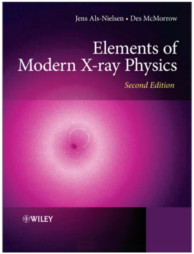
PHYS 570 - Introduction to Synchrotron Radiation



Term:	Fall 2016
Meetings:	Monday & Wednesday 17:00-18:15
Location:	212 Stuart Building
Instructor:	Carlo Segre
Office:	106A Life Sciences
Phone:	312.567.3498
email:	segre@iit.edu
Book:	*Elements of Modern X-Ray Physics, 2nd ed.*, J. Als-Nielsen and D. McMorrow (Wiley, 2011)
Web Site:	http://csrri.iit.edu/∼segre/phys570/16F

Course objectives

- Understand the means of production of synchrotron x-ray radiation

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- Be able to write a General User Proposal in the format used by the Advanced Photon Source

Course syllabus

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- Visits to Advanced Photon Source (outside class, not required)
  - All students who plan to attend will need to request badges from APS
  - Go to the APS User Portal, https://www1.aps.anl.gov/Users-Information and register as a new user.
  - Use MRCAT (Sector 10) as location of experiment
  - Use Carlo Segre as local contact
  - State that your beamtime will be in the second week of October

Course grading

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Weekly or bi-weekly

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Grading scale

A	–	80%	to	100%
B	–	65%	to	80%
C	–	50%	to	65%
E	–	0%	to	50%

Topics to be covered (at a minimum)

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Resources for the course

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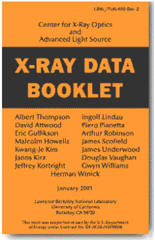

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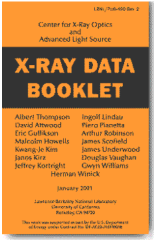

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- Hephaestus from the Demeter suite:
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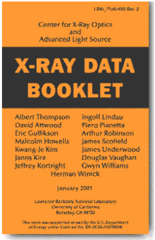

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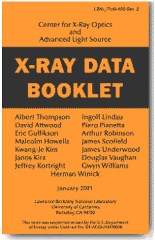

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- X-ray Oriented Programs:
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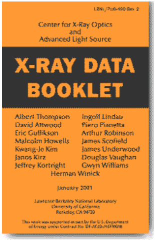

Today's outline - August 22, 2016

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Reading Assignment: Chapter 1.1–1.6; 2.1–2.2

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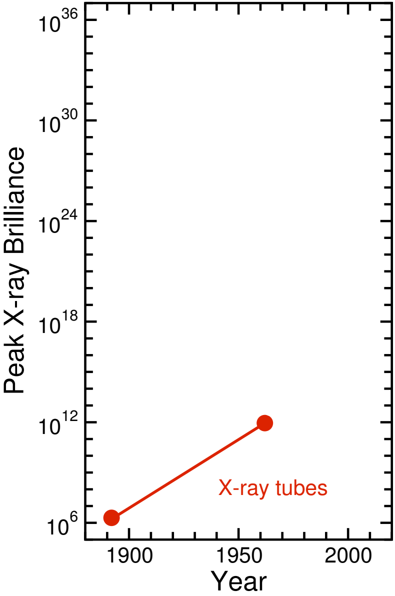

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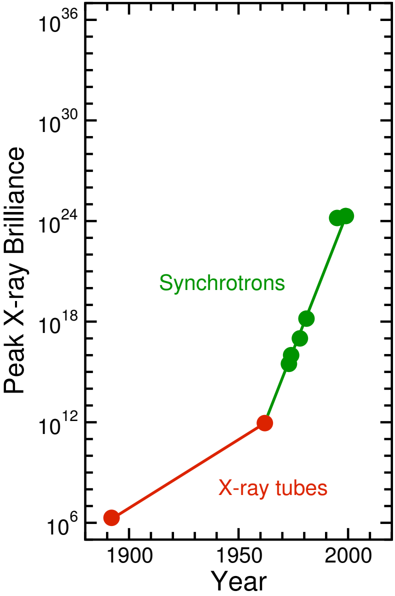

