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Final Exam, Tuesday, May 5, 2015, Life Sciences 240
4 sessions: 09:00-11:00; 11:00-13:00; 14:00-16:00; 16:00-18:00

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Send me your presentation in Powerpoint or PDF format before before your session

Problem 5.9

Consider a simple model of the surface roughness of a crystal in which all of the lattice sites of the $z = 0$ layer are fully occupied by atoms, but the next layer out ($z = -1$) has a site occupancy of η, with $\eta \leq 1$, the $z = -2$ layer an occupancy of η^2, etc. Show that midway between the Bragg points, the so-called anti-Bragg points, the intensity of the crystal truncation rods is given by

$$I^{CTR} = \frac{(1-\eta)^2}{4(1+\eta)^2}$$

What effect does a small, but finite, value of the roughness parameter η have on I^{CTR}?

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evaluating these geometric sums in the usual way

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$$
\begin{aligned}
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&= A(\vec{Q}) \left[\frac{1}{1 - e^{i2\pi l}} + \frac{\eta e^{-i2\pi l}}{1 - \eta e^{-i2\pi l}} \right]
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$$= A(\vec{Q}) \frac{(1+\eta) - 2\eta}{2(1+\eta)} = A(\vec{Q}) \frac{(1-\eta)}{2(1+\eta)}$$

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if the coverage is very low, this can be approximated using the first term of a binomial expansion

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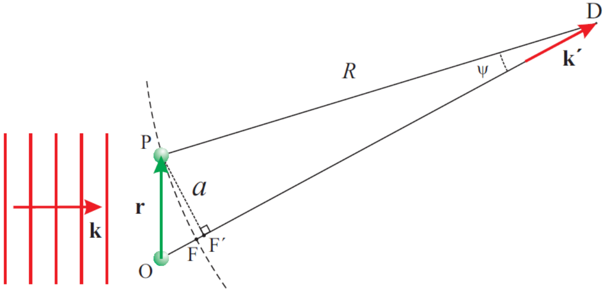
Thus any small value of the roughness parameter will dampen the intensity of the peaks

Phase difference in scattering



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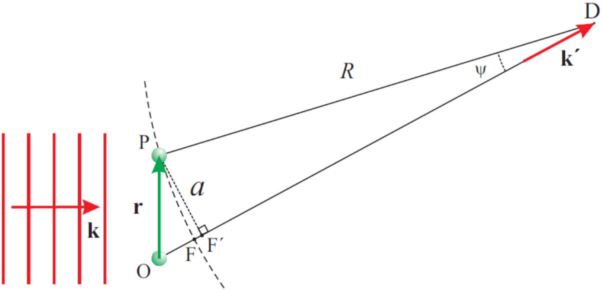
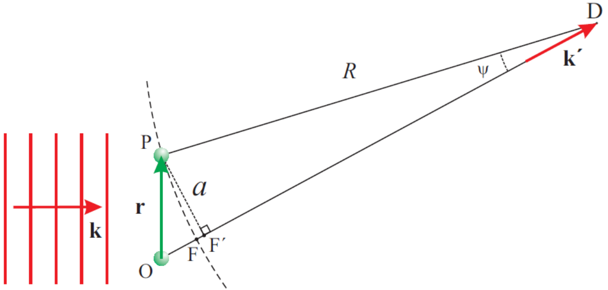


All imaging can be broken into a three step process

Phase difference in scattering



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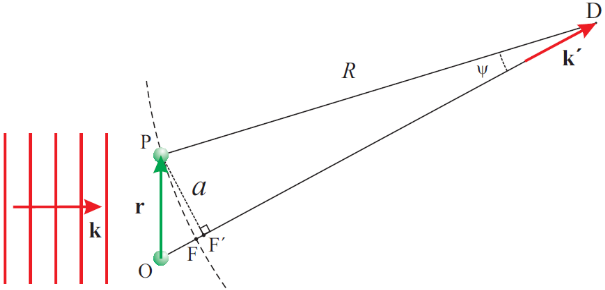

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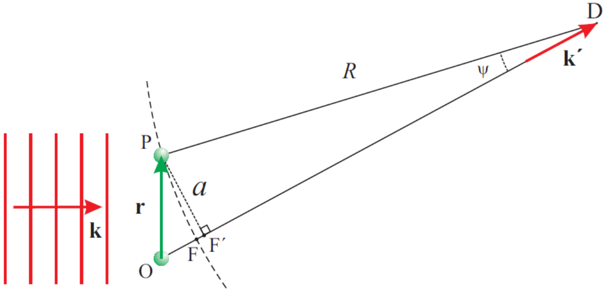

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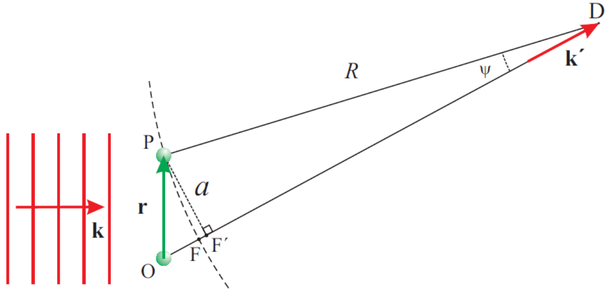

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The spherical waves scattered off the two points will travel different distances

