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- Models of Surfaces
- Reflectivity from the MRCAT mirror

HW #02

1. Knowing that the photoelectric absorption of an element scales as the inverse of the energy cubed, calculate:

(a) the absorption coefficient at 10keV for copper when the value at 5keV is 1698.3 cm^{-1};

(b) The actual absorption coefficient of copper at 10keV is 1942.1 cm^{-1}, why is this so different than your calculated value?

2. A 30 cm long, ionization chamber, filled with 80% helium and 20% nitrogen gases at 1 atmosphere, is being used to measure the photon rate (photons/sec) in a synchrotron beamline at 12 keV. If a current of 10 nA is measured, what is the photon flux entering the ionization chamber?

3. A 5 cm deep ionization chamber is used to measure the fluorescence from a sample containing arsenic (As). Using any noble gases or nitrogen, determine a gas fill (at 1 atmosphere) for this chamber which absorbs at least 60% of the incident photons. How does this change if you are measuring the fluorescence from ruthenium (Ru)?

HW #02

4. Calculate the critical angle of reflection of 10 keV and 30 keV x-rays for:

(a) A slab of glass (SiO_2);

(b) A thick chromium mirror;

(c) A thick platinum mirror.

(d) If the incident x-ray beam is 2 mm high, what length of mirror is required to reflect the entire beam for each material?

5. Calculate the fraction of silver (Ag) fluorescence x-rays which are absorbed in a 1 mm thick silicon (Si) detector and the charge pulse expected for each absorbed photon. Repeat the calculation for a 1 mm thick germanium (Ge) detector.

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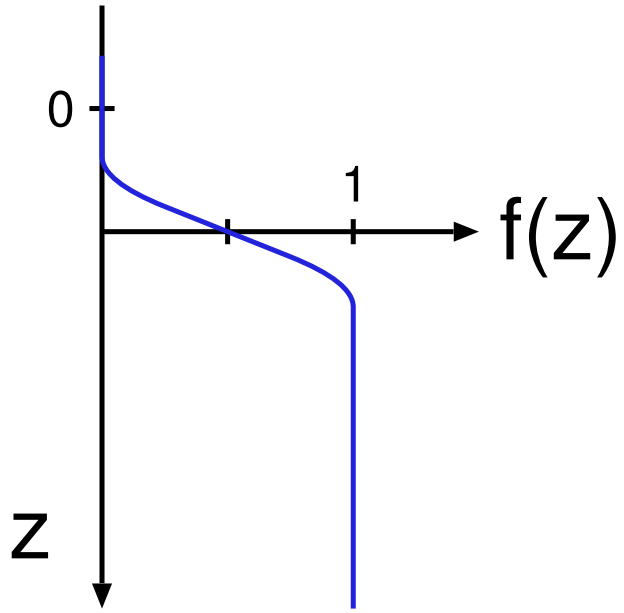

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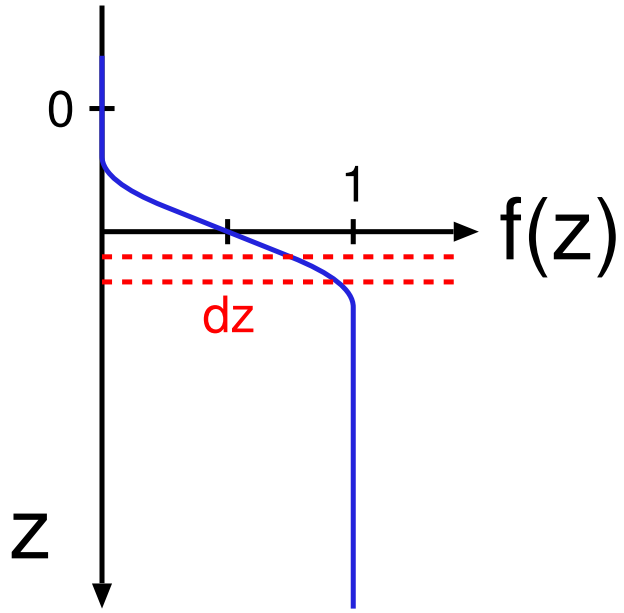

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The reflectivity of this kind of interface can be calculated best in the kinematical limit ($Q > Q_c$).

The density profile of the interface can be described by the function $f(z)$ which approaches 1 as $z \to \infty$.

The reflectivity can be computed as the superposition of the reflectivity of a series of infinitesmal slabs of thickness dz at a depth z.

Reflectivity of a Graded Interface

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Calculating the full reflection coefficient relative to the Fresnel reflection coefficient

$$\frac{R(Q)}{R_F(Q)} = \left|\int_{-\infty}^{\infty}\left(\frac{df}{dz}\right)e^{iQz}dz\right|^2$$

The Error Function - a Specific Case

The error function is often chosen as a model for the density gradient

$$f(z) = erf(\frac{z}{\sqrt{2}\sigma}) = \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{2}\sigma} e^{-t^2} dt$$

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$$Q = k \sin\theta, \qquad Q' = k' \sin\theta'$$

Rough Surfaces

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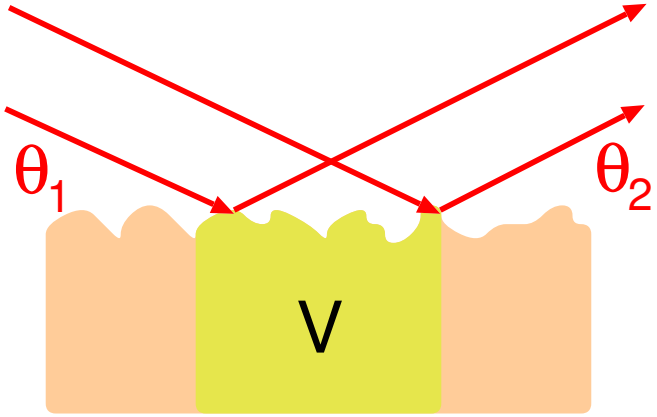

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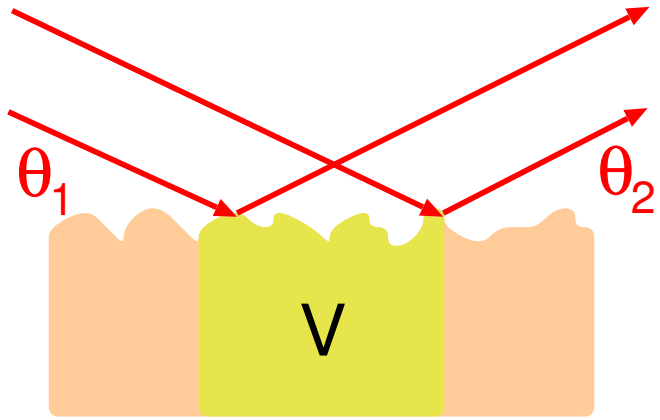

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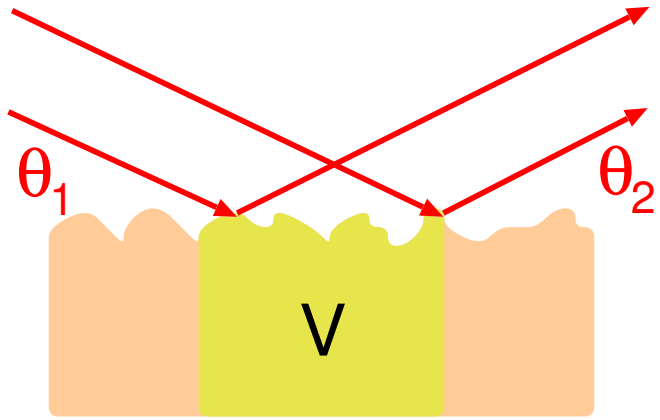

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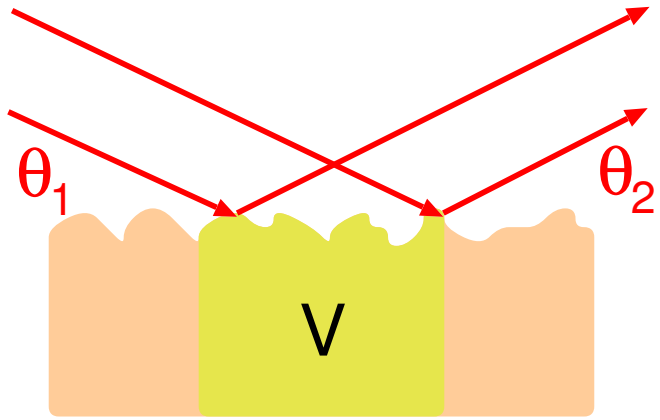

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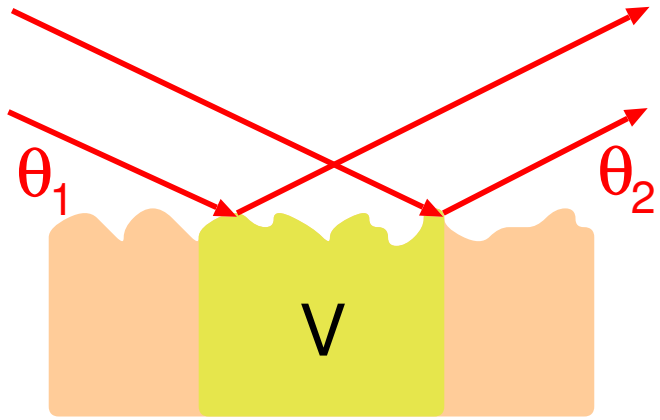

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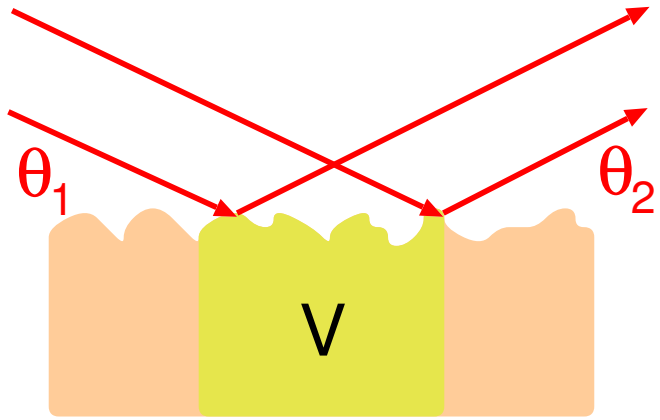

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The actual scattering cross section is the square of this integral

$$\frac{d\sigma}{d\Omega} = \left(\frac{r_o \rho}{Q_z}\right)^2 \int_S \int_{S'} e^{iQ_z(h(x,y)-h(x',y'))} e^{iQ_x(x-x')} e^{iQ_y(y-y')} dxdydx'dy'$$

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where $A_o/\sin\theta_1$ is just the illuminated surface area and the term in the angled brackets is an ensemble average over all possible choices of the origin within the illuminated area.

