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- Reflectivity papers

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Homework Assignment #03:
Chapter 3: 1, 3, 4, 6, 8
due Thursday, February 26, 2015

Layering in liquid films

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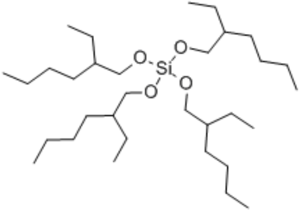

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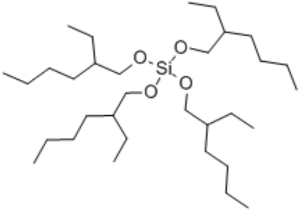

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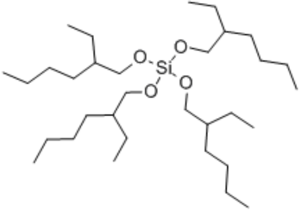

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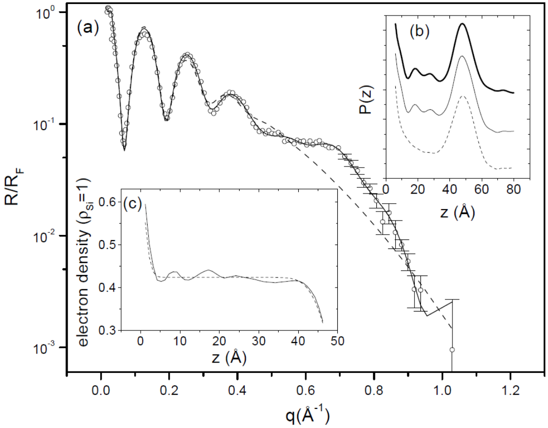

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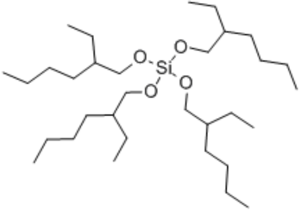

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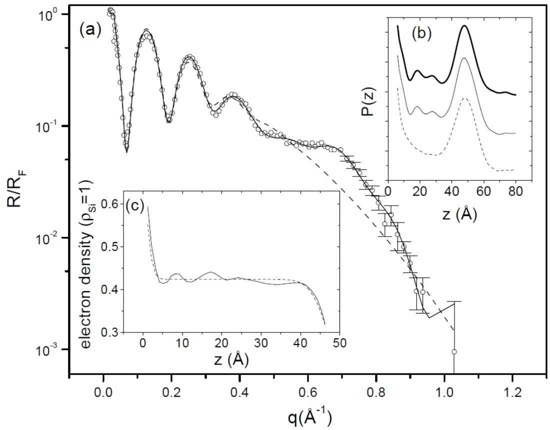

Deviations from uniform density are used to fit experimental reflectivity

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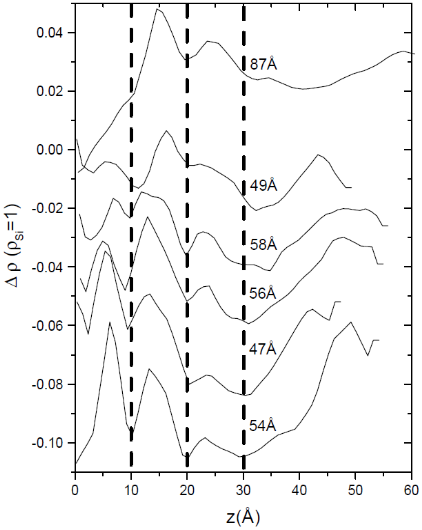

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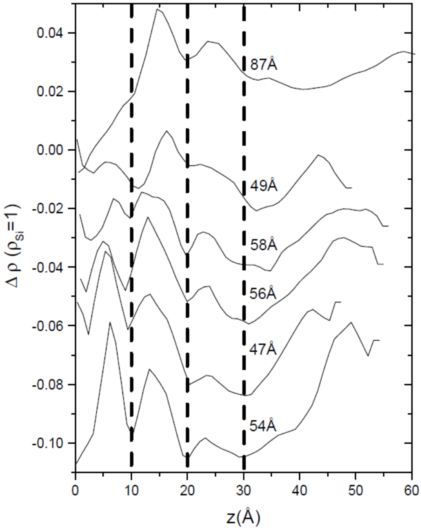