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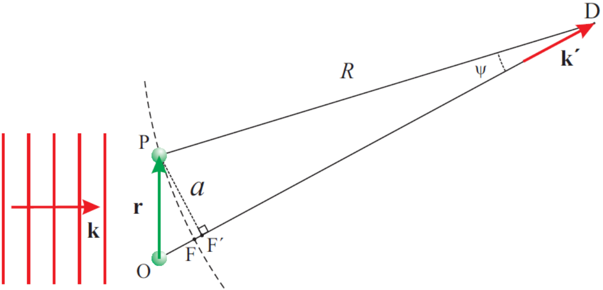

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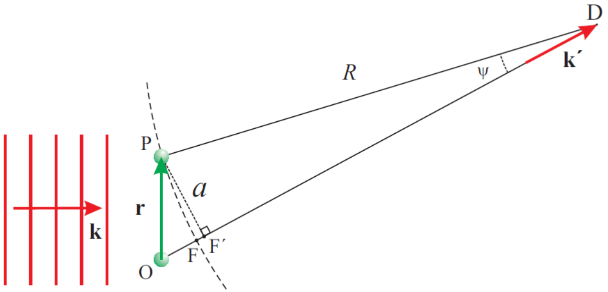
The spherical waves scattered off the two points will travel different distances

In the far field, the phase difference is $\vec{Q} \cdot \vec{r}$ with $\vec{Q} = \vec{k} - \vec{k'}$

Franuhofer, Fresnel, and contact regimes



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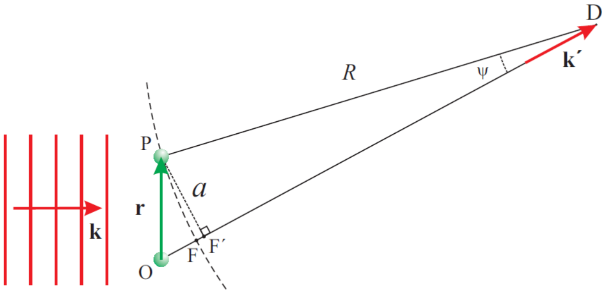

In the far field (Fraunhofer) regime, the phase difference is $\approx \vec{k'} \cdot \vec{r}$

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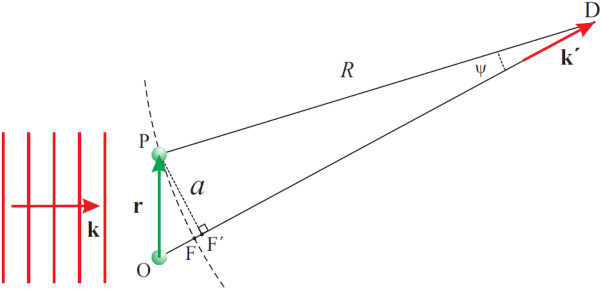
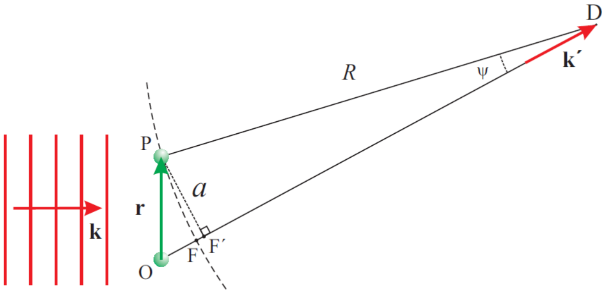


In the far field (Fraunhofer) regime, the phase difference is $\approx \vec{k'} \cdot \vec{r}$

The error in difference computed with the far field approximation is

$$\Delta = R - R\cos\psi$$

Franuhofer, Fresnel, and contact regimes



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$$\begin{aligned}\Delta &= R - R\cos\psi \\ &\approx R(1-(1-\psi^2/2))\end{aligned}$$

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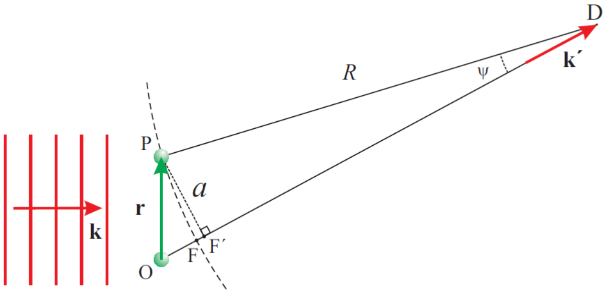

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The error in difference computed with the far field approximation is

$$\begin{aligned}\Delta &= R - R\cos\psi \\ &\approx R(1-(1-\psi^2/2)) \\ &= a^2/(2R)\end{aligned}$$

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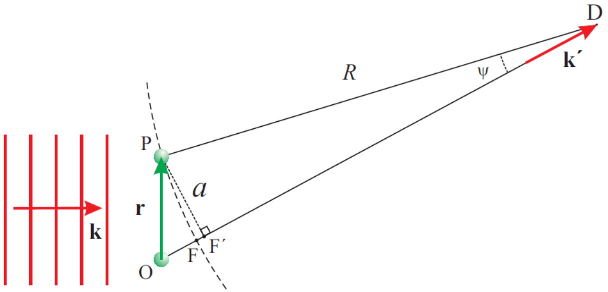