Scattering Cross Section

If we assume that $h(x,y) - h(x',y')$ depends only on the relative difference in position, $x - x'$ and $y - y'$ the four dimensional integral collapses to the product of two two dimensional integrals

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{r_o\rho}{Q_z}\right)^2 \int_{S'} dx'dy' \int_S \left\langle e^{iQ_z(h(0,0)-h(x,y))} \right\rangle e^{iQ_x x} e^{iQ_y y} dxdy$$

$$= \left(\frac{r_o\rho}{Q_z}\right)^2 \frac{A_o}{\sin\theta_1} \int \left\langle e^{iQ_z(h(0,0)-h(x,y))} \right\rangle e^{iQ_x x} e^{iQ_y y} dxdy$$

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Finally, it is assumed that the statistics of the height variation are Gaussian and

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{r_o\rho}{Q_z}\right)^2 \frac{A_o}{\sin\theta_1} \int e^{-Q_z^2\left\langle[h(0,0)-h(x,y)]^2\right\rangle/2} e^{iQ_x x} e^{iQ_y y} dxdy$$

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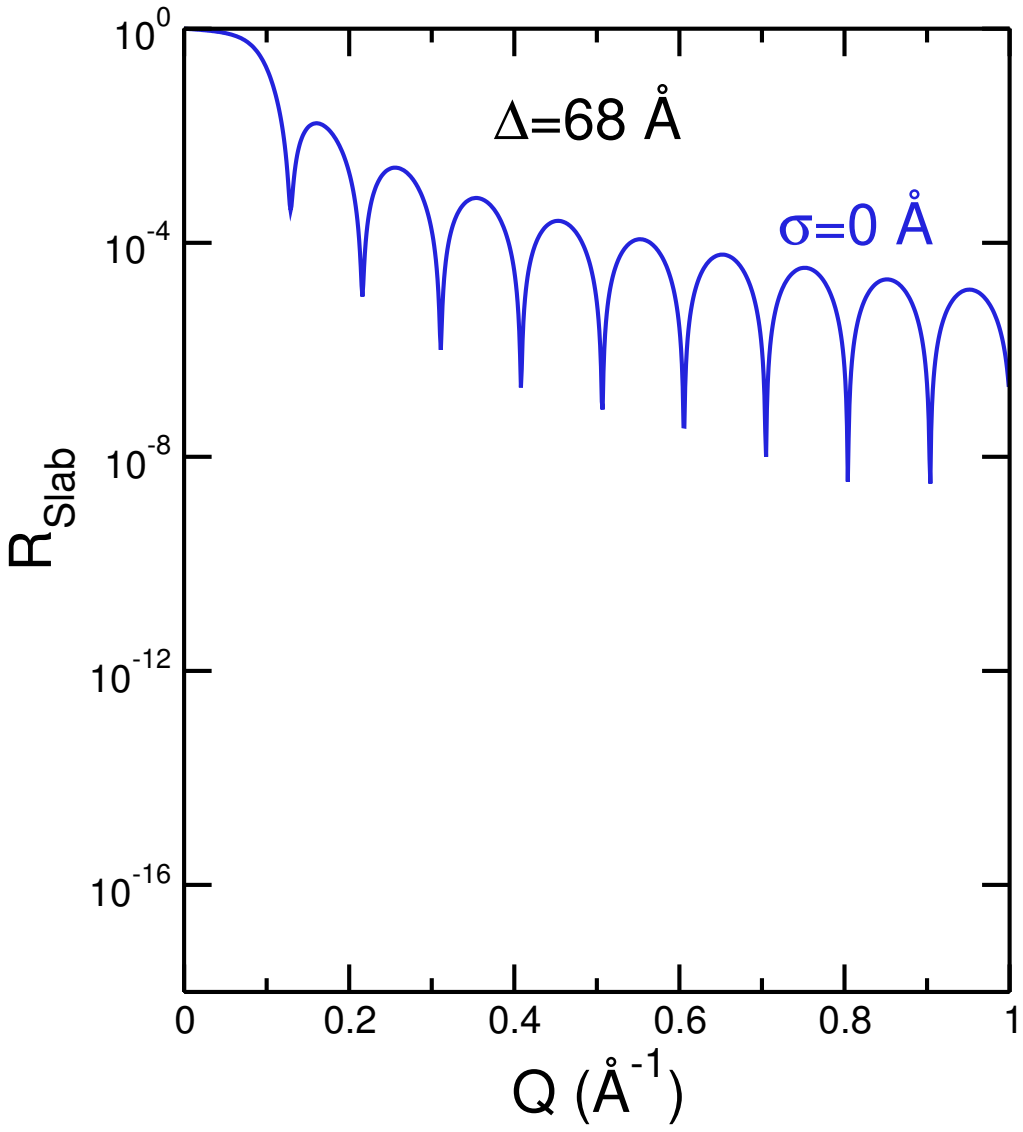

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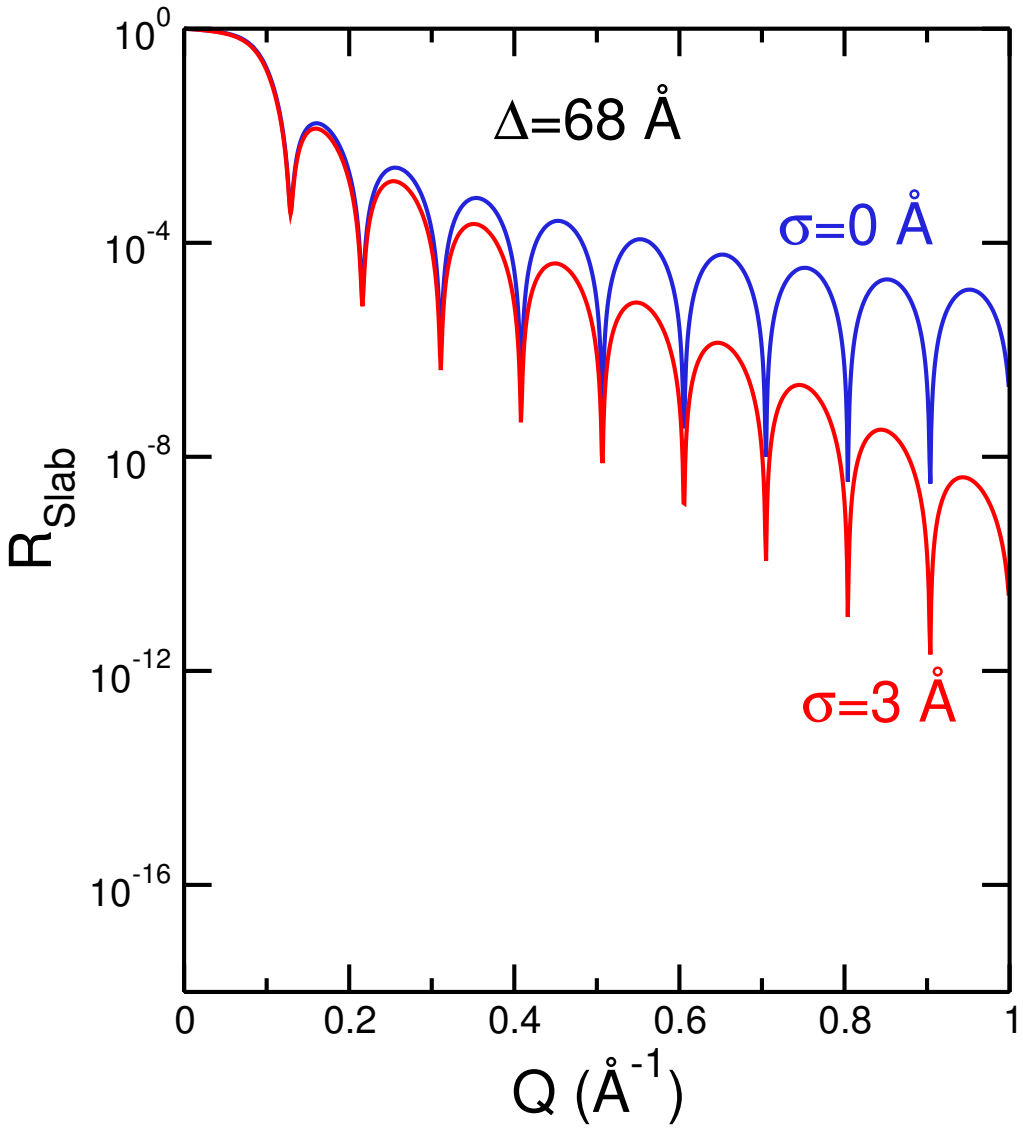