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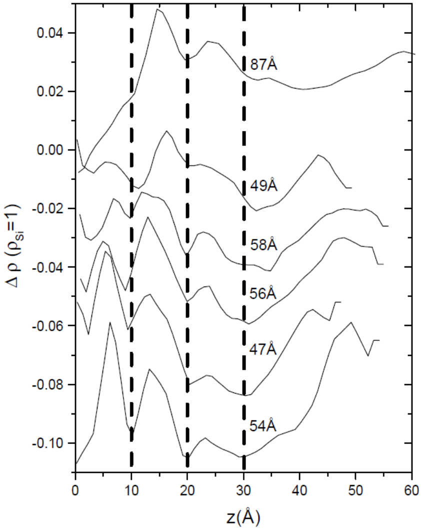

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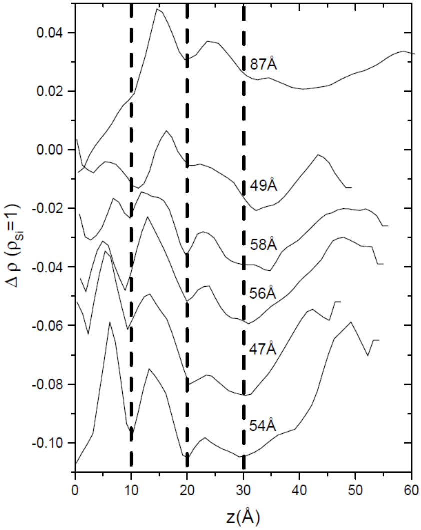

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A broad peak appears at free surface indicating that ordering requires a hard smooth surface.

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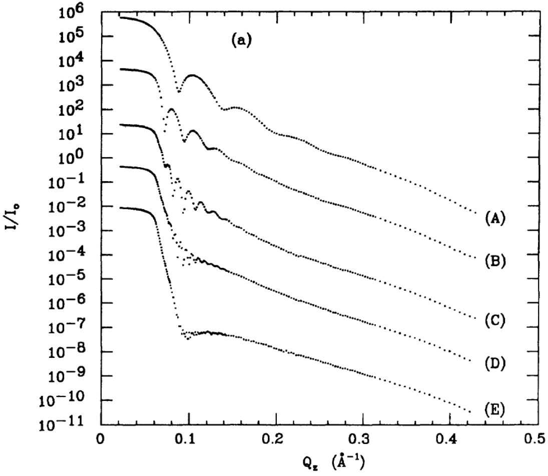

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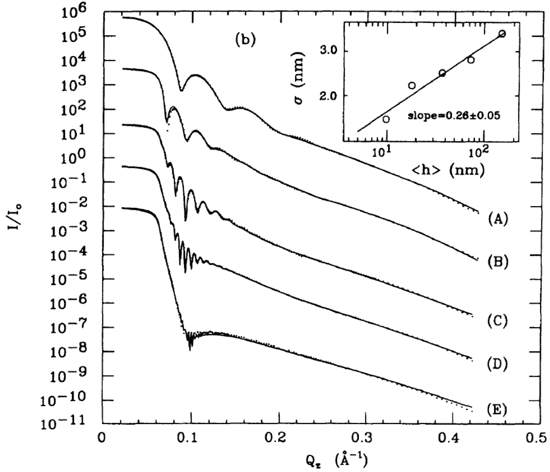

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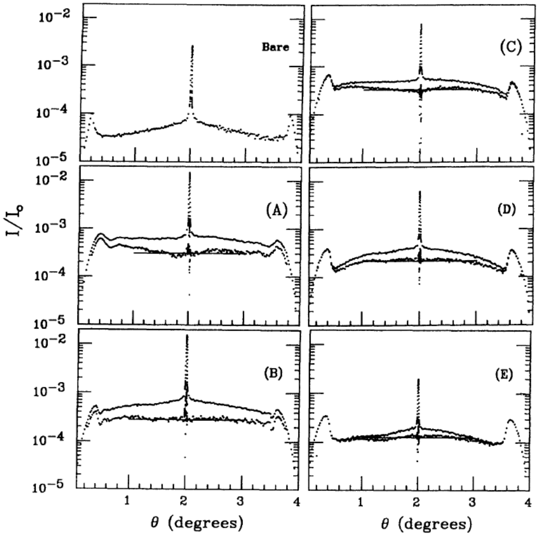

