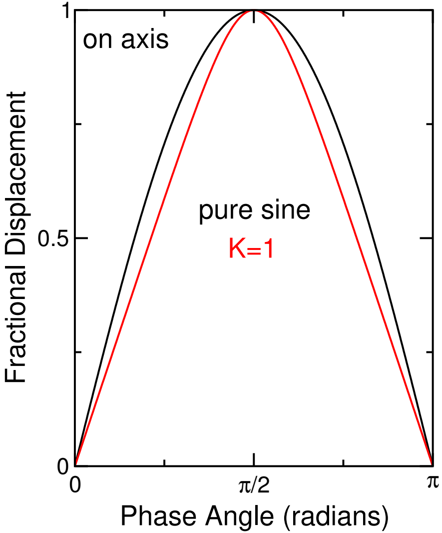

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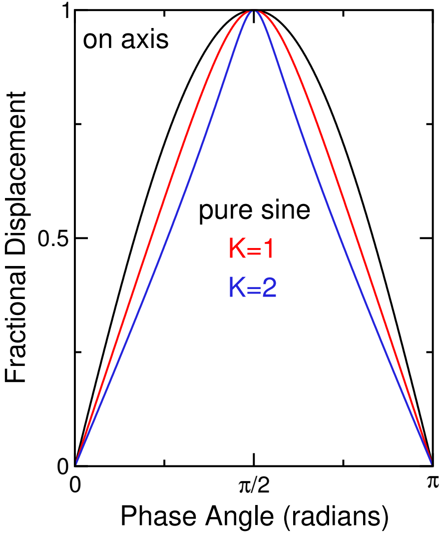
Similarly, for $K = 2$



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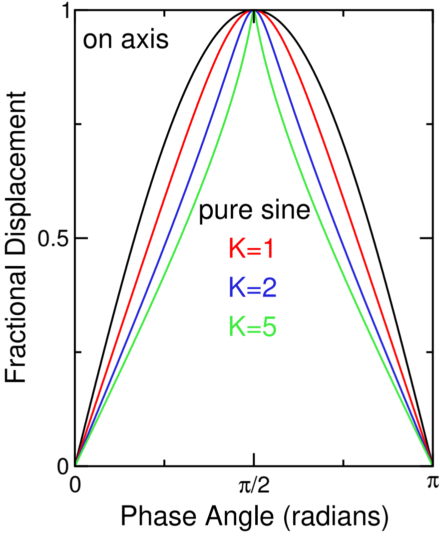
Similarly, for $K = 2$ and $K = 5$, the deviation becomes more pronounced.



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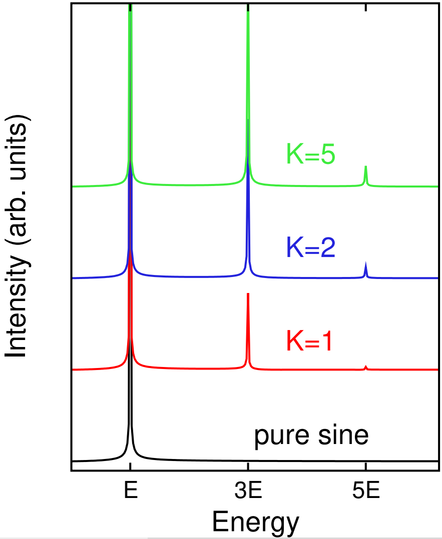
Similarly, for $K = 2$ and $K = 5$, the deviation becomes more pronounced. This shows how higher harmonics must be present in the radiation as seen by the observer.



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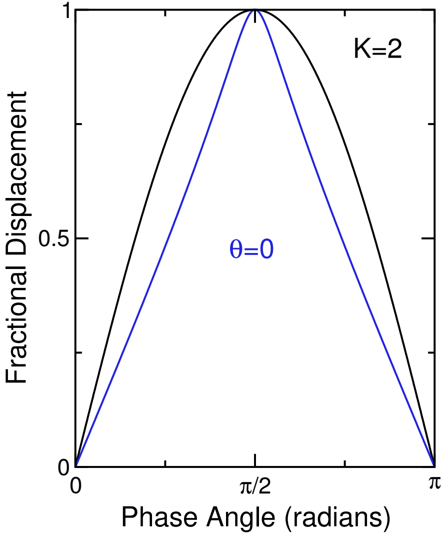

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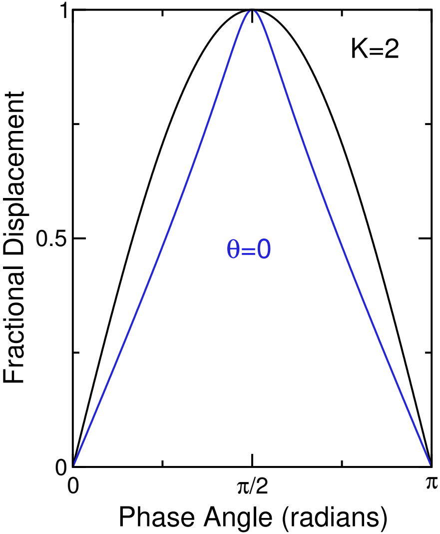

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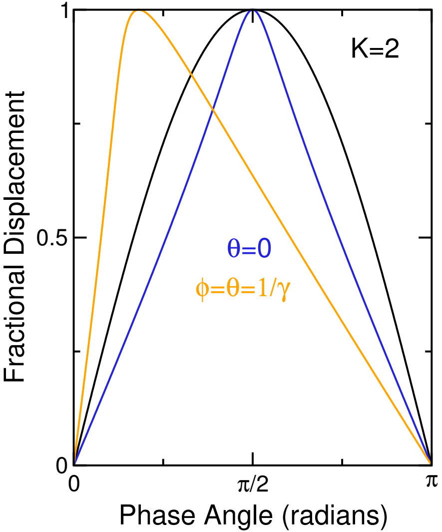