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$$R \gg a^2/\lambda \qquad \text{Fraunhofer}$$

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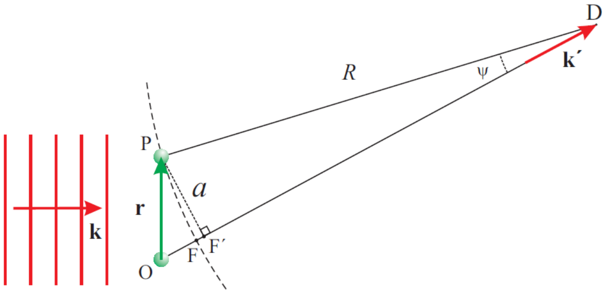

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$R \gg a^2/\lambda$ Fraunhofer

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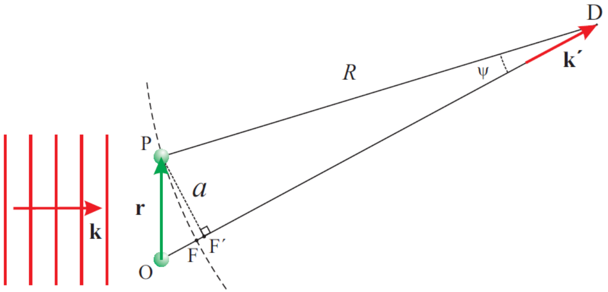

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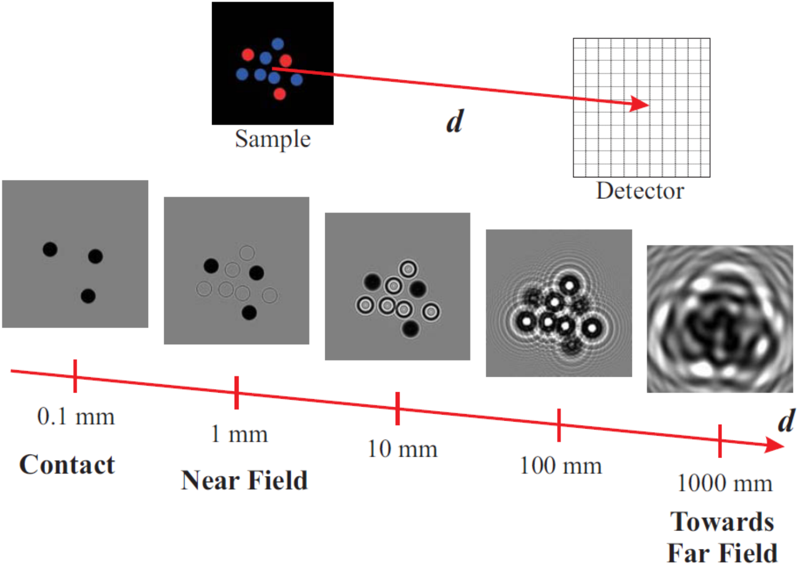
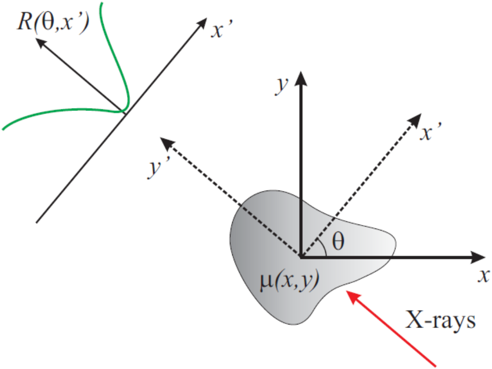
$R \gg a^2/\lambda$	Fraunhofer
$R \approx a^2/\lambda$	Fresnel
$R \ll a^2/\lambda$	Contact

Contact to far-field imaging



Fourier slice theorem

$$I = I_0 e^{-\int \mu(x,y) dy'}$$



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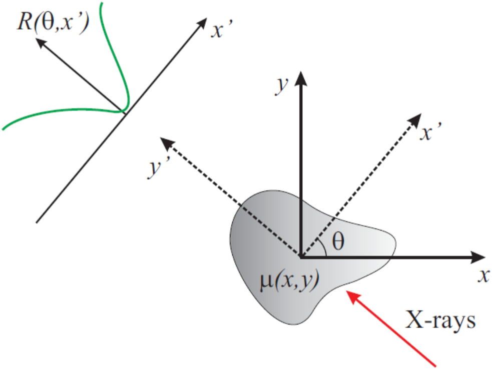

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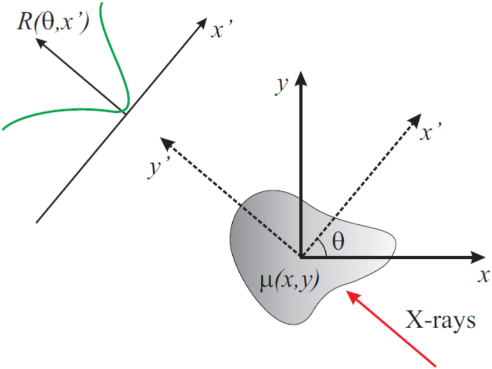