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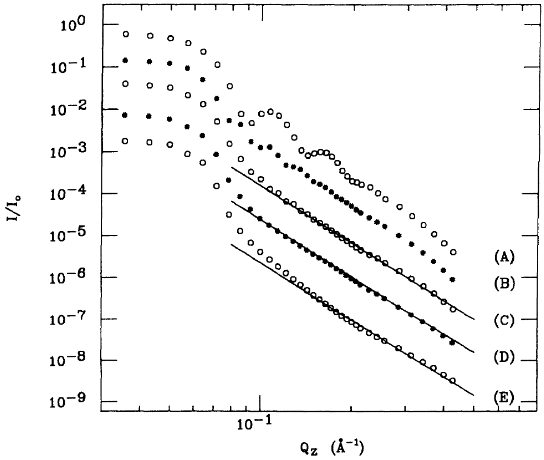

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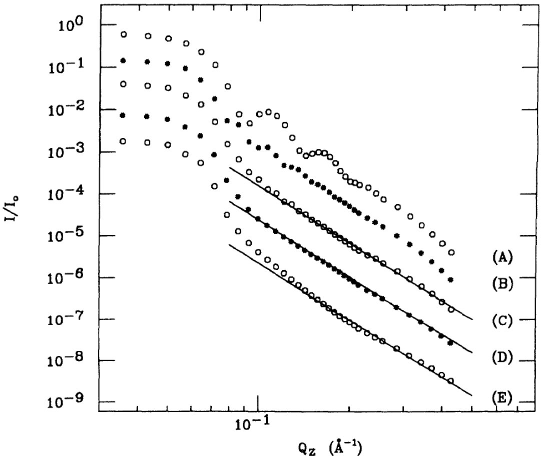

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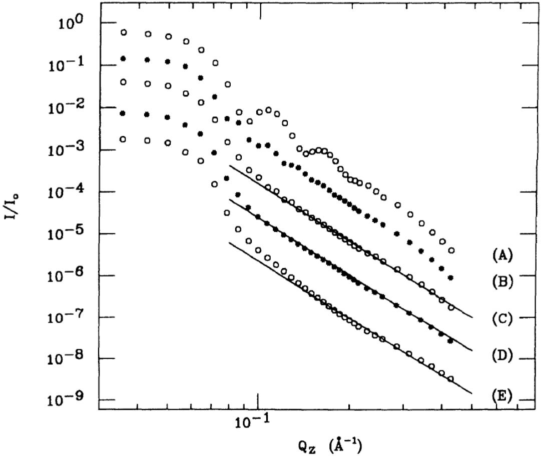

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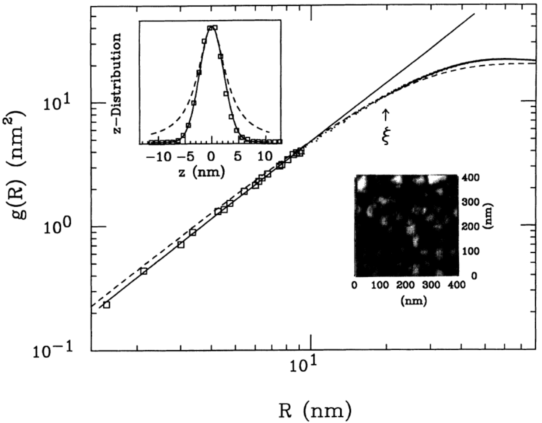