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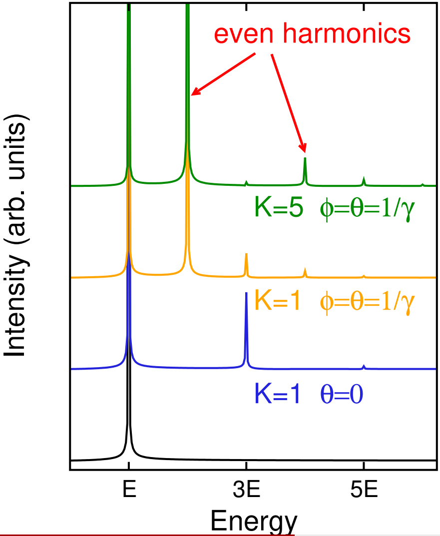

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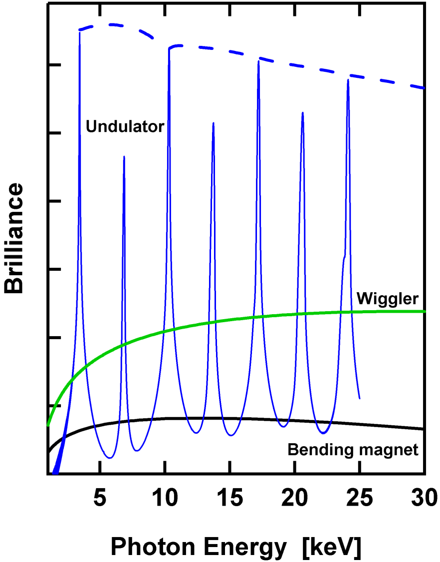
The last term introduces an antisymmetric term which skews the function and leads to the presence of forbidden harmonics (2^{nd}, 4^{th}, etc) in the radiation from the undulator compared to the on-axis radiation.

Spectral comparison



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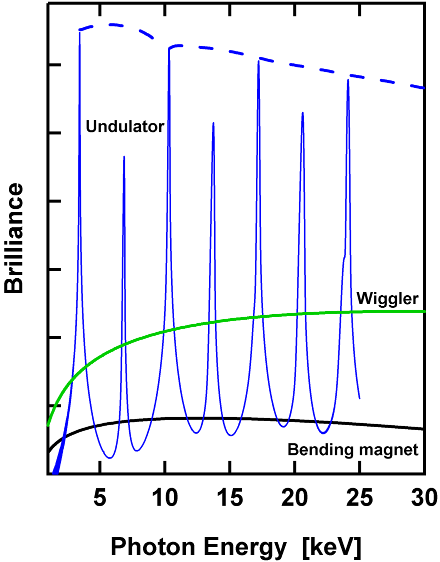

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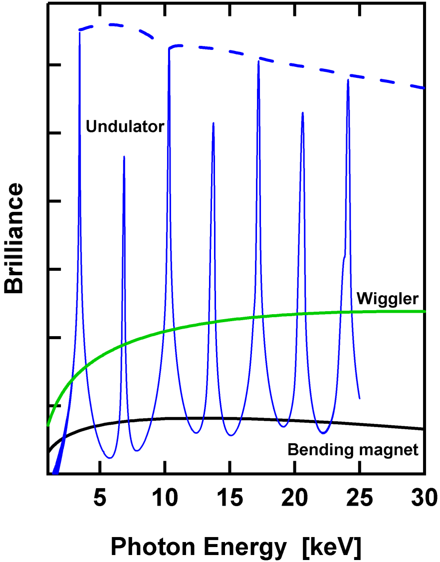

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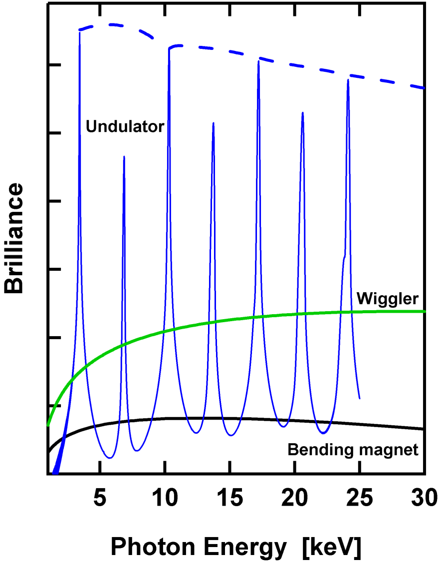

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- Harmonics can be tuned in energy (dashed lines)

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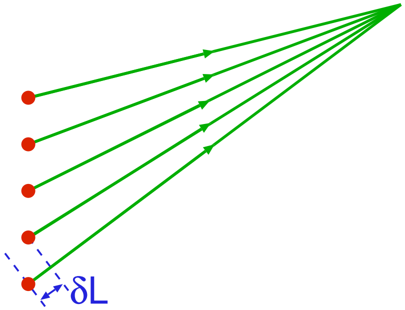
A diffraction grating consists of N coherent sources whose emission is detected at a single point.