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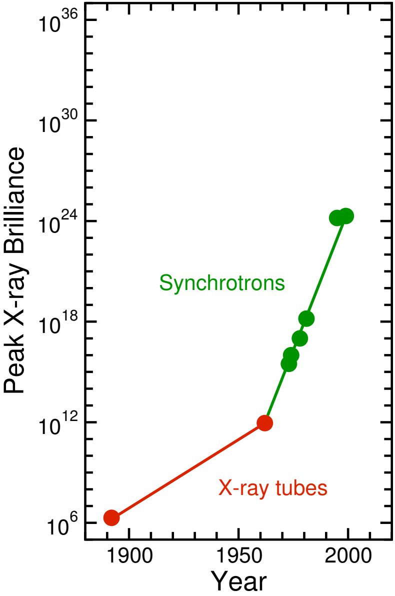

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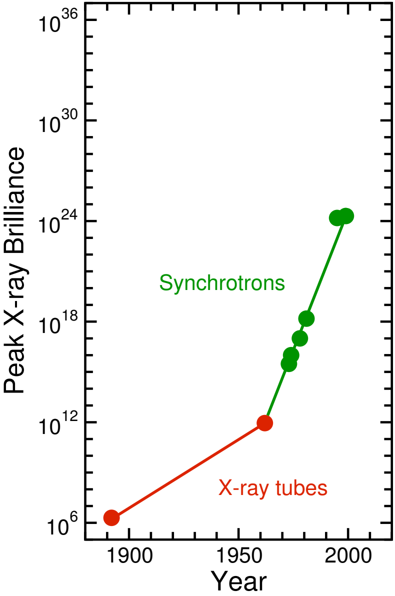

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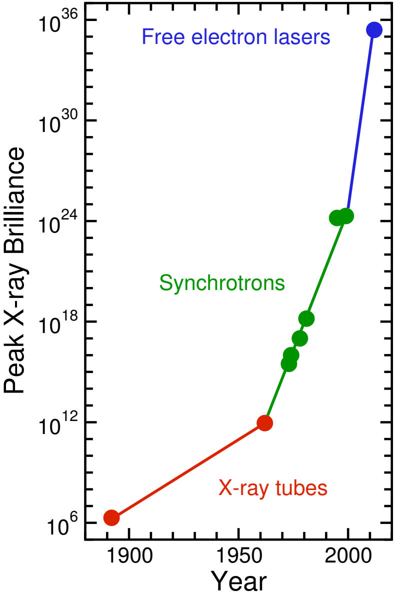

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- 4^{th} generation are free electron lasers (LCLS, XFEL)

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$$\begin{aligned}
\lambda &= hc/\mathcal{E} \\
&= (4.1357 \times 10^{-15}\,\text{eV}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})/\mathcal{E} \\
&= (4.1357 \times 10^{-18}\,\text{keV}\cdot\text{s})(2.9979 \times 10^{18}\,\text{Å/s})/\mathcal{E} \\
&= 12.398\,\text{Å}\cdot\text{keV}/\mathcal{E} \qquad \text{to give units of Å}
\end{aligned}$$

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We will only discuss the first three.

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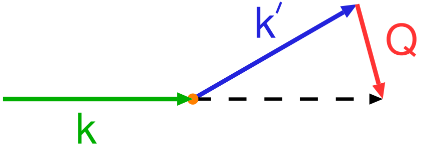

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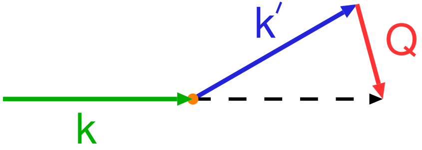

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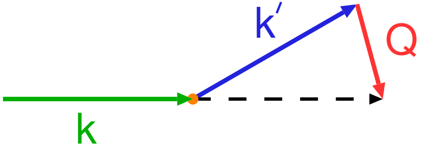

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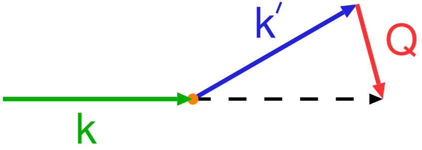

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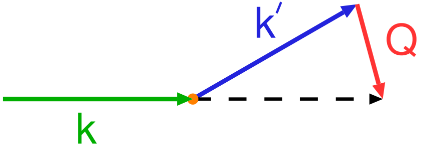

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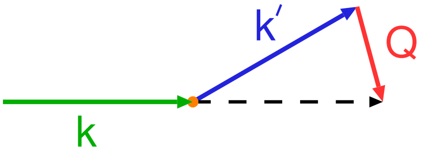

