Today's Outline - February 17, 2015

Today's Outline - February 17, 2015

- How to write a GU proposal

Today's Outline - February 17, 2015

- How to write a GU proposal
- Mirrors

Today's Outline - February 17, 2015

- How to write a GU proposal
- Mirrors
- Refractive optics

Today's Outline - February 17, 2015

- How to write a GU proposal
- Mirrors
- Refractive optics

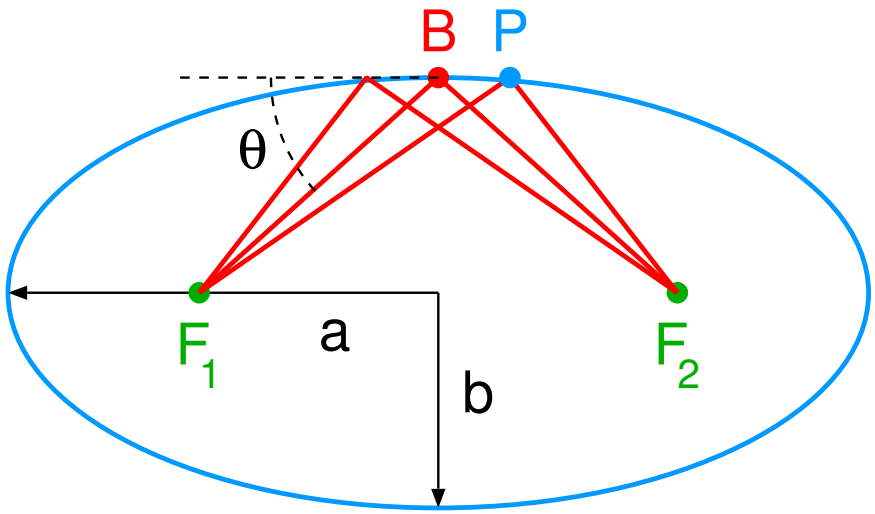
Homework Assignment #03:
Chapter3: 1, 3, 4, 6, 8
due Thursday, February 26, 2015

Writing a General User Proposal

1. Log into the APS site
2. Start a general user proposal
3. Add an Abstract
4. Choose a beam line
5. Answer the 6 questions

Tangential Focusing Mirror

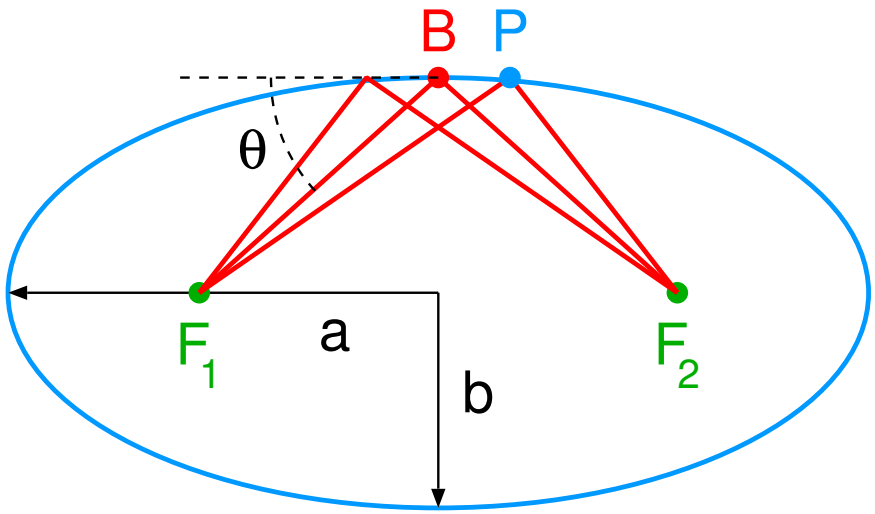
The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus.



Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.

$$F_1P + F_2P = 2a$$

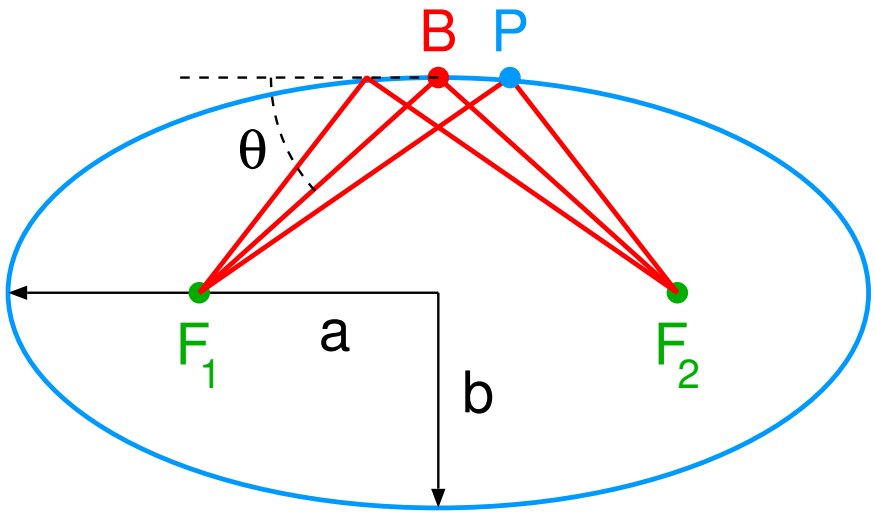


Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.

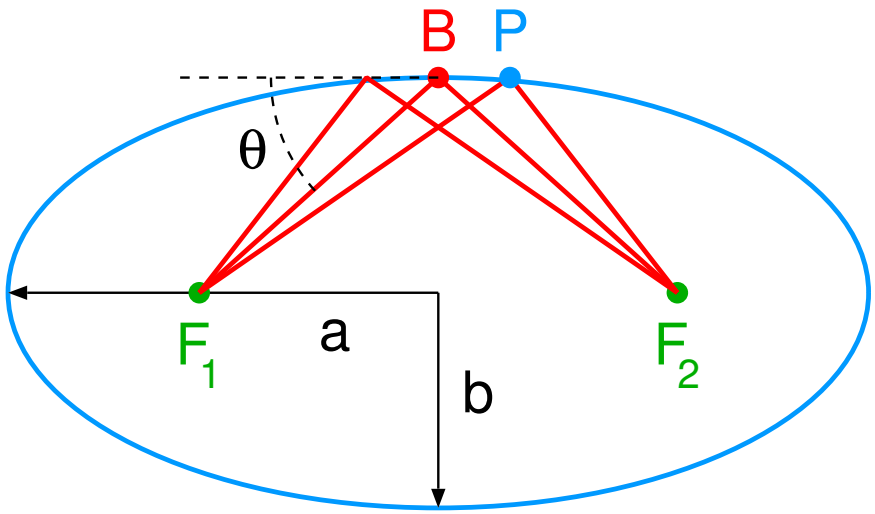
$$F_1P + F_2P = 2a$$

$$F_1B = F_2B = a$$



Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.



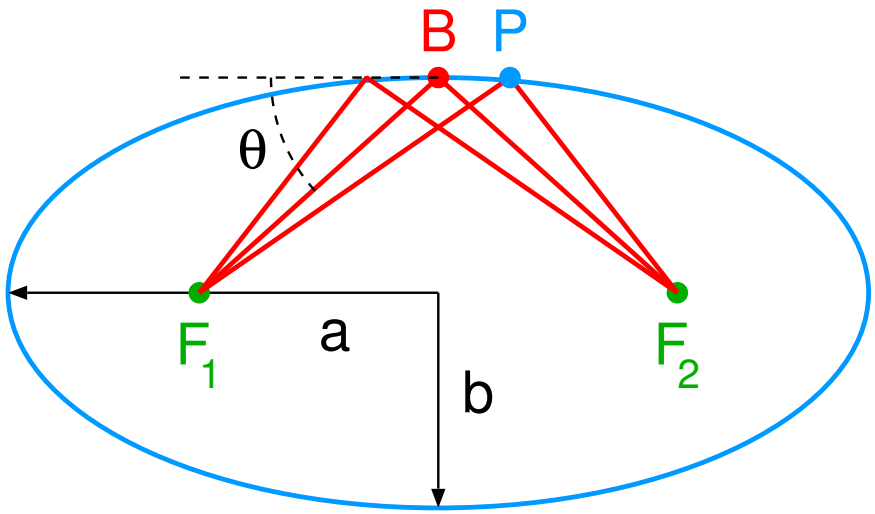
$$F_1P + F_2P = 2a$$

$$F_1B = F_2B = a$$

$$\sin\theta = \frac{b}{a}$$

Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.



$$F_1P + F_2P = 2a$$

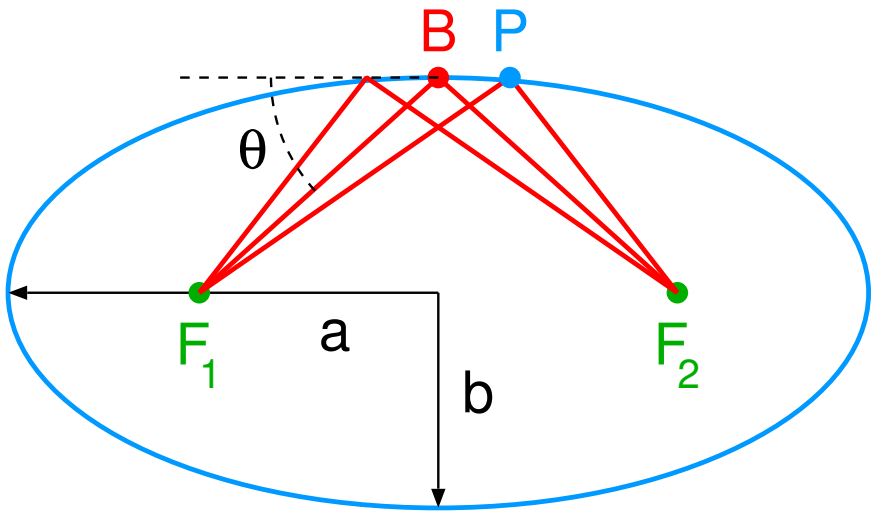
$$F_1B = F_2B = a$$

$$\sin\theta = \frac{b}{a}$$

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.



$$F_1P + F_2P = 2a$$

$$F_1B = F_2B = a$$

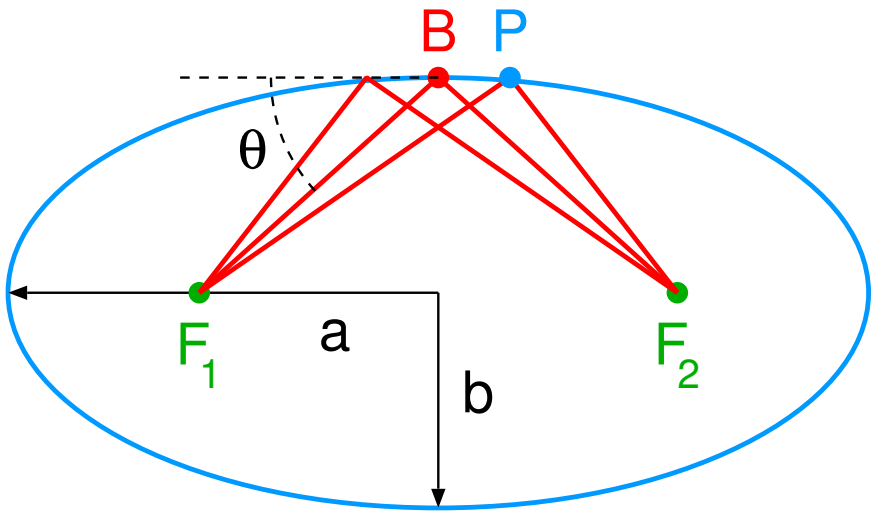
$$\sin\theta = \frac{b}{a}$$

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{2}{a}$$

$$f = \frac{a}{2}$$

Tangential Focusing Mirror

The shape of an ideal mirror is an ellipse, where any ray coming from one focus will be projected to the second focus. Consider a 1:1 focusing mirror. For an ellipse the sum of the distances from any point on the ellipse to the foci is a constant.



$$F_1P + F_2P = 2a$$

$$F_1B = F_2B = a$$

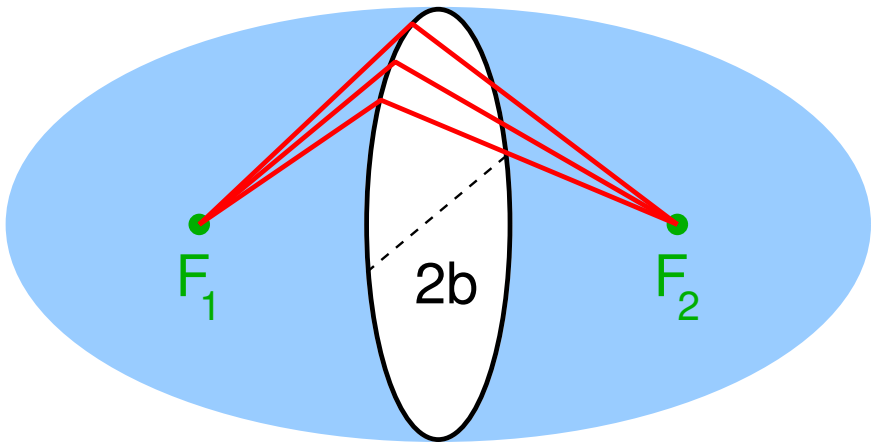
$$\sin\theta = \frac{b}{a} = \frac{b}{2f}$$