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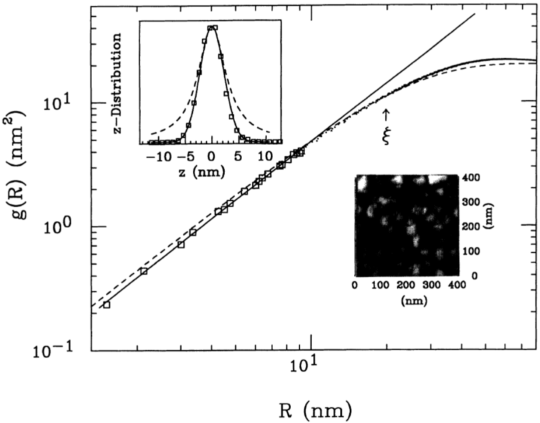

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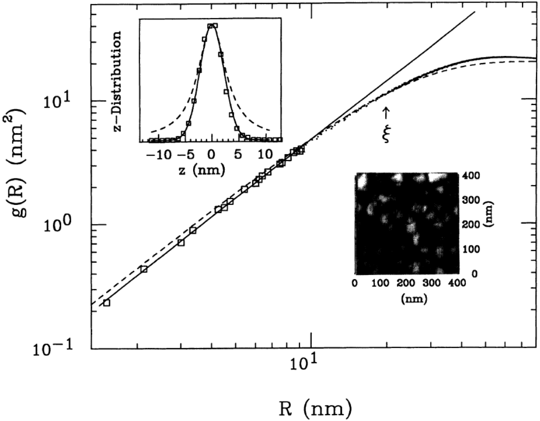

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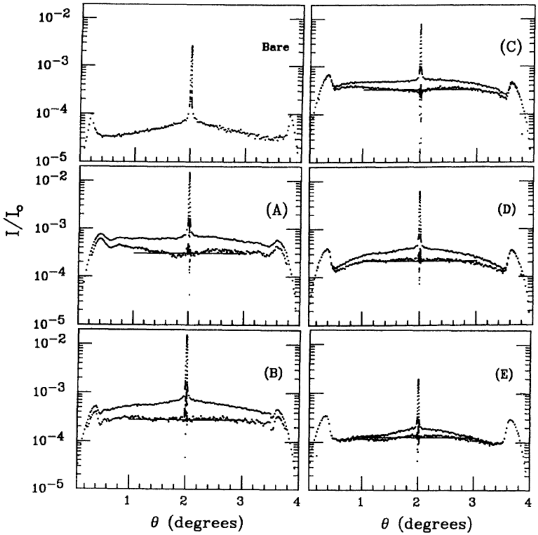

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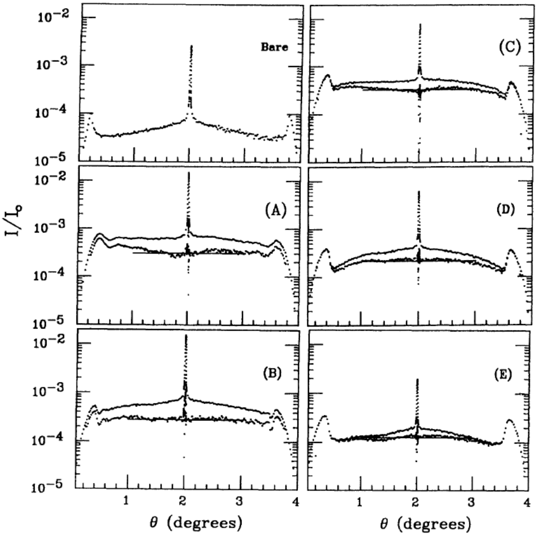

Thus $z_s = h/\beta = 2.7$ and diffraction data confirm $\xi = 19.9\langle h\rangle^{1/2.7}$ Å

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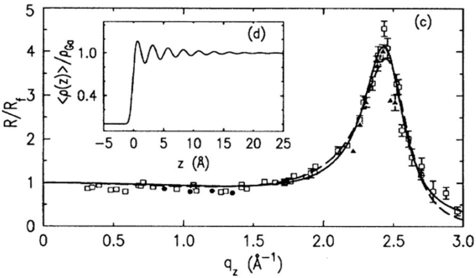