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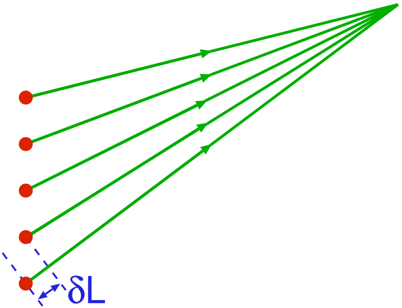

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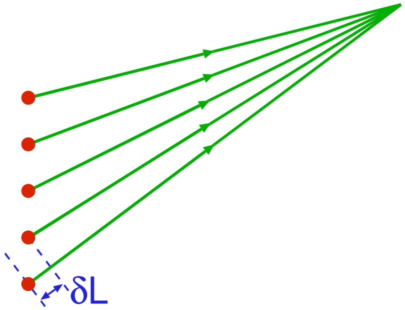

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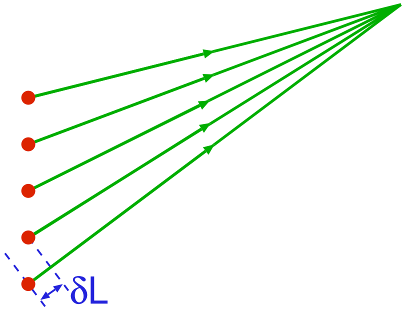

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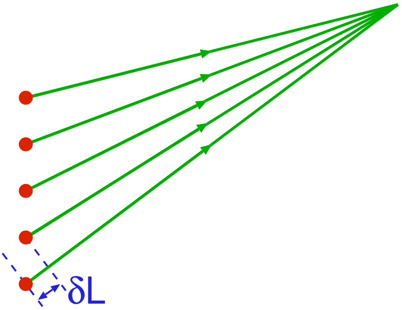

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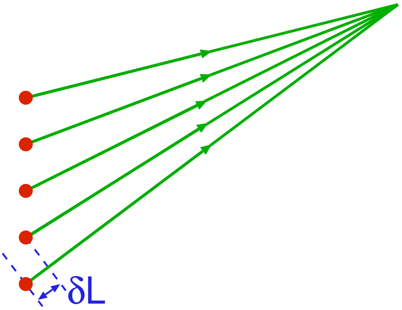

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Geometric series

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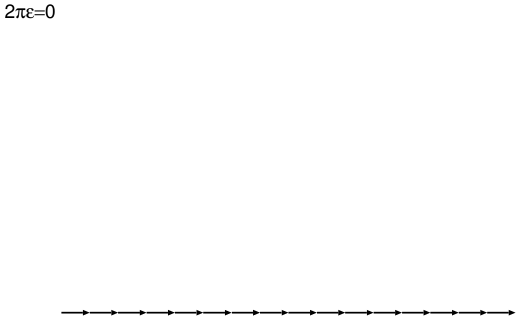

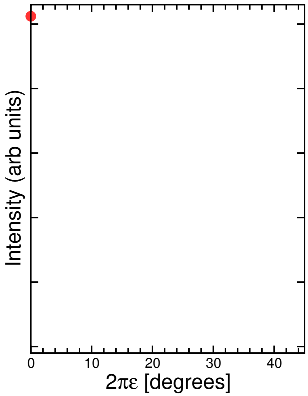

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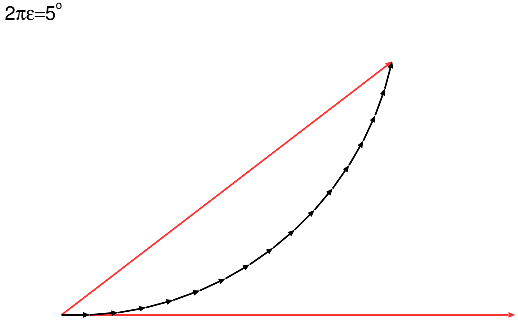

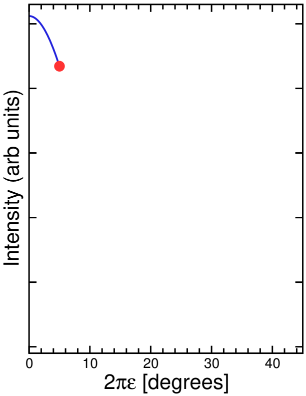

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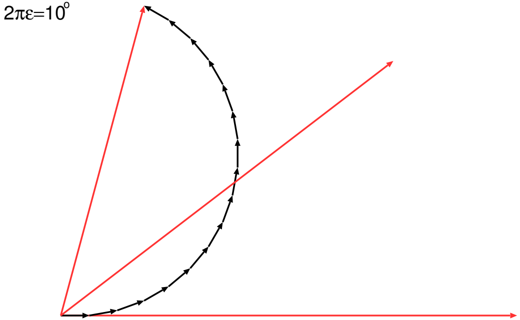

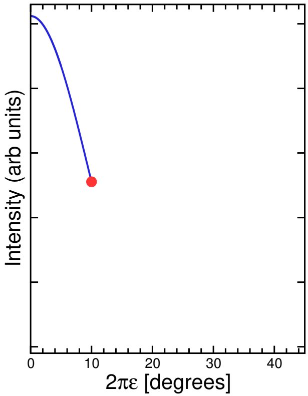

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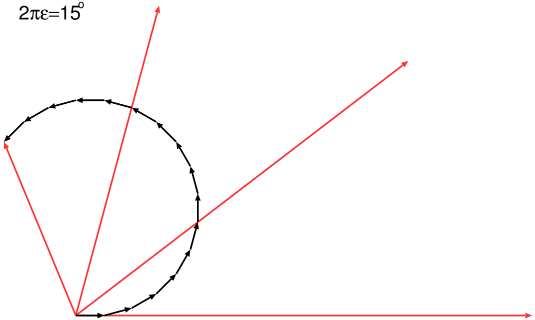

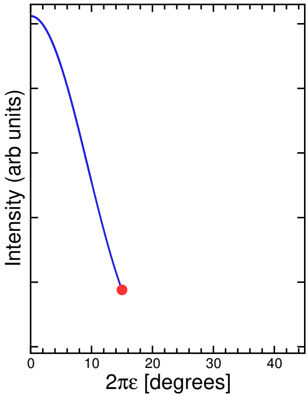

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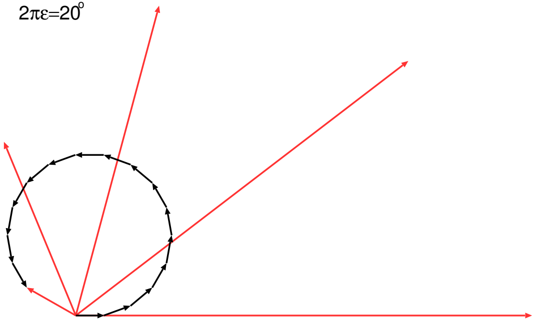