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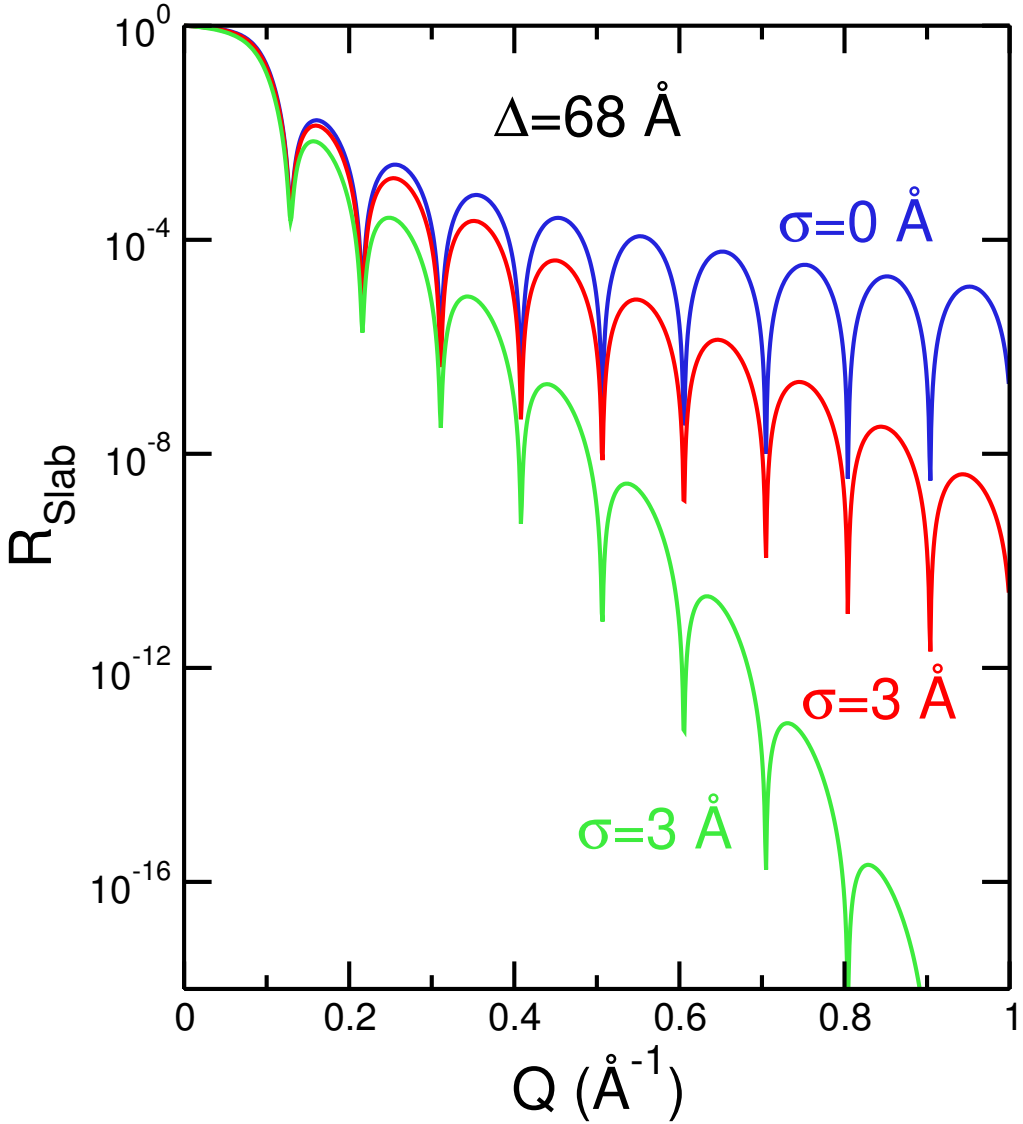

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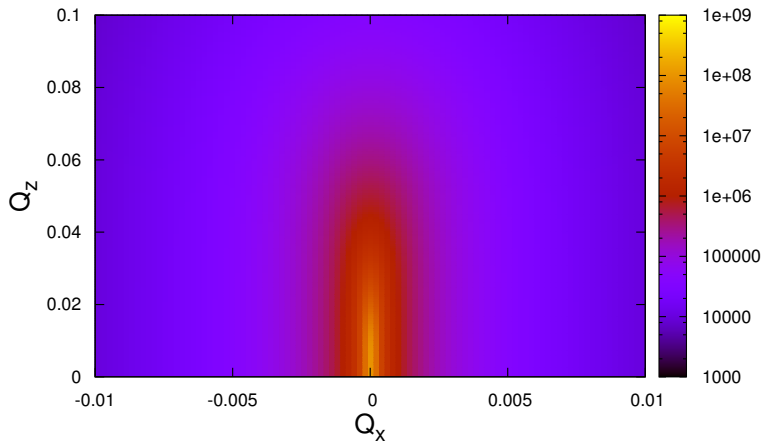

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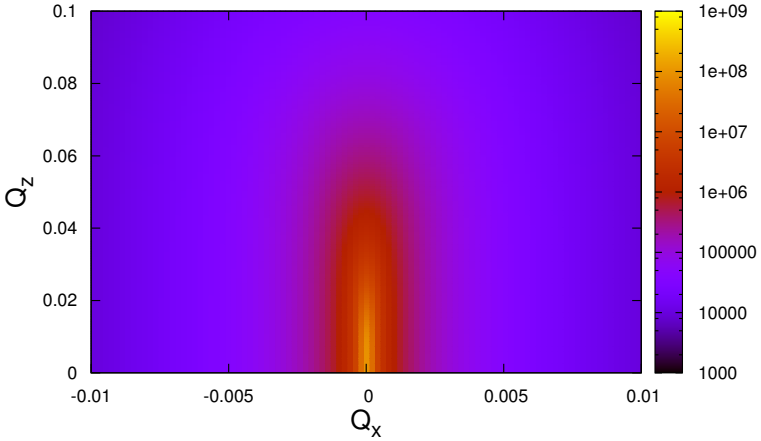

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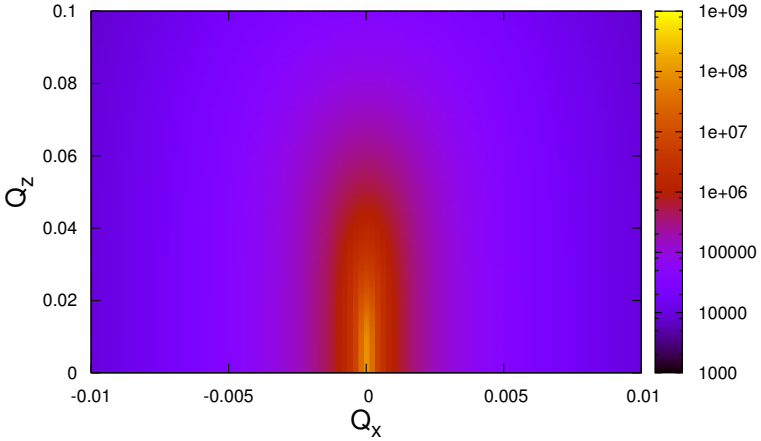

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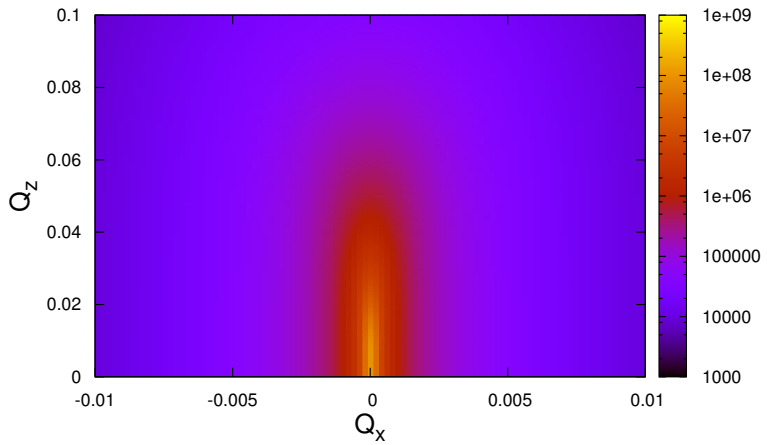

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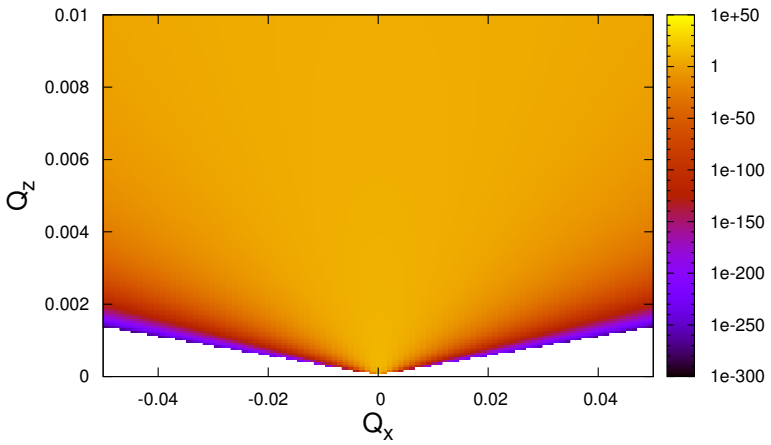

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\left(\frac{d\sigma}{d\Omega}\right) &= \left(\frac{r_o\rho}{Q_z}\right)^2 \frac{A_o}{\sin\theta_1} e^{-Q_z^2\sigma^2} \int e^{Q_z^2 C(x,y)} e^{iQ_x x} e^{iQ_y y} dxdy \\
&= \left(\frac{r_o\rho}{Q_z}\right)^2 \frac{A_o}{\sin\theta_1} e^{-Q_z^2\sigma^2} \int \left[e^{Q_z^2 C(x,y)} - 1 + 1\right] e^{iQ_x x} e^{iQ_y y} dxdy \\
&= \left(\frac{d\sigma}{d\Omega}\right)_{Fresnel} e^{-Q_z^2\sigma^2} + \left(\frac{r_o\rho}{Q_z}\right)^2 \frac{A_o}{\sin\theta_1} e^{-Q_z^2\sigma^2} F_{diffuse}(\vec{Q})
\end{aligned}$$

