• Structure factors

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Homework Assignment #04: Chapter 4: 2, 4, 6, 7, 10 due Tuesday, March 24, 2015

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=  $f(\vec{G}) \left( 1 + e^{i\pi(h+k)} + e^{i\pi(k+l)} + e^{i\pi(h+l)} \right)$   
=  $f(\vec{G}) \times \begin{cases} 4 & h+k, k+l, h+l = 2n \\ 0 & \text{otherwise} \end{cases}$ 



This is a face centered cubic structure with two atoms in the basis which leads to 8 atoms in the conventional unit cell. These are located at



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$$F_{hkl}^{diamond} = f(\vec{G}) \Big( 1 + e^{i\pi(h+k)} + e^{i\pi(k+l)} + e^{i\pi(h+l)} + e^{i\pi(h+l)/2} + e^{i\pi(3h+3k+l)/2} + e^{i\pi(3h+3k+3l)/2} + e^{i\pi(3h+k+3l)/2} \Big)$$



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$$\begin{aligned} F_{hkl}^{diamond} &= f(\vec{G}) \Big( 1 + e^{i\pi(h+k)} + e^{i\pi(k+l)} \\ &+ e^{i\pi(h+l)} + e^{i\pi(h+k+l)/2} + e^{i\pi(3h+3k+l)/2} \\ &+ e^{i\pi(h+3k+3l)/2} + e^{i\pi(3h+k+3l)/2} \Big) \end{aligned}$$

This is non-zero when h,k,l all even and h + k + l = 4n or h,k,l all odd





 $\leftarrow \mathsf{bcc}$ 



$$\leftarrow \mathsf{bcc}$$





$$\leftarrow$$
 bcc





 $\leftarrow \mathsf{diamond}$ 







The sphere radius is set by the length of the  $\vec{k}$  and  $\vec{k'}$  vectors which characterize the incident and scattered x-rays.

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http://phillips-lab.biochem.wisc.edu/software.html









The scattering vector,  $\vec{Q}$ , terminates on the origin of the reciprocal lattice, as does  $\vec{k}$ .



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If the Ewald sphere (circle) lies on a reciprocal lattice point, a reflection can occur for that specific orientation of  $\vec{k}$  to the reciprocal lattice (the physical crystal).



In directions of  $\vec{k'}$  (detector position) where there is no reciprocal lattice point, there can be no diffraction peak.



If the crystal is rotated slightly with respect to the incident beam,  $\vec{k}$ , there may be no Bragg reflections possible at all.

# Polychromatic Radiation



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# Polychromatic Radiation



If  $\Delta \vec{k}$  is large enough, there may be more than one reflection lying on the Ewald sphere.

With an area detector, there may then be multiple reflections appear for a particular orientation (very common with protein crystals where the unit cell is very large).

# Multiple Scattering



Alternatively, scattering can occur internal to the crystal along  $\vec{k}_{int}$  and then along another reciprocal lattice vector  $\vec{G}$  to the detector at  $\vec{k'}$ .

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Alternatively, scattering can occur internal to the crystal along  $\vec{k}_{int}$  and then along another reciprocal lattice vector  $\vec{G}$  to the detector at  $\vec{k'}$ .

This is the cause of monochromator glitches which sometimes remove intensity but can also add intensity.

# Laue Diffraction



The Laue diffraction technique uses a wide range of radiation from  $\vec{k}_{min}$  to  $\vec{k}_{max}$ 

These define two Ewald spheres and a volume between them such that any reciprocal lattice point which lies in the volume will meet the Laue condition for reflection.





 $F 4_1/d \,\overline{3} \, 2/m$ 

 $m\overline{3}m$ 

2  $x, \overline{y}, \overline{z}$ 

 $3 \overline{x}, y, \overline{z}$ 

 $4 \ \overline{x}, \overline{y}, z$  $5 \ z, x, y$ 

 $6 \overline{z}, \overline{x}, y$ 

7  $z, \overline{x}, \overline{y}$ 8  $\overline{z}, x, \overline{y}$ 

9 y, z, x

10  $\overline{y}, z, \overline{x}$ 

No. 227



 $33 \frac{1}{4} - y, \frac{1}{4} - z, \frac{1}{4} - x$  $34 \frac{1}{4} + y, \frac{1}{4} - z, \frac{1}{4} + x$  $35 \frac{1}{4} + y, \frac{1}{4} + z, \frac{1}{4} - x$  $36 \frac{1}{4} - y, \frac{1}{4} + z, \frac{1}{4} + x$ 