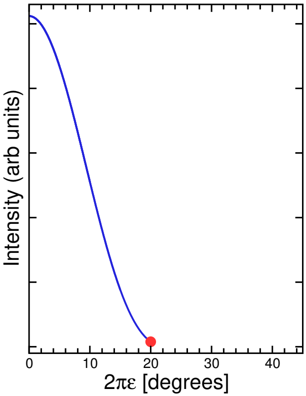

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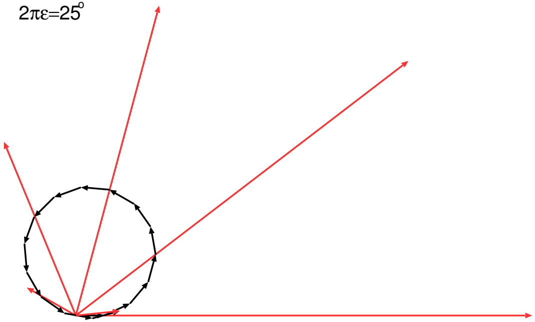

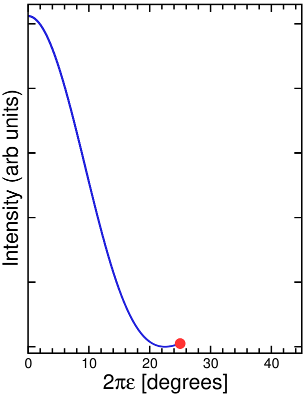

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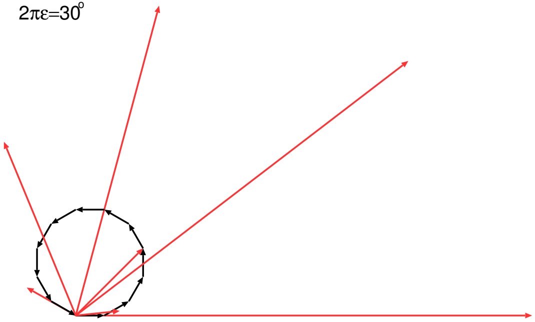

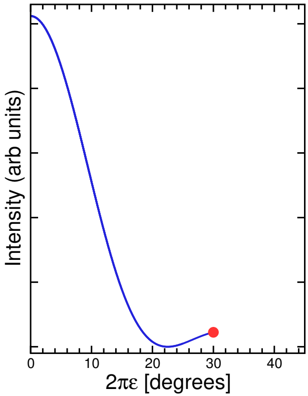

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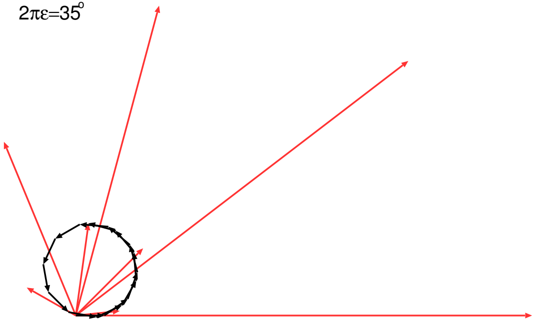

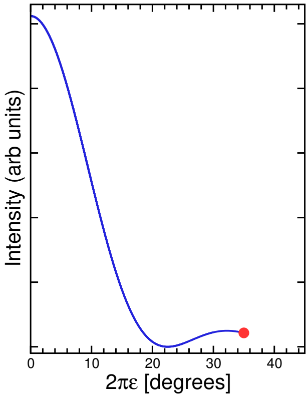

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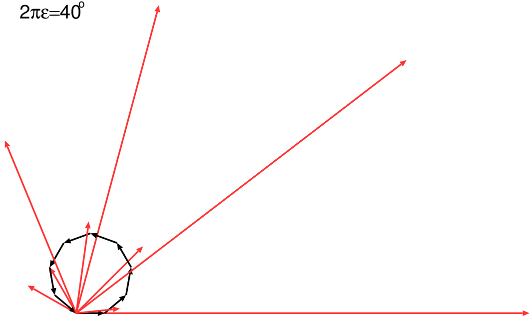

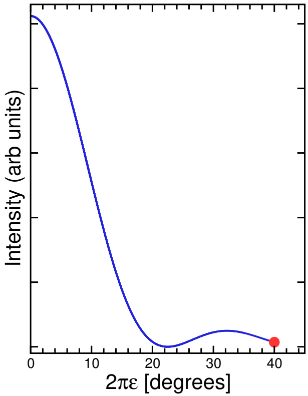

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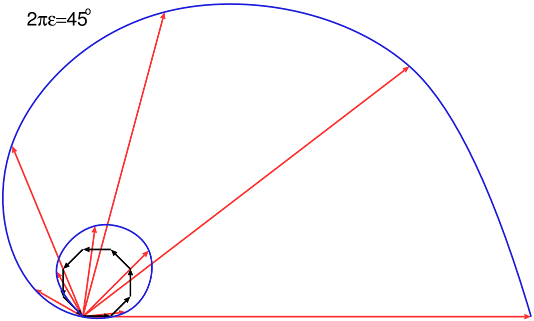