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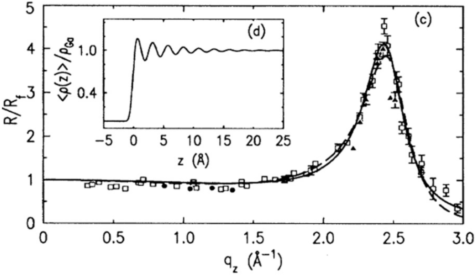

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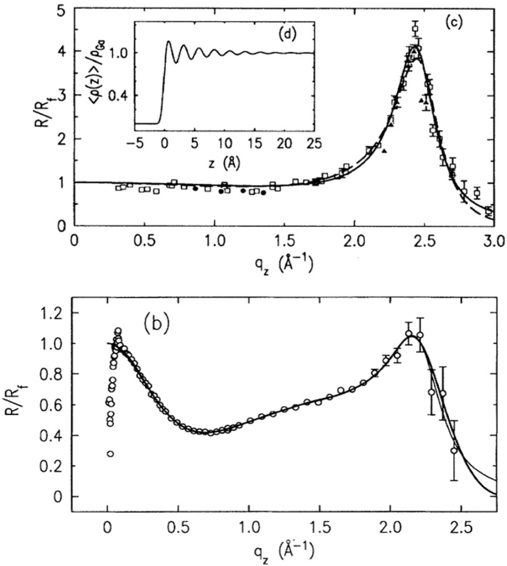

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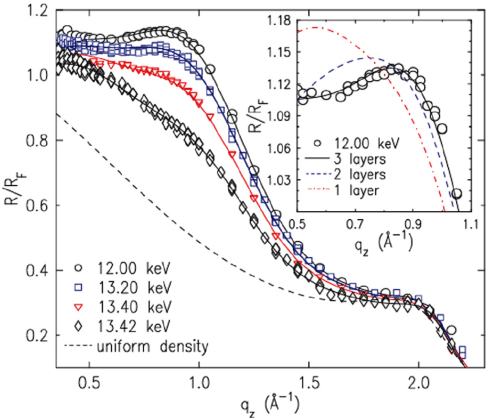

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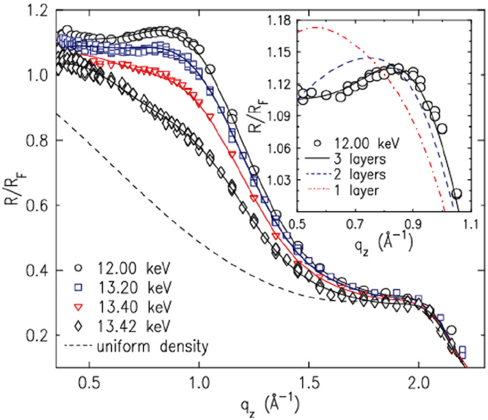
surface layer is rich in Bi (95%), second layer is deficient (25%), and third layer is rich in Bi (53%) once again



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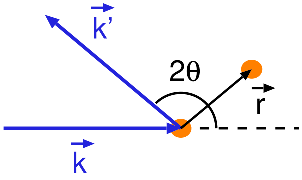

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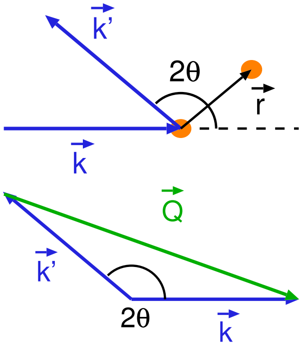

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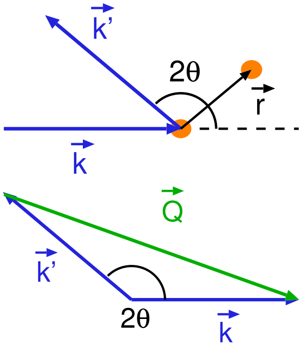

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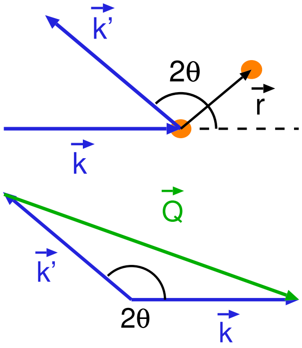

