Today's Outline - February 05, 2015

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- Reflection from a thin slab

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Reading Assignment: Chapter 3.5–3.8

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Homework Assignment #02:
Problems on Blackboard
due Thursday, February 12, 2015

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No class on Tuesday, February 10, 2015

Review of Interface Effects

We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium.

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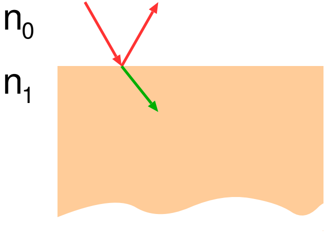

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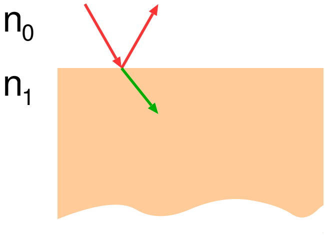
We have covered the interface boundary conditions which govern the transmission and reflection of waves at a change in medium. These result in the Fresnel equations which we rewrite here in terms of the momentum transfer.



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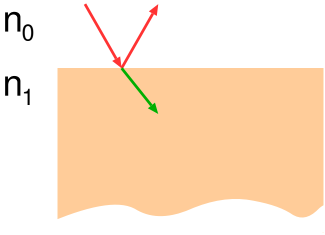

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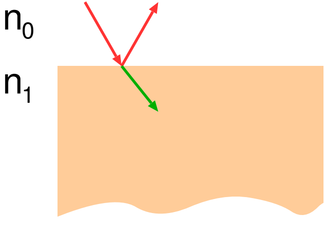

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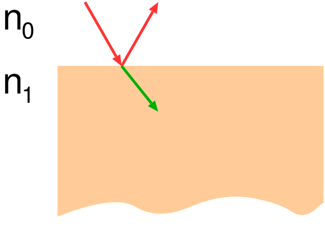

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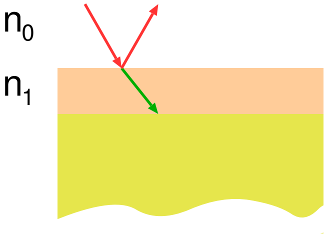

$$r = \frac{Q - Q'}{Q + Q'}$$

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We have assumed that the transmitted wave eventually attenuates to zero in all cases due to absorption. We now consider what happens if there is a second interface encountered by the transmitted wave before it dies away. That is, a thin slab of material on top of an infinite substrate

Reflection and Transmission Coefficients

For a slab of thickness Δ on a substrate, the transmission and reflection coefficients at each interface are labeled:

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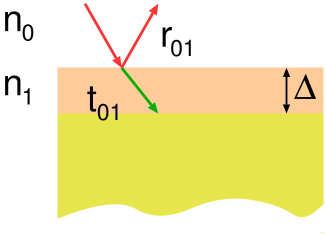

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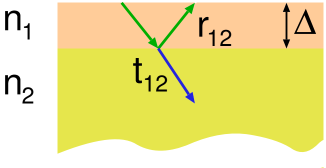

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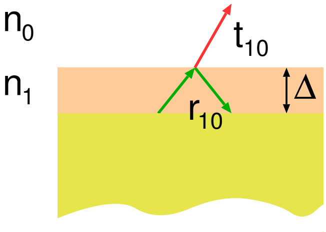

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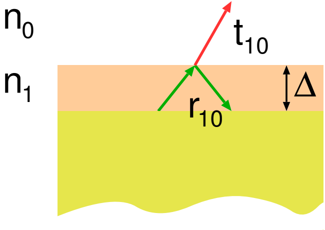

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Build the composite reflection coefficient from all possible events

Overall Reflection from a Slab

The composite reflection coefficient for each ray emerging from the top surface is computed

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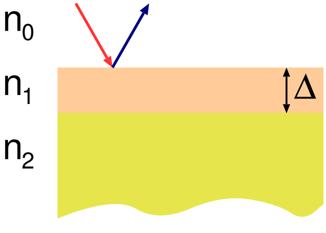

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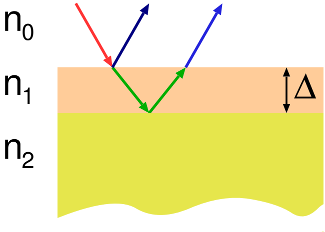

$$r_{01}$$

$$+$$

$$t_{01} r_{12} t_{10}$$

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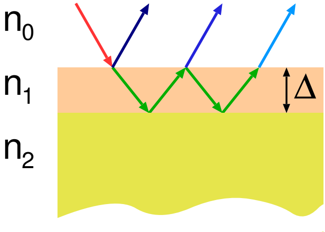

$$r_{01}$$
$$+$$
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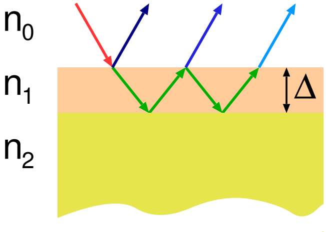

$$r_{01} + t_{01} r_{12} t_{10} + t_{01} r_{12} r_{10} r_{12} t_{10}$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin\alpha_1)\Delta}$$

