Today's Outline - August 31, 2016

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- The bending magnet source

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- Fundamental wavelength from an undulator

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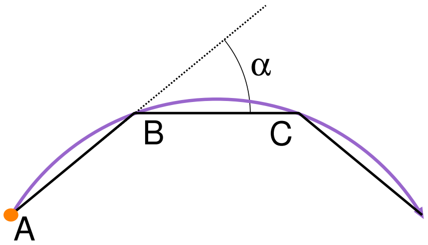
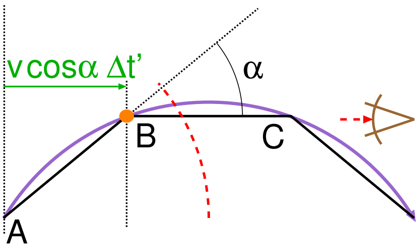
Homework Assignment #01:
Chapter Chapter 2: 2,3,5,6,8
due Monday, September 12, 2016

Off-axis emission



Consider the emission from segment AB, which is not along the line toward the observer.

Off-axis emission



Consider the emission from segment AB, which is not along the line toward the observer. While on the AB segment, the electron moves only a distance $v\cos\alpha\Delta t'$ in the direction of the BC segment.

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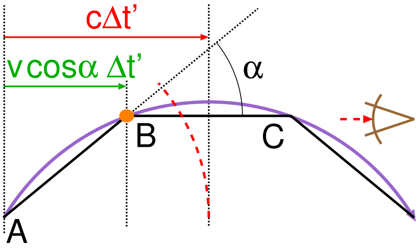

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The light pulse emitted at A still travels $c\Delta t'$, in the same time.

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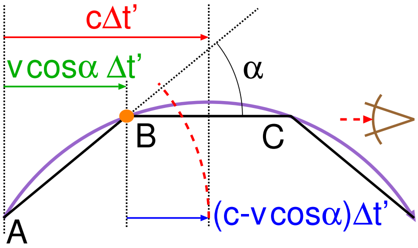

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The light pulse emitted at B is therefore, a distance $(c - v\cos\alpha)\Delta t'$ behind the pulse emitted at A.

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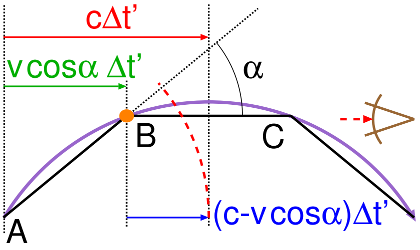

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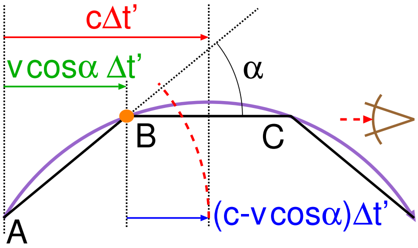

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$$\Delta t = \frac{(c - v\cos\alpha)\Delta t'}{c}$$

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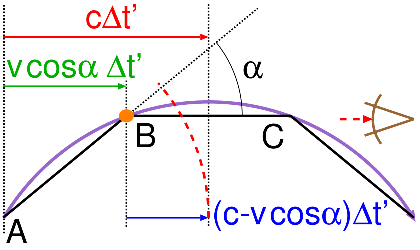

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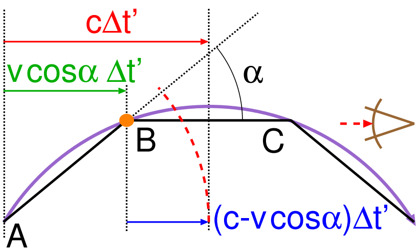

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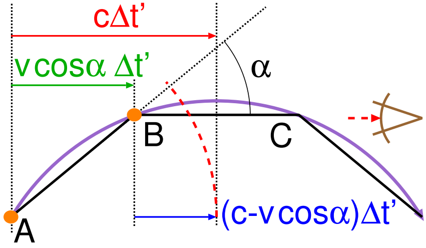
$$\Delta t = \frac{(c - v\cos\alpha)\Delta t'}{c} = \left(1 - \frac{v}{c}\cos\alpha\right)\Delta t' = (1 - \beta\cos\alpha)\Delta t'$$

Corrected Doppler shift



$$\Delta t = (1 - \beta \cos\alpha)\Delta t'$$

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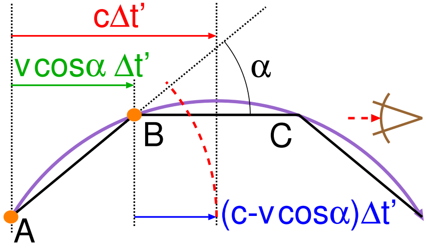

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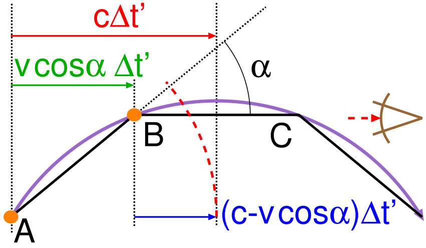
Since α is very small:

Corrected Doppler shift



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$$\cos\alpha \approx 1 - \frac{\alpha^2}{2}$$

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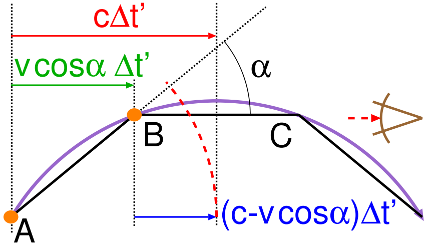

$$\Delta t = (1 - \beta \cos\alpha)\Delta t'$$

Since α is very small:

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and γ is very large, we have

$$\frac{\Delta t}{\Delta t'} \approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{\alpha^2}{2}\right)$$

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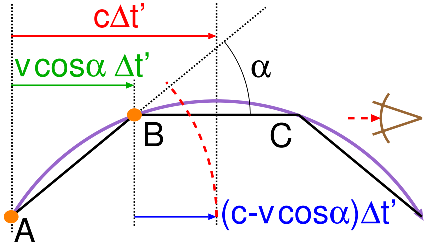

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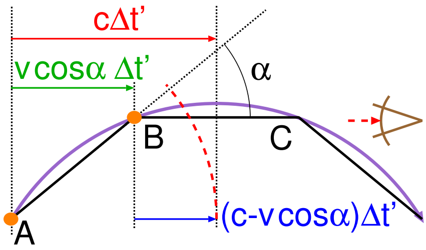