Bounded Correlations

If the correlations remain bounded as $r \to \infty$

$$g(x,y) = 2\left\langle h^2 \right\rangle - 2\left\langle h(0,0)h(x.y)\right\rangle = 2\sigma^2 - 2C(x,y)$$

where

$$C(x,y) = \sigma^2 e^{-(r/\xi)^{2h}}$$

$$\begin{aligned}
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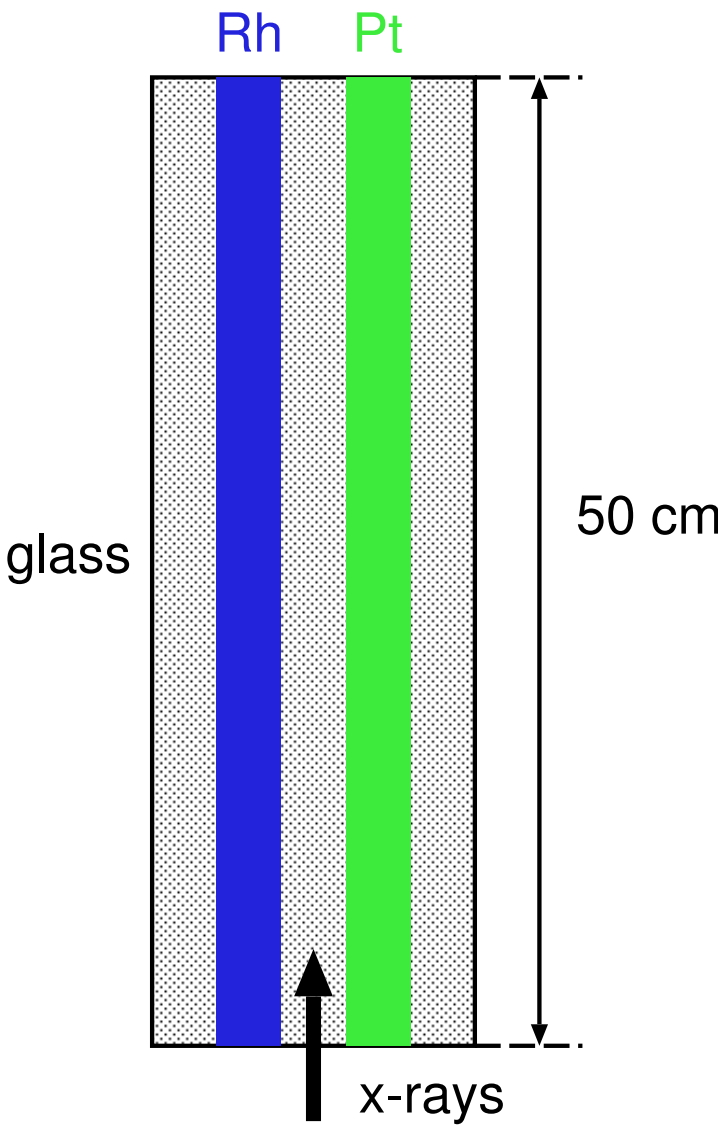
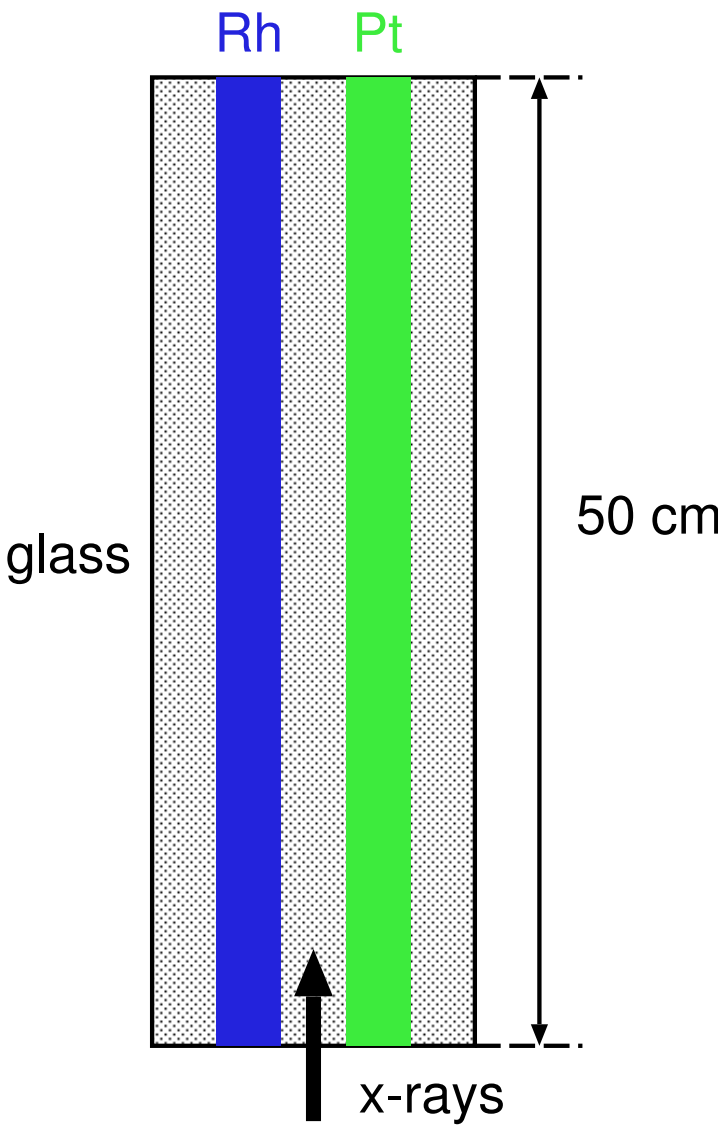
\end{aligned}$$

And the scattering exhibits both a specular peak, reduced by uncorrelated roughness, and diffuse scattering from the correlated portion of the surface