y, z, x 35  $\frac{1}{2} + y,$ y, z, x 36  $\frac{1}{4} - y,$  $\frac{1}{4} + x, \frac{1}{4} - z, \frac{1}{4} + y$  37  $\overline{x}, z, \overline{y}$  $\frac{1}{4} - x, \frac{1}{4} - z, \frac{1}{4} + y$  38  $\overline{x}, \overline{z} y$  $\frac{1}{4} - x, \frac{1}{4} - z, \frac{1}{4} - y$  39  $\overline{x}, z, \overline{y}$  $\frac{1}{4} - x, \frac{1}{4} + z, \frac{1}{4} + y$  40  $\overline{x}, \overline{z} \overline{y}$  $\frac{1}{4} - z, \frac{1}{4} + y, \frac{1}{4} + x$  41  $\overline{z}, \overline{y}, x$  $\frac{1}{4} - z, \frac{1}{4} - y, \frac{1}{4} + x$  42  $\overline{z}, \overline{y}, \overline{x}$  $\frac{1}{4} - z, \frac{1}{4} - y, \frac{1}{4} + x$  42  $\overline{z}, \overline{y}, \overline{x}$  $\frac{1}{4} + z, \frac{1}{4} - y, \frac{1}{4} + x, 44 \overline{z}, \overline{y}, \overline{x}$  $\frac{1}{4} - z, \frac{1}{4} - y, \frac{1}{4} + x, 44 \overline{z}, \overline{y}, \overline{x}$ 

 $\begin{array}{c} 19 & \frac{1}{4} - z_{-} \frac{1}{4} - y_{+} \frac{1}{4} - x & 43 \pm y_{+} x \\ 20 & \frac{1}{4} + z_{+} \frac{1}{4} - y_{+} \frac{1}{4} + x & 44 \pm y_{+} x \\ 21 & \frac{1}{4} - y_{+} \frac{1}{4} - x_{+} \frac{1}{4} + z & 45 y_{+} x_{-} \\ 22 & \frac{1}{4} + y_{+} \frac{1}{4} - x_{+} \frac{1}{4} + z & 45 y_{+} x_{+} \\ 23 & \frac{1}{4} - y_{+} \frac{1}{4} - x_{+} \frac{1}{4} - z & 47 y_{+} x_{+} \\ 24 & \frac{1}{4} + y_{+} \frac{1}{4} - x + 4 + y_{+} \frac{1}{4} - z & 45 y_{+} x_{+} \\ + (0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, 0) \end{array}$ 



C. Segre (IIT)

 $Fd\overline{3}m$ 

#### Wyckoff Positions of Group 195 (P23)

Multiplicity	Wyckoff letter	Site symmetry	Coordinates
12	j	1	(x,y,z) (-x,-y,z) (-x,y,-z) (x,-y,-z) (z,x,y) (z,-x,-y) (-z,-x,y) (-z,x,-y) (y,z,x) (-y,z,-x) (y,-z,-x) (-y,-z,x)
6	i	2	(x,1/2,1/2) (-x,1/2,1/2) (1/2,x,1/2) (1/2,-x,1/2) (1/2,1/2,x) (1/2,1/2,-x)
6	h	2	(x,1/2,0) (-x,1/2,0) (0,x,1/2) (0,-x,1/2) (1/2,0,x) (1/2,0,-x)
6	g	2	(x,0,1/2) (-x,0,1/2) (1/2,x,0) (1/2,-x,0) (0,1/2,x) (0,1/2,-x)
6	f	2	(x,0,0) (-x,0,0) (0,x,0) (0,-x,0) (0,0,x) (0,0,-x)
4	е	.3.	(x,x,x) (-x,-x,x) (-x,x,-x) (x,-x,-x)
3	d	222	(1/2,0,0) (0,1/2,0) (0,0,1/2)
3	с	222	(0,1/2,1/2) (1/2,0,1/2) (1/2,1/2,0)
1	b	23.	(1/2,1/2,1/2)
1	а	23.	(0,0,0)