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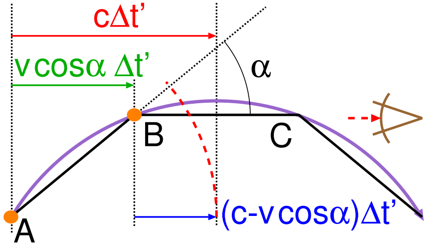

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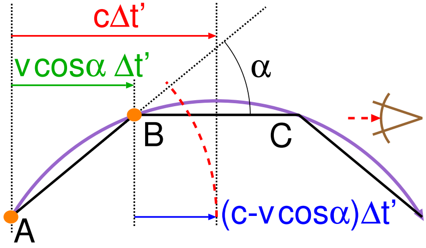

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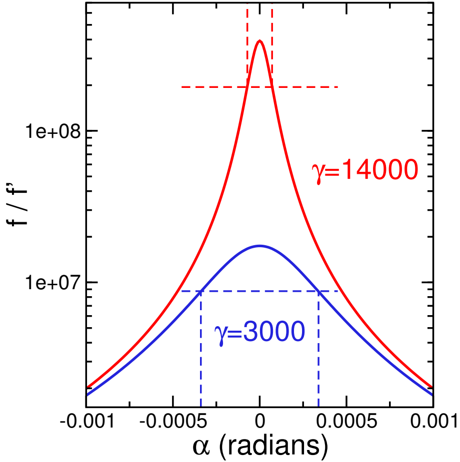
$$\frac{\Delta t}{\Delta t'} \approx \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1 + \alpha^2\gamma^2}{2\gamma^2}$$

called the time compression ratio.

Radiation opening angle

The Doppler shift is defined in terms of the time compression ratio

$$\frac{f}{f'} = \frac{\Delta t'}{\Delta t} = \frac{2\gamma^2}{1+\alpha^2\gamma^2}$$



- For APS and NSLS parameters the Doppler blue shift is between 10^7 and 10^9

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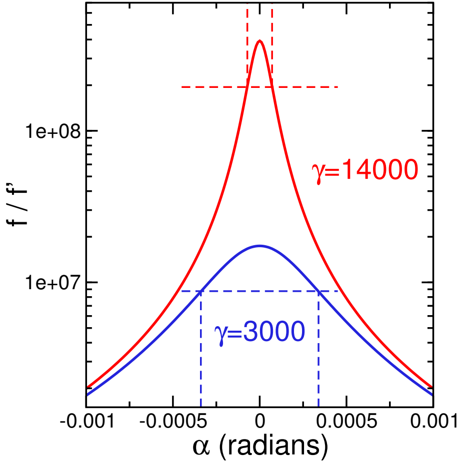

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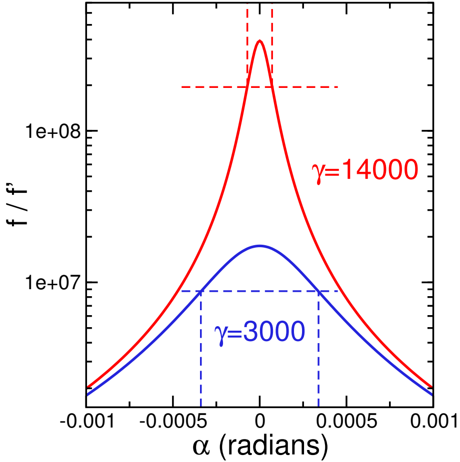
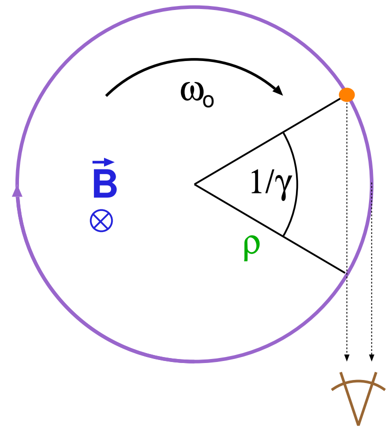


- For APS and NSLS parameters the Doppler blue shift is between 10^7 and 10^9
- The intesection of the horizontal and vertical dashed lines indicate where $\alpha = \pm 1/\gamma$ and f/f' is one half of it's maximum value
- The highest energy emitted radiation appears within a cone of half angle $1/\gamma$

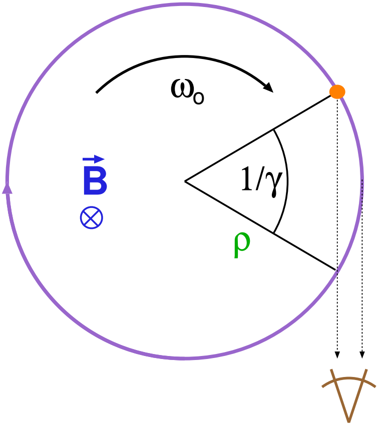
Curved arc emission



But in the limit, the compression ratio:

$$\left.\frac{\Delta t}{\Delta t'}\right|_{\Delta t \to 0}$$

Curved arc emission

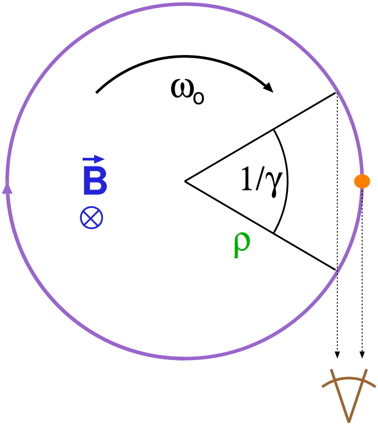


But in the limit, the compression ratio:

$$\left.\frac{\Delta t}{\Delta t'}\right|_{\Delta t \to 0} = \frac{dt}{dt'} = 1 - \beta \cos\alpha$$

so we need to treat the electron path as a continuous arc.