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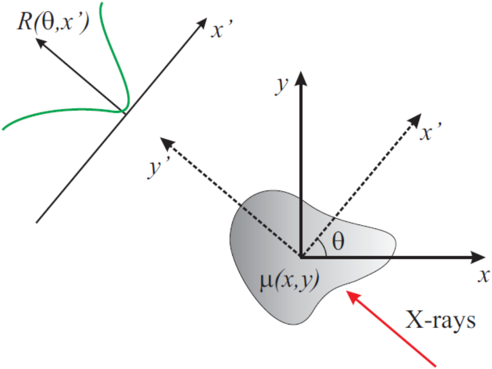
$$P(q_x) = \int p(x)e^{iq_x x}dx$$



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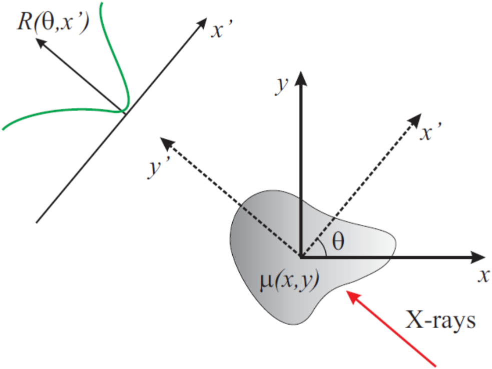

$$F(q_x, q_y) = \int\int f(x,y) e^{iq_x x + q_y y} dxdy$$

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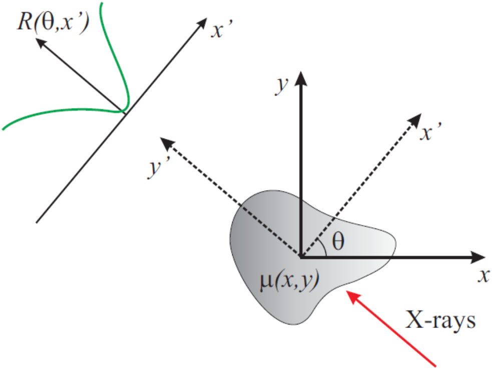

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$$F(q_x, q_y = 0) = \int \left[\int f(x,y)dy\right] e^{iq_x x}dx$$

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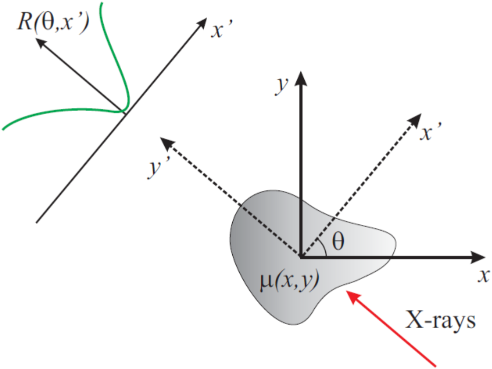


$$F(q_x,q_y) = \int\int f(x,y)e^{iq_x x + q_y y}dxdy$$

$$F(q_x,q_y=0) = \int\left[\int f(x,y)dy\right]e^{iq_x x}dx = \int p(x)e^{iq_x x}dx$$