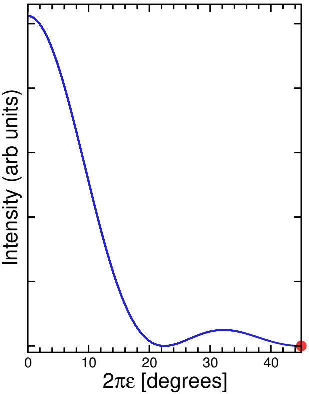

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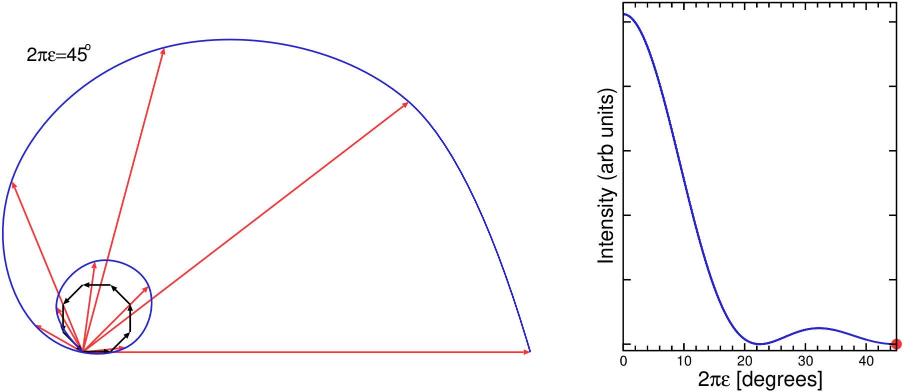
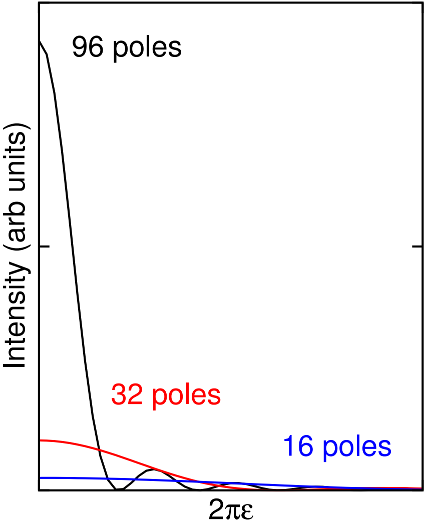


With the height and width of the peak dependent on the number of poles.

Undulator coherence



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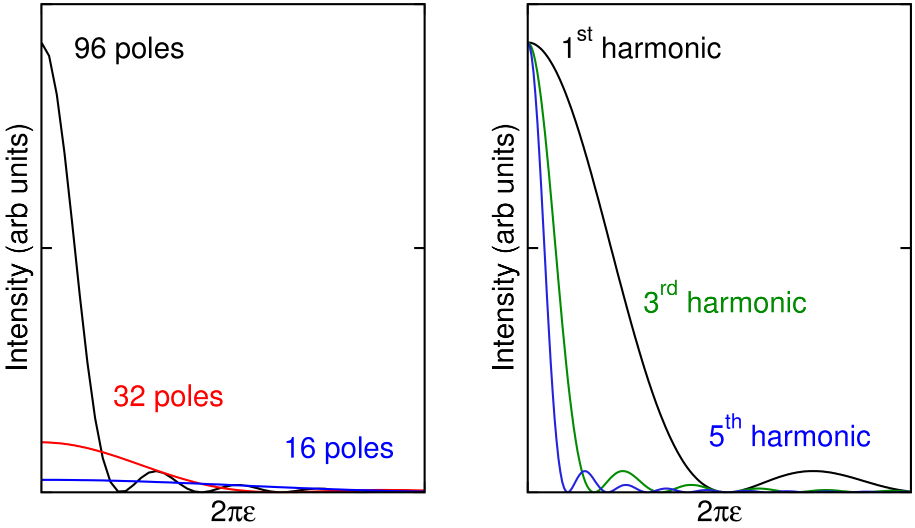
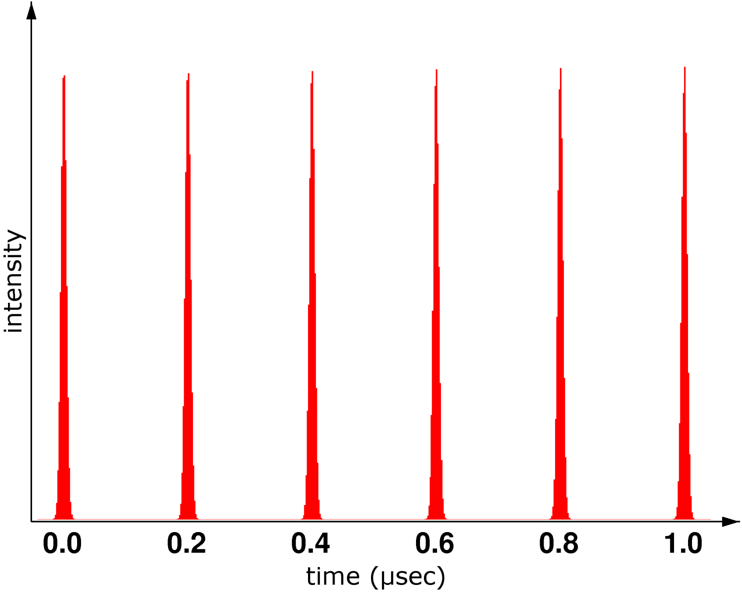


Synchrotron time structure



Emittance

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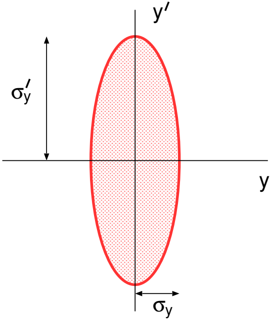

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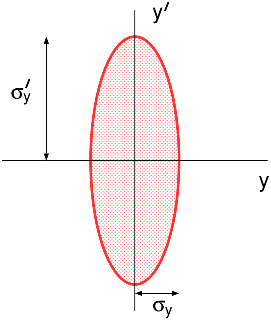