Curved arc emission



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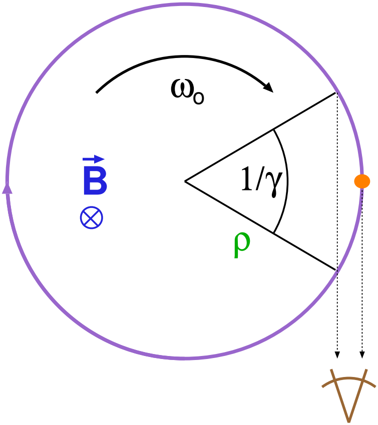
An electron moving in a constant magnetic field describes a circular path

Curved arc emission



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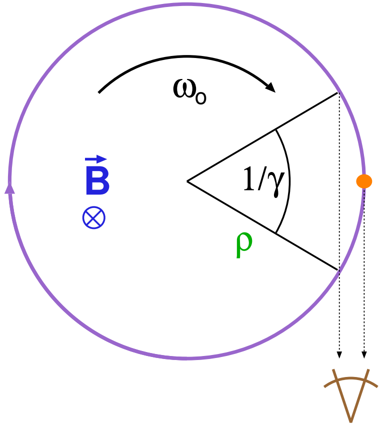
$$F_{Lorentz} = evB$$

Curved arc emission



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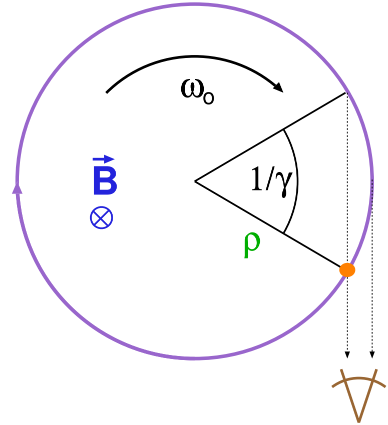
Curved arc emission



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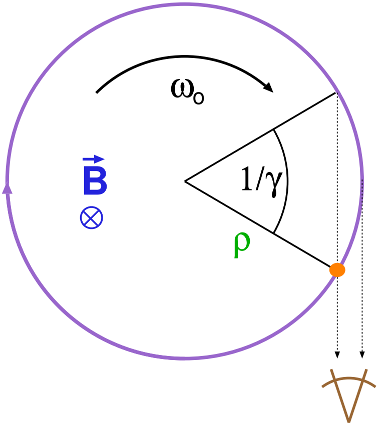
$$evB = m\frac{v^2}{\rho}$$



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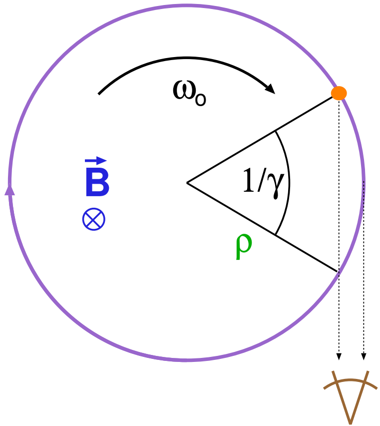
$$F_{Lorentz} = evB \qquad a = \frac{dp}{dt} = \frac{v^2}{\rho}$$

$$evB = m\frac{v^2}{\rho} \qquad \longrightarrow \qquad mv = p = \rho e B$$

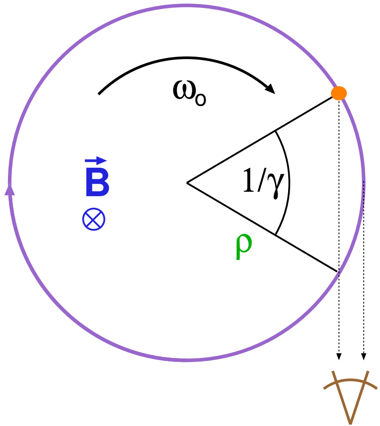
Electron bending radius



$$mv = p = \rho e B$$

but the electron is relativistic so we must correct the momentum to retain consistent laws of physics $p \rightarrow \gamma mv$

Electron bending radius



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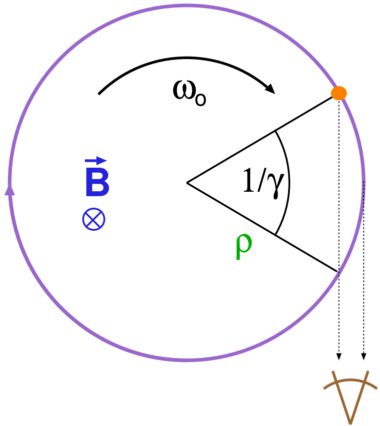

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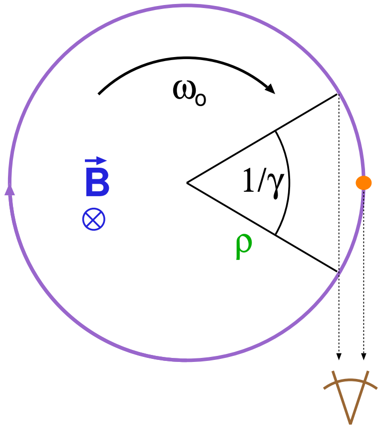
at a synchrotron $\gamma \gg 1$ so $v \approx c$

Electron bending radius



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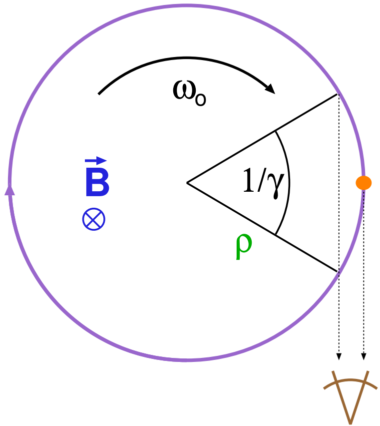
$$\gamma mc \approx \rho e B$$

Electron bending radius



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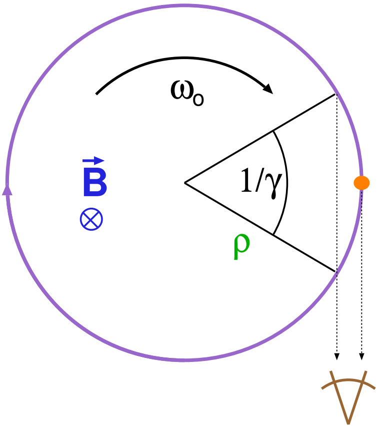
$$\gamma m c \approx \rho e B \quad \longrightarrow \quad \gamma m c^2 \approx \rho e c B$$