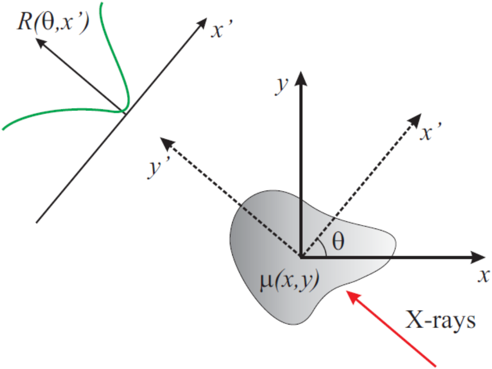
Fourier slice theorem

$$I = I_0 e^{-\int \mu(x,y)dy'}$$

$$\log_{10}\left(\frac{I_0}{I}\right) = \int \mu(x,y)dy'$$

$$p(x) = \int f(x,y)dy$$

$$P(q_x) = \int p(x)e^{iq_x x}dx$$



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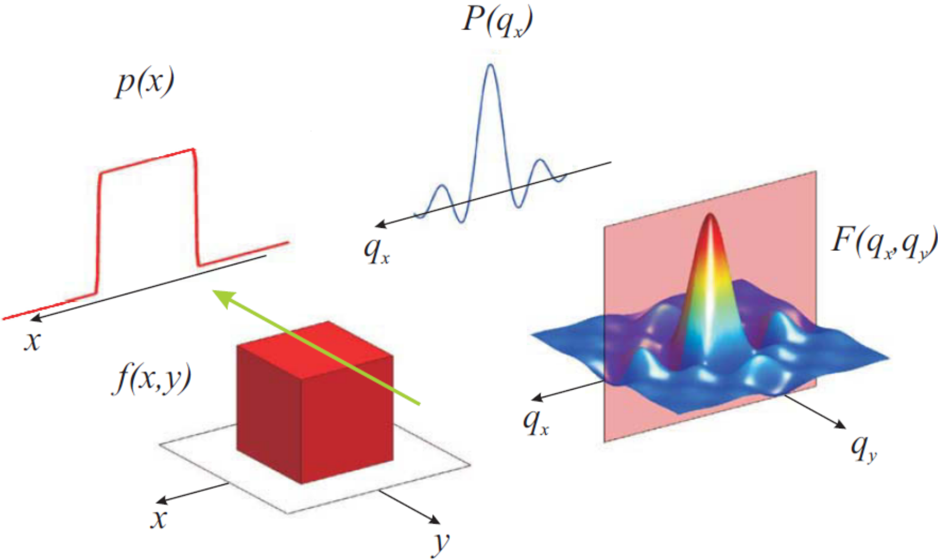
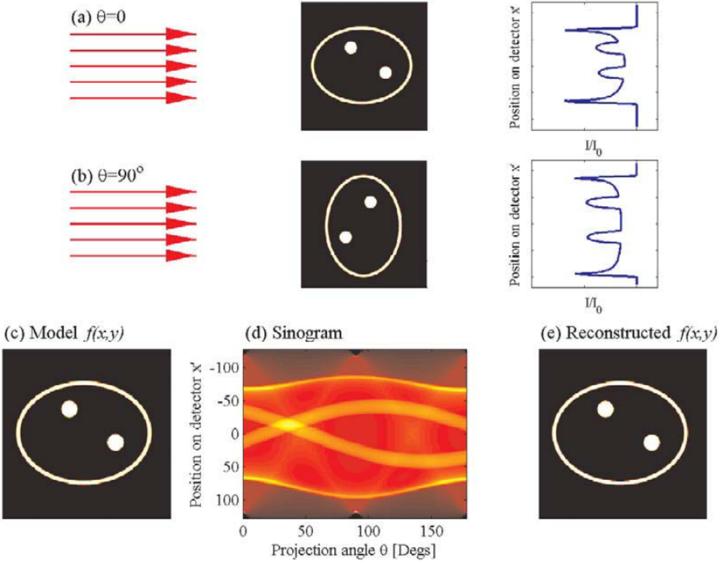
Fourier transform reconstruction

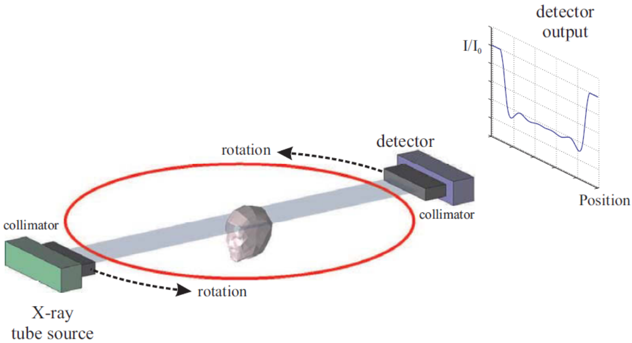


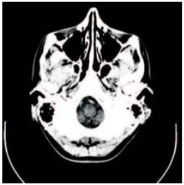
Sinograms



(a) θ=0
Position on detector x'
I/I0
(b) θ=90°
Position on detector x'
I/I0
(c) Model f(x,y)
(d) Sinogram
Position on detector x'
-100
-50
0
50
100
0
50
100
150
Projection angle θ [Degs]
(e) Reconstructed f(x,y)

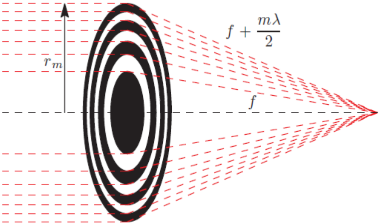
Medical tomography



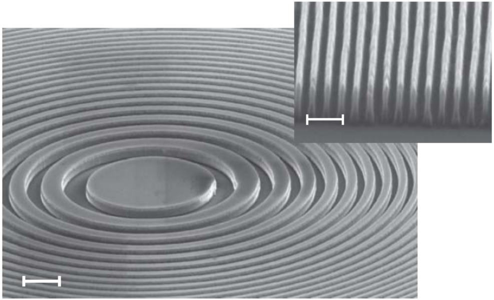


Microscopy

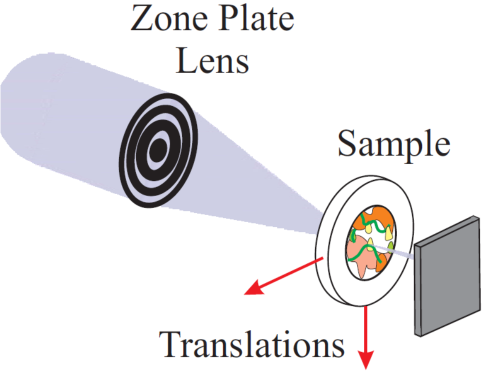
(a)



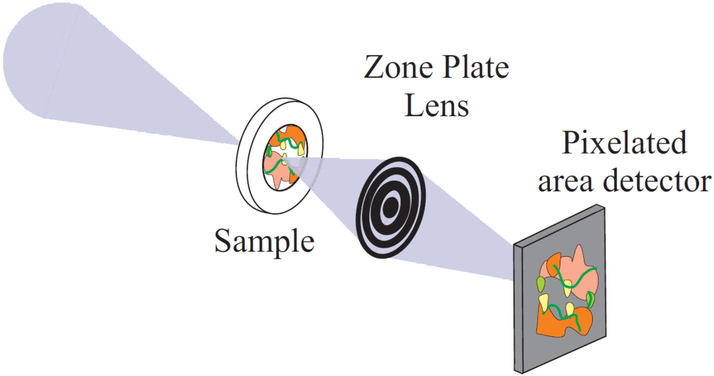
(b)