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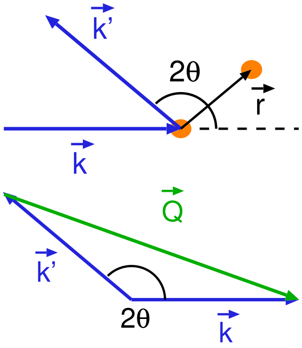

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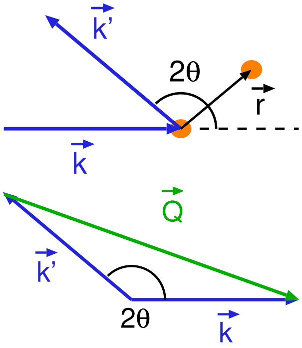

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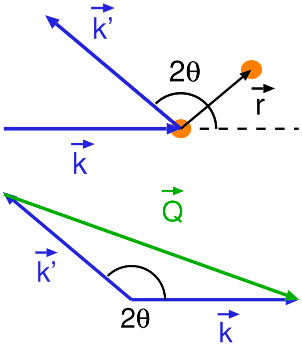

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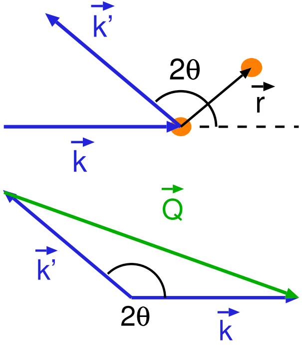
Scattering from two electrons

Consider systems where there is only weak scattering, with no multiple scattering effects. We begin with the scattering of x-rays from two electrons.



$$\vec{Q} = (\vec{k} - \vec{k'})$$

$$|\vec{Q}| = 2k\sin\theta = \frac{4\pi}{\lambda}\sin\theta$$

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We will now look at the consequences of this orientation and generalize to more than two electrons

Two electrons — fixed orientation

The expression

$$I(\vec{Q}) = 2r_o^2 \left(1 + \cos(\vec{Q} \cdot \vec{r})\right)$$

assumes that the two electrons have a specific, fixed orientation. In this case the intensity as a function of Q is.

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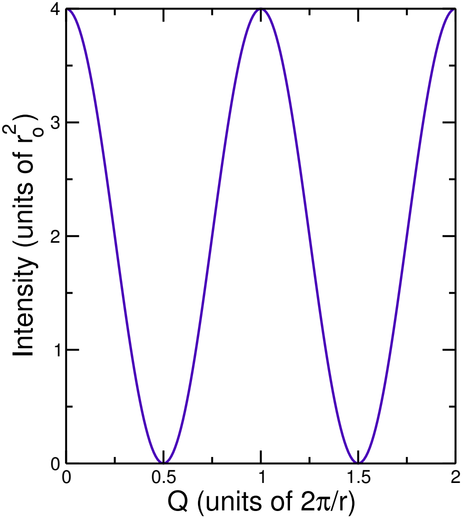

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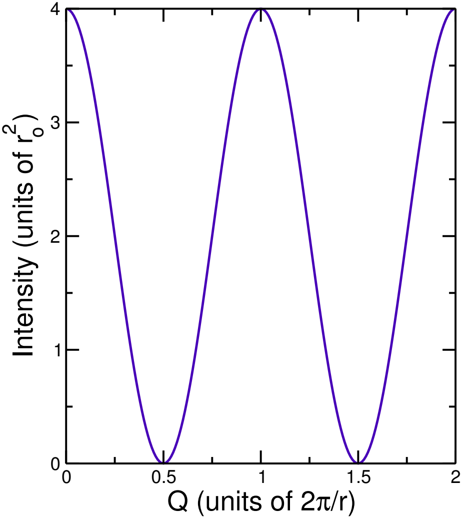
Fixed orientation is not the usual case, particularly for solution and small-angle scattering.