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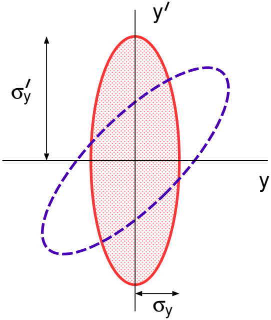

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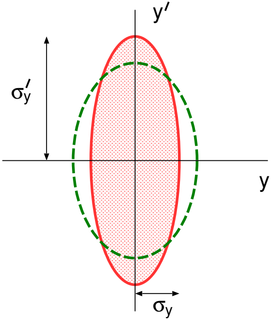

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For photon emission from a single electron in a 2m undulator at 1Å

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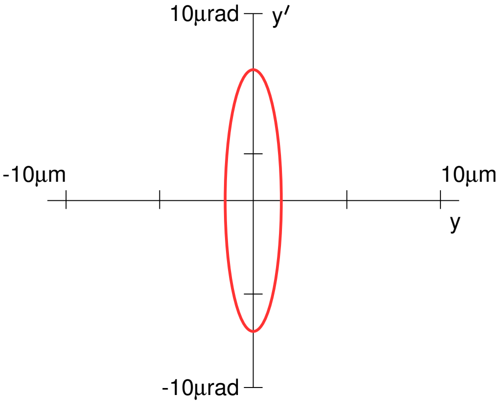

$$\sigma_\gamma = \frac{\sqrt{L\lambda}}{4\pi} = 1.3\mu\text{m}$$

$$\sigma'_\gamma = \sqrt{\frac{\lambda}{L}} = 7.1\mu\text{rad}$$

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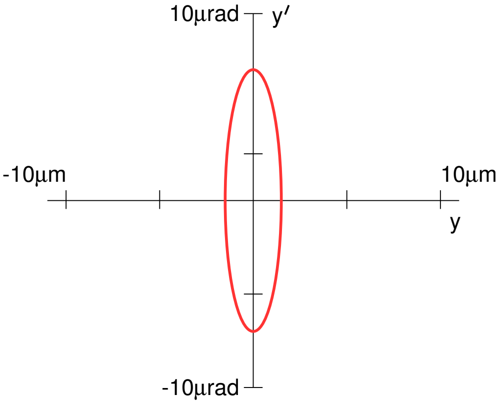

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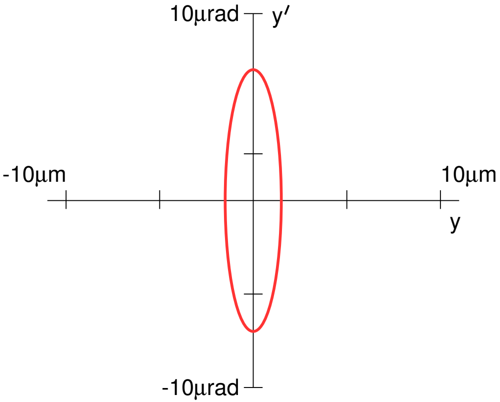

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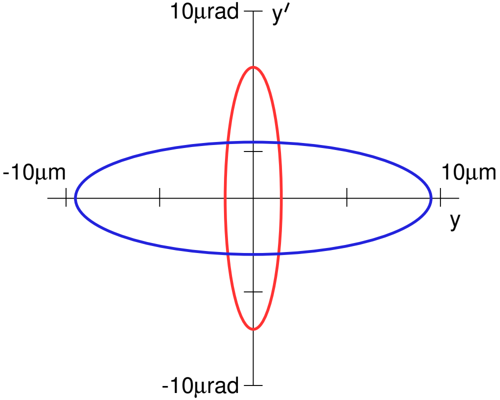

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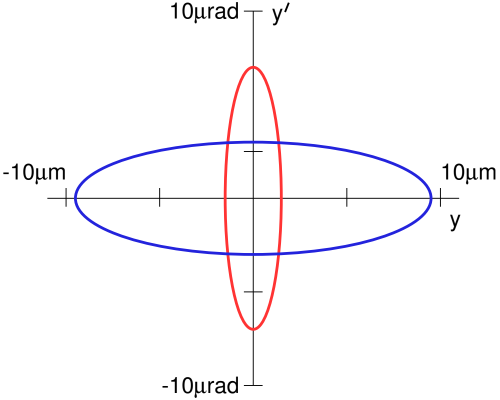

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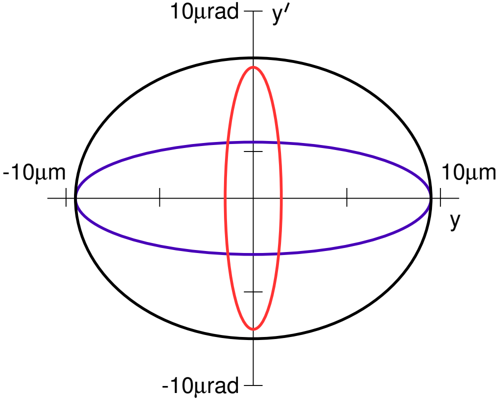

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Evolution of APS parameters

Parameter	1995	2001	2005
σ_x	334 μm	352 μm	280 μm
σ'_x	24 μrad	22 μrad	11.6 μrad
σ_y	89 μm	18.4 μm	9.1 μm
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The next big upgrade (slated for 2020) will make the beam more square in space and by choosing the undulator correctly, a higher performance insertion device.