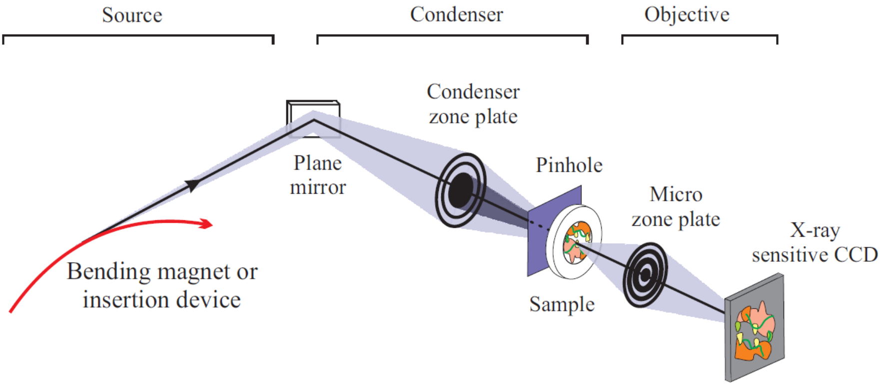
Microscopy



Microscopy



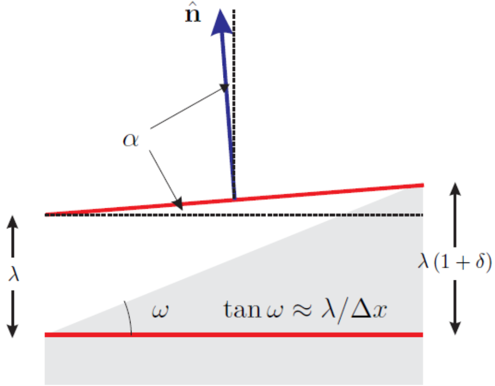
Microscopy


Source
Condenser
Objective
Condenser zone plate
Plane mirror
Pinhole
Micro zone plate
X-ray sensitive CCD
Sample
Bending magnet or insertion device

Angular deviation from refraction

When x-rays cross an interface that is not normal to their direction, there is refraction

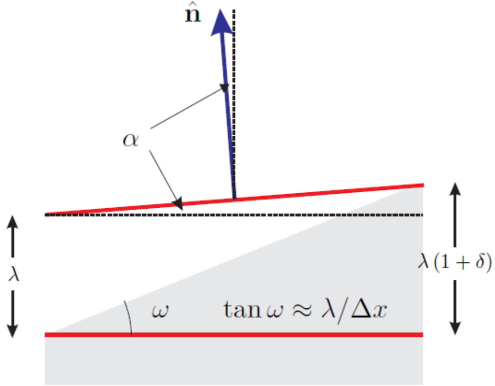
Angular deviation from refraction



When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

Angular deviation from refraction



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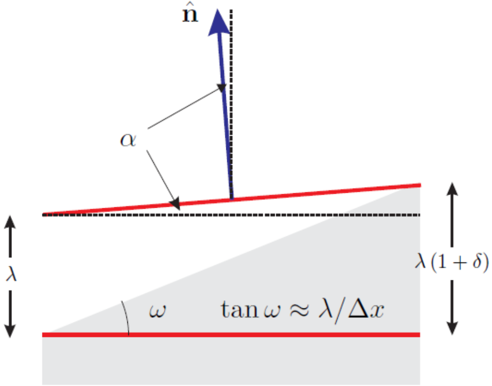

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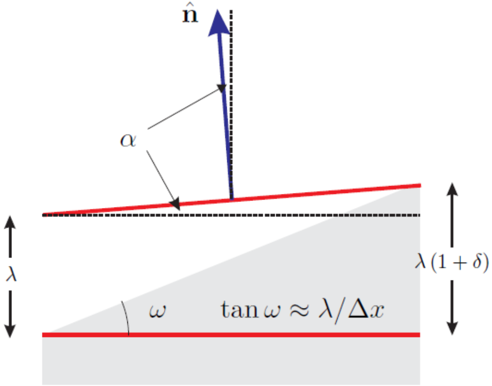
$$\lambda_n = \frac{\lambda}{n}$$

Angular deviation from refraction

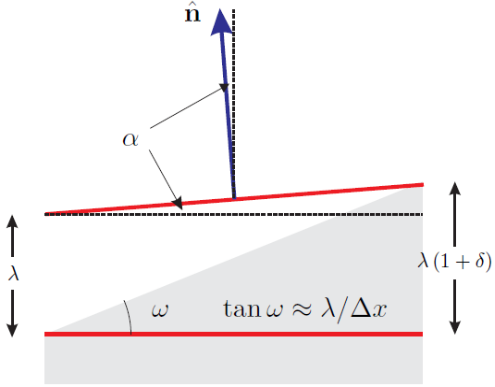


When x-rays cross an interface that is not normal to their direction, there is refraction