Electron bending radius



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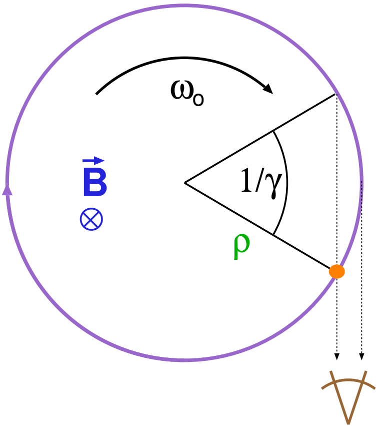
$$\gamma mc \approx \rho e B \quad \longrightarrow \quad \gamma mc^2 \approx \rho e c B$$

since $\mathcal{E} = mc^2$ and $c = 2.998 \times 10^8 \text{m/s}^2$ we have

Electron bending radius



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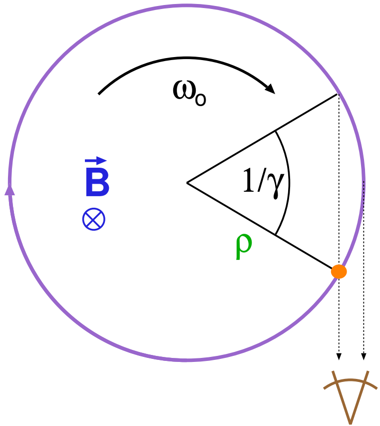
Electron bending radius



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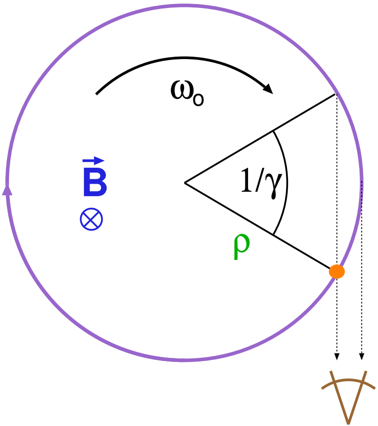
Electron bending radius



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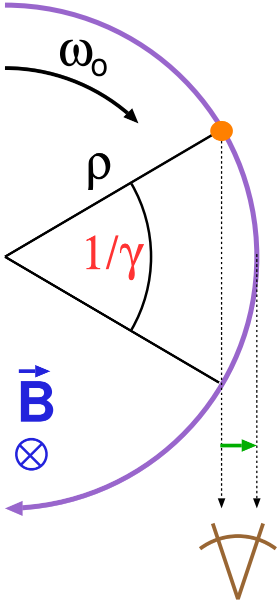
$$\rho = \frac{\mathcal{E}[\text{J}]}{ecB[\text{T}]} = \frac{\mathcal{E}[\text{eV}]}{cB[\text{T}]} = 3.3\frac{\mathcal{E}[\text{GeV}]}{B[\text{T}]}$$

Curved arc emission



The observer, looking in the plane of the circular trajectory,

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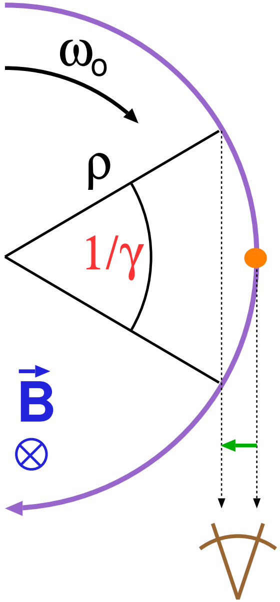

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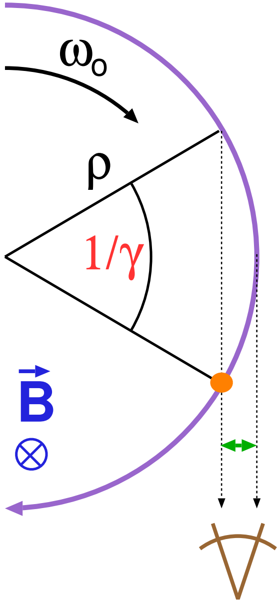

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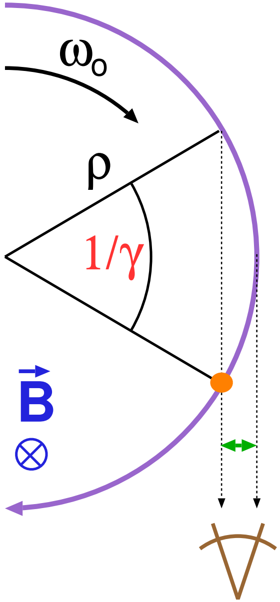

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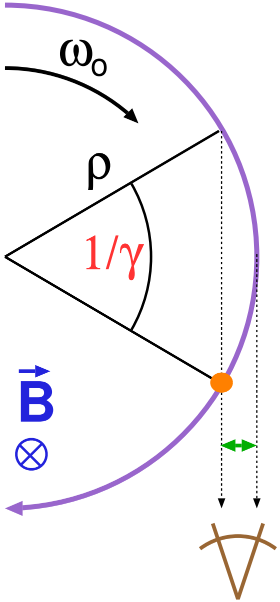

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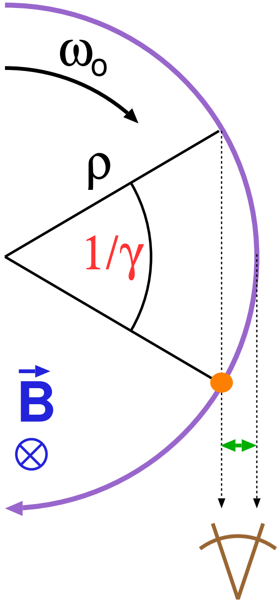

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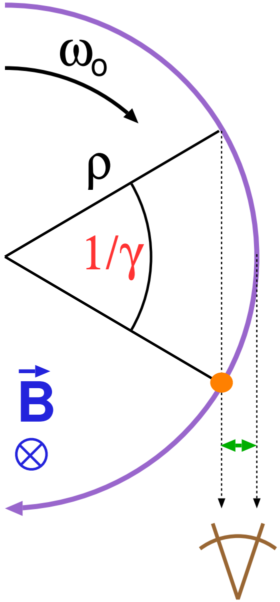

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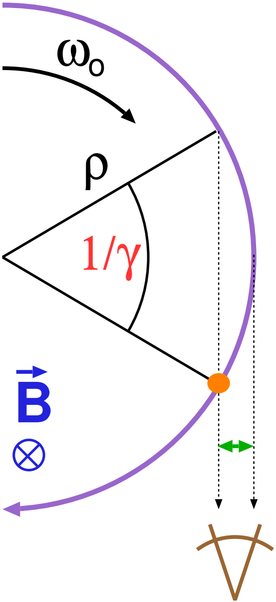

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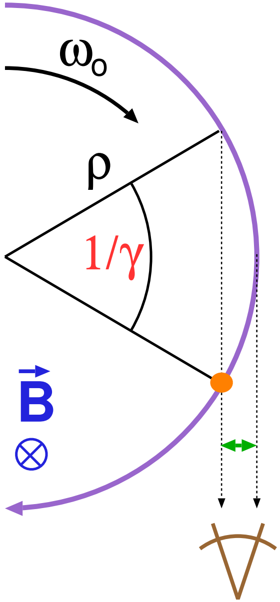

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The Fourier transform of this pulse is the spectrum of the radiation from the bending magnet.