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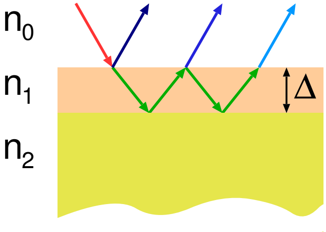

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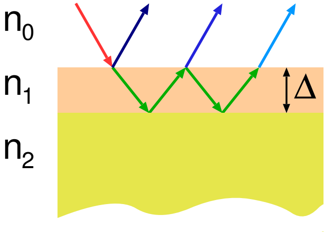

$$r_{01}$$
$$+$$
$$t_{01} r_{12} t_{10} \cdot p^2$$
$$+$$
$$t_{01} r_{12} r_{10} r_{12} t_{10}$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

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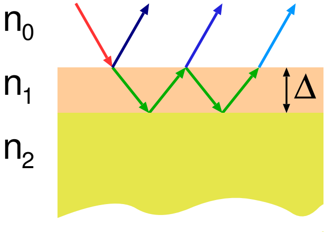

$$r_{01}$$
$$+$$
$$t_{01} r_{12} t_{10} \cdot p^2$$
$$+$$
$$t_{01} r_{12} r_{10} r_{12} t_{10} \cdot p^4$$

Inside the medium, the x-rays are travelling an additional 2Δ per traversal. This adds a phase shift of

$$p^2 = e^{i2(k_1 \sin \alpha_1)\Delta} = e^{iQ_1 \Delta}$$

which multiplies the reflection coefficient with each pass through the slab

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The composite reflection coefficient can now be expressed as a sum

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$$= \frac{Q_0^2 + 2Q_0Q_1 + Q_1^2}{(Q_0 + Q_1)^2} = \frac{(Q_0 + Q_1)^2}{(Q_0 + Q_1)^2} = 1$$

Reflection Coefficient of a Slab

Starting with the reflection coefficient of the slab obtained earlier

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In the case of $n_0 = n_2$ there is the further simplification of $r_{12} = -r_{01}$.

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Reflection Coefficient of a Slab

Starting with the reflection coefficient of the slab obtained earlier

$$r_{slab} = r_{01} + t_{01} t_{10} r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

Using the identity

$$t_{01} t_{10} = 1 - r_{01}^2$$

$$= r_{01} + \left(1 - r_{01}^2\right) r_{12} p^2 \frac{1}{1 - r_{10} r_{12} p^2}$$

Expanding over a common denominator and recalling that $r_{10} = -r_{01}$.

$$= \frac{r_{01} + r_{01}^2 r_{12} p^2 + \left(1 - r_{01}^2\right) r_{12} p^2}{1 - r_{10} r_{12} p^2}$$

In the case of $n_0 = n_2$ there is the further simplification of $r_{12} = -r_{01}$.

$$r_{slab} = \frac{r_{01} + r_{12} p^2}{1 + r_{01} r_{12} p^2} = \frac{r_{01}\left(1 - p^2\right)}{1 - r_{01}^2 p^2}$$

Kiessig Fringes

$$p^2 = e^{iQ_1\Delta}$$

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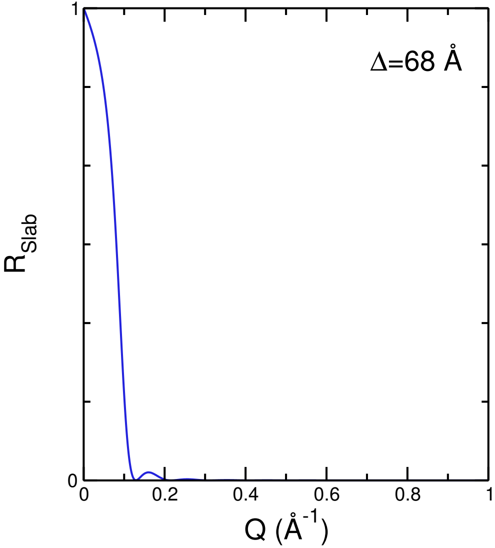

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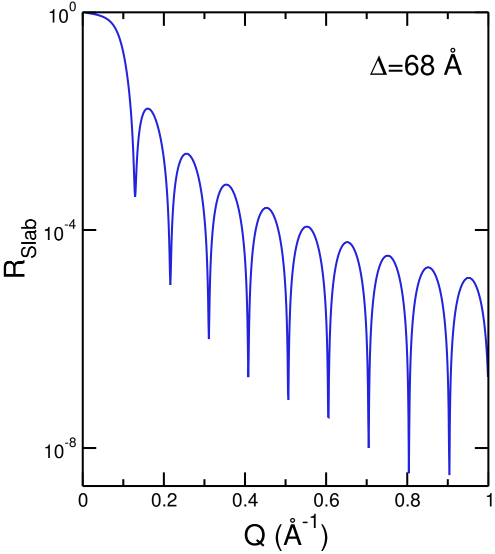