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{2}{a}$$

$$f = \frac{a}{2}$$

Saggital Focusing Mirror

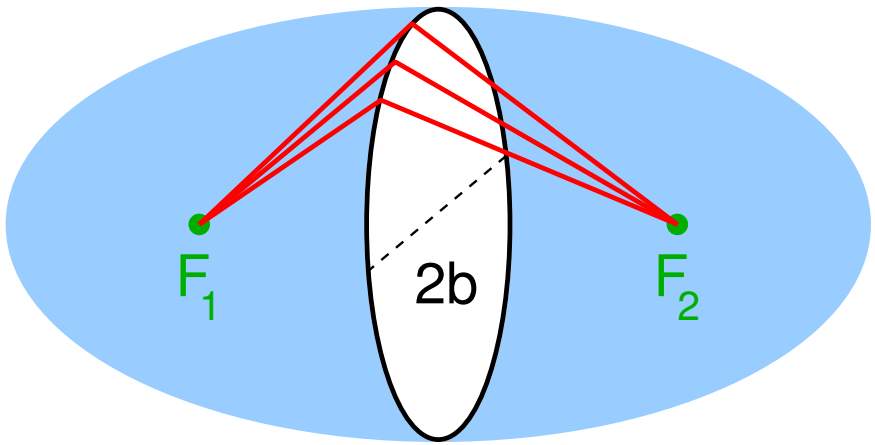
Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of saggital focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.



Saggital Focusing Mirror

Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of saggital focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.

$$\rho_{saggital} = b = 2f \sin\theta$$

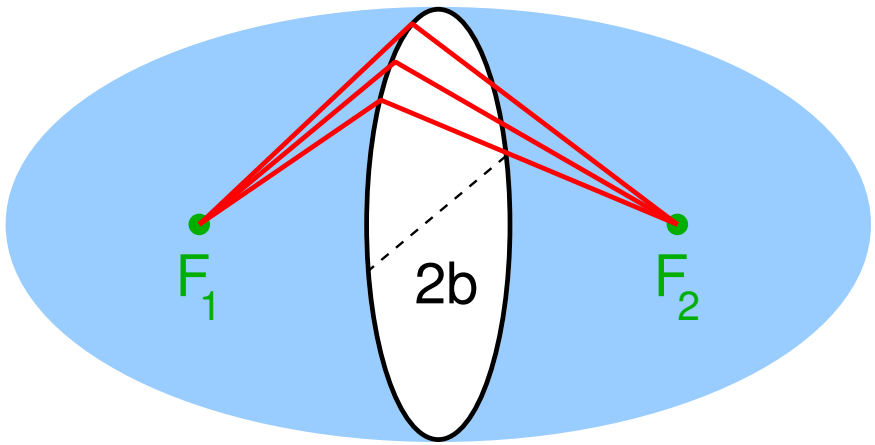


Saggital Focusing Mirror

Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of saggital focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.

$$\rho_{saggital} = b = 2f \sin\theta$$

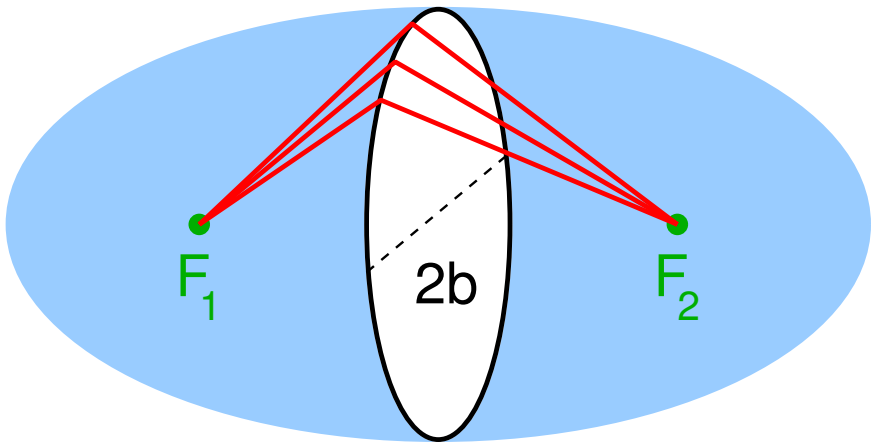
The tangential focus is also usually approximated by a circular cross-section with radius



Saggital Focusing Mirror

Ellipses are hard figures to make, so usually, they are approximated by circles. In the case of saggital focusing, an ellipsoid of revolution with diameter $2b$, is used for focusing.

$$\rho_{saggital} = b = 2f \sin\theta$$



The tangential focus is also usually approximated by a circular cross-section with radius

$$\rho_{tangential} = a = \frac{2f}{\sin\theta}$$

Types of Focusing Mirrors

A simple mirror such as the one at MRCAT consists of a polished glass slab with two "legs".

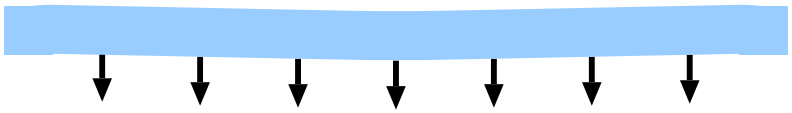
Types of Focusing Mirrors

A simple mirror such as the one at MRCAT consists of a polished glass slab with two "legs". A force is applied mechanically to push the legs apart and bend the mirror to a radius as small as $R = 500\text{m}$.

Types of Focusing Mirrors

A simple mirror such as the one at MRCAT consists of a polished glass slab with two "legs". A force is applied mechanically to push the legs apart and bend the mirror to a radius as small as $R = 500$m.

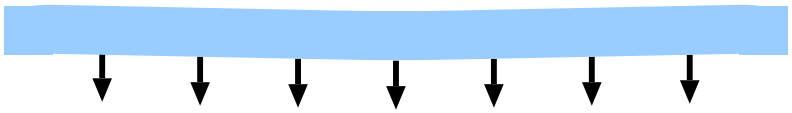
The biomorph mirror is designed to obtain a smaller form error than a simple bender through the use of multiple actuators tuned experimentally.



Types of Focusing Mirrors

A simple mirror such as the one at MRCAT consists of a polished glass slab with two "legs". A force is applied mechanically to push the legs apart and bend the mirror to a radius as small as $R = 500$m.

The biomorph mirror is designed to obtain a smaller form error than a simple bender through the use of multiple actuators tuned experimentally.



A cost effective way to focus in both directions is a toroidal mirror which has a fixed bend in the transverse direction but which can be bent longitudinally to change the vertical focus.

Dual Focusing Options

Dual Focusing Options

- Toroidal mirror — simple, moderate focus, good for initial focusing element, easy to distort beam

Dual Focusing Options

- Toroidal mirror — simple, moderate focus, good for initial focusing element, easy to distort beam
- Saggittal focusing crystal & vertical focusing mirror — adjustable in both directions, good for initial focusing element

Dual Focusing Options

- Toroidal mirror — simple, moderate focus, good for initial focusing element, easy to distort beam
- Saggittal focusing crystal & vertical focusing mirror — adjustable in both directions, good for initial focusing element
- Kirkpatrick-Baez mirror pair — in combination with an initial focusing element, good for final small focal spot and variable energy

Dual Focusing Options

- Toroidal mirror — simple, moderate focus, good for initial focusing element, easy to distort beam
- Saggittal focusing crystal & vertical focusing mirror — adjustable in both directions, good for initial focusing element
- Kirkpatrick-Baez mirror pair — in combination with an initial focusing element, good for final small focal spot and variable energy
- K-B mirrors & zone plates — in combination with an initial focusing element, gives smallest focal spot, but hard to vary energy

Refractive Optics

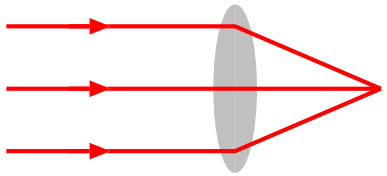
Just as with visible, light, it is possible to make refractive optics for x-rays

Refractive Optics

Just as with visible, light, it is possible to make refractive optics for x-rays

visible light:

$$n \sim 1.2 - 1.5$$
$$f \sim 0.1\text{m}$$

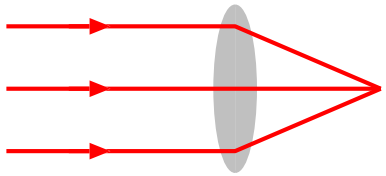


Refractive Optics

Just as with visible, light, it is possible to make refractive optics for x-rays

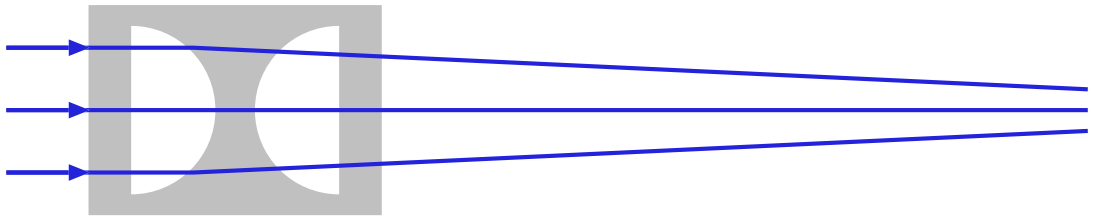
visible light:

$$n \sim 1.2 - 1.5$$
$$f \sim 0.1\text{m}$$



x-rays:

$$n \approx 1 - \delta,\ \delta \sim 10^{-5}$$
$$f \sim 100\text{m!}$$

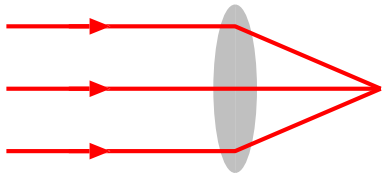


Refractive Optics

Just as with visible, light, it is possible to make refractive optics for x-rays

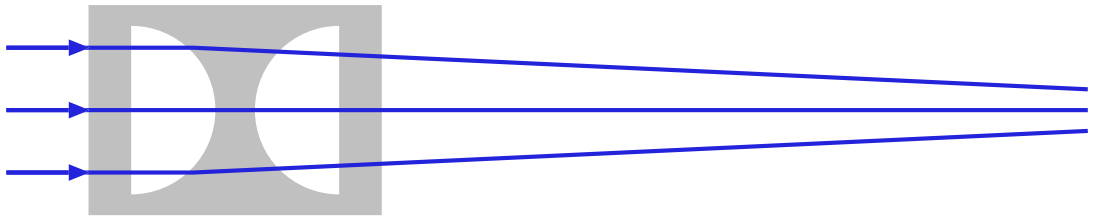
visible light:

$$n \sim 1.2 - 1.5$$
$$f \sim 0.1\text{m}$$



x-rays:

$$n \approx 1 - \delta,\ \delta \sim 10^{-5}$$
$$f \sim 100\text{m!}$$



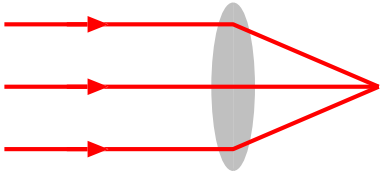
x-ray lenses are complementary to those for visible light

Refractive Optics

Just as with visible, light, it is possible to make refractive optics for x-rays

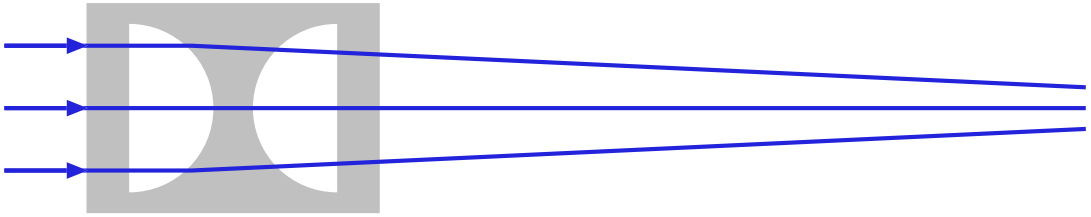
visible light:

$$n \sim 1.2 - 1.5$$
$$f \sim 0.1\text{m}$$

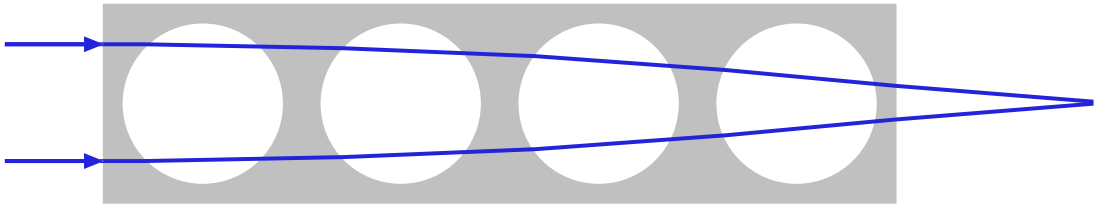


x-rays:

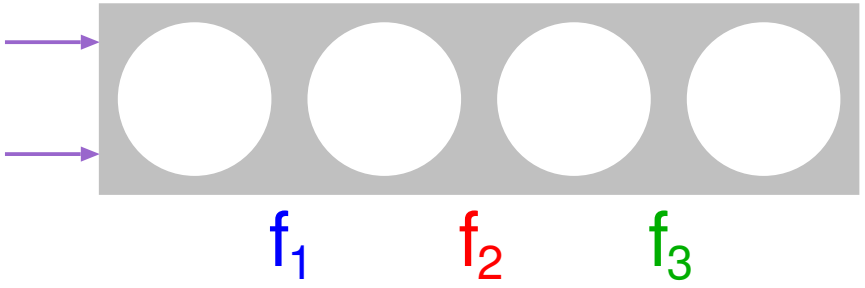
$$n \approx 1 - \delta,\ \delta \sim 10^{-5}$$
$$f \sim 100\text{m!}$$



x-ray lenses are complementary to those for visible light
getting manageable focal distances requires making compound lenses

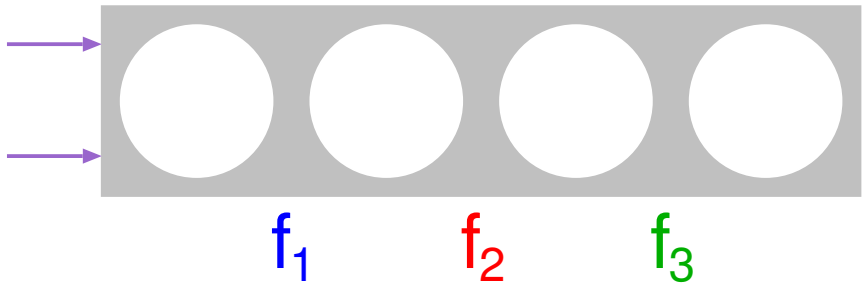


Focal Length of Compound Lens



Start with a 3-element compound lens, calculate effective focal length

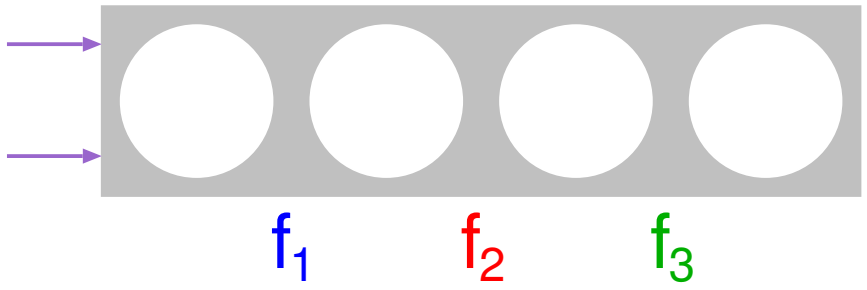
Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

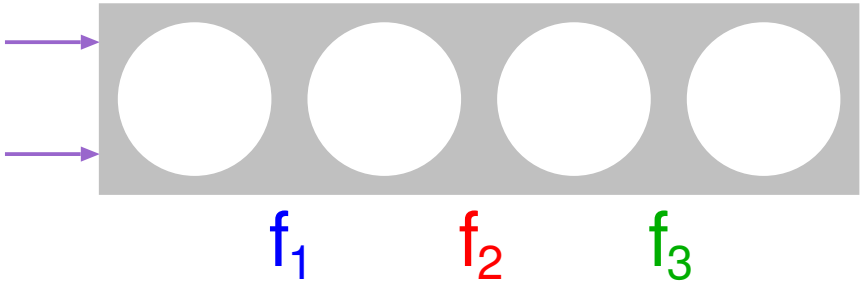
Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

Focal Length of Compound Lens

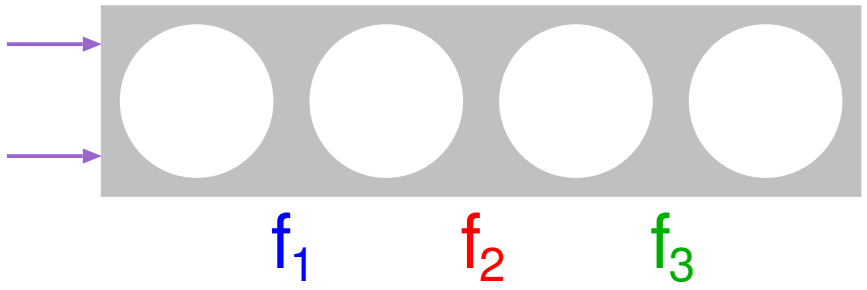


Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1}$$

Focal Length of Compound Lens



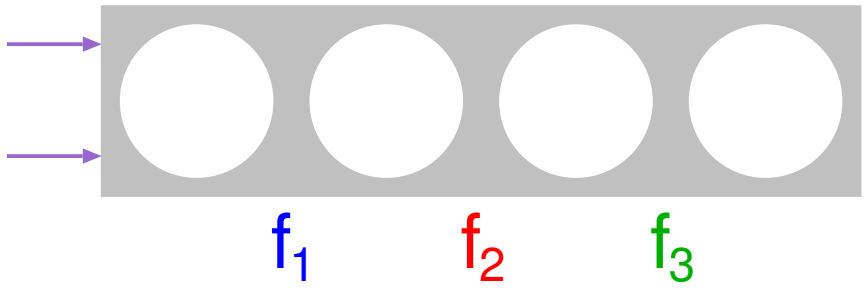
$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

Focal Length of Compound Lens



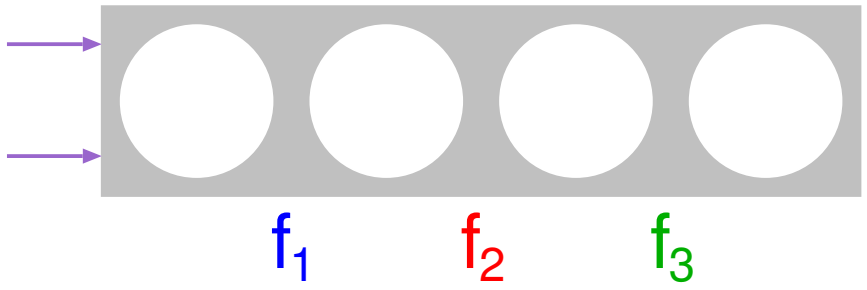
$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

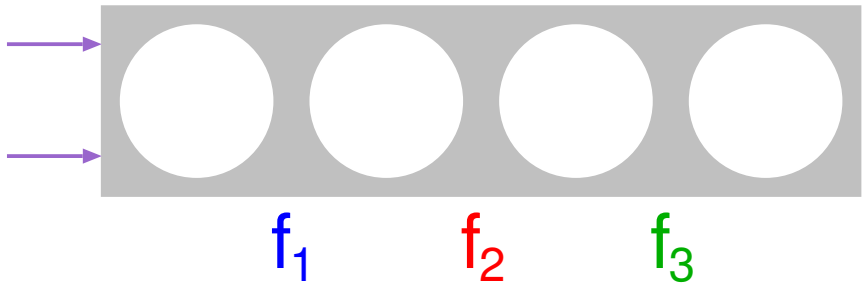
$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

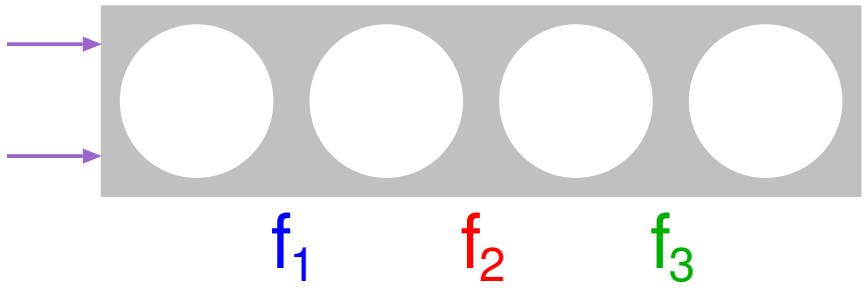
$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{1}{f}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

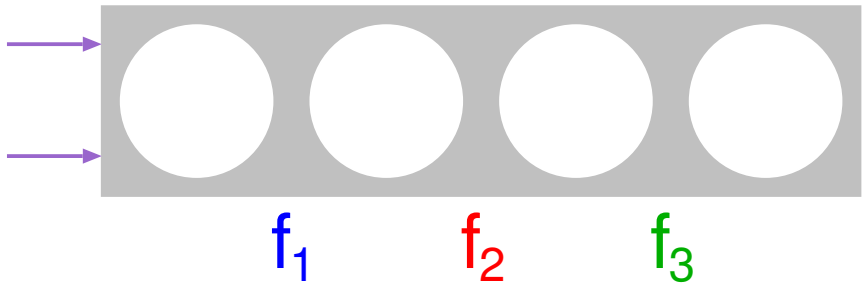
$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{1}{f} \rightarrow i_2 = \frac{f}{2}$$

Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{1}{f} \rightarrow i_2 = \frac{f}{2}$$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2}$$

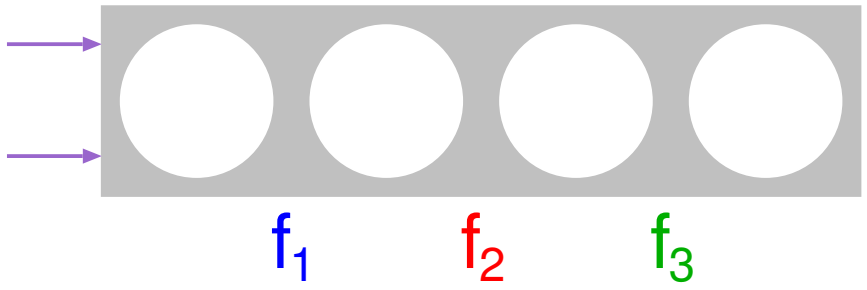
Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

similarly for the third lens, $o_3 = -i_2$

Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{1}{f} \rightarrow i_2 = \frac{f}{2}$$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{2}{f}$$

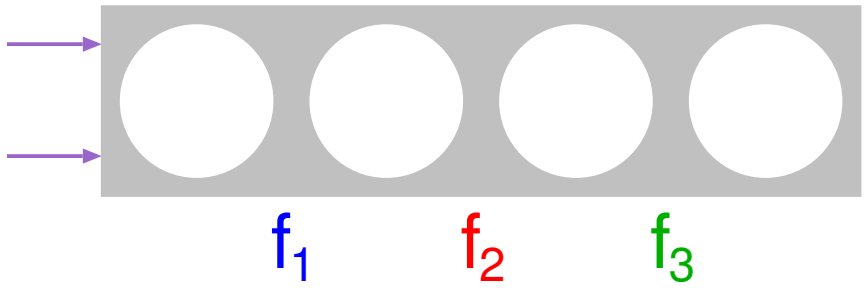
Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

similarly for the third lens, $o_3 = -i_2$

Focal Length of Compound Lens



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{1}{f} \rightarrow i_2 = \frac{f}{2}$$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{2}{f} \rightarrow i_2 = \frac{f}{3}$$

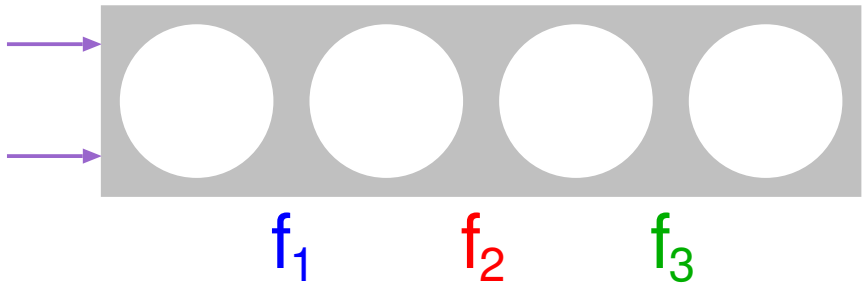
Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$f_1 = f$, $o_1 = \infty$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

similarly for the third lens, $o_3 = -i_2$

Focal Length of Compound Lens



Start with a 3-element compound lens, calculate effective focal length assuming each lens has the same focal length, f

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$f_1 = f$, $o_1 = \infty$

$$\frac{1}{i_1} = \frac{1}{f_1} + \frac{1}{o_1} \rightarrow \frac{1}{i_1} = \frac{1}{f} \rightarrow i_1 = f$$

for the second lens, the image i_1 is a virtual object, $o_2 = -i_1$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{1}{f} \rightarrow i_2 = \frac{f}{2}$$

similarly for the third lens, $o_3 = -i_2$

$$\frac{1}{i_2} = \frac{1}{f_2} + \frac{1}{o_2} \rightarrow \frac{1}{i_2} = \frac{1}{f} + \frac{2}{f} \rightarrow i_2 = \frac{f}{3}$$

so for N lenses $f_{eff} = f/N$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.