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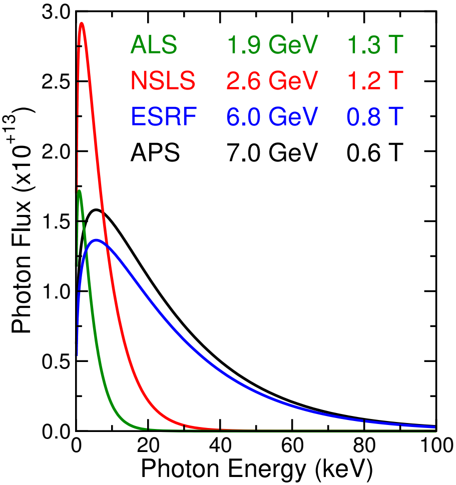
$$\mathcal{E}_c[\text{keV}] = 0.665\mathcal{E}^2[\text{GeV}]B[\text{T}]$$

Bending magnet spectrum

When the radiation pulse time is Fourier transformed, we obtain the spectrum of a bending magnet.



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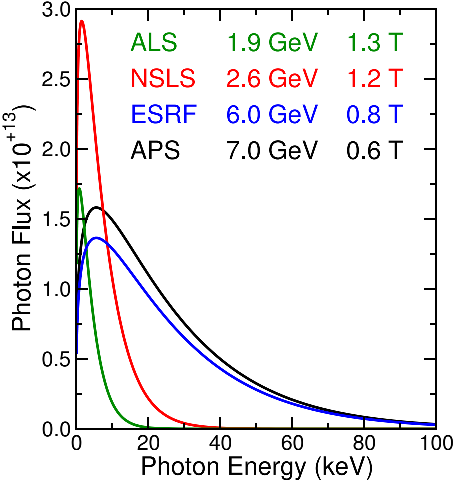

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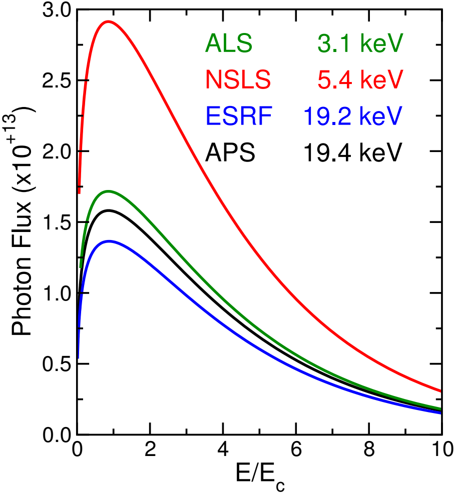

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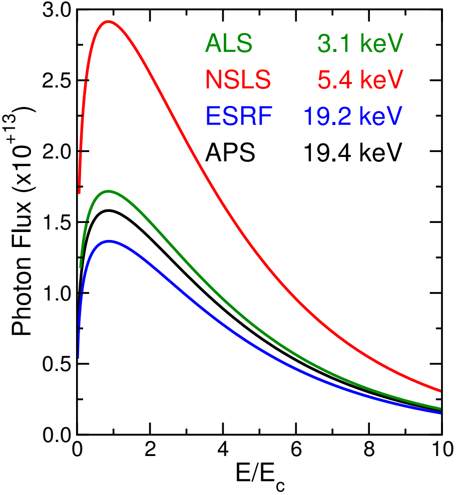
$$1.33\times10^{13}\mathcal{E}^2\, I \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\right)$$

where $K_{2/3}$ is a modified Bessel function of the second kind.



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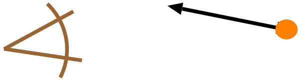
Polarization

A bending magnet also produces circularly polarized radiation



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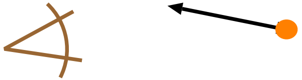

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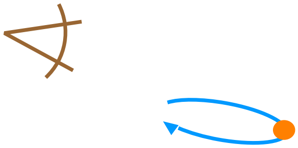

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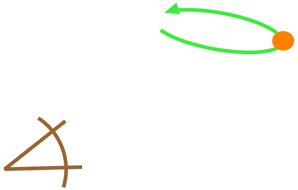
- From above, the motion looks like an arc in the clockwise direction

Polarization

A bending magnet also produces circularly polarized radiation



- If the observer is in the plane of the electron orbit, the electron motion looks like a half period of linear sinusoidal motion
- From above, the motion looks like an arc in the clockwise direction
- From below, the motion looks like an arc in the counterclockwise direction

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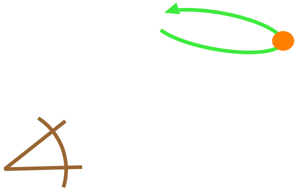

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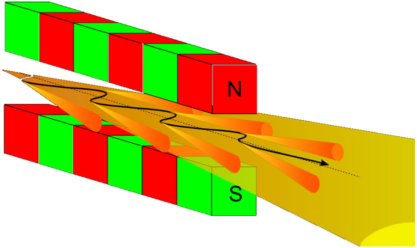
The result is circularly polarized radiation above and below the on-axis radiation.