#### Wyckoff Positions of Group 227 (Fd-3m) [origin choice 1]

Multiplicity	Wyckoff letter	Site	Coordinates			
manapricity		symmetry	(0,0,0) + (0,1/2,1/2) + (1/2,0,1/2) + (1/2,1/2,0) +			
192	i	1	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			
96	h	2	$ \begin{array}{l} (118_{37},y+114) & (718_{7},y+112_{2},y+34) & (318_{37},y+112_{37},y+34) & (518_{7},yy+114) \\ (y+114_{1},118_{37}) & (y+34,718_{7},y+12) & (y+112_{37},y+312) & (y+114_{57},518_{7}) \\ (y+114_{1},118_{57}) & (y+112_{7},y+34,718_{1}) & (y+12_{7},y+34,38_{7},y+12) & (518_{37},y+114_{7},518) \\ (118_{7},y+114_{7}) & (318_{37},y+34) & (y+12_{7},112_{7},118_{7},y+114_{7},112_{1}) & (518_{37},y+114_{7},y) \\ (y,118_{7},y+114_{7}) & (128_{7},y+34) & (y+12_{7},718_{7},y+314) & (y,518_{37},y+114) \\ (y+114_{37},118_{1}) & (y+34) & (y+12_{7},718_{7},y+314) & (y+12_{7},718_{7},y+314) \\ (y+114_{37},118_{1}) & (y+34) & (y+12_{7},718_{7},y+12_{7},118) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{7},718) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{7},718) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{7},718) & (y+12_{7},718) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{37},118) & (y+34) & (y+12_{7},118) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{37},118) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{37},118) & (y+14) \\ (y+114_{37},118) & (y+34) & (y+12_{37},118) & (y+14) \\ (y+114_{37},118) & (y+14) & (y+14) \\ (y+14) & (y+14) & (y+14) \\ (y+14) & (y+14) & (y+14) & (y+14) & (y+14) \\ (y+14) & (y+14) & (y+14) & (y+14) & (y+14) \\ (y+14) & (y+14) & (y+14) & (y+14) \\ (y+14) & (y+14)$			
96	g	m	$\begin{array}{l} (x,x) & (x,x) + 1/2z+1/2) & (x+1/2x+1/2,z) & (x+1/2,x,z+1/2) \\ (z,x) & (z+1/2,x,x+1/2) & (z,z+1/2,x) \\ (x,x) & (x+1/2,z+1/2,x) & (x+1/2,z,x+1/2) \\ (x+3/4x+1/4,z+3/4) & (x+1/4,z+1/4) & (x+1/4z+3/4z+3/4) & (x+3/4z+1/4) \\ (x+3/4z+1/4,x+3/4) & (x+1/4,z+3/4x+1/4,z+1/4,x+1/4) \\ (x+3/4x+1/4,x+3/4) & (x+1/4,x+3/4x+3/4) & (x+1/4x+3/4x+1/4) \\ (x+1/4,x+3/4) & (z+1/4,x+3/4x+3/4) & (z+1/4,x+1/4) \\ (z+1/4,x+1/4) & (z+1/4,x+1/4) \\ \end{array}$			
48	f	2.m m	(x,0,0) (-x,1/2,1/2) (0,x,0) (1/2,-x,1/2) (0,0,x) (1/2,1/2,-x) (3/4,x+1/4,3/4) (1/4,-x+1/4,1/4) (x+3/4,1/4,3/4) (-x+3/4,3/4,1/4) (3/4,1/4,-x+3/4) (1/4,3/4,x+3/4)			
32	е	.3m	(x,x) (-x,-x+1/2,x+1/2) (-x+1/2,x+1/2,-x) (x+1/2,-x,-x+1/2) (x+3/4,x+1/4,-x+3/4) (-x+1/4,-x+1/4) (x+1/4,-x+3/4,x+3/4) (-x+3/4,x+3/4,x+1/4)			
16	d	3m	(5/8,5/8,5/8) (3/8,7/8,1/8) (7/8,1/8,3/8) (1/8,3/8,7/8)			
16	с	3m	(1/8,1/8,1/8) (7/8,3/8,5/8) (3/8,5/8,7/8) (5/8,7/8,3/8)			
8	b	-43m	(1/2,1/2,1/2) (1/4,3/4,1/4)			
8	а	-43m	(0,0,0) (3/4,1/4,3/4)			

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Zn nanoparticles formed by ion implantation. SAXS measured after irradiation with high energy  $Xe^{+14}$  ions using 18 keV x-rays

Zn nanoparticles formed by ion implantation. SAXS measured after irradiation with high energy  $\rm Xe^{+14}$  ions using 18 keV x-rays



Expt. geometry

Zn nanoparticles formed by ion implantation. SAXS measured after irradiation with high energy  $Xe^{+14}$  ions using 18 keV x-rays



Expt. geometry

Unirradiated

Zn nanoparticles formed by ion implantation. SAXS measured after irradiation with high energy  $\rm Xe^{+14}$  ions using 18 keV x-rays



Expt. geometry

Irradiated || x-rays

Zn nanoparticles formed by ion implantation. SAXS measured after irradiation with high energy  $\rm Xe^{+14}$  ions using 18 keV x-rays



Expt. geometry

Irradiated || x-rays

Irradiated  $\perp$  x-rays

"Shape elongation of embedded Zn nanoparticles induced by swift heavy ion irradiation: A SAXS study", H. Amekura, K. Kono, N. Okubo, and N. Ishikawa, *Phys. Status Solidi B* **252**, 165-169 (2015).

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SAXS intensity for  $\parallel$  and  $\perp$  x-ray incidence

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SAXS intensity for  $\parallel$  and  $\perp$  x-ray incidence

Interparticle distance as a function of irradiation fluence

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