The MRCAT Mirror

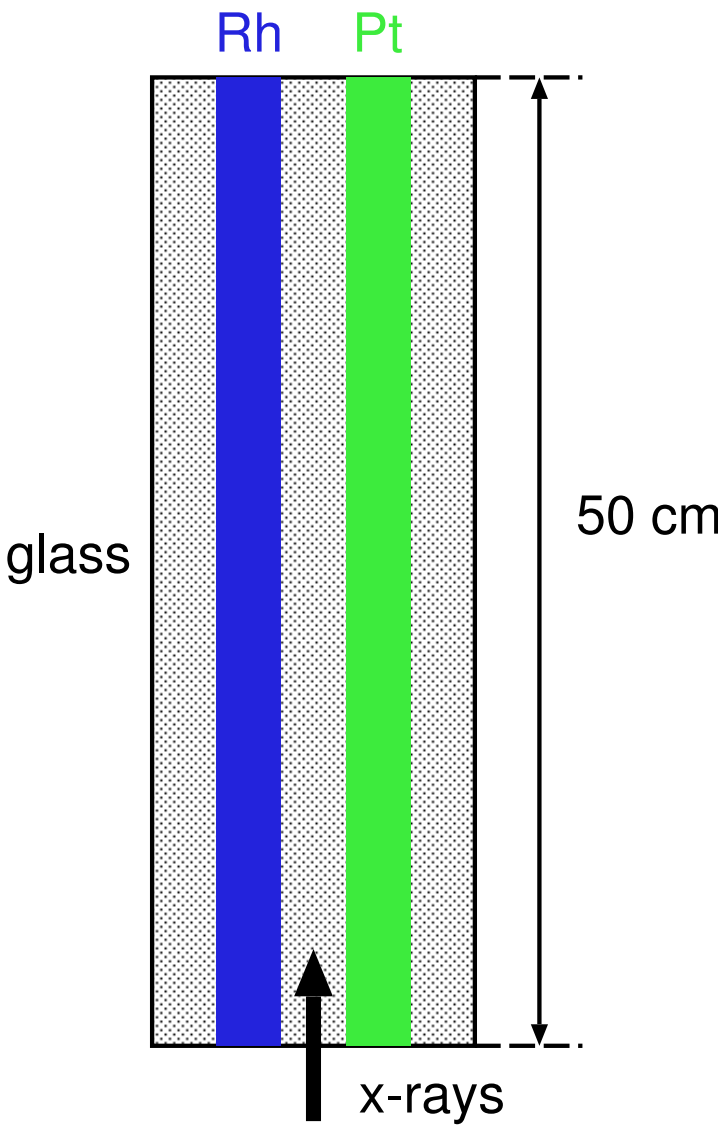


The MRCAT Mirror



Ultra low expansion glass polished to a few Å roughness

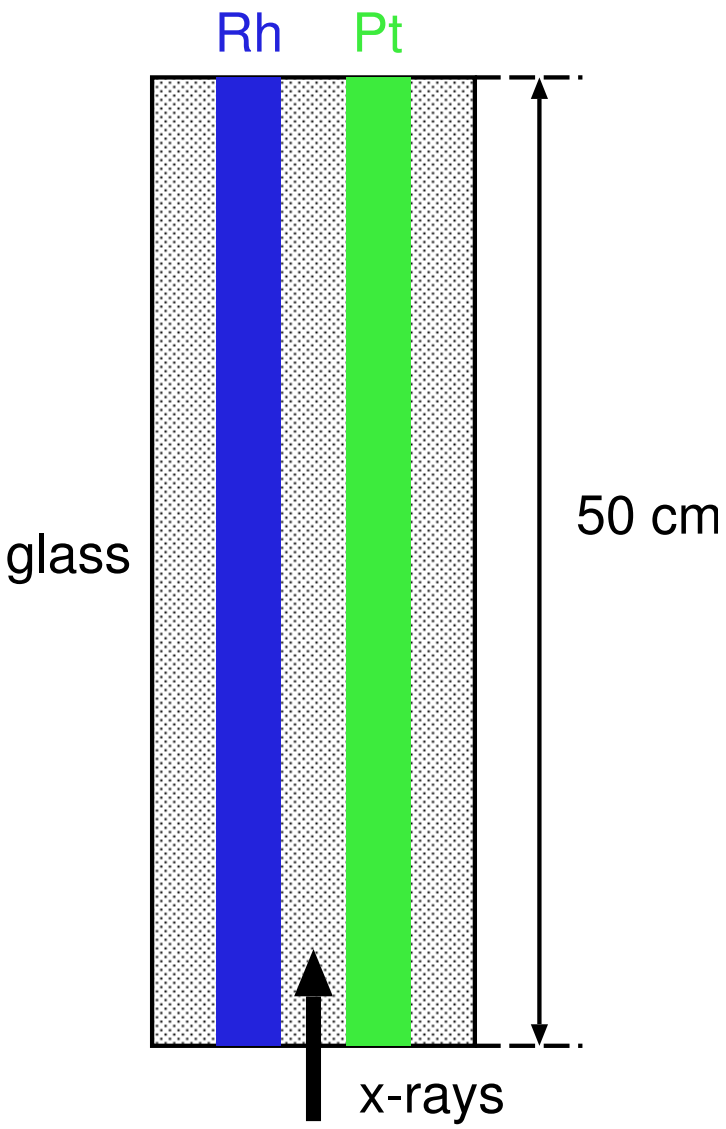
The MRCAT Mirror



Ultra low expansion glass polished to a few Å roughness

One platinum stripe and one rhodium stripe deposited along the length of the mirror on top of a chromium buffer layer

The MRCAT Mirror



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A mounting system which permits angular positioning to less than 1/100 of a degree as well as horizontal and vertical motions

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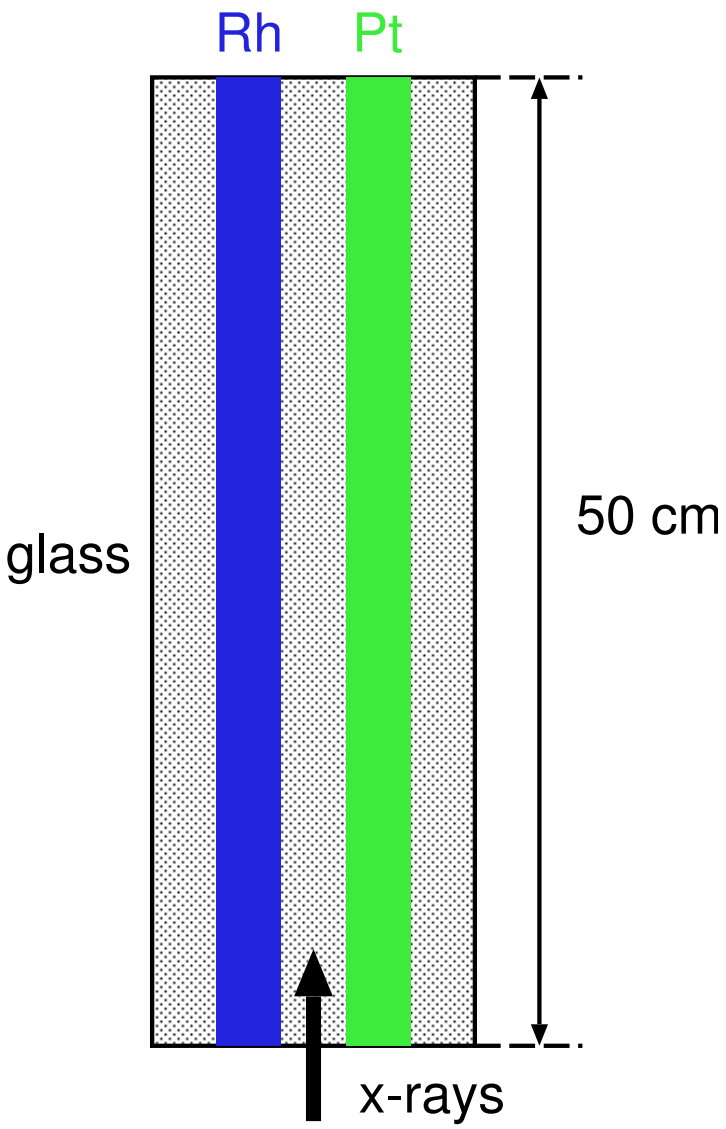


Ultra low expansion glass polished to a few Å roughness

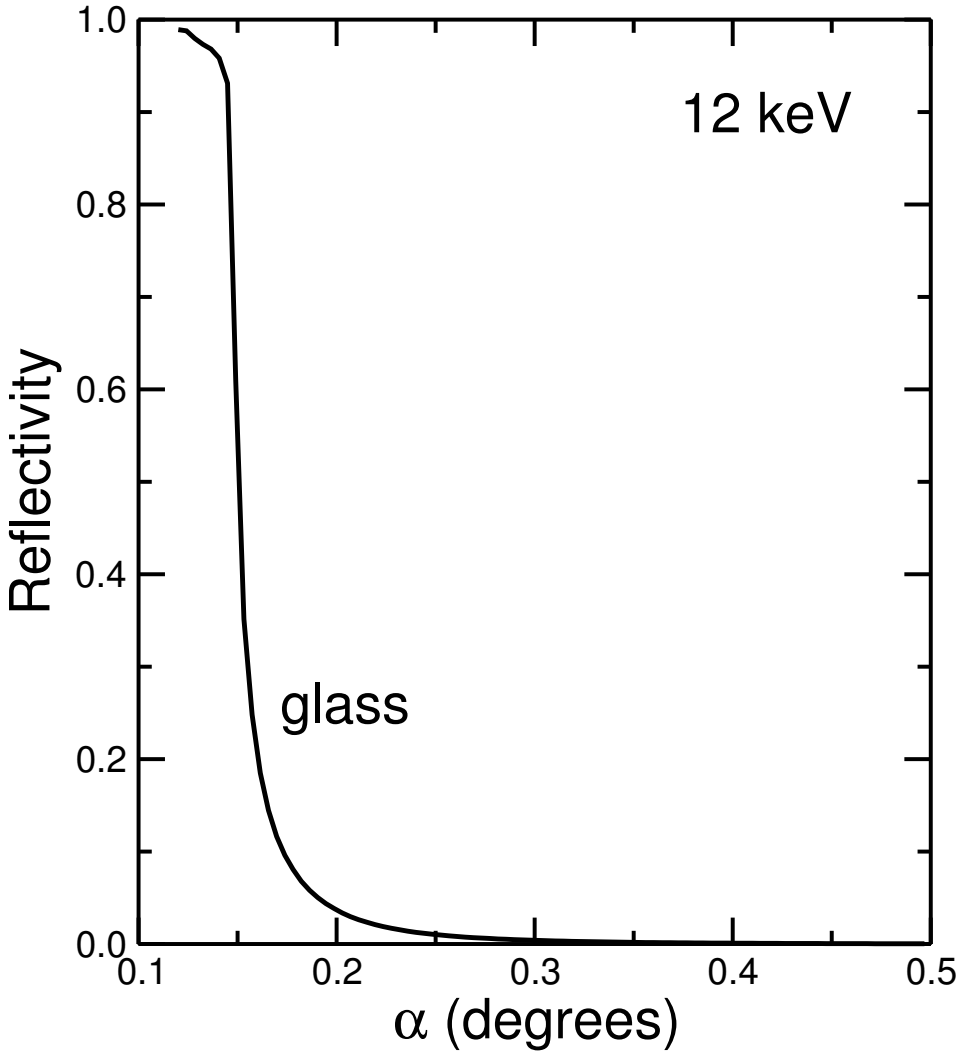
One platinum stripe and one rhodium stripe deposited along the length of the mirror on top of a chromium buffer layer

A mounting system which permits angular positioning to less than 1/100 of a degree as well as horizontal and vertical motions

A bending mechanism to permit vertical focusing of the beam to $\sim 60\ \mu\mathrm{m}$