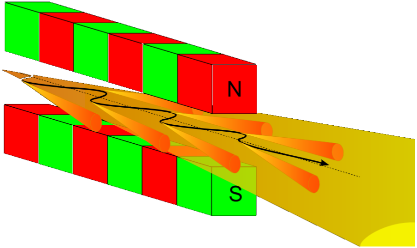
Wigglers and undulators

Wiggler



Wigglers and undulators

Wiggler



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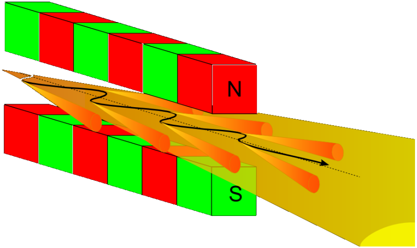

Like bending magnet except:

- larger $\vec{B} \rightarrow$ higher E_c

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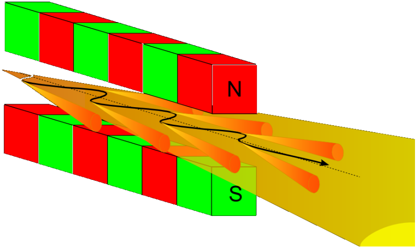

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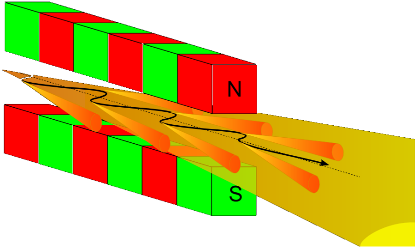
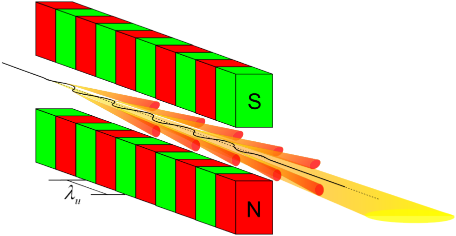


Undulator



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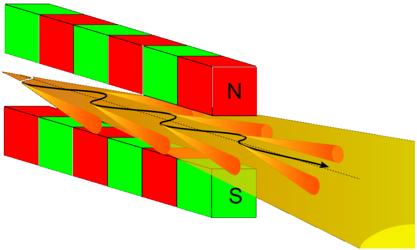

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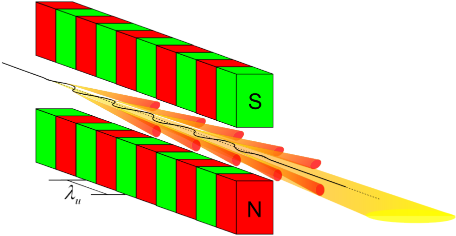

Like bending magnet except:

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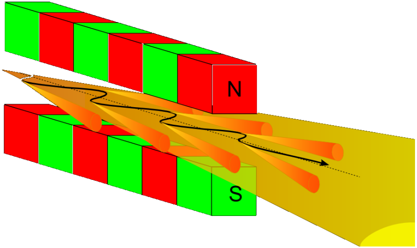
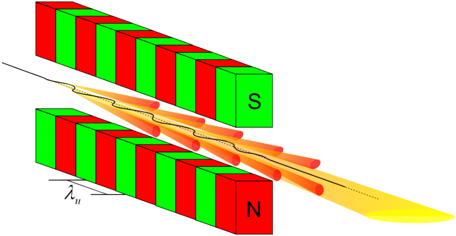
Different from bending magnet:

Wigglers and undulators

Wiggler



Undulator



Like bending magnet except:

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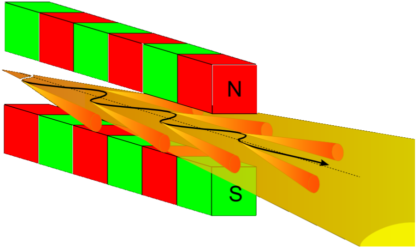
Different from bending magnet:

- shallow bends $\rightarrow$ smaller source

Wigglers and undulators

Wiggler



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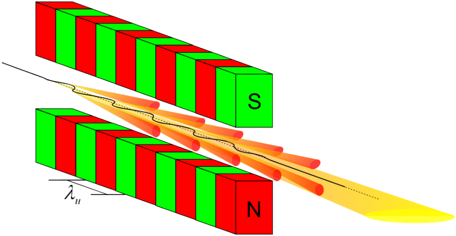

Like bending magnet except:

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Different from bending magnet:

- shallow bends $\rightarrow$ smaller source
- interference $\rightarrow$ peaked spectrum

Wiggler radiation

- The electron's trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet

$$Power[\text{kW}] = 1.266\mathcal{E}_e^2[\text{GeV}]B^2[\text{T}]L[\text{m}]I[\text{A}]$$

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- The magnetic field varies along the length of the wiggler and is higher than that in a bending magnet, having an average value of $B_{rms} = B_0/\sqrt{2}$
- This results in a significantly higher power load on all downstream components

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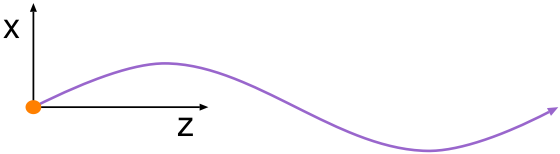
Undulator characterization



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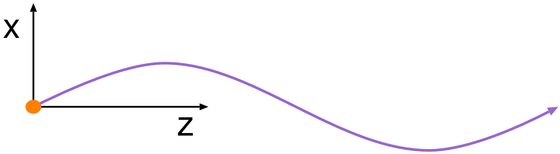
Undulator characterization



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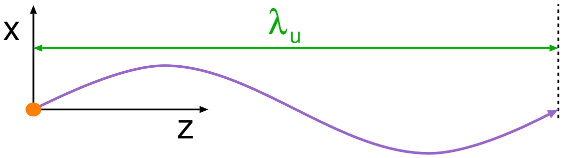

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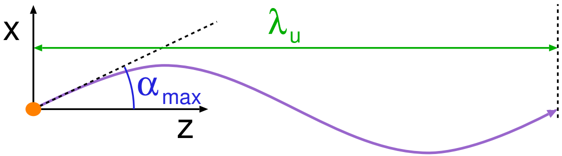

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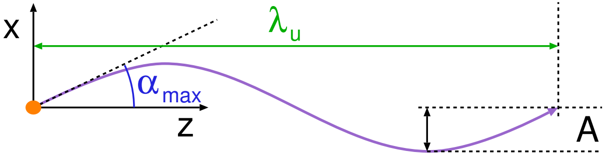

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$$x = A\sin(k_u z)$$

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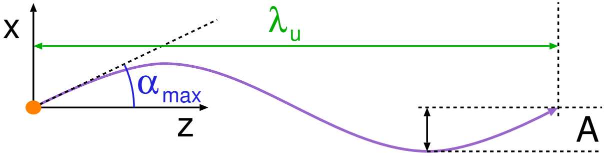