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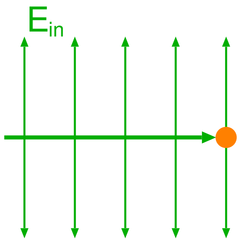
Start with the scattering from a single electron, then build up to more complexity

Thomson scattering

Assumptions:



Thomson scattering

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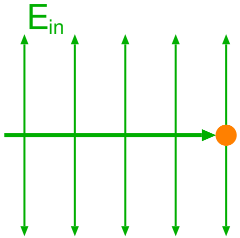

Thomson scattering

Assumptions:

- incident x-ray plane wave
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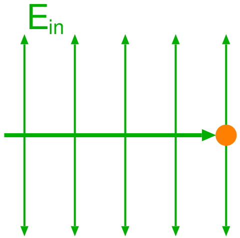

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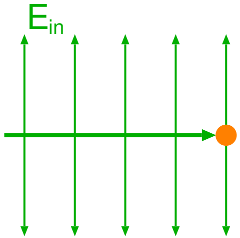

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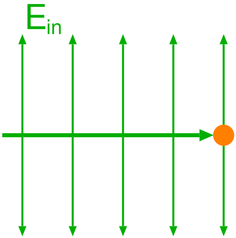

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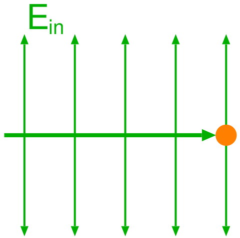

The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

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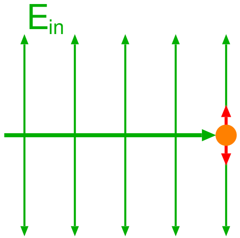

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The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

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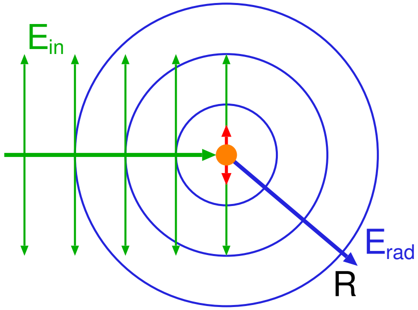

The electron is exposed to the incident electric field $E_{in}(t')$ and is accelerated

The acceleration of the electron, $a_x(t')$, results in the radiation of a spherical wave with the same frequency

The observer at R "sees" a scattered electric field $E_{rad}(R, t)$ at a later time $t = t' + R/c$

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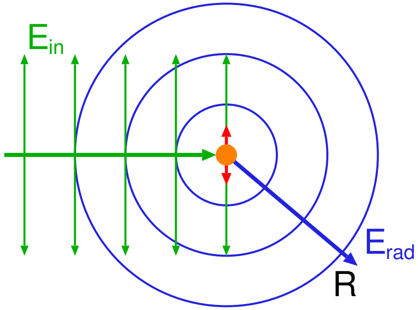

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Using this, calculate the elastic scattering cross-section

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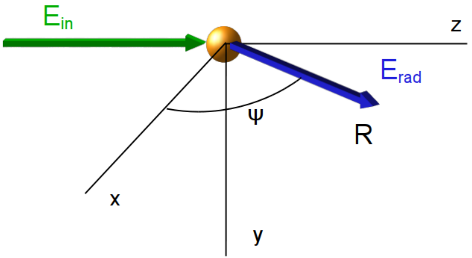

$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi$$

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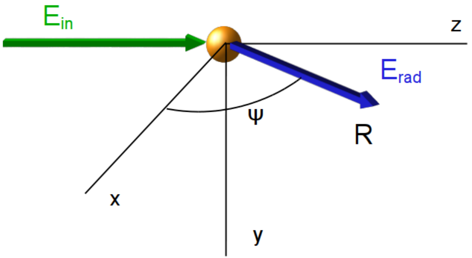
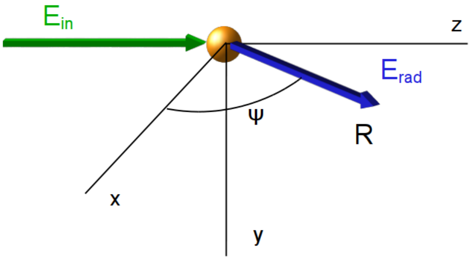


$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi \qquad \textit{where} \qquad t' = t - R/c$$