The angle of refraction α can be calculated

$$\lambda_n = \frac{\lambda}{n} = \frac{\lambda}{1-\delta}$$

Angular deviation from refraction



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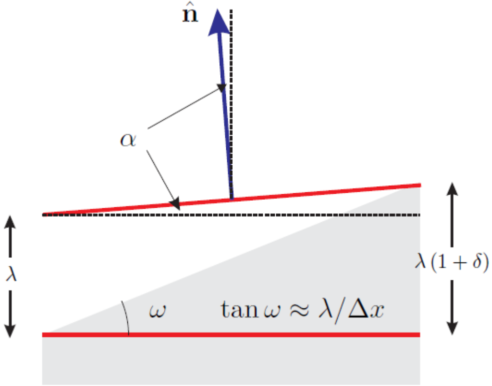

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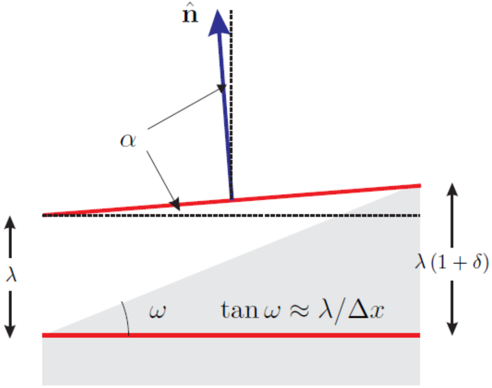

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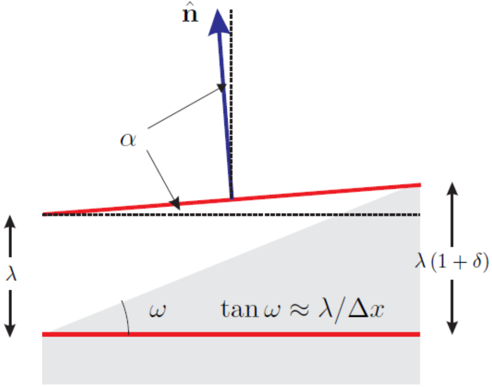

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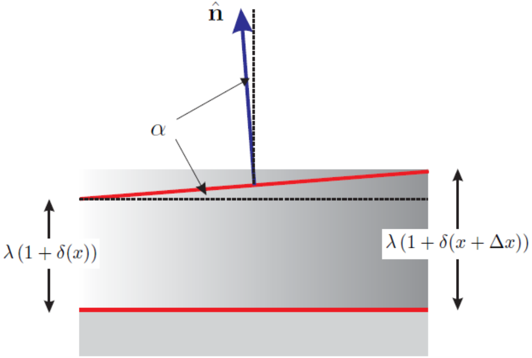
Angular deviation from graded density

In a similar way, there is an angular deviation when the material density varies normal to the propagation direction

Angular deviation from graded density



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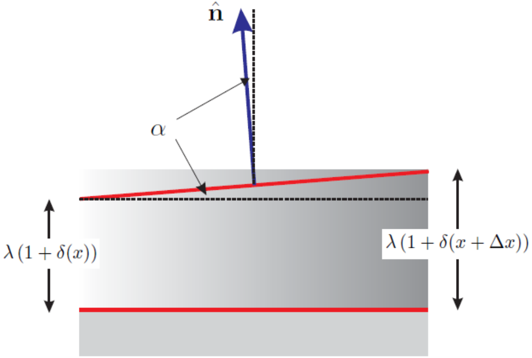

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$$\alpha = \frac{\lambda(1+\delta(x+\Delta x)) - \lambda(1+\delta(x))}{\Delta x}$$

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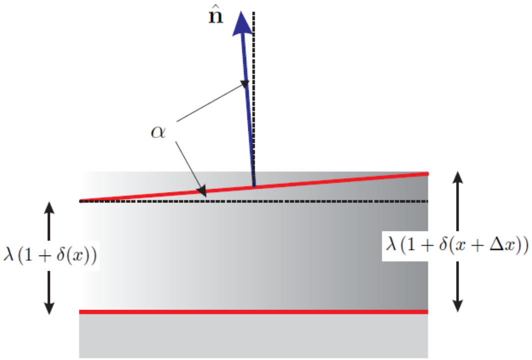

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$$\delta(x+\Delta x) \approx \delta(x) + \Delta x \frac{\partial \delta(x)}{\partial x}$$

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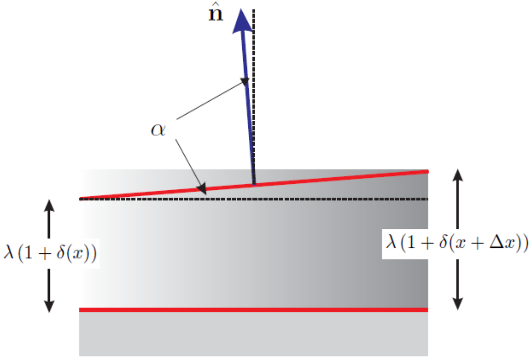

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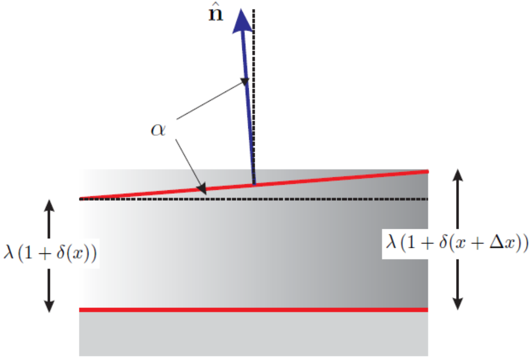