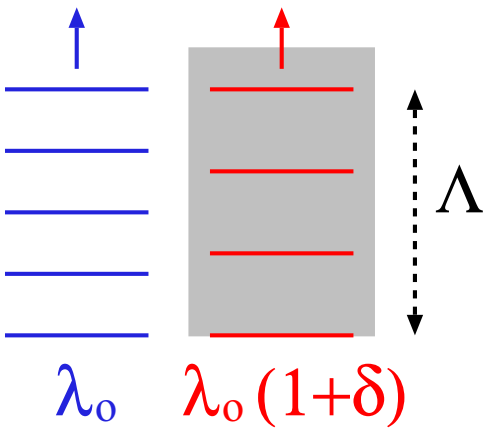
Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.

consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

Rephasing Distance

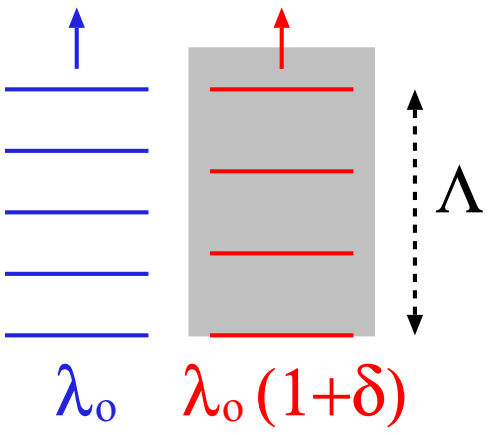
A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



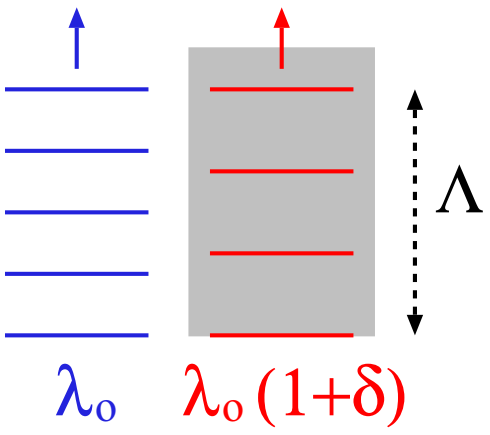
consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

if the two waves start in phase, they will be in phase once again after a distance

$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

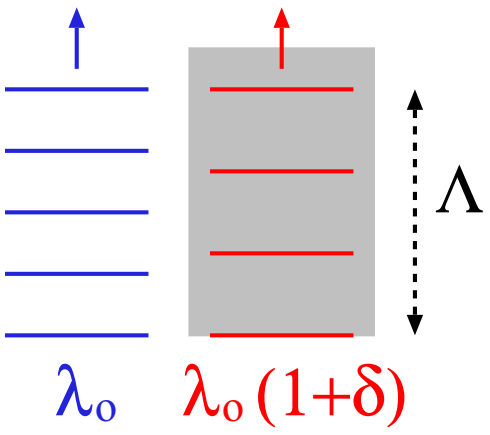
if the two waves start in phase, they will be in phase once again after a distance

$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

$$N\lambda_o + \lambda_o = N\lambda_o + N\delta\lambda_o$$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

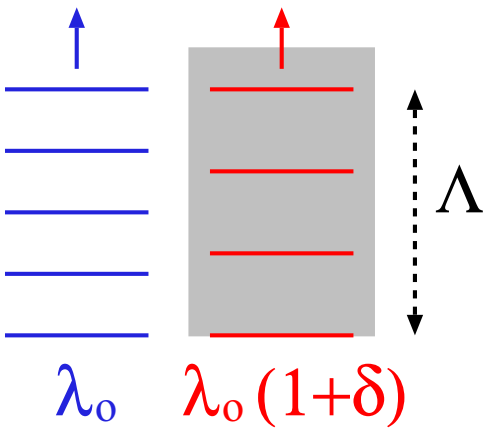
if the two waves start in phase, they will be in phase once again after a distance

$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

$$N\lambda_o + \lambda_o = N\lambda_o + N\delta\lambda_o \quad \longrightarrow \quad \lambda_o = N\delta\lambda_o$$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

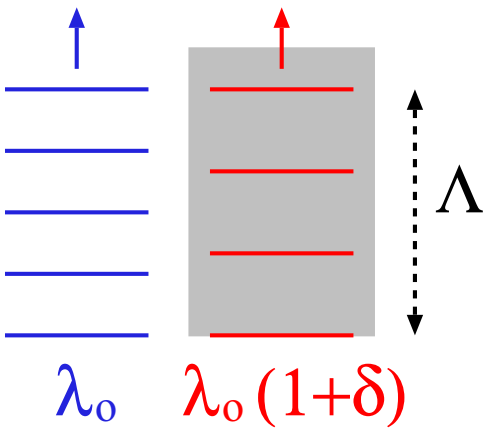
if the two waves start in phase, they will be in phase once again after a distance

$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

$$N\lambda_o + \lambda_o = N\lambda_o + N\delta\lambda_o \quad \longrightarrow \quad \lambda_o = N\delta\lambda_o \quad \longrightarrow \quad N = \frac{1}{\delta}$$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

if the two waves start in phase, they will be in phase once again after a distance

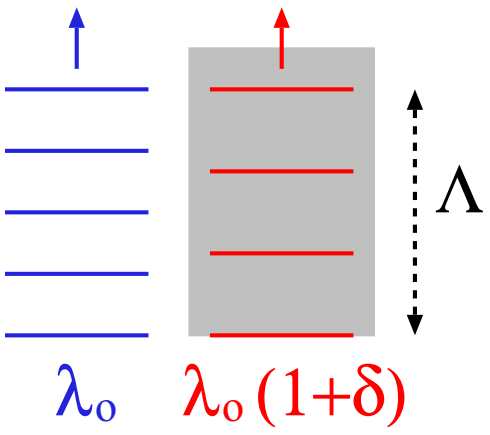
$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

$$N\lambda_o + \lambda_o = N\lambda_o + N\delta\lambda_o \quad \longrightarrow \quad \lambda_o = N\delta\lambda_o \quad \longrightarrow \quad N = \frac{1}{\delta}$$

$$\Lambda = N\lambda_o = \frac{\lambda_o}{\delta}$$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

if the two waves start in phase, they will be in phase once again after a distance

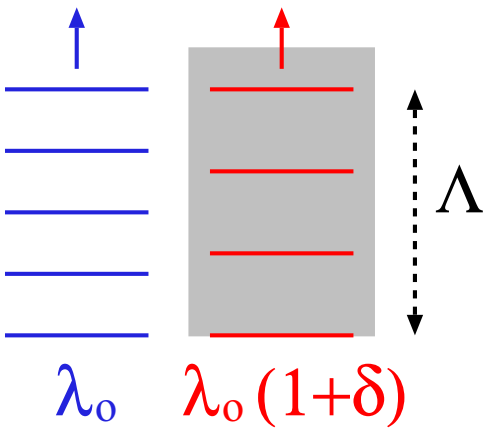
$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

$$N\lambda_o + \lambda_o = N\lambda_o + N\delta\lambda_o \quad \longrightarrow \quad \lambda_o = N\delta\lambda_o \quad \longrightarrow \quad N = \frac{1}{\delta}$$

$$\Lambda = N\lambda_o = \frac{\lambda_o}{\delta} = \frac{2\pi}{\lambda_o r_o \rho}$$

Rephasing Distance

A spherical surface is not the ideal lens as it introduces aberrations. Derive the ideal shape for perfect focusing of x-rays.



consider two waves, one traveling inside the solid and the other in vacuum, $\lambda = \lambda_o/(1-\delta) \approx \lambda_o(1+\delta)$

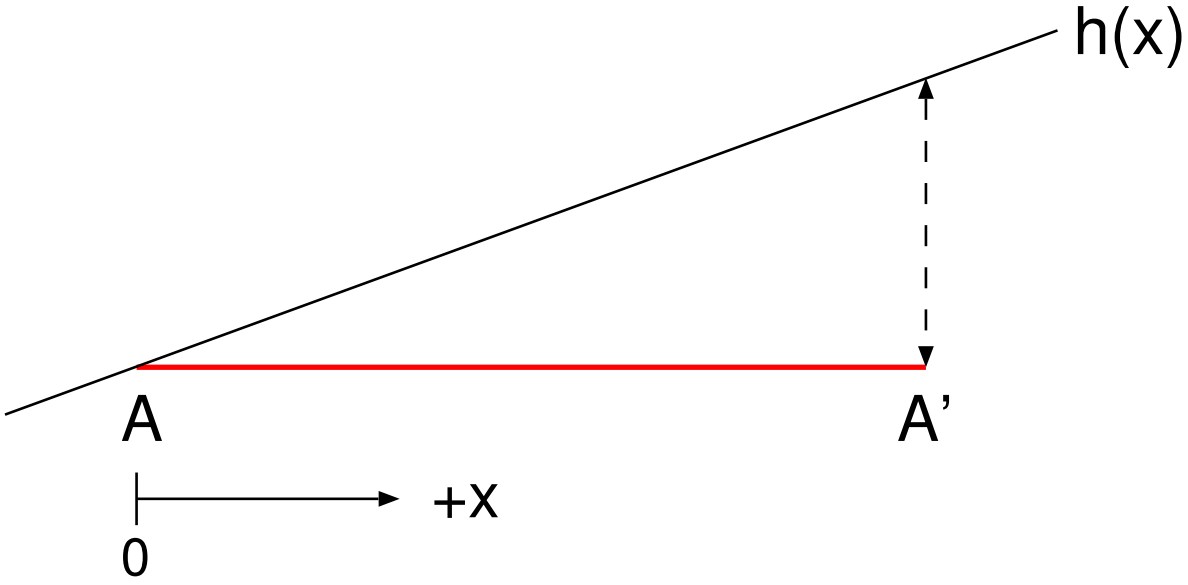
if the two waves start in phase, they will be in phase once again after a distance

$$\Lambda = (N+1)\lambda_o = N\lambda_o(1+\delta)$$

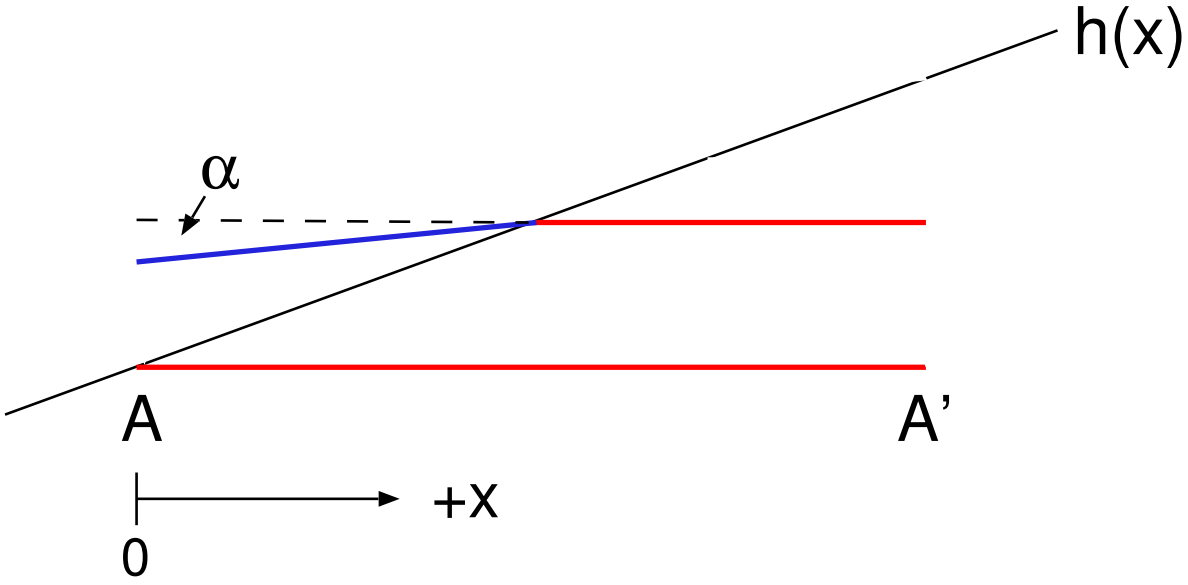
$$N\lambda_o + \lambda_o = N\lambda_o + N\delta\lambda_o \quad \longrightarrow \quad \lambda_o = N\delta\lambda_o \quad \longrightarrow \quad N = \frac{1}{\delta}$$

$$\Lambda = N\lambda_o = \frac{\lambda_o}{\delta} = \frac{2\pi}{\lambda_o r_o \rho} \approx 10\mu\text{m}$$