Mirror Performance I

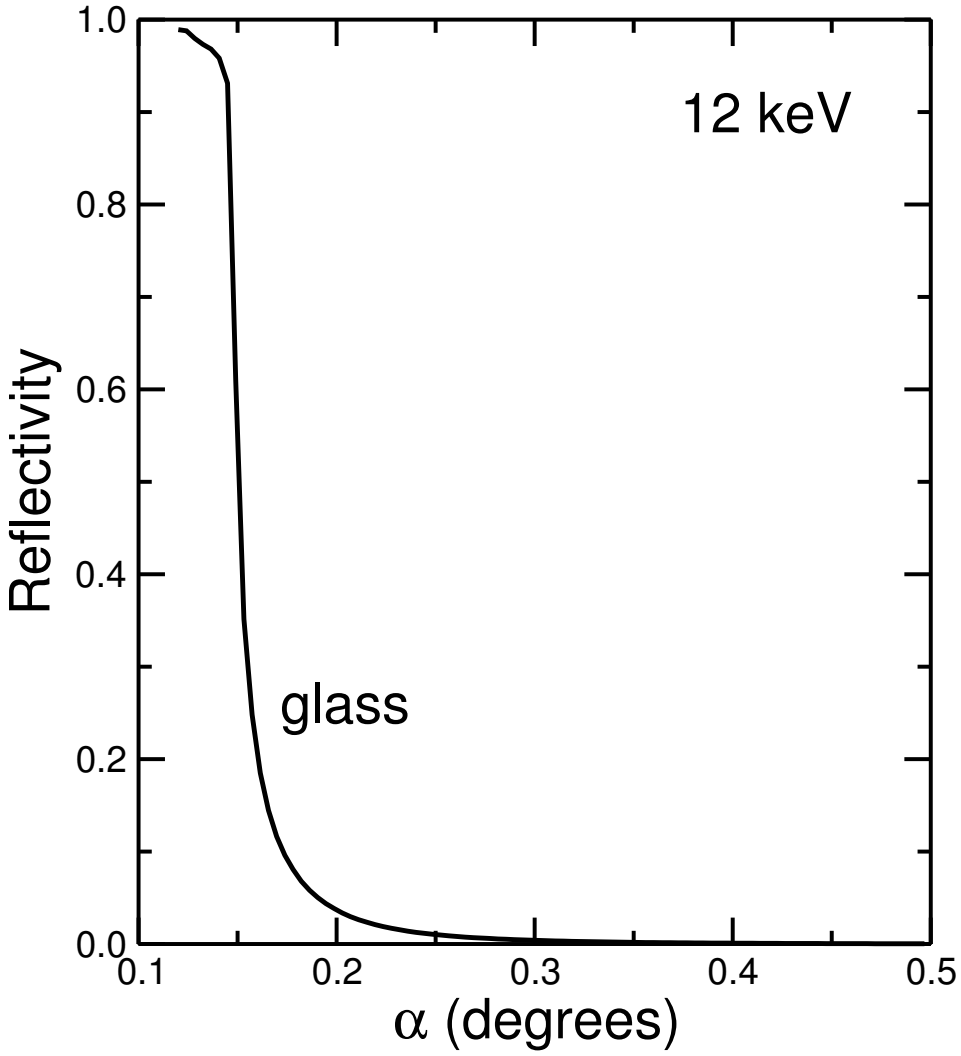
When illuminated with 12 keV x-rays on the glass "stripe", the reflectivity is measured as:



Mirror Performance I

When illuminated with 12 keV x-rays on the glass "stripe", the reflectivity is measured as:

With the Rh stripe, the thin slab reflection is evident and the critical angle is significantly higher.



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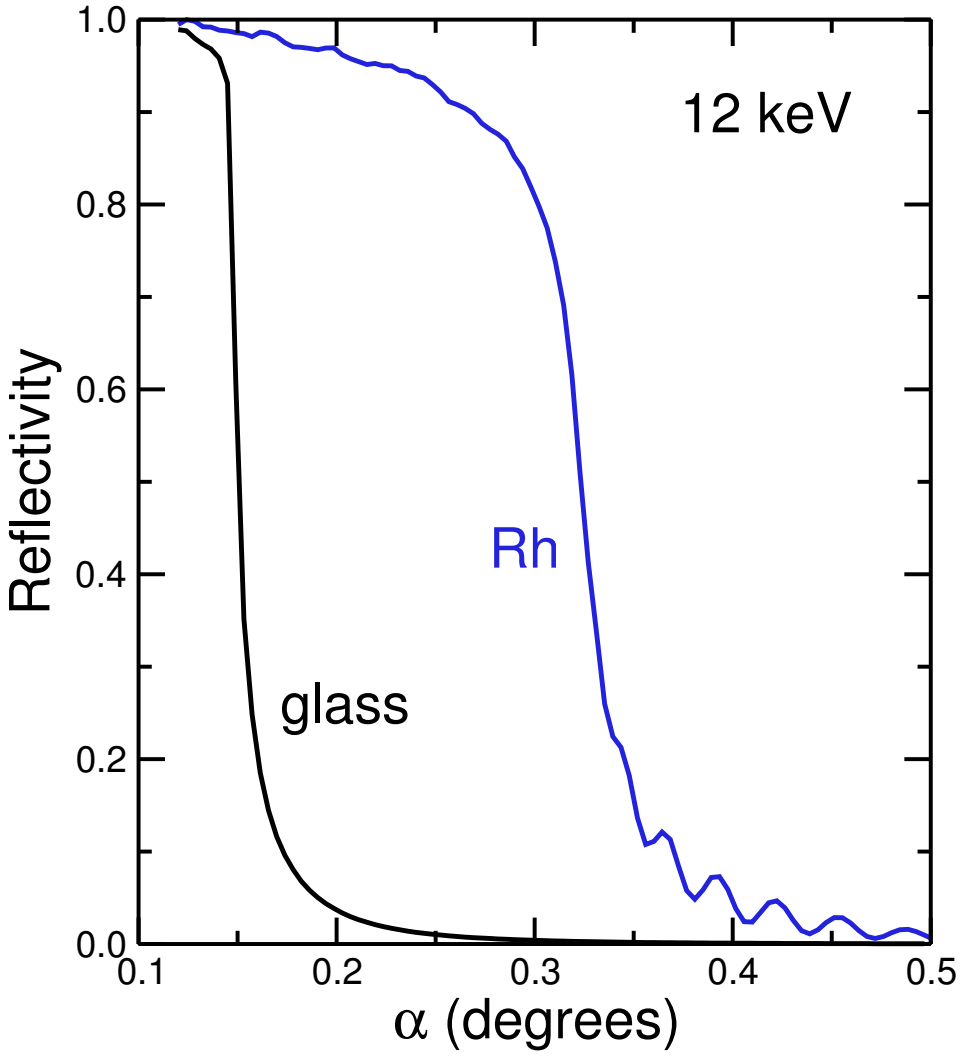

Mirror Performance I

When illuminated with 12 keV x-rays on the glass "stripe", the reflectivity is measured as:

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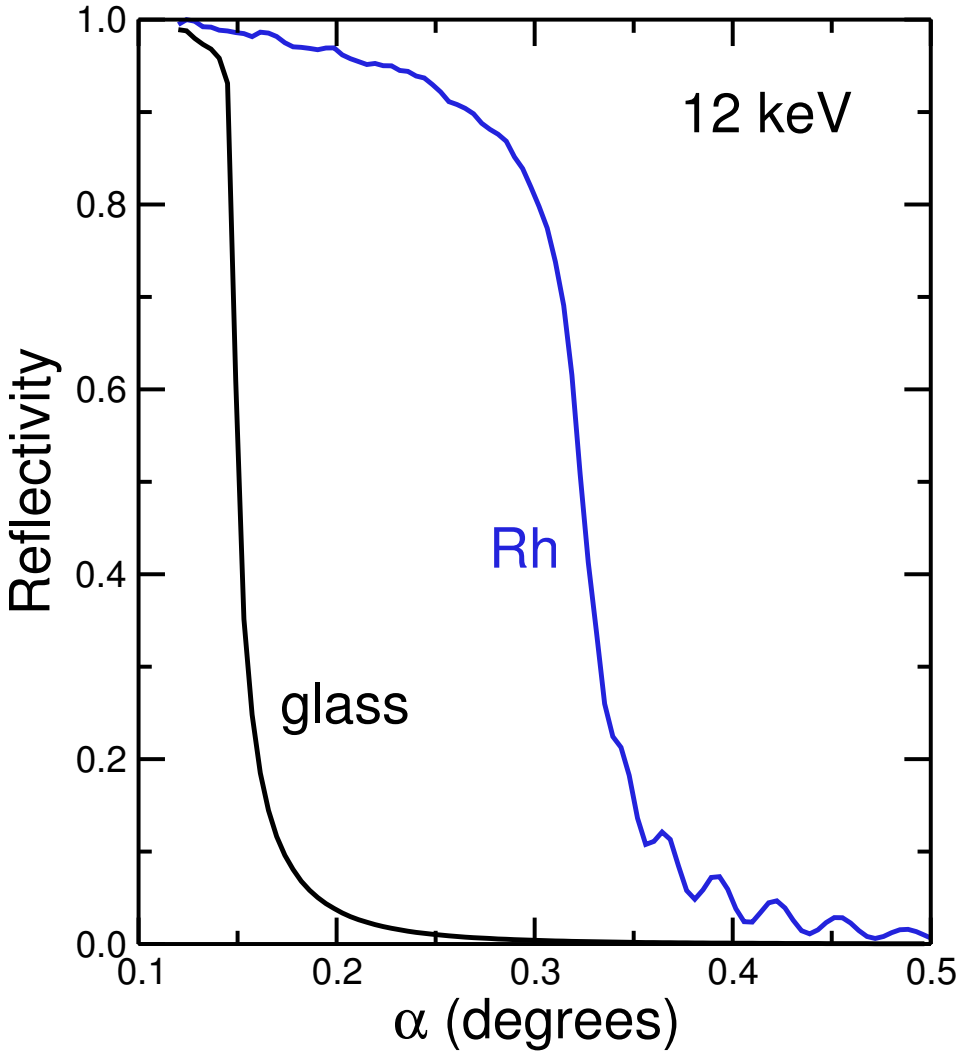
The Pt stripe gives a higher critical angle still but a lower reflectivity and it looks like an infinite slab.