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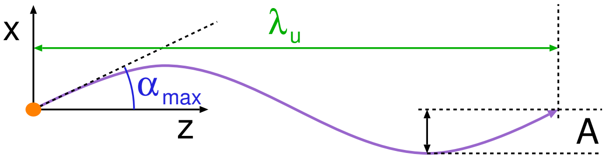

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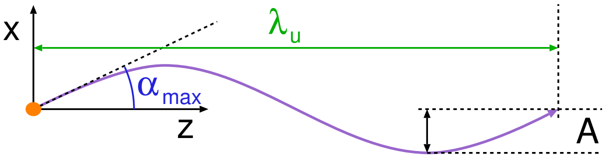

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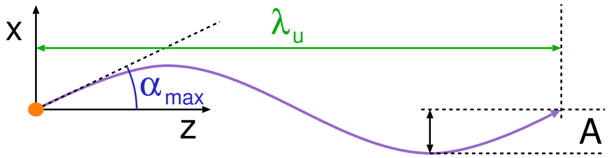

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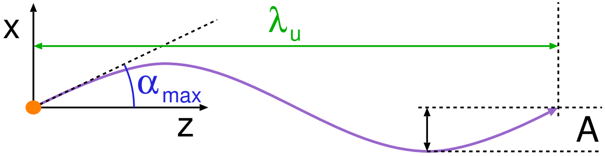

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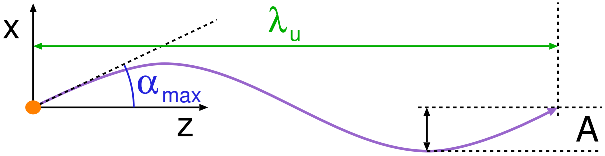
Define a dimensionless quantity, K which scales α_{max} to the natural opening angle of the radiation, $1/\gamma$

Undulator characterization



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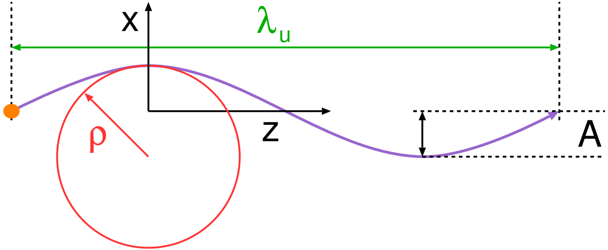
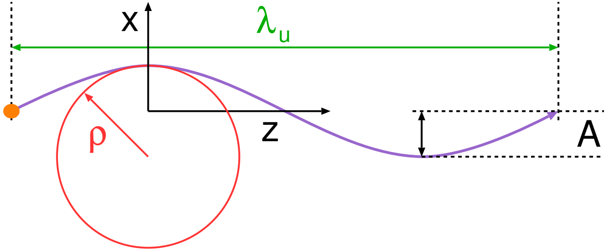
$$K = \alpha_{max}\gamma$$

Circular path approximation



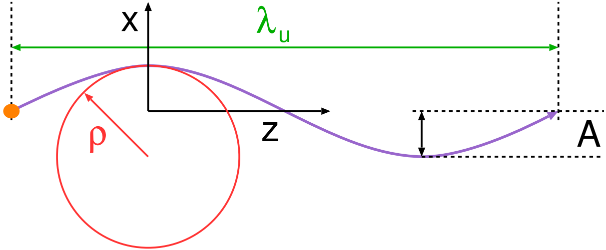
Consider the trajectory of the electron along one period of the undulator.

Circular path approximation



Consider the trajectory of the electron along one period of the undulator. Since the curvature is small, the path can be approximated by an arc or a circle of radius ρ whose origin lies at $x = -(\rho - A)$ and $z = 0$.

Circular path approximation



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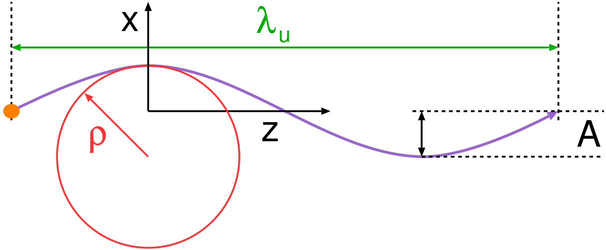

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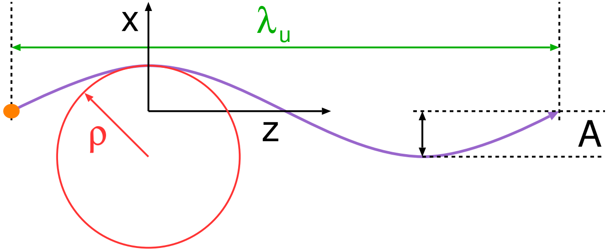

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$$\rho^2 = [x + (\rho - A)]^2 + z^2$$
$$x + (\rho - A) = \sqrt{\rho^2 - z^2}$$

Radius of curvature

From the equation for a circle:

$$x = A - \rho + \sqrt{\rho^2 - z^2}$$

Radius of curvature

From the equation for a circle:

$$\begin{aligned} x &= A - \rho + \sqrt{\rho^2 - z^2} \\ &= A - \rho + \rho\sqrt{1 - \frac{z^2}{\rho^2}} \end{aligned}$$

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From the equation for a circle:

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\begin{aligned}
x &= A - \rho + \sqrt{\rho^2 - z^2} \\
&= A - \rho + \rho\sqrt{1 - \frac{z^2}{\rho^2}} \\
&\approx A - \rho + \rho\left(1 - \frac{1}{2}\frac{z^2}{\rho^2}\right)
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For the undulating path:

$$\begin{aligned} x &= A\cos(k_u z) \\ &\approx A\left(1 - \frac{k_u^2 z^2}{2}\right) \\ &\approx A - \frac{A k_u^2 z^2}{2} \end{aligned}$$

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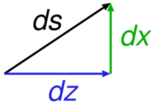
Combining, we have

$$\frac{z^2}{2\rho} = \frac{A k_u^2 z^2}{2} \quad \longrightarrow \quad \frac{1}{\rho} = A k_u^2 \quad \longrightarrow \quad \rho = \frac{1}{A k_u^2} = \frac{\lambda_u^2}{4\pi^2 A}$$

Electron path length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



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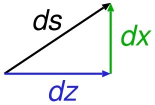