Ideal Interface Profile

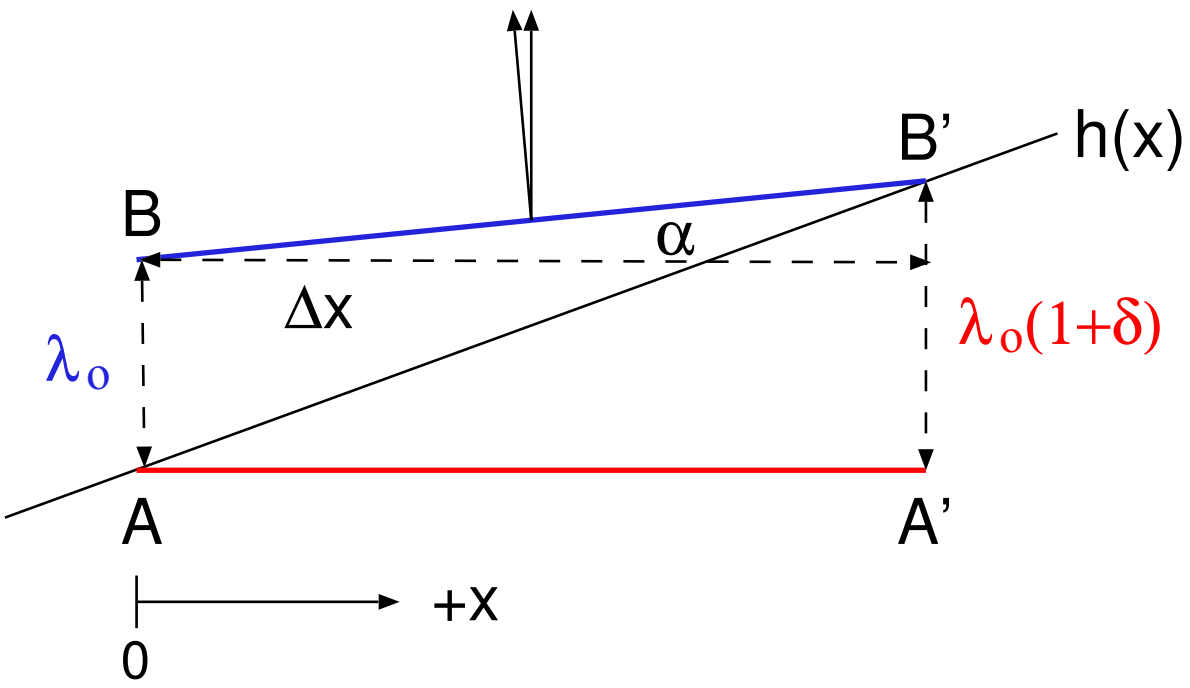


Ideal Interface Profile



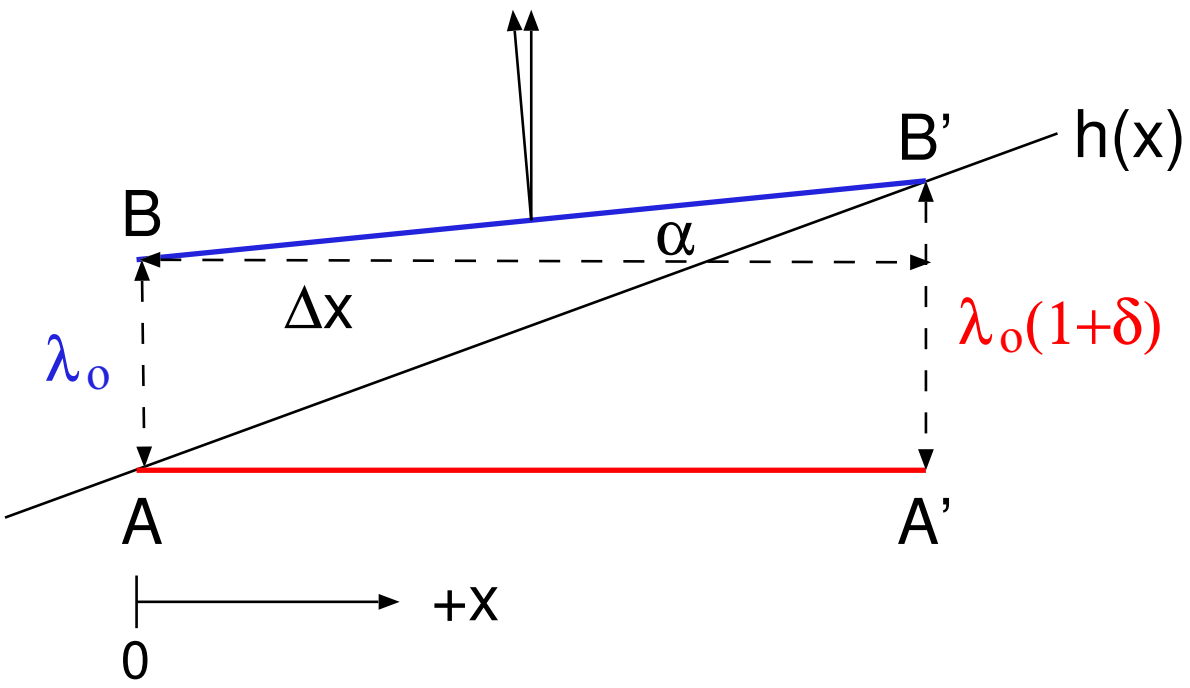
The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α.

Ideal Interface Profile



The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α. Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

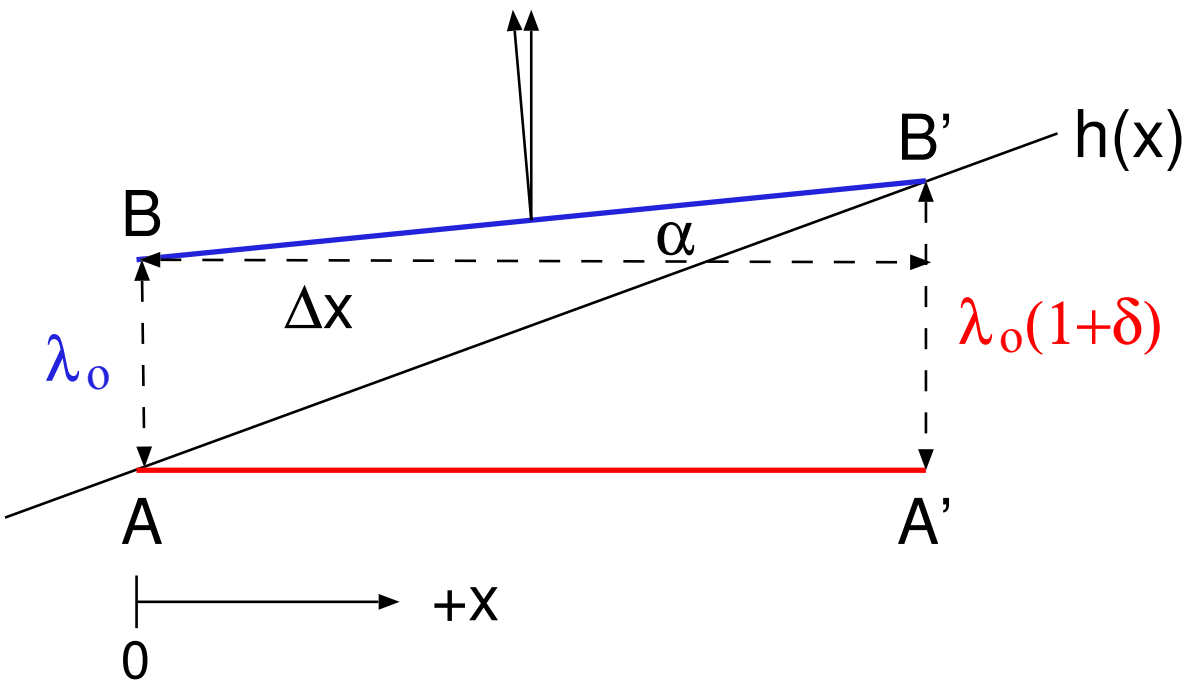
Ideal Interface Profile



The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α. Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

$$\lambda_o\,(1+\delta) = h'(x)\Delta x$$

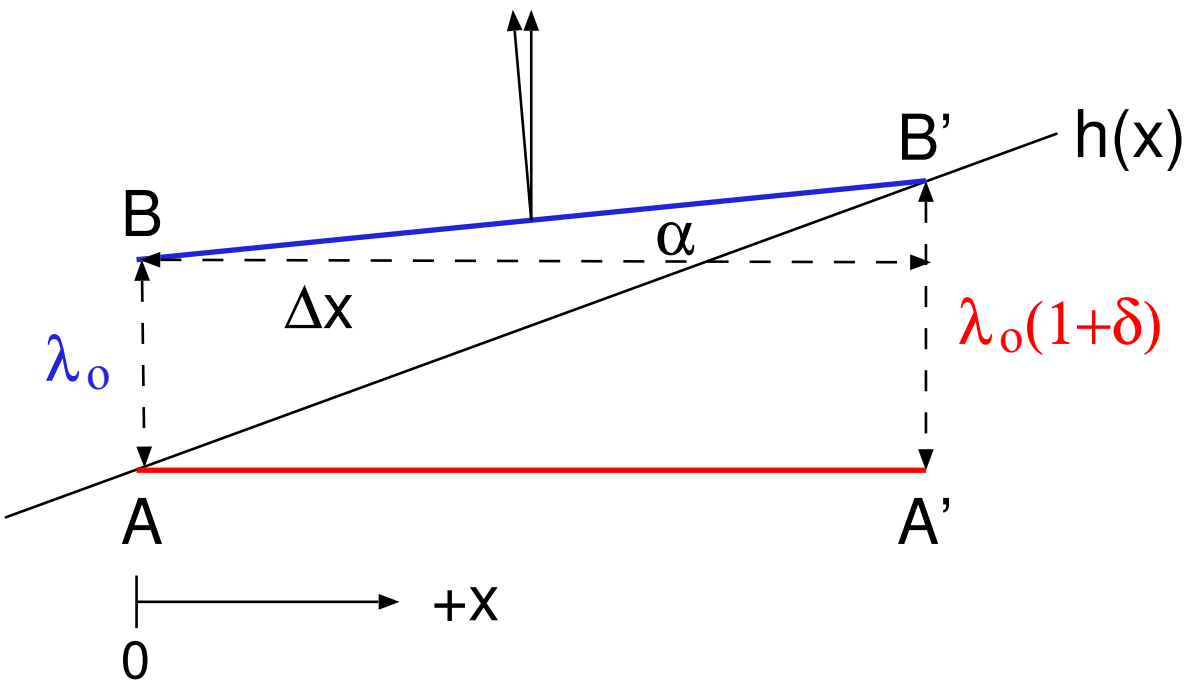
Ideal Interface Profile



The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α. Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

$$\lambda_o\,(1+\delta) = h'(x)\Delta x \quad \longrightarrow \quad \Delta x \approx \frac{\lambda_o}{h'(x)}$$