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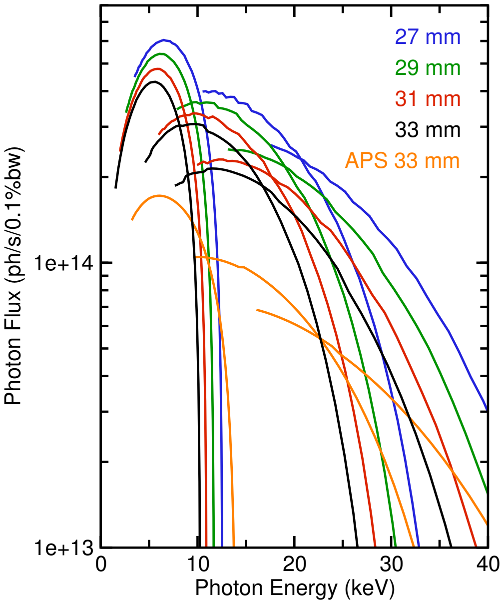

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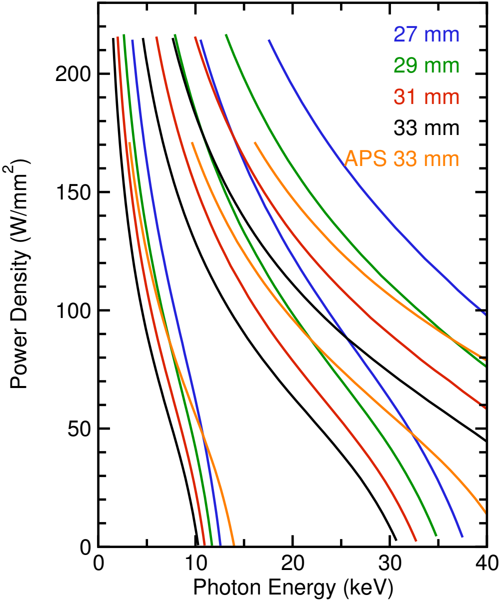

Energy recovery linacs

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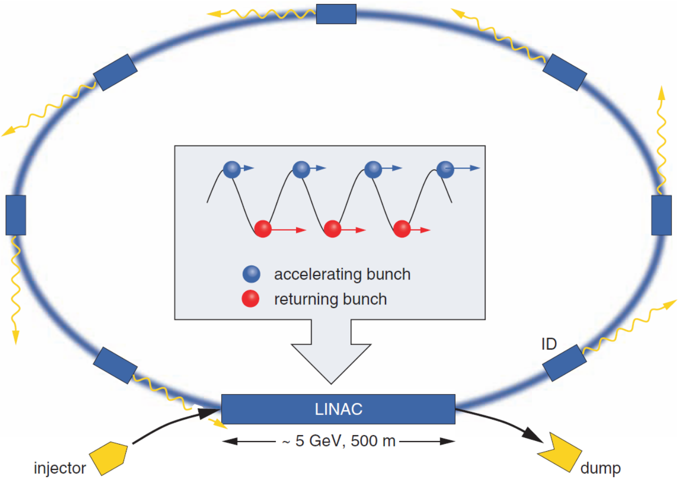
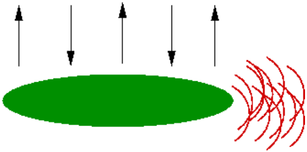


Free electron laser



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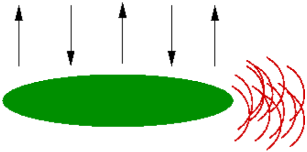

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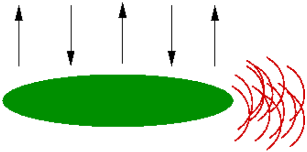

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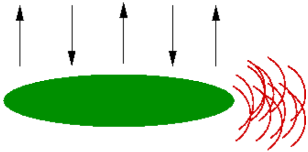

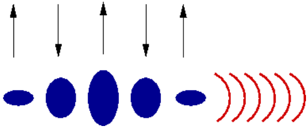

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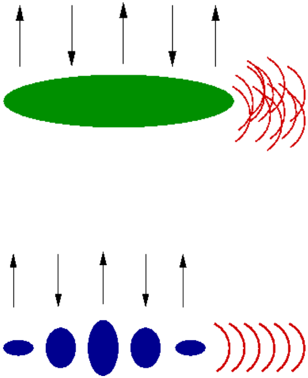

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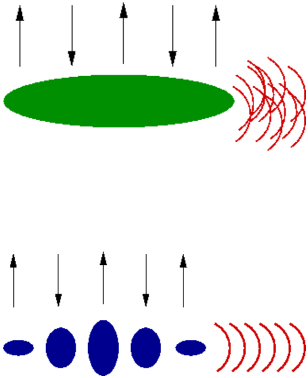

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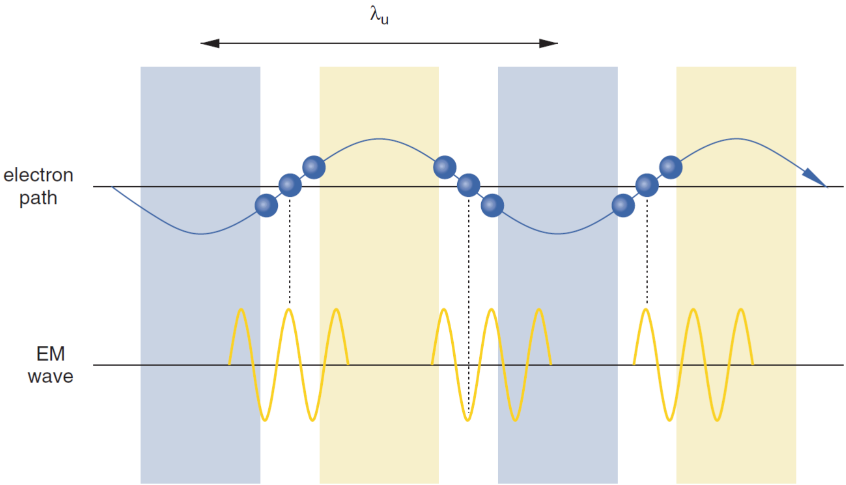
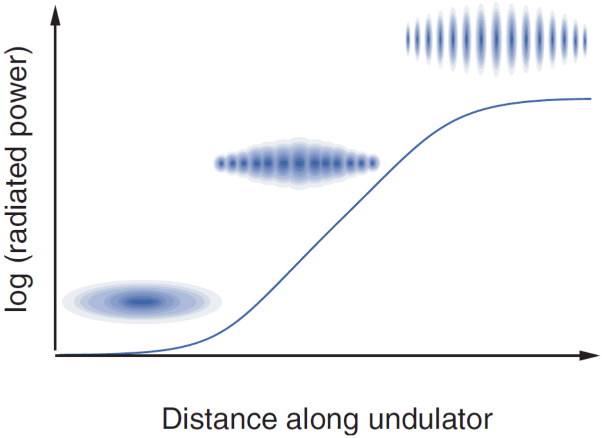
- Over course of 100 m, electric field of photons, feeds back on electron bunch
- Microbunches form with period of FEL (and radiation in electron frame)
- Each microbunch emits coherently with neighboring ones