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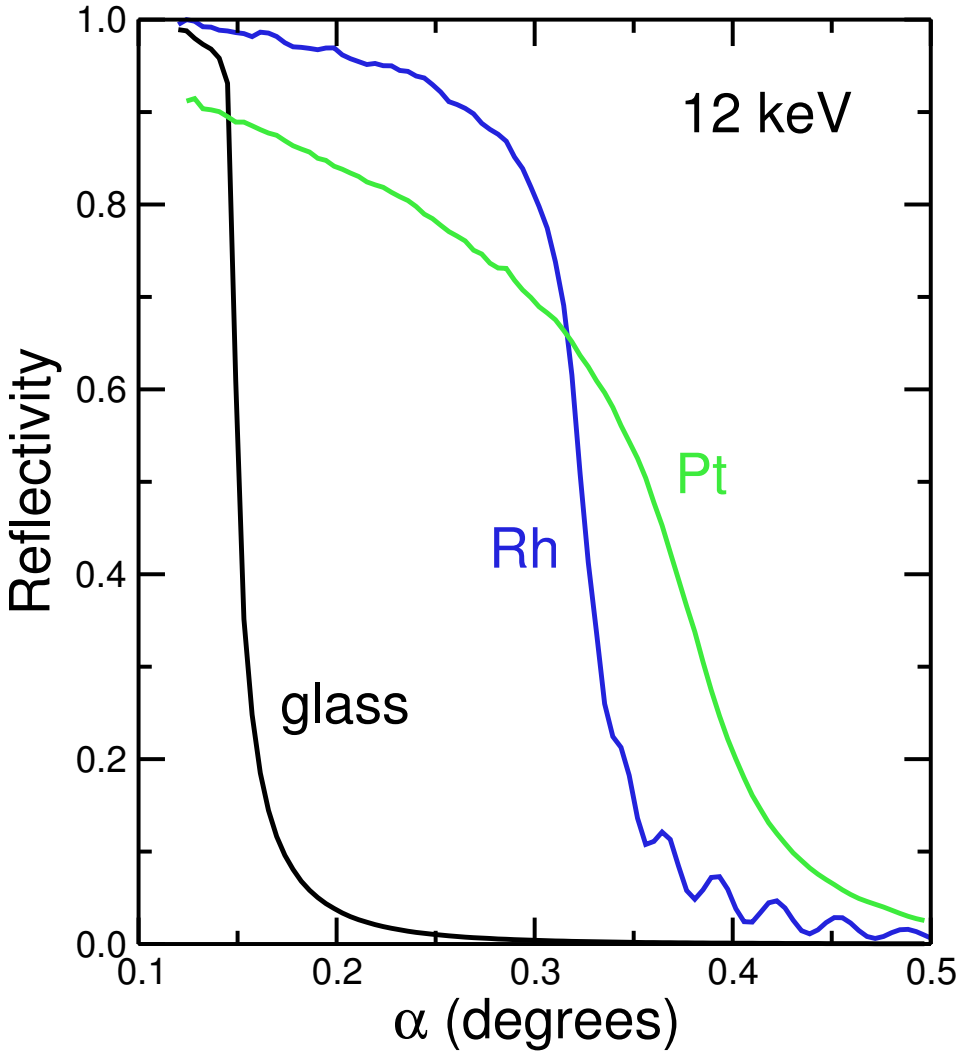
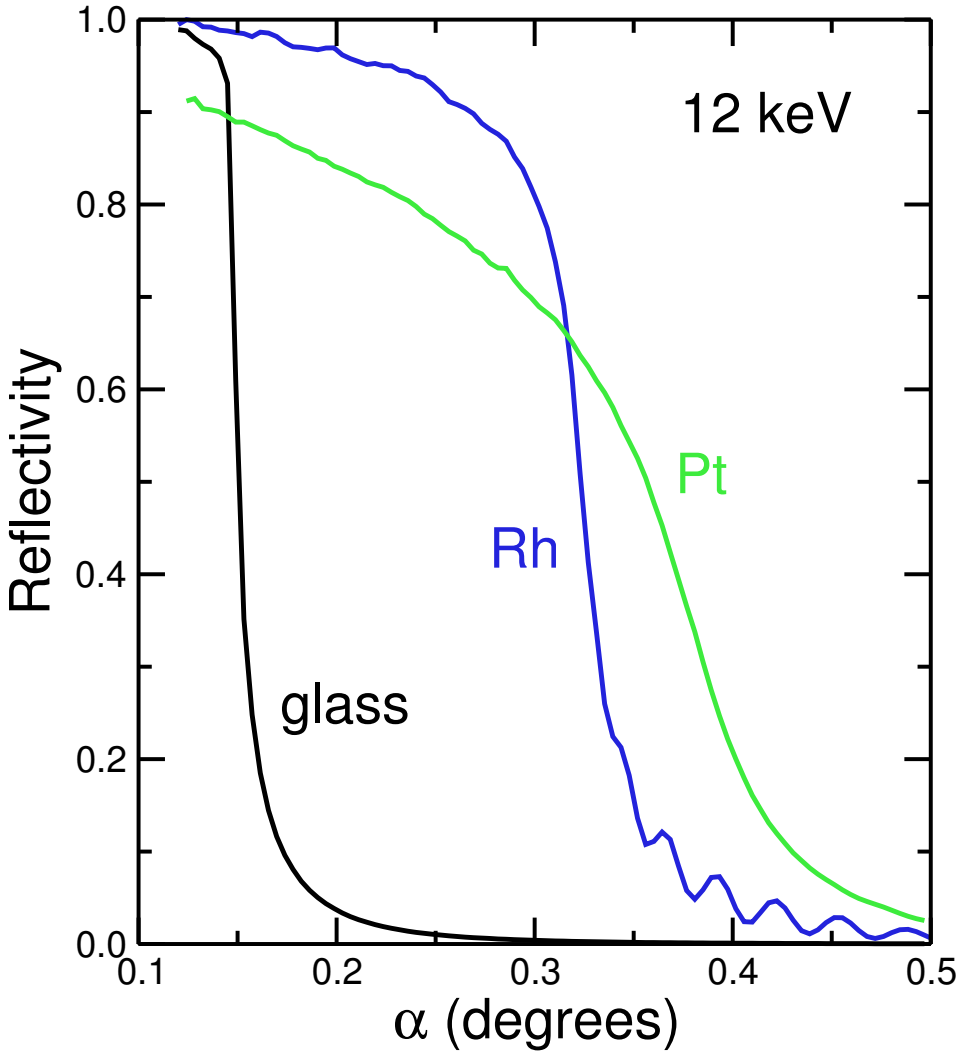


Mirror Performance I

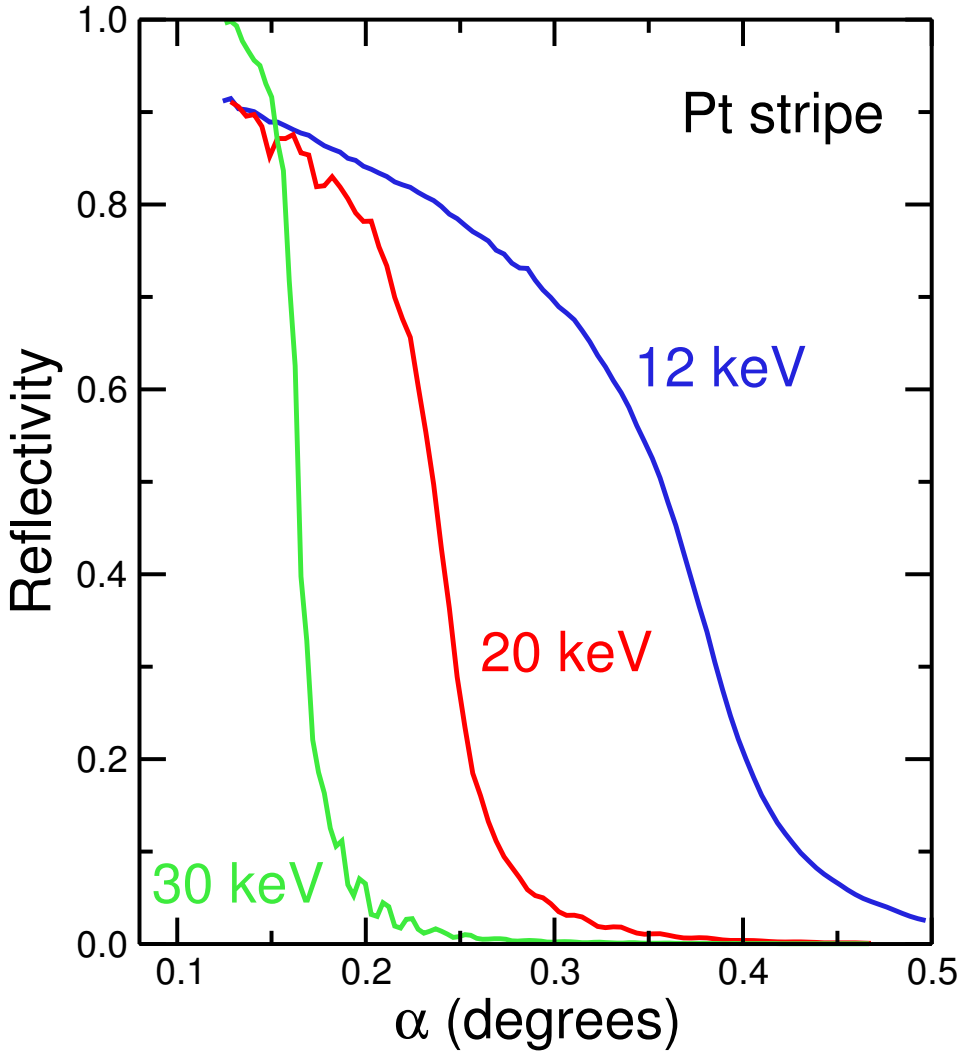
When illuminated with 12 keV x-rays on the glass "stripe", the reflectivity is measured as:

With the Rh stripe, the thin slab reflection is evident and the critical angle is significantly higher.

The Pt stripe gives a higher critical angle still but a lower reflectivity and it looks like an infinite slab. Why?