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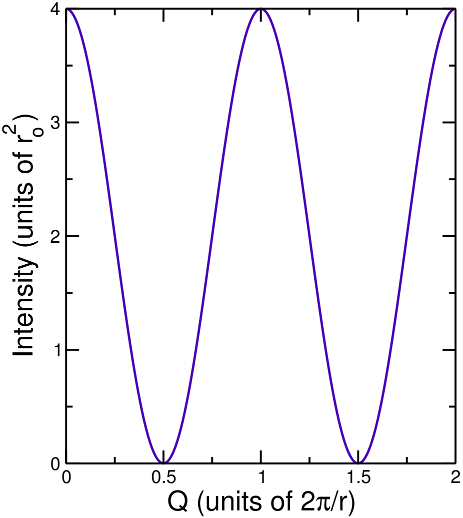
Randomly oriented electrons

Recall that when we had a fixed orientation of the two electrons, we had and intensity variation as $\cos(Qr)$.



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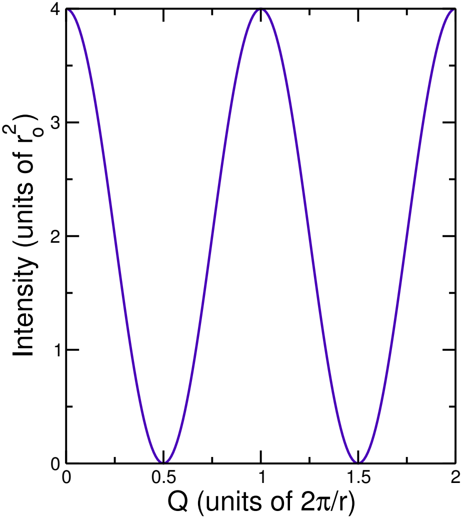

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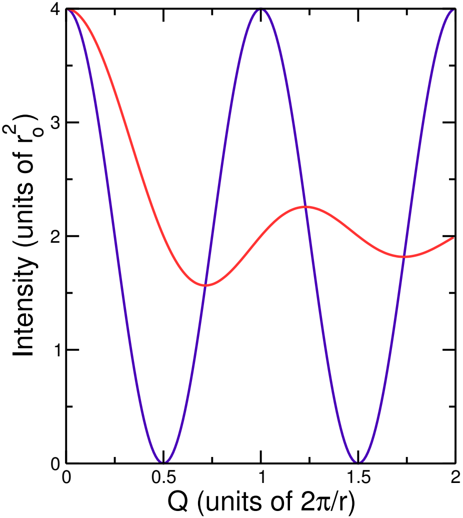

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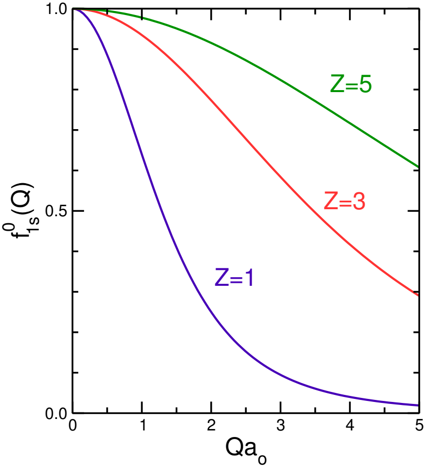

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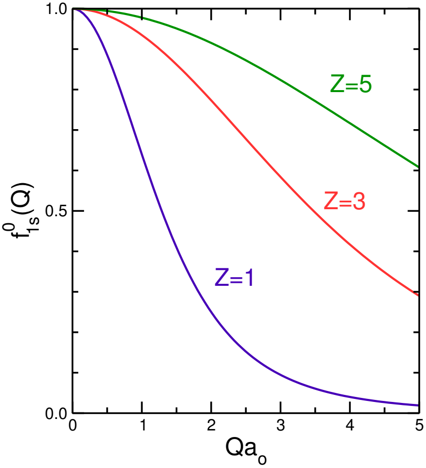