Thomson scattering



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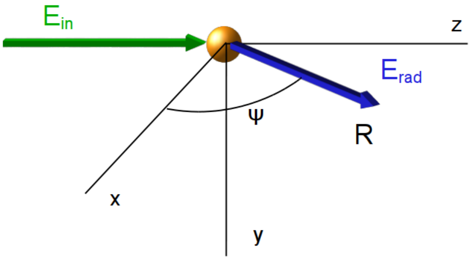

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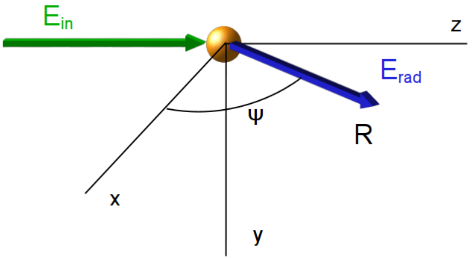

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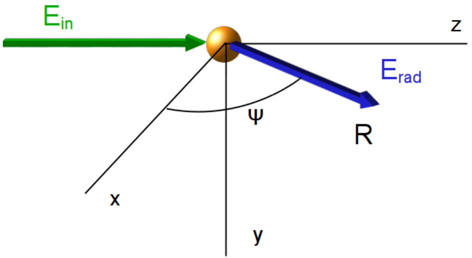

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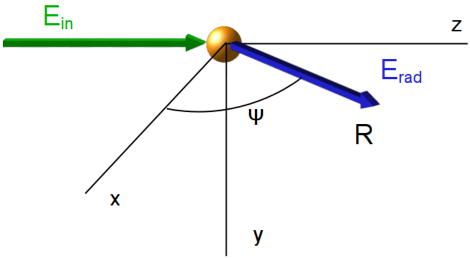

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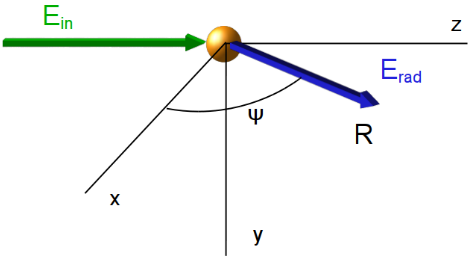

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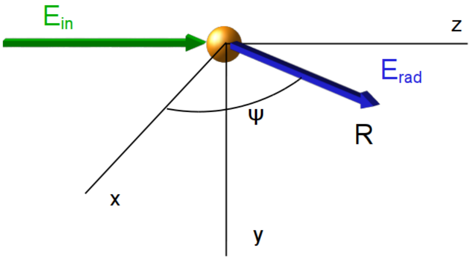

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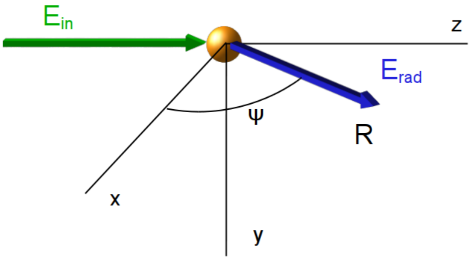

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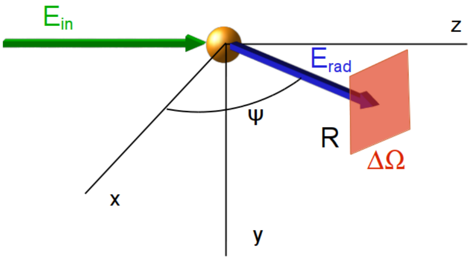
$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} = 2.82\times 10^{-5}\text{Å}$$

Scattering cross-section



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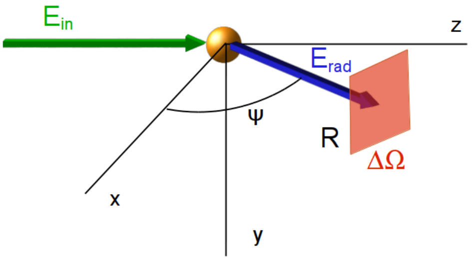

Detector of solid angle $\Delta\Omega$ at a distance R from electron

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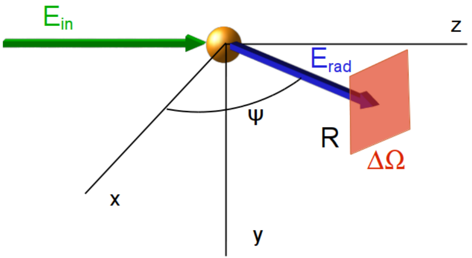