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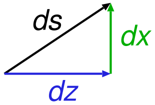

$$\begin{aligned} ds &= \sqrt{(dx)^2 + (dz)^2} \\ &= \sqrt{1 + \left(\frac{dx}{dz}\right)^2} dz \end{aligned}$$

Electron path length

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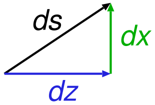

$$ds = \sqrt{(dx)^2 + (dz)^2}$$
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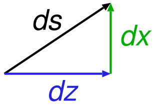

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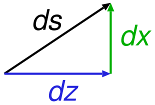

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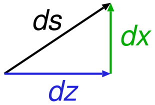

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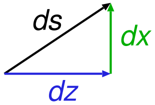

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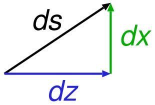

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$$= \lambda_u \left(1 + \frac{A^2 k_u^2}{4}\right) = \lambda_u \left(1 + \frac{1}{4}\frac{K^2}{\gamma^2}\right)$$

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Combining the above expressions yields

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$$K = 0.934 \cdot 3.3[\text{cm}] \cdot 0.6[\text{T}]$$

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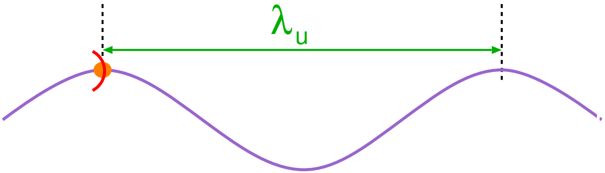
$$K = 0.934 \cdot 3.3[\text{cm}] \cdot 0.6[\text{T}] = 1.85$$

Undulator wavelength

Consider an electron traveling through the undulator and emitting radiation at the first maximum excursion from the center.



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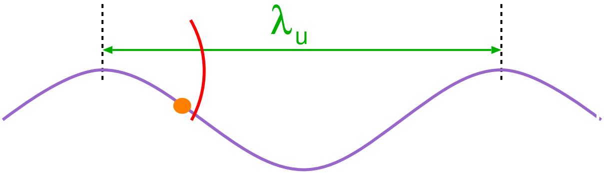

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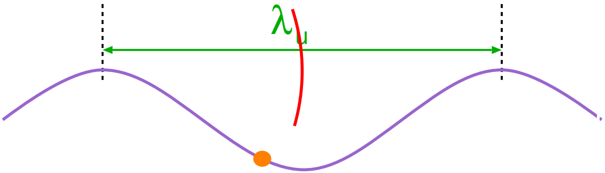

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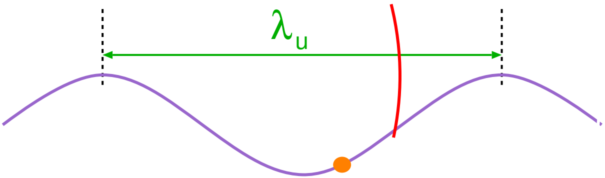

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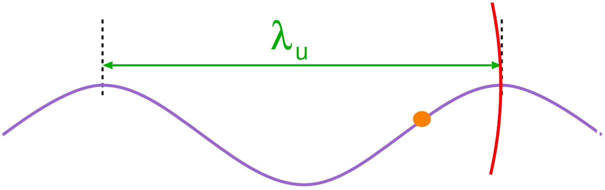

The emitted wave travels slightly faster than the electron.

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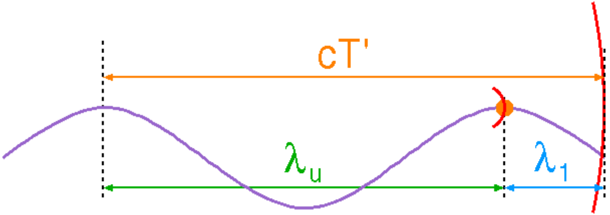

The emitted wave travels slightly faster than the electron.
It moves cT' in the time the electron travels a distance λ_u along the undulator.

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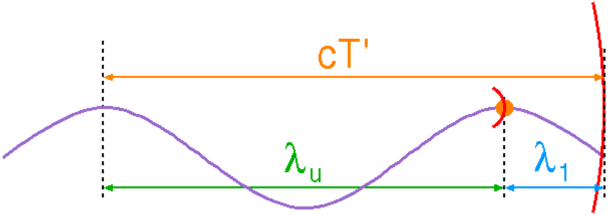

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The observer sees radiation with a compressed wavelength,

λ_1

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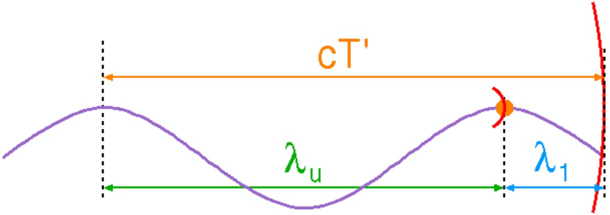

The emitted wave travels slightly faster than the electron.
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The observer sees radiation with a compressed wavelength,

$$\lambda_1 = cT' - \lambda_u$$

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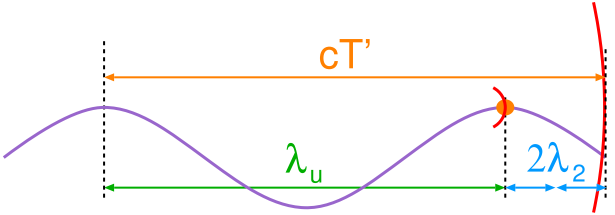

The emitted wave travels slightly faster than the electron.
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The observer sees radiation with a compressed wavelength, along with harmonics which satisfy the same condition.

$$n\lambda_n = cT' - \lambda_u$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

Over the time T' the electron actually travels a distance $S\lambda_u$, so that

$$T' = \frac{S\lambda_u}{v}$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

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$$S \approx 1 + \frac{K^2}{4\gamma^2}$$

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$$= \lambda_u \left(\left[1 + \frac{K^2}{4\gamma^2} \right] \frac{1}{\beta} - \cos\theta \right)$$

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$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right)$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

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$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right) = \frac{\lambda_u}{2\gamma^2} \left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2 \right)$$

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2\right)$$

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2\right)$$

regrouping terms

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2\right)$$

regrouping terms

$$\approx \frac{\lambda_u}{2\gamma^2}\left(2\gamma^2\left[\frac{1}{\beta} - 1\right] + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

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The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

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