Mirror Performance II



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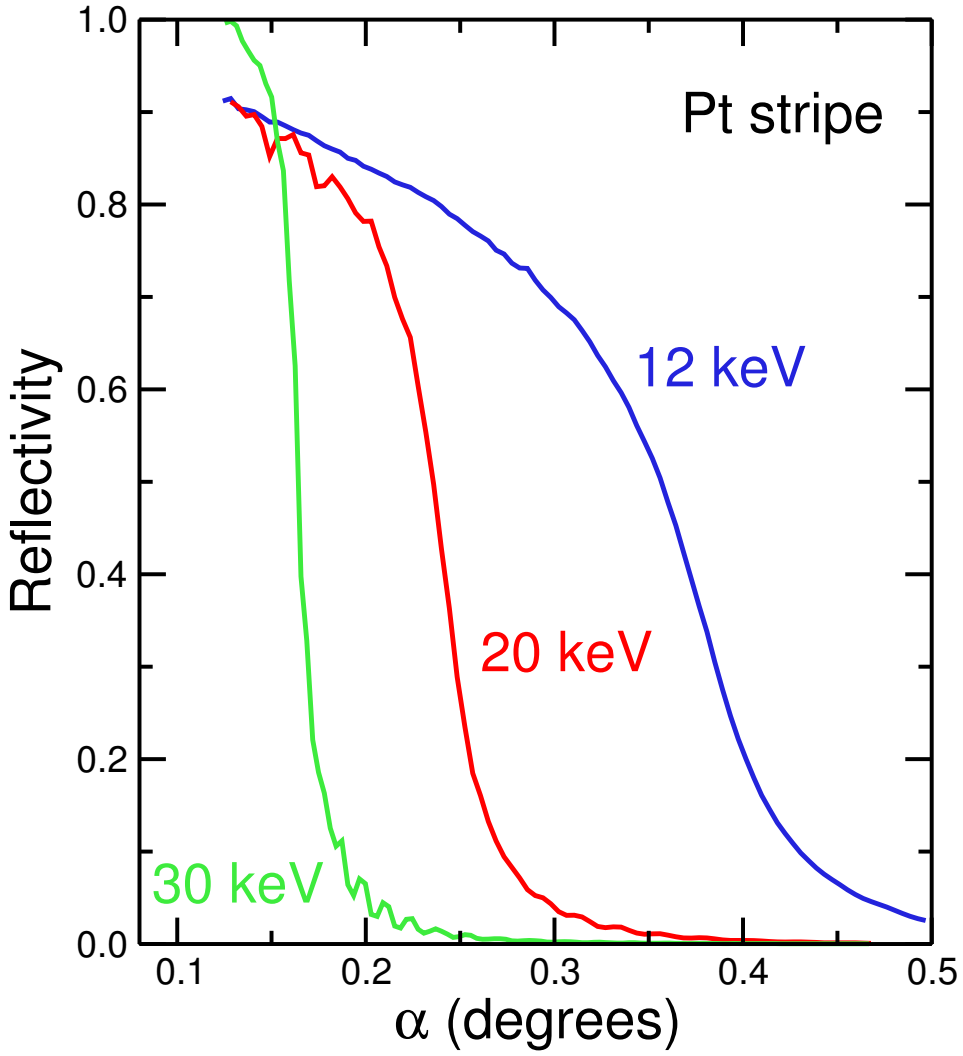
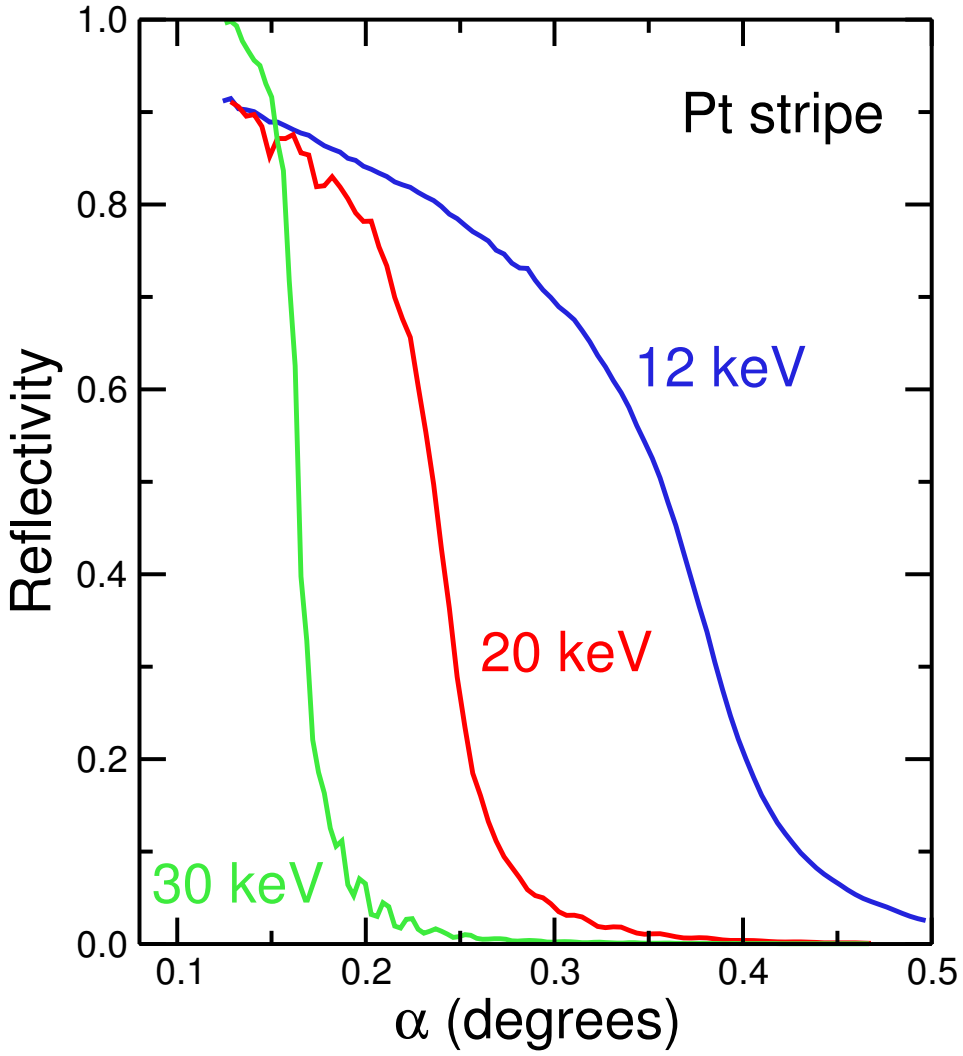


As we move up in energy the critical angle for the Pt stripe drops.

Mirror Performance II



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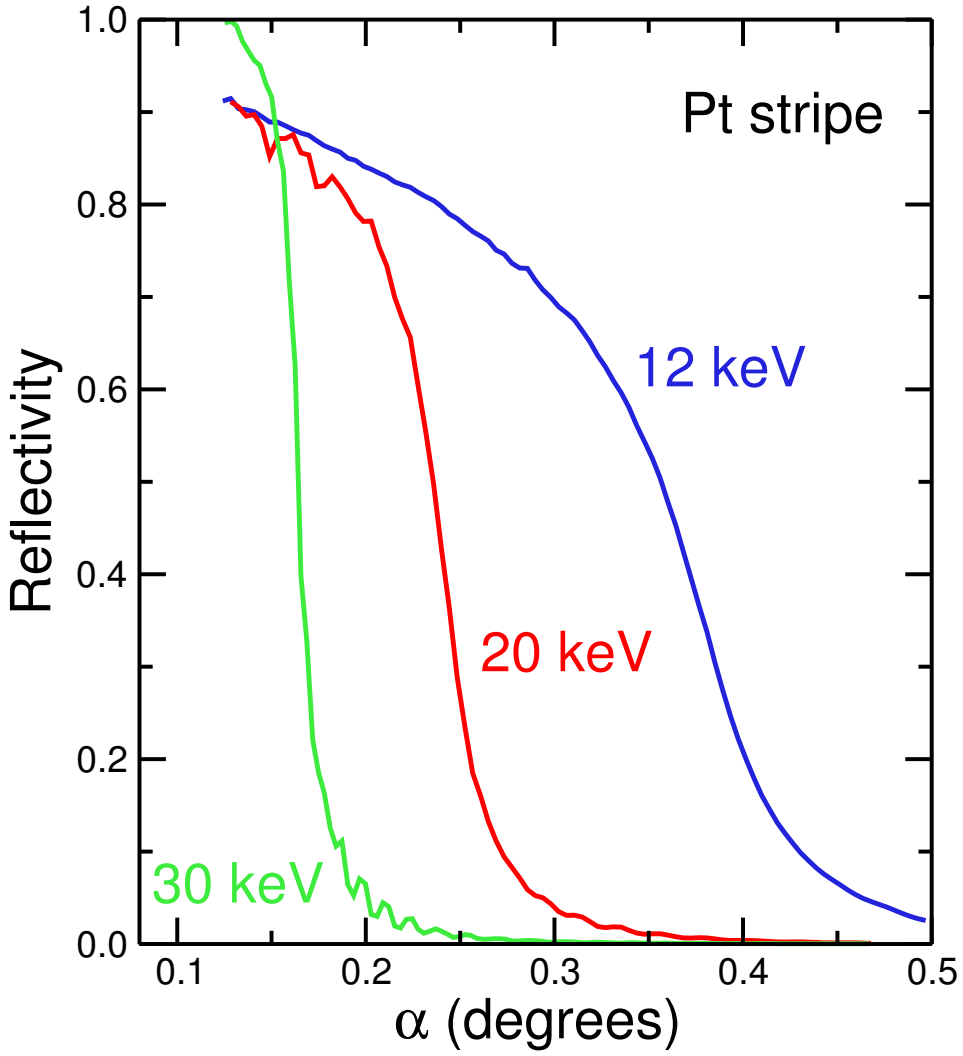
The reflectivity at low angles improves as we are well away from the Pt absorption edges at 11,565 eV, 13,273 eV, and 13,880 eV.

Mirror Performance II



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As energy rises, the Pt layer begins to show the reflectivity of a thin slab.