Ideal Interface Profile

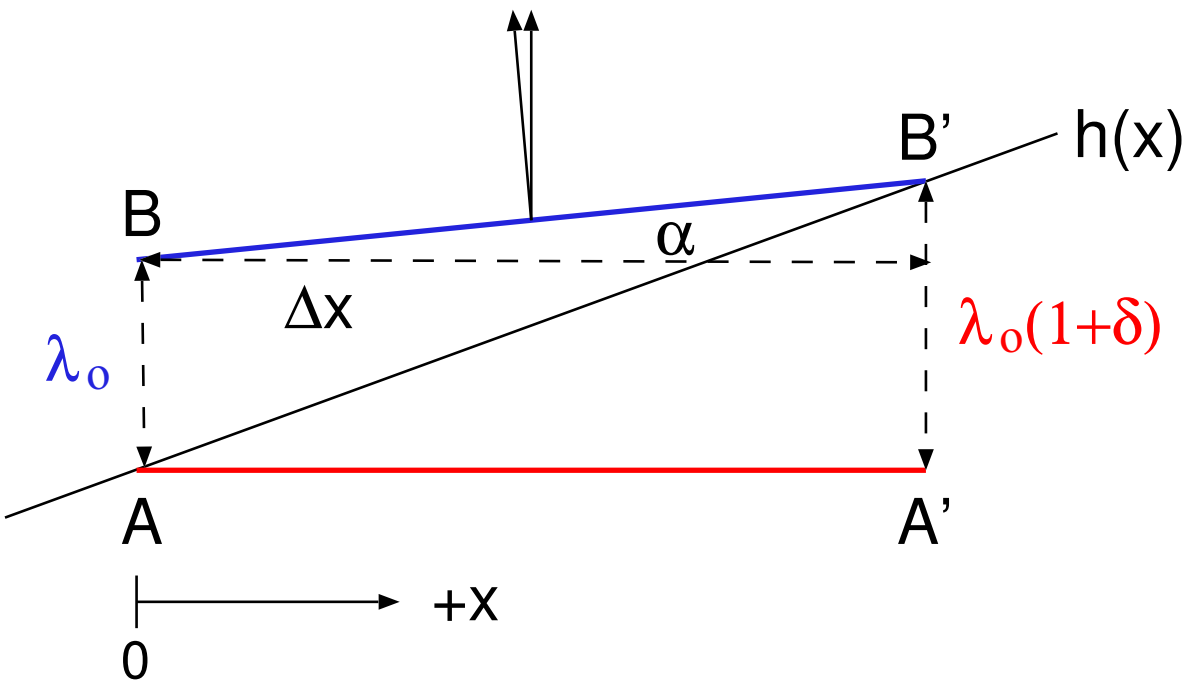


The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α. Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

$$\lambda_o\,(1+\delta) = h'(x)\Delta x \quad \longrightarrow \quad \Delta x \approx \frac{\lambda_o}{h'(x)}$$

$$\alpha(x) = \frac{\lambda_o \delta}{\Delta x}$$

Ideal Interface Profile

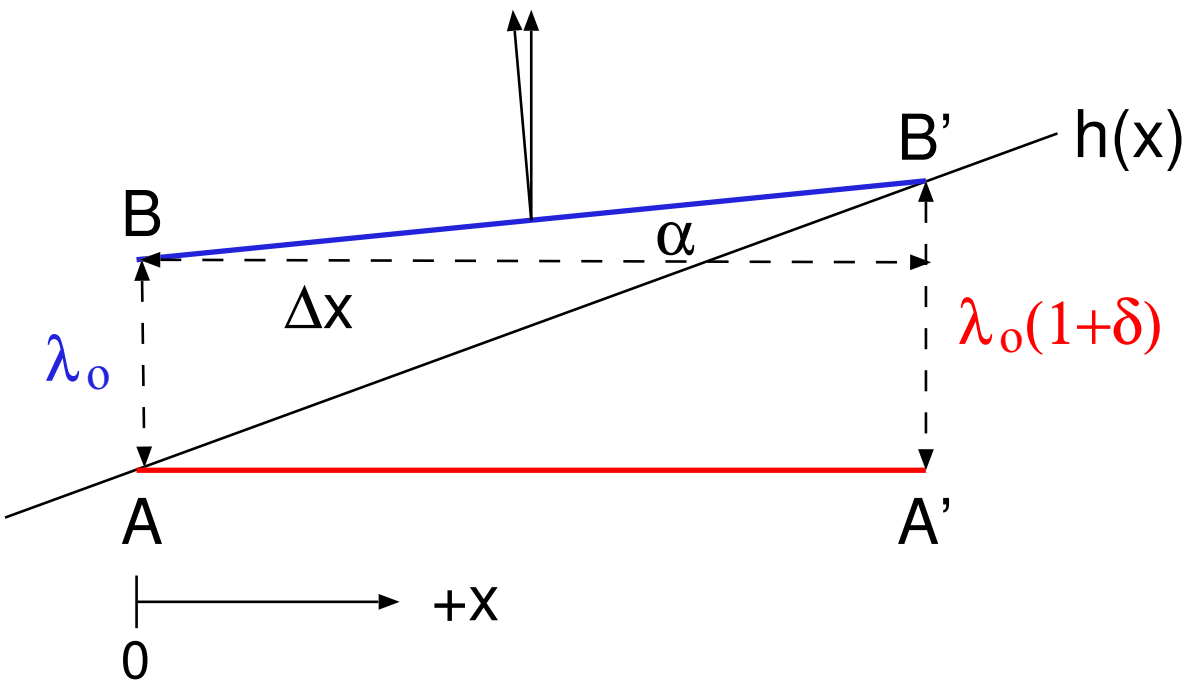


The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α. Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

$$\lambda_o\,(1+\delta) = h'(x)\Delta x \quad \longrightarrow \quad \Delta x \approx \frac{\lambda_o}{h'(x)}$$

$$\alpha(x) = \frac{\lambda_o \delta}{\Delta x} = h'(x)\delta$$

Ideal Interface Profile

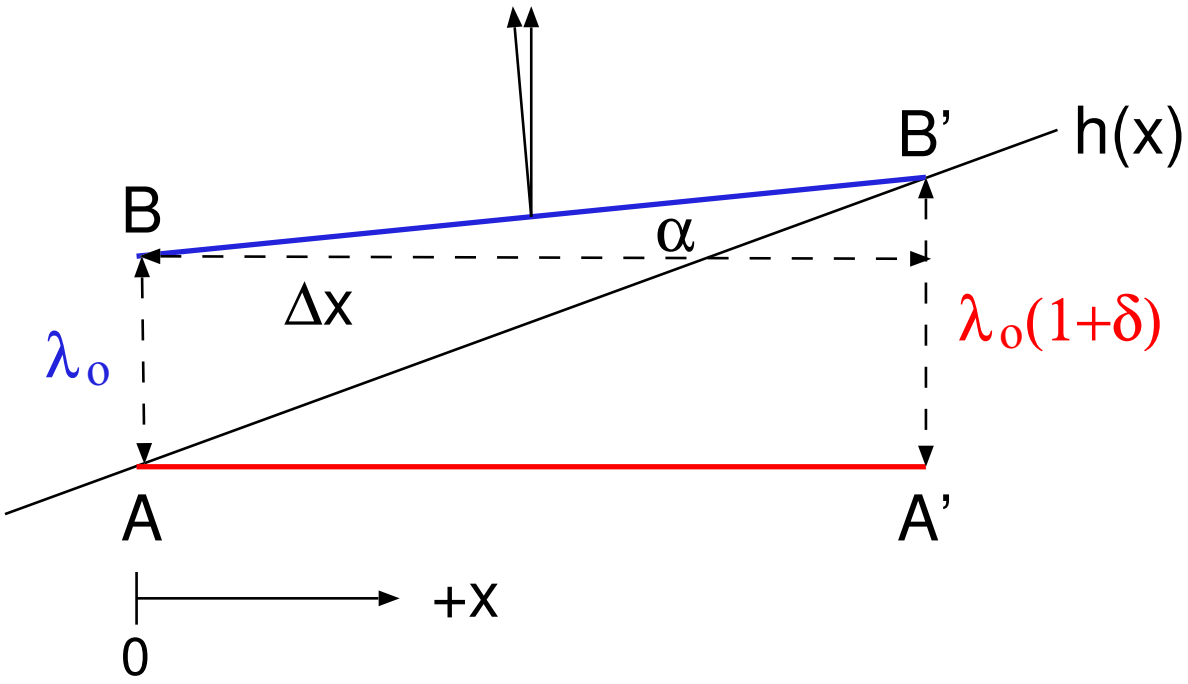


The wave exits the material into vacuum through a surface of profile $h(x)$, and is twisted by an angle α. Follow the path of two points on the wave-front, A and A' as they propagate to B and B'.

$$\lambda_o\,(1+\delta) = h'(x)\Delta x \quad \longrightarrow \quad \Delta x \approx \frac{\lambda_o}{h'(x)}$$

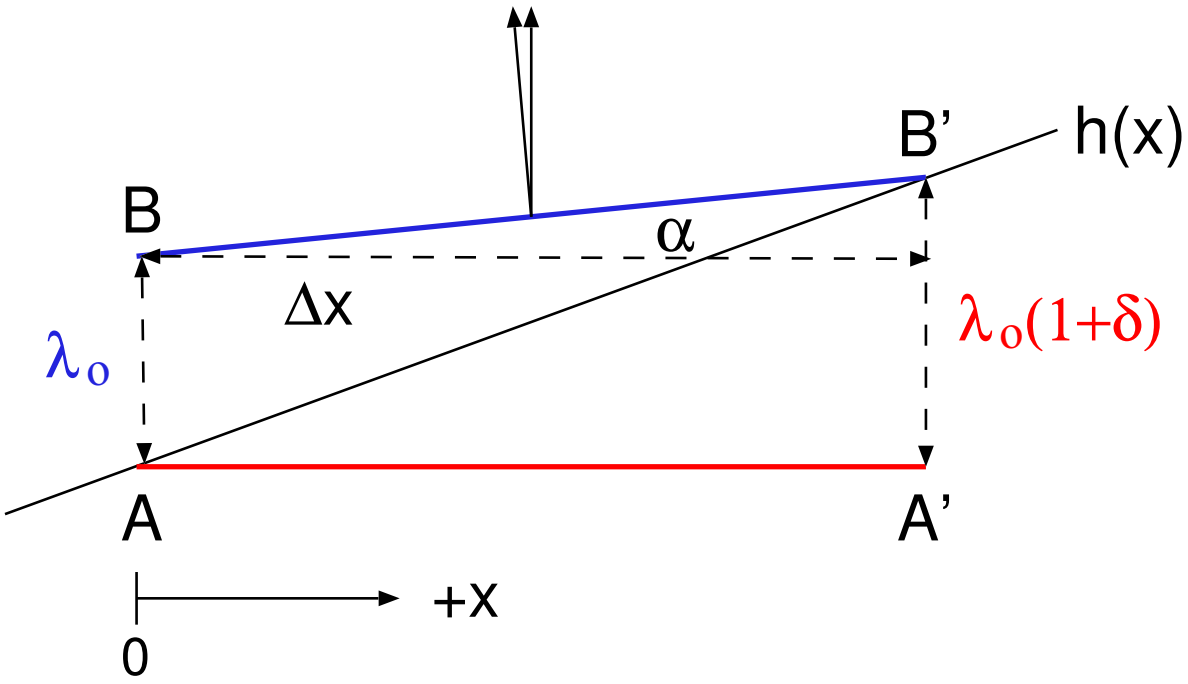
$$\alpha(x) = \frac{\lambda_o \delta}{\Delta x} = h'(x)\delta = h'(x)\frac{\lambda_o}{\Lambda}$$