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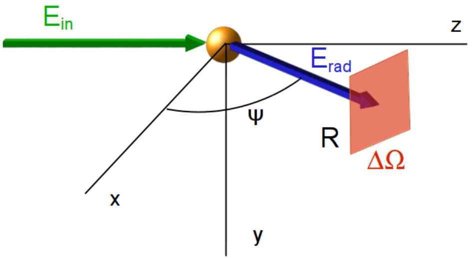

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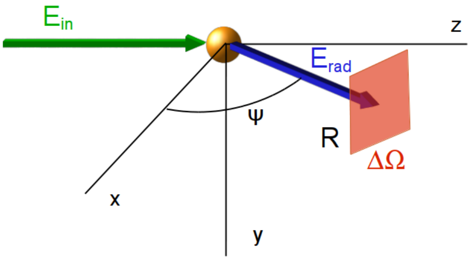

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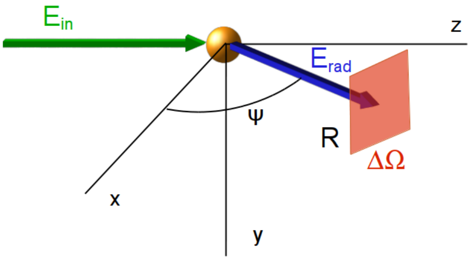

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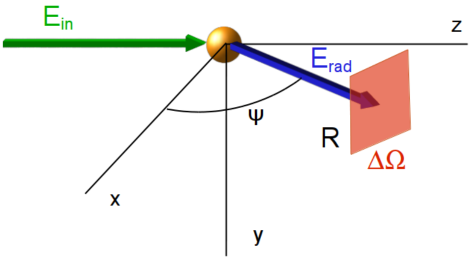

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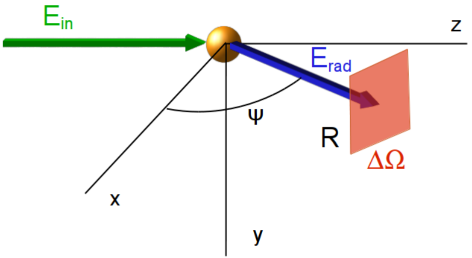

Differential cross-section is obtained by normalizing

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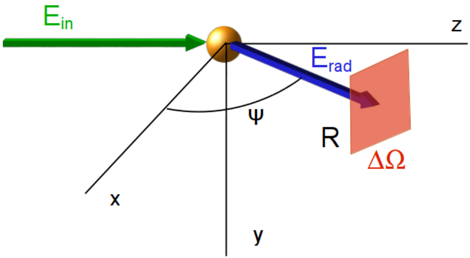

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$$\frac{d\sigma}{d\Omega}$$

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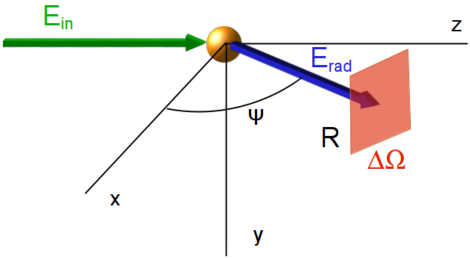

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$$\frac{d\sigma}{d\Omega} = \frac{I_{sc}}{\Phi_0 \Delta\Omega}$$

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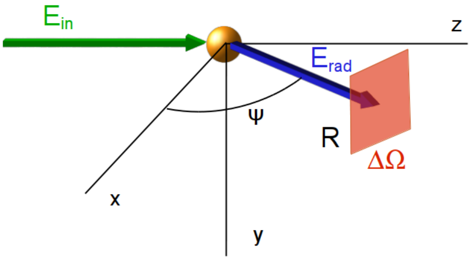

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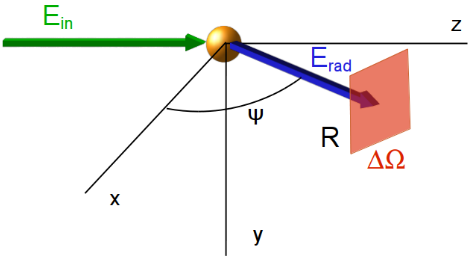