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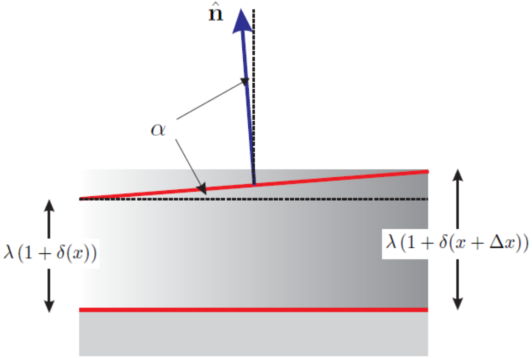

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$$\delta(x+\Delta x) \approx \delta(x) + \Delta x \frac{\partial \delta(x)}{\partial x}$$

$$\alpha_{gradient} = \lambda \frac{\partial \delta(x)}{\partial x} \qquad \text{compare to} \qquad \alpha_{\text{refrac}} = \lambda \frac{\delta}{\Delta x}$$

Phase shift from angular deviation

This deviation of the x-ray beam, leads to a phase shift $\phi(\vec{r}) = \vec{k}' \cdot \vec{r}$ at a specific position along the original propagation direction z

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Phase shift from angular deviation

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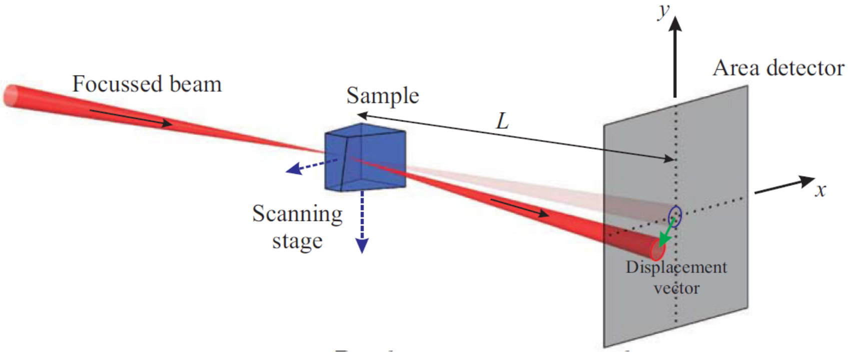
Thus the angular deviation, in each of the x and y directions in the plane perpendicular to the original propagation direction becomes

By measuring the angular deviation as a function of position in a sample, one can reconstruct the phase shift $\phi(x, y)$ due to the sample by integration.

Phase contrast experiment



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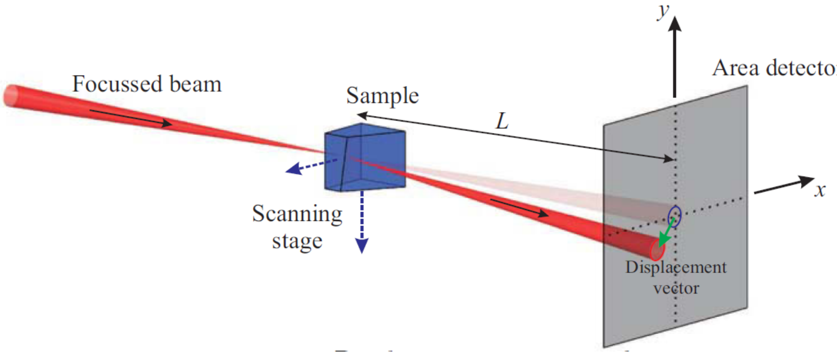
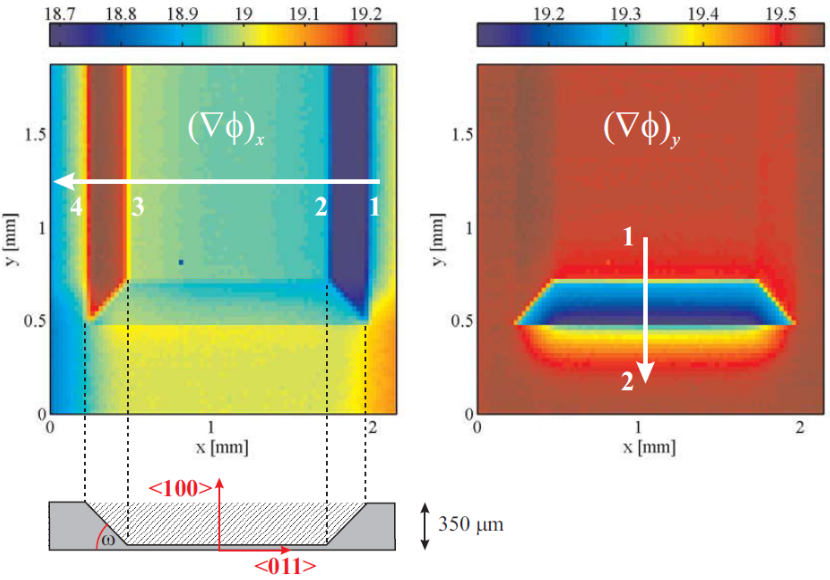


Imaging a silicon trough



Imaging blood cells