Ideal Interface Profile



If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

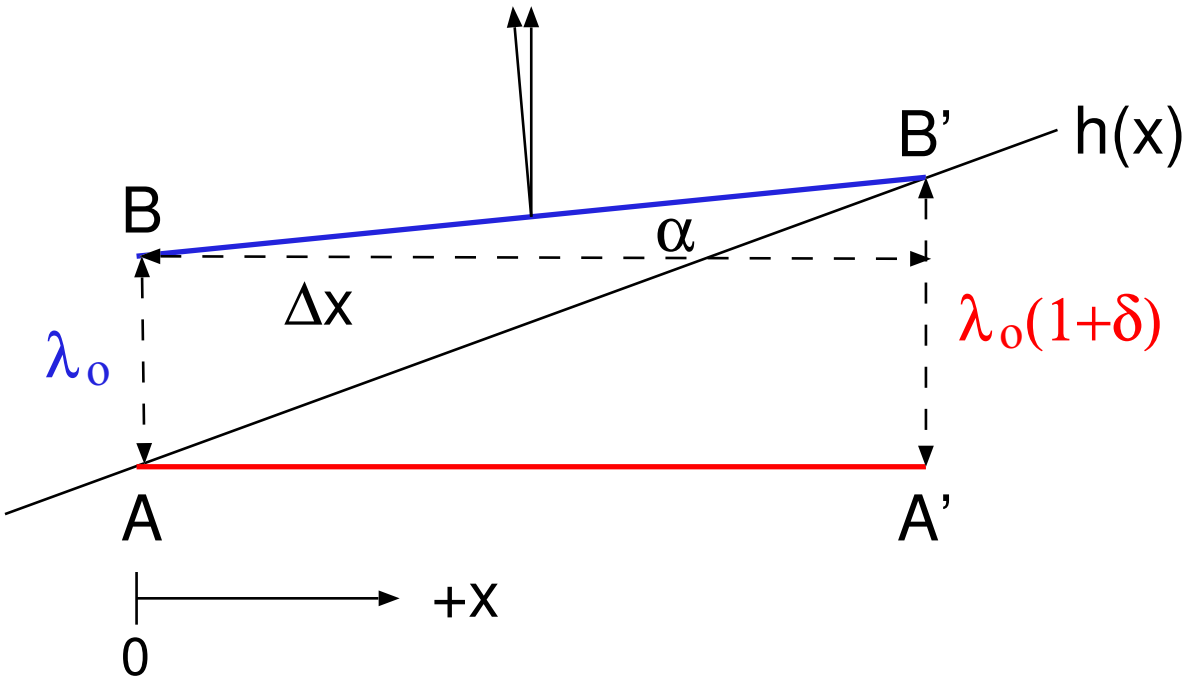
Ideal Interface Profile



If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

$$\alpha(x) = \frac{x}{f}$$

Ideal Interface Profile



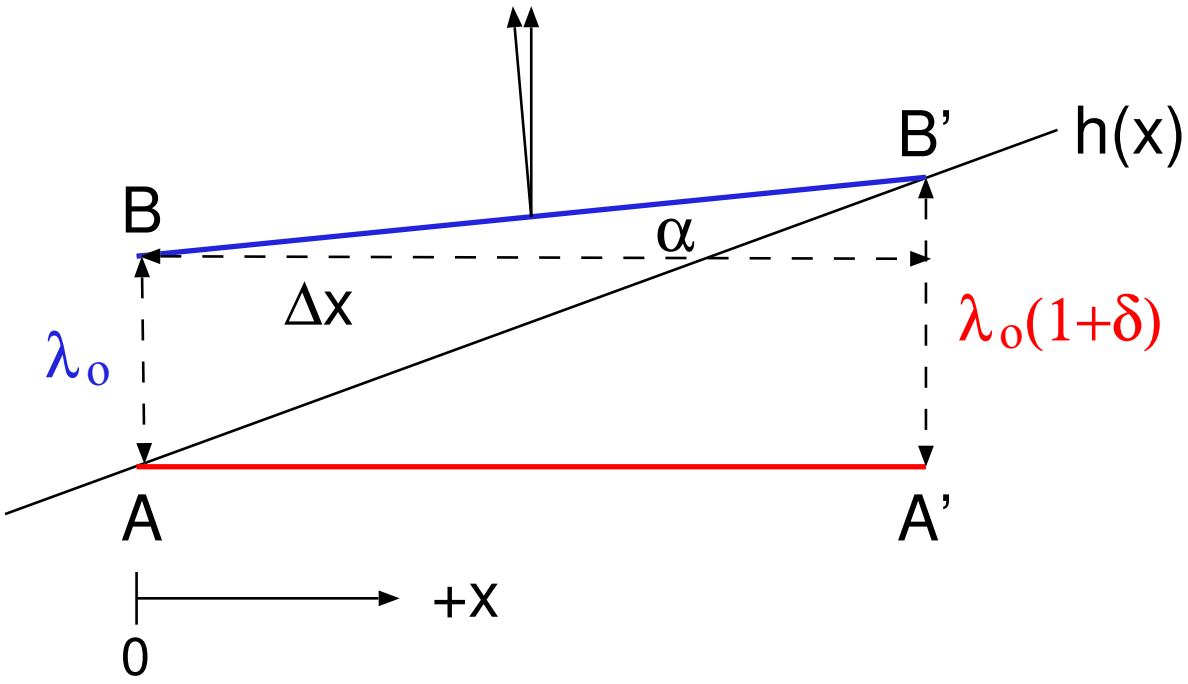
If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

$$\alpha(x) = \frac{x}{f}$$

combining, we have

$$\frac{\lambda_o h'(x)}{\Lambda} = \frac{x}{f}$$

Ideal Interface Profile



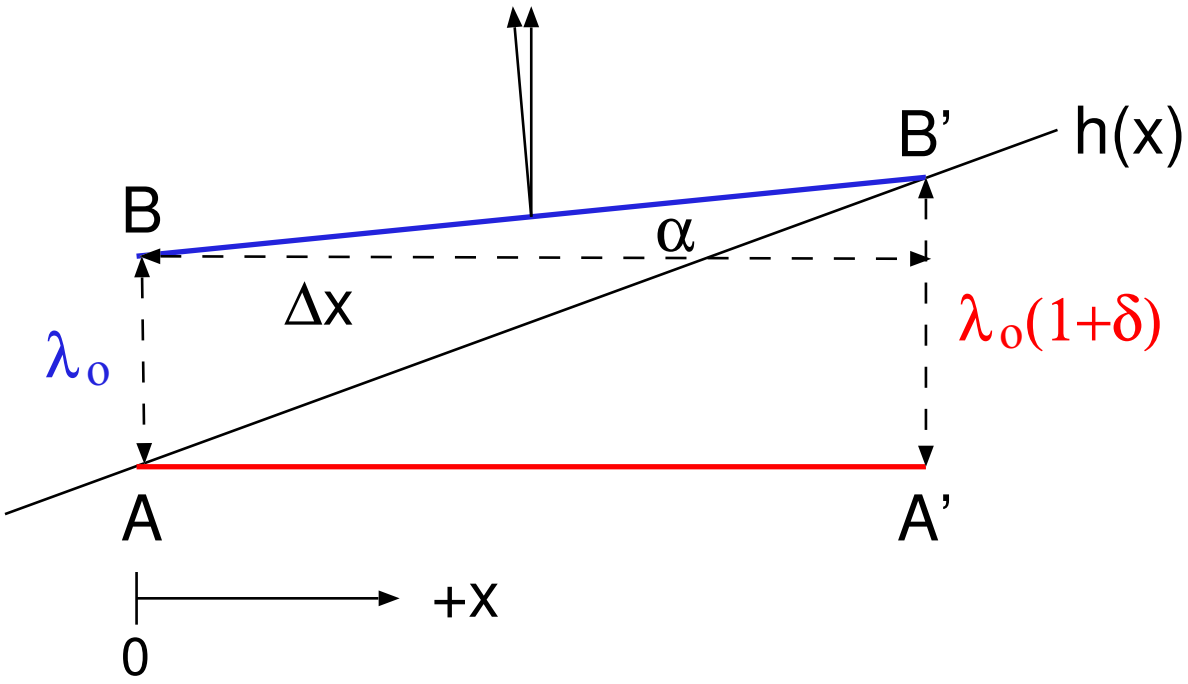
If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

$$\alpha(x) = \frac{x}{f}$$

combining, we have

$$\frac{\lambda_o h'(x)}{\Lambda} = \frac{x}{f} \longrightarrow \frac{h'(x)}{\Lambda} = \frac{x}{f\lambda_o}$$

Ideal Interface Profile



If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

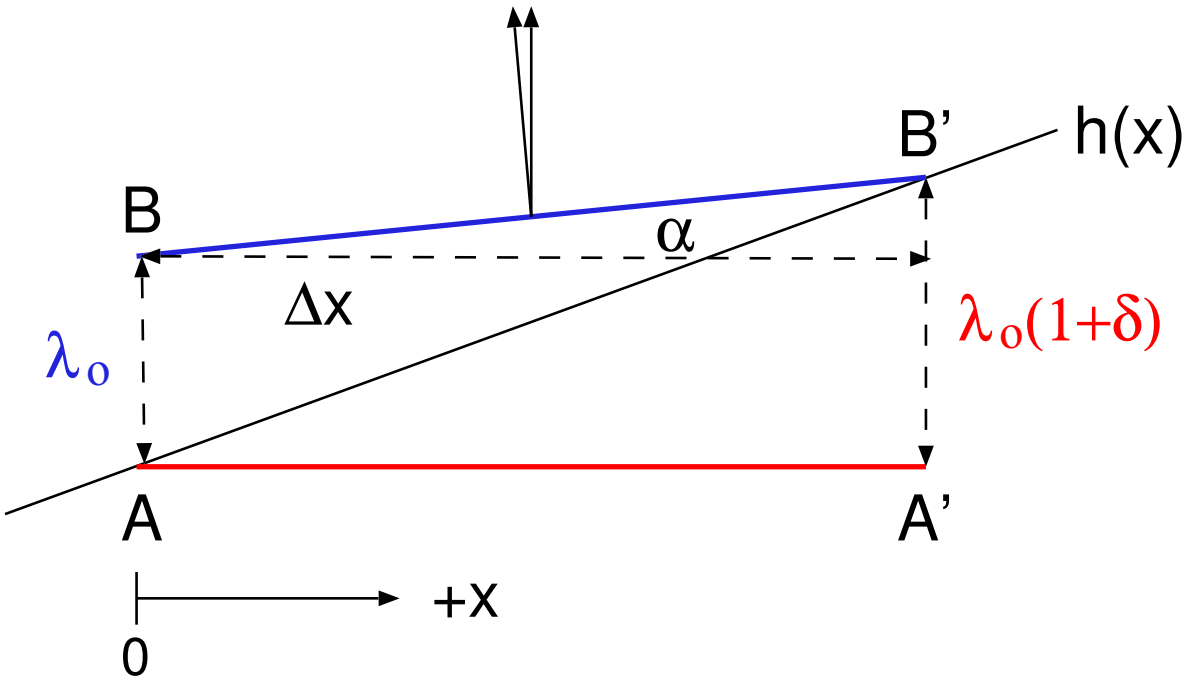
$$\alpha(x) = \frac{x}{f}$$

combining, we have

$$\frac{\lambda_o h'(x)}{\Lambda} = \frac{x}{f} \longrightarrow \frac{h'(x)}{\Lambda} = \frac{x}{f\lambda_o}$$

this can be easily integrated

Ideal Interface Profile



If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

$$\alpha(x) = \frac{x}{f}$$

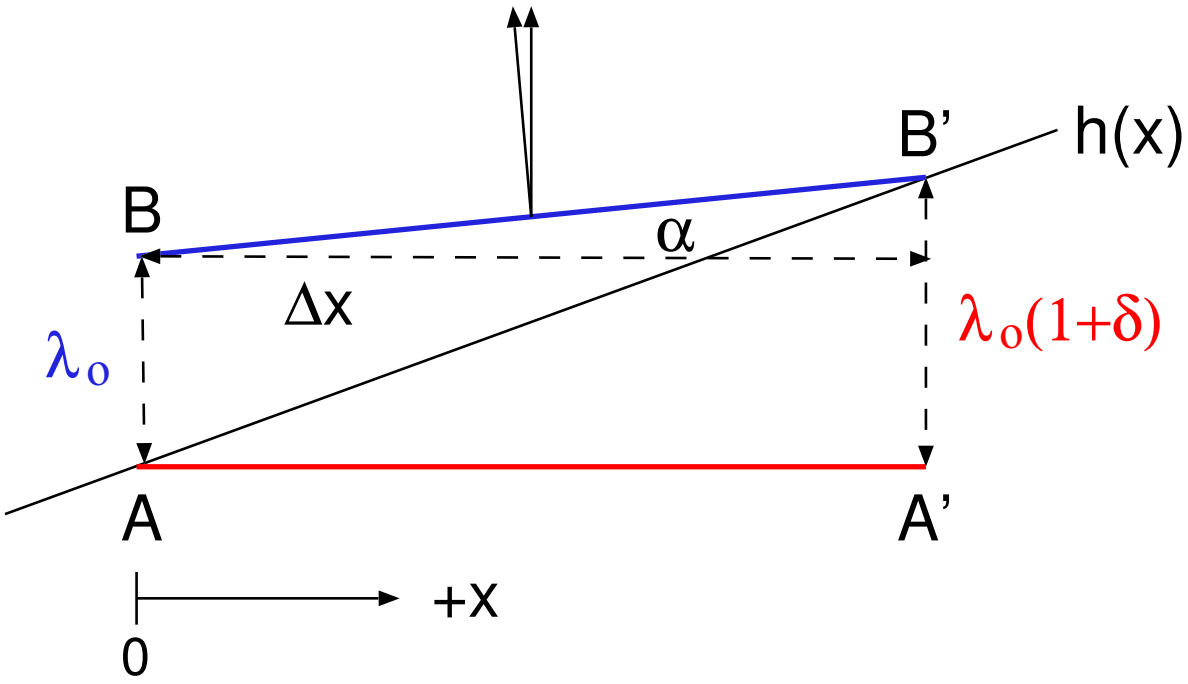
combining, we have

$$\frac{\lambda_o h'(x)}{\Lambda} = \frac{x}{f} \longrightarrow \frac{h'(x)}{\Lambda} = \frac{x}{f\lambda_o}$$

this can be easily integrated

$$\frac{h(x)}{\Lambda} = \frac{x^2}{2f\lambda_o}$$

Ideal Interface Profile



If the desired focal length of this lens is f, the wave must be redirected at an angle which, depends on the distance from the optical axis

$$\alpha(x) = \frac{x}{f}$$

combining, we have

$$\frac{\lambda_o h'(x)}{\Lambda} = \frac{x}{f} \longrightarrow \frac{h'(x)}{\Lambda} = \frac{x}{f\lambda_o}$$

this can be easily integrated

$$\frac{h(x)}{\Lambda} = \frac{x^2}{2f\lambda_o} = \left[\frac{x}{\sqrt{2f\lambda_o}}\right]^2$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)}$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta} \frac{x^2}{h(x)}$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

If the surface is a circle instead of a parabola

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

If the surface is a circle instead of a parabola

$$h(x) = \sqrt{R^2 - x^2}$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

If the surface is a circle instead of a parabola

$$h(x) = \sqrt{R^2 - x^2} \qquad \text{and for } x \ll R \qquad h(x) \approx R\left(1 - \frac{1}{2}\frac{x^2}{R^2}\right)$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

If the surface is a circle instead of a parabola

$$h(x) = \sqrt{R^2 - x^2} \qquad \text{and for } x \ll R \qquad h(x) \approx R\left(1 - \frac{1}{2}\frac{x^2}{R^2}\right)$$

$$x^2 = R^2 - h^2(x) \approx R^2$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