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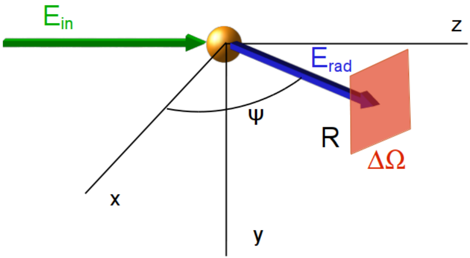

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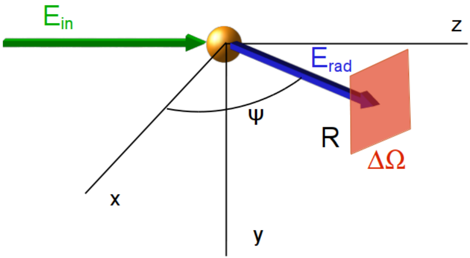

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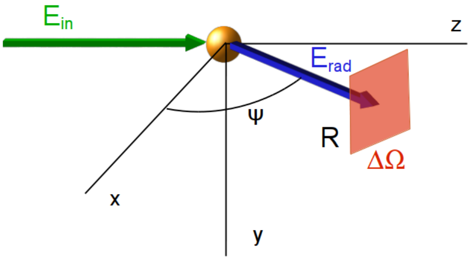

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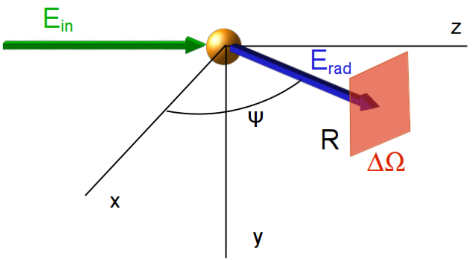

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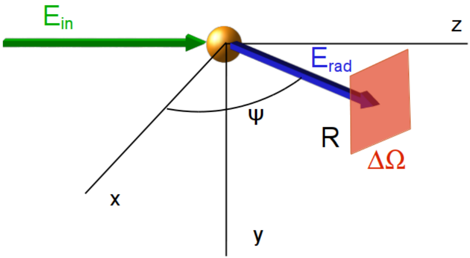

Integrate to obtain the total Thomson scattering cross-section from an electron.

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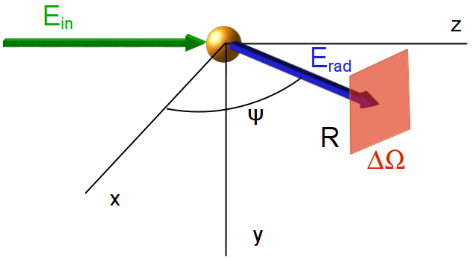

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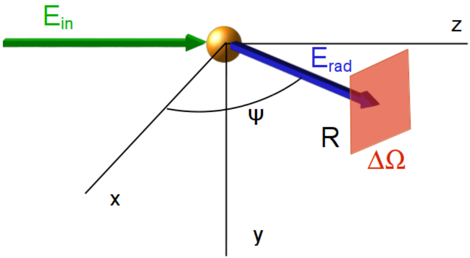

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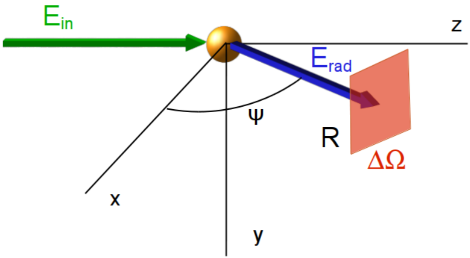

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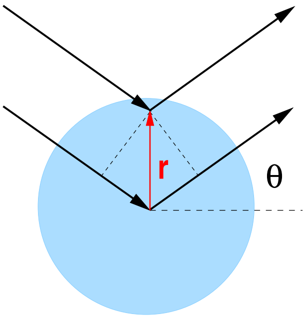
$$\text{Polarization factor} = \begin{cases} 1 \\ \sin^2\Psi \\ \frac{1}{2}\left(1+\sin^2\Psi\right) \end{cases}$$

Atomic scattering

If we have a charge distribution instead of a single electron, the scattering is more complex



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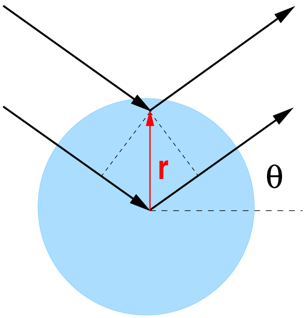