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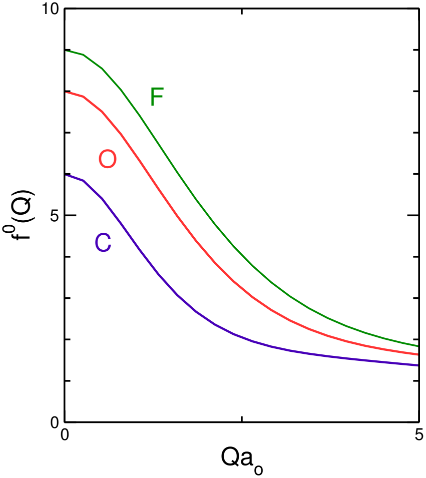
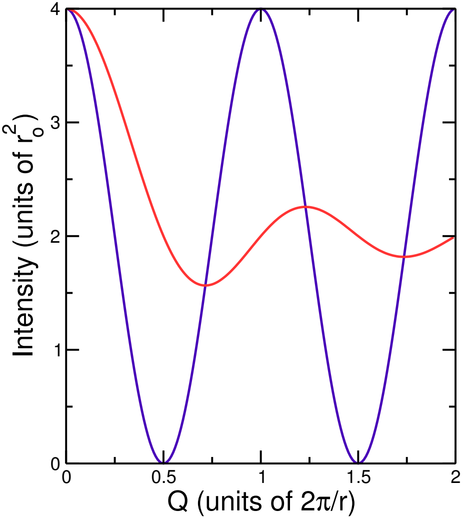


$$f^0\left(Q/4\pi\right) = \sum_{j=1}^{4} a_j e^{-b_j \sin^2\theta/\lambda^2} + c = \sum_{j=1}^{4} a_j e^{-b_j (Q/4\pi)^2} + c$$

Two hydrogen atoms

Previously we derived the scattering intensity from two localized electrons both fixed and randomly oriented to the x-rays.



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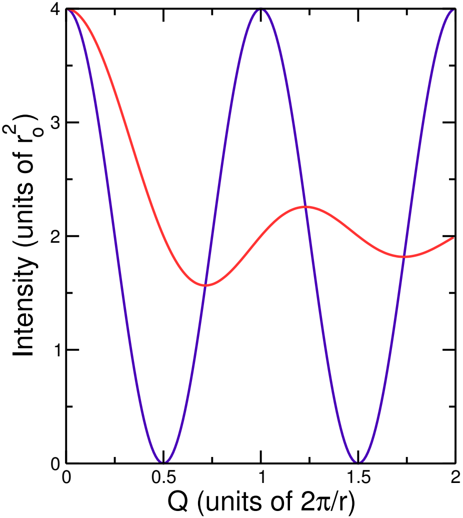

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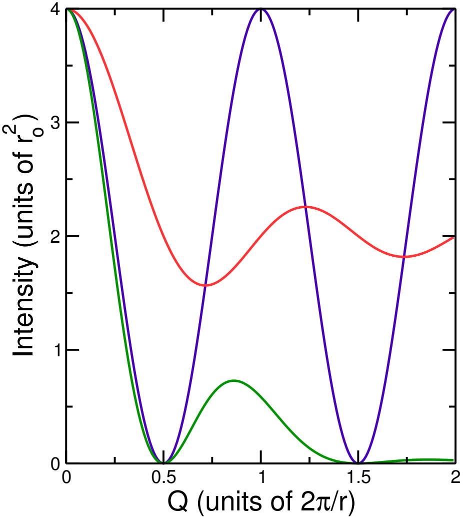

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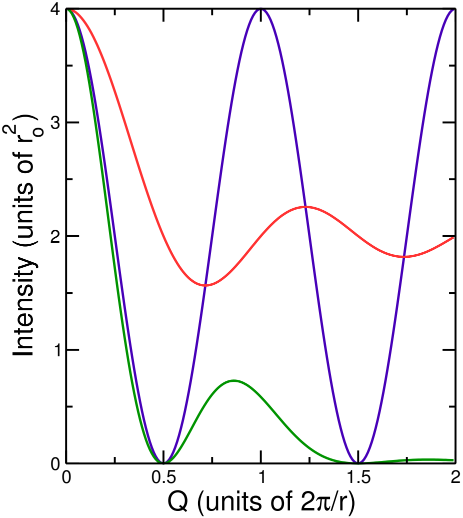

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