If the surface is a circle instead of a parabola

$$h(x) = \sqrt{R^2 - x^2} \qquad \text{and for } x \ll R \qquad h(x) \approx R\left(1 - \frac{1}{2}\frac{x^2}{R^2}\right)$$

$$x^2 = R^2 - h^2(x) \approx R^2 \qquad \text{so we have} \qquad f \approx \frac{R}{2\delta}$$

Focal length of circular lens

From the previous expression for the ideal parabolic surface, the focal length can be written in terms of the surface profile.

$$f = \frac{x^2 \Lambda}{2\lambda_o h(x)} = \frac{1}{2\delta}\frac{x^2}{h(x)} \qquad \text{or alternatively} \qquad f = \frac{1}{\delta}\frac{x}{h'(x)}$$

If the surface is a circle instead of a parabola

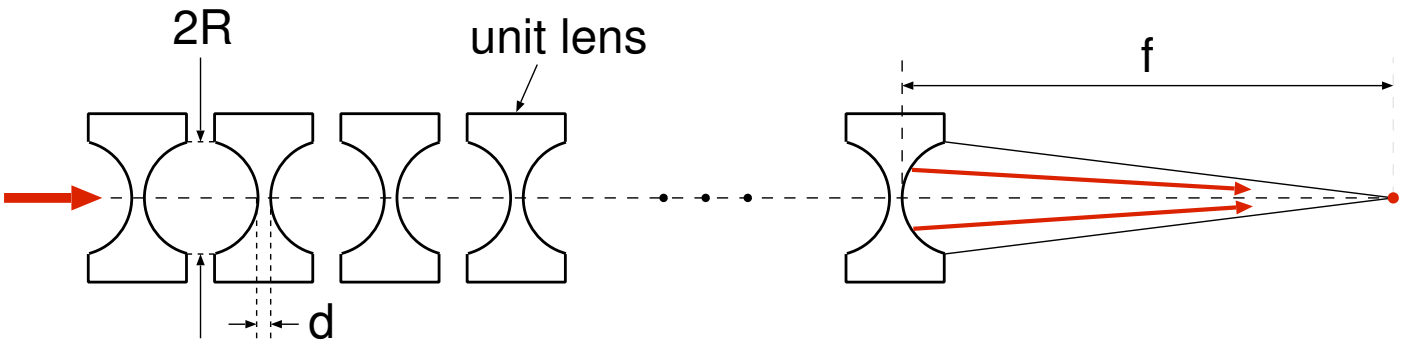
$$h(x) = \sqrt{R^2 - x^2} \qquad \text{and for } x \ll R \qquad h(x) \approx R\left(1 - \frac{1}{2}\frac{x^2}{R^2}\right)$$

$$x^2 = R^2 - h^2(x) \approx R^2 \qquad \text{so we have} \qquad f \approx \frac{R}{2\delta}$$

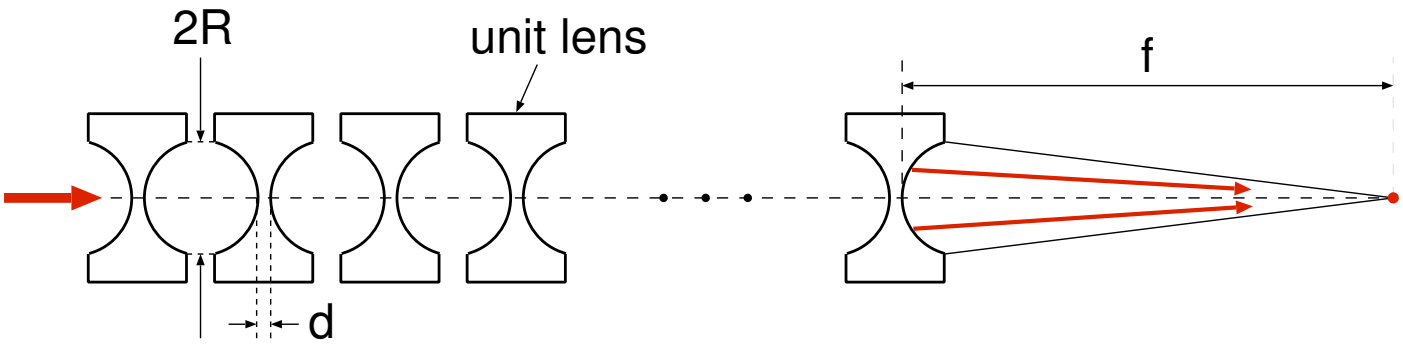
for N circular lenses

$$f_n \approx \frac{R}{2N\delta}$$

Focussing by a beryllium lens



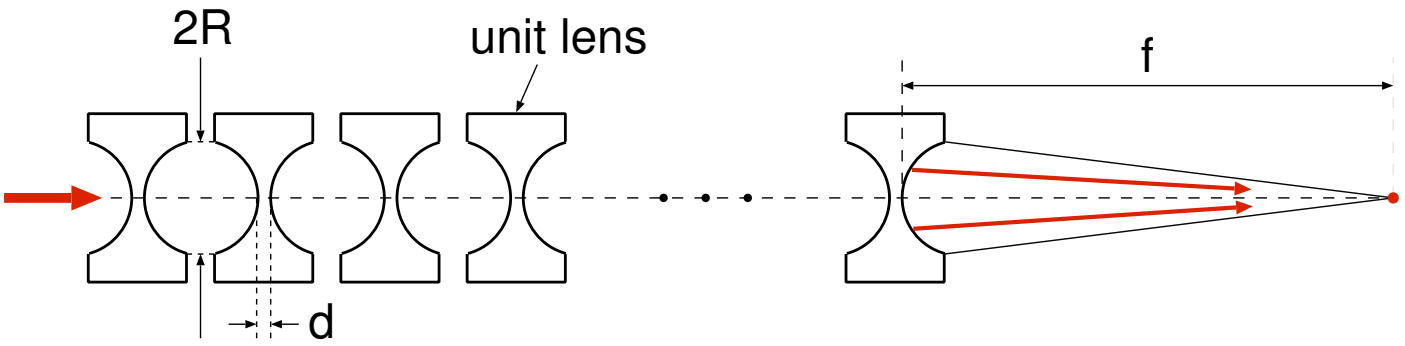
Focussing by a beryllium lens



For 50 holes of radius $R = 1$mm in beryllium (Be) at $E = 10$keV, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

$$f_N = \frac{R}{2N\delta}$$

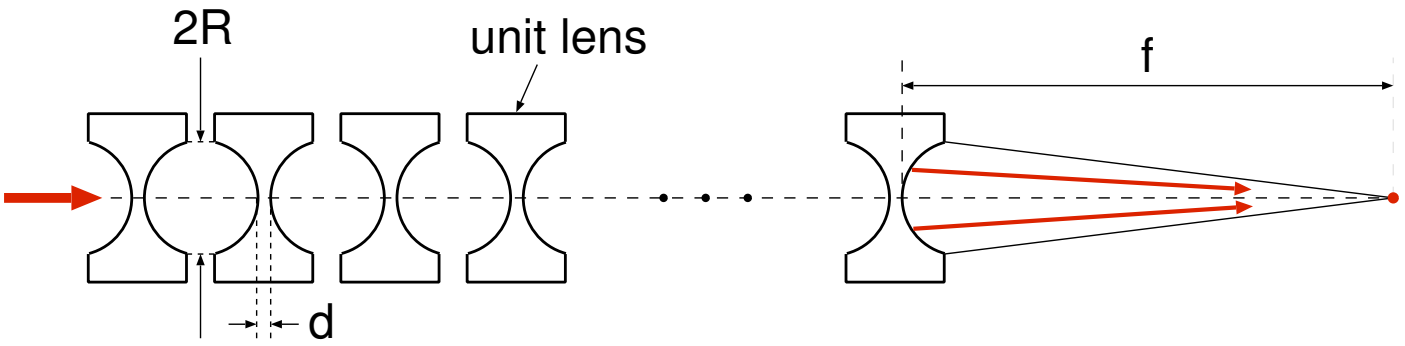
Focussing by a beryllium lens



For 50 holes of radius $R = 1$mm in beryllium (Be) at $E = 10$keV, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

$$f_N = \frac{R}{2N\delta} = \frac{1 \times 10^{-3}\text{m}}{2(50)(3.41 \times 10^{-6})}$$

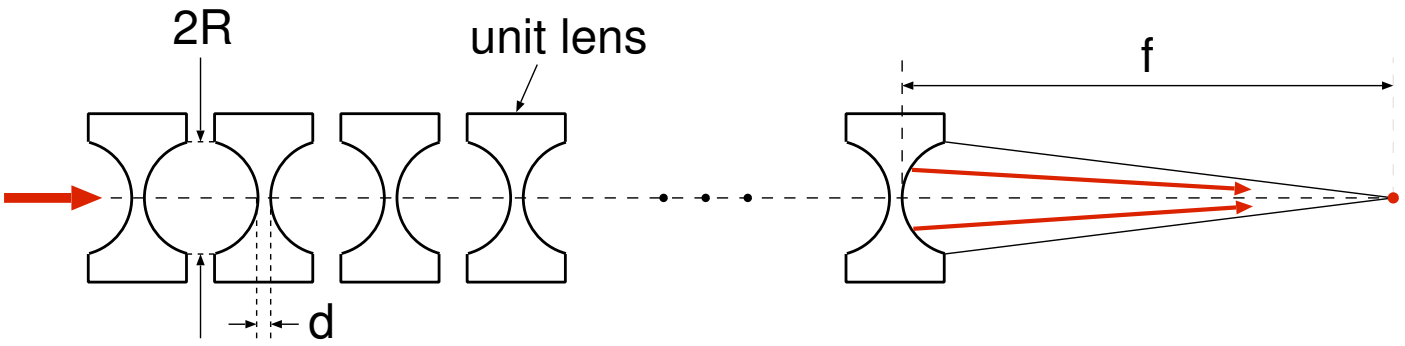
Focussing by a beryllium lens



For 50 holes of radius $R = 1$mm in beryllium (Be) at $E = 10$keV, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

$$f_N = \frac{R}{2N\delta} = \frac{1 \times 10^{-3}\text{m}}{2(50)(3.41 \times 10^{-6})} = 2.93\text{m}$$

Focussing by a beryllium lens

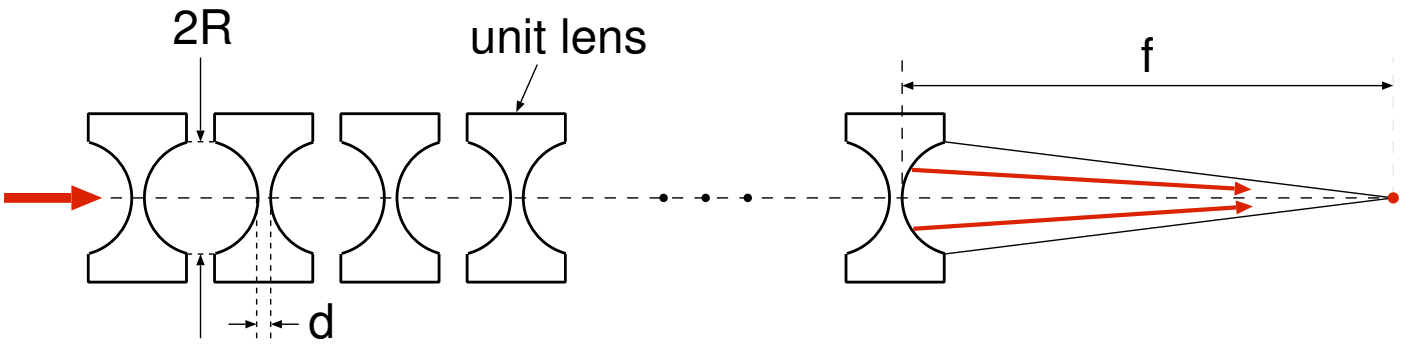


For 50 holes of radius $R = 1$mm in beryllium (Be) at $E = 10$keV, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

$$f_N = \frac{R}{2N\delta} = \frac{1 \times 10^{-3}\text{m}}{2(50)(3.41 \times 10^{-6})} = 2.93\text{m}$$

depending on the wall thickness of the lenslets, the transmission can be up to 74%

Focussing by a beryllium lens



For 50 holes of radius $R = 1$mm in beryllium (Be) at $E = 10$keV, we can calculate the focal length, knowing $\delta = 3.41 \times 10^{-6}$

$$f_N = \frac{R}{2N\delta} = \frac{1 \times 10^{-3}\text{m}}{2(50)(3.41 \times 10^{-6})} = 2.93\text{m}$$

depending on the wall thickness of the lenslets, the transmission can be up to 74%

H.R. Beguiristain, J.T. Cremer, M.A. Piestrup, C.K. Gary, and R.H. Pantell, *Optics Letters*, **27**, *778 (2007).*