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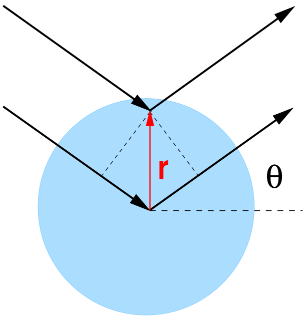

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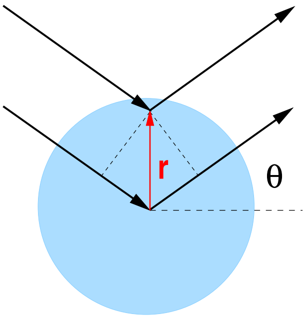

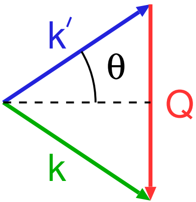

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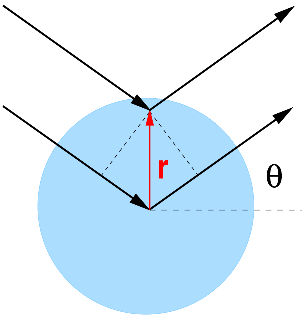

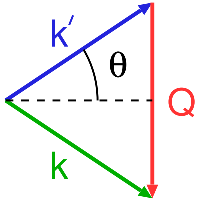

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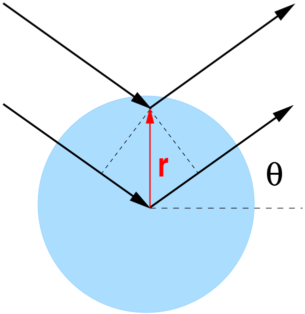

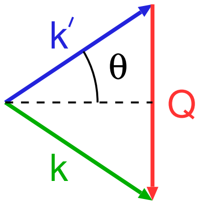

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$$|\mathbf{Q}| = 2\,|\mathbf{k}|\,sin\theta = \frac{4\pi}{\lambda} sin\theta$$

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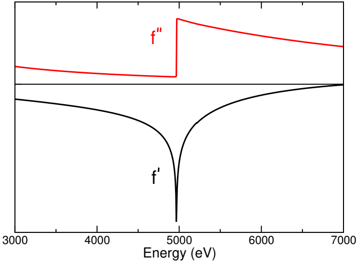

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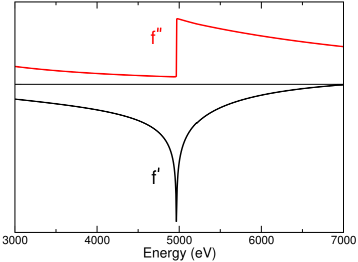

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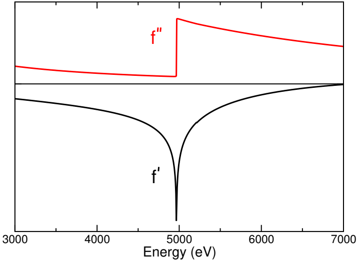
The volume element at $\mathbf{r}$ contributes $-r_0\rho(\mathbf{r})d^3r$ with phase factor $e^{i\mathbf{Q}\cdot\mathbf{r}}$ for an entire atom, integrate to get the atomic form factor $f^0(\mathbf{Q})$:

$$-r_0 f^0(\mathbf{Q}) = -r_0 \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d^3r$$

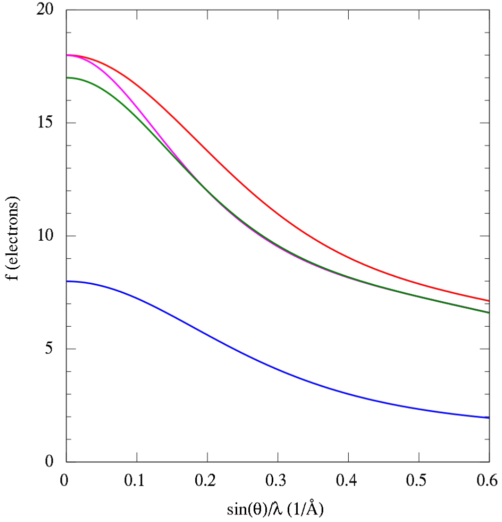
Electrons which are tightly bound cannot respond like a free electron. This results in a depression of the atomic form factor, called f' and a lossy term near an ionization energy, called f''. Together these are the anomalous corrections to the atomic form factor.