Self-amplified spontaneous emission



FEL emission



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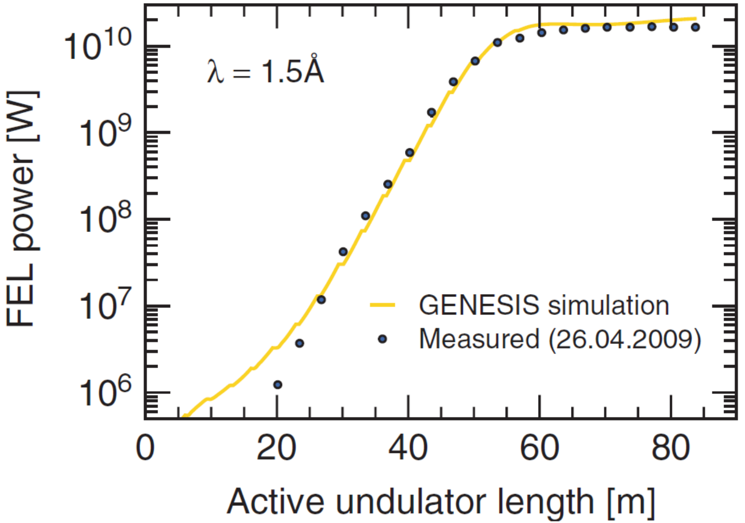

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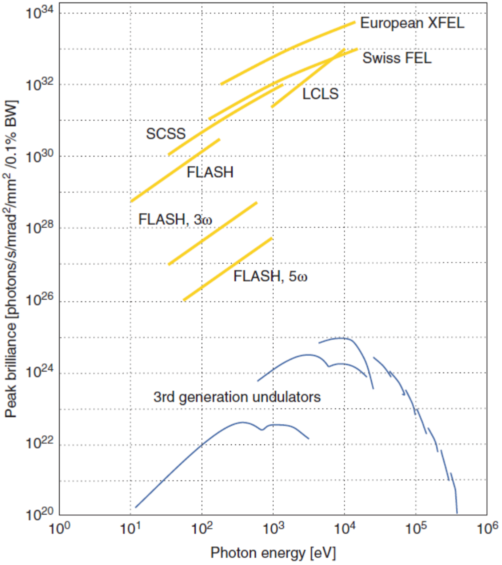
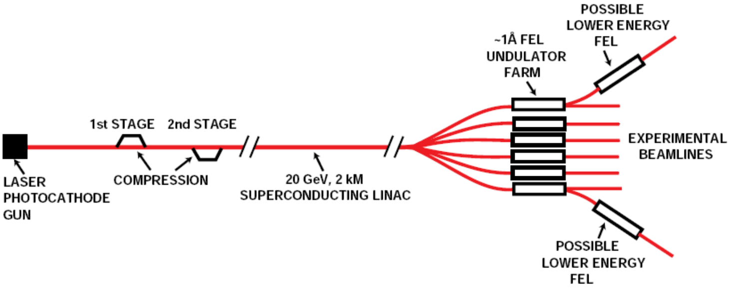
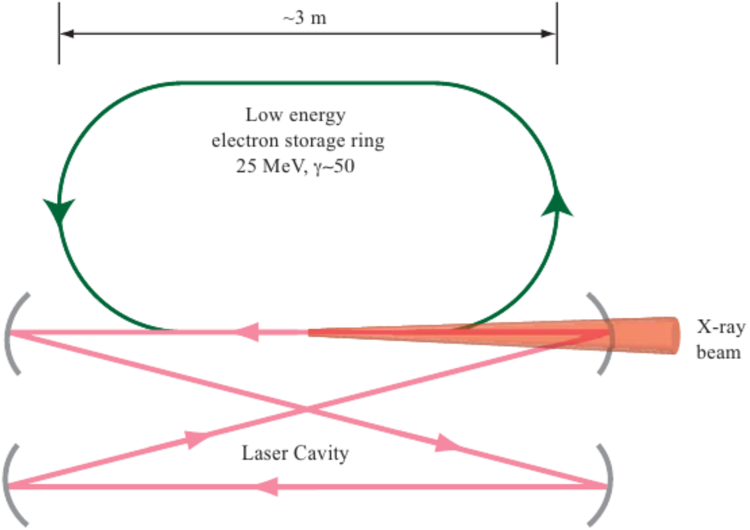


FEL layout



POSSIBLE
LOWER ENERGY
FEL
~1Å FEL
UNDULATOR
FARM
1st STAGE
2nd STAGE
EXPERIMENTAL
BEAMLINES
LASER
PHOTOCATHODE
GUN
COMPRESSION
20 GeV, 2 kM
SUPERCONDUCTING LINAC
POSSIBLE
LOWER ENERGY
FEL

Compact sources



Lyncean CLS