the total atomic scattering factor is

$$f(\mathbf{Q}, \hbar\omega) = f^0(\mathbf{Q}) + f'(\hbar\omega) + if''(\hbar\omega)$$

Atomic form factor



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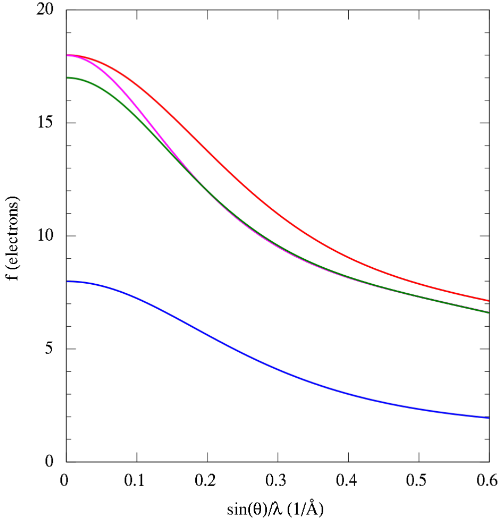

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$$\mathbf{Q} = \frac{4\pi}{\lambda} \sin\theta$$

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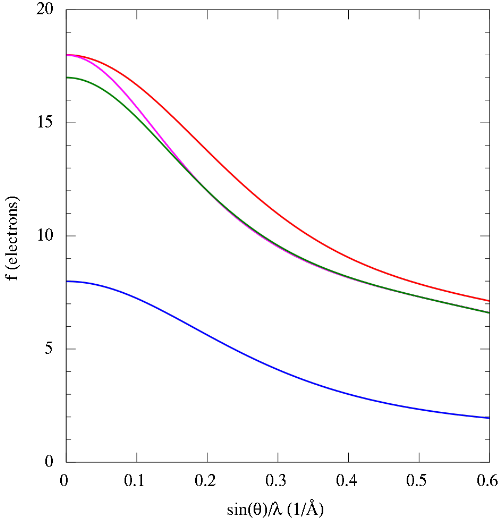

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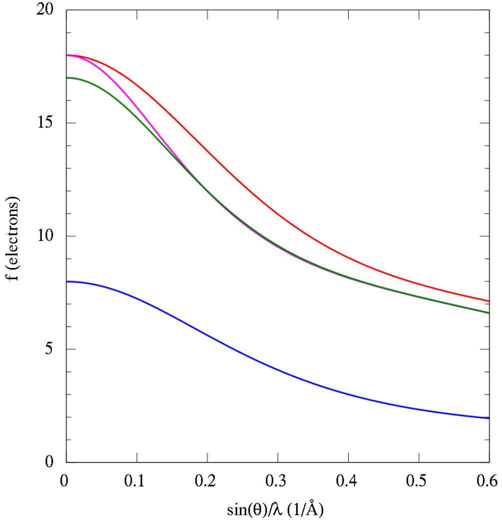

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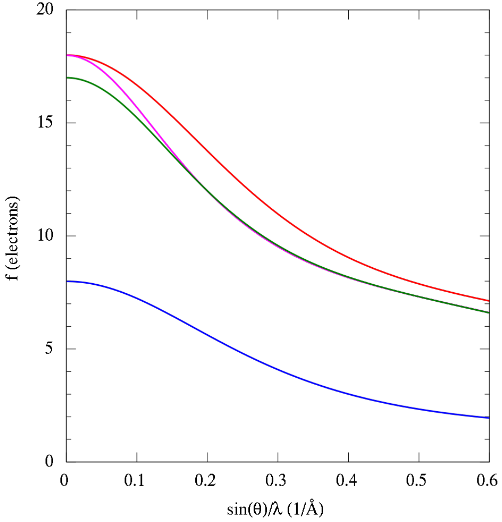

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O, Cl, Cl^-, Ar

Scattering from atoms: all effects

Scattering from an atom is built up from component quantities:

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$-\ r_0$ $= -r_0$

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polarization factor $$P = \begin{cases} 1 \\ \sin^2\Psi \\ \frac{1}{2}(1+\sin^2\Psi) \end{cases}$$

$$-r_0 f(\mathbf{Q}, \hbar\omega)\sin^2\Psi = -r_0\left[f^0(\mathbf{Q}) + f'(\hbar\omega) + if''(\hbar\omega)\right]\sin^2\Psi$$