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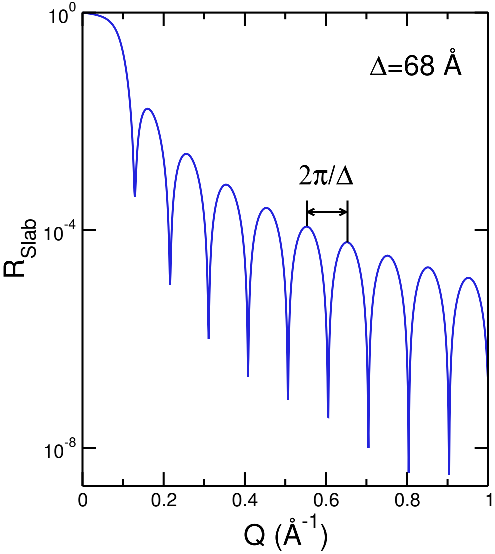
$$2\pi/\Delta = 0.092\text{Å}^{-1}$$



Kinematical Reflection from a Thin Slab

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Recall the reflection coefficient for a thin slab. If the slab is thin and we are well above the critical angle

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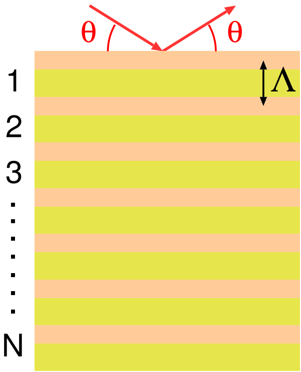
Multilayers in the Kinematical Regime



N repetitions of a bilayer of thickness Λ composed of two materials, A and B which have a density contrast ($\rho_A > \rho_B$).

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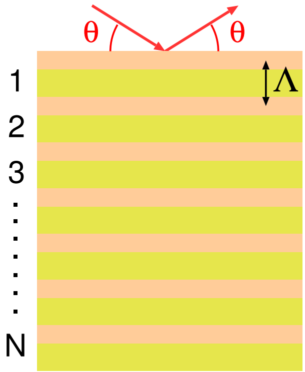

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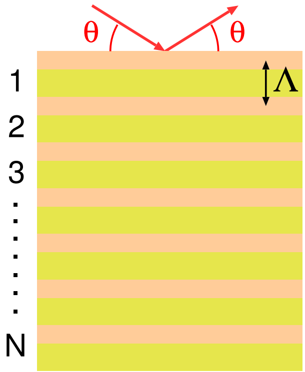

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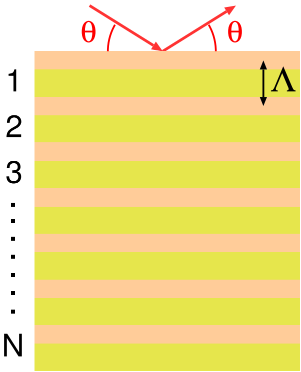

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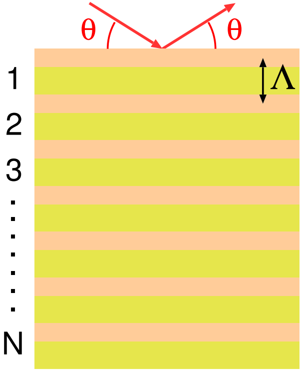

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Form a stack of N bilayers

$$r_N(\zeta) = \sum_{\nu=0}^{N-1} r_1(\zeta) e^{i2\pi\zeta\nu} e^{-\beta\nu}$$

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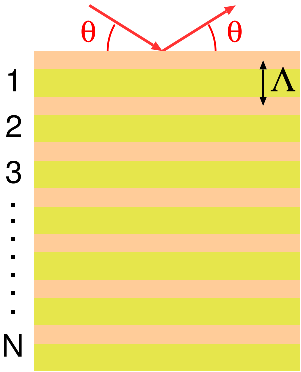

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The reflectivity from a single bilayer can be evaluated using the reflectivity developed for a slab but replacing the density of the slab material with the difference in densities of the bilayer components and assuming that material A is a fraction Γ of the bilayer thickness

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$$r_1(\zeta) = -i\frac{\lambda r_o \rho_{AB}}{\sin\theta} \int_{-\Gamma\Lambda/2}^{+\Gamma\Lambda/2} e^{i2\pi\zeta z/\Lambda} dz$$

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$$r_1 = -2ir_o\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}$$

Absorption Coefficient of a Bilayer

The total reflectivity for the multilayer is therefore:

$$r_N = -2ir_o\rho_{AB}\left(\frac{\Lambda^2\Gamma}{\zeta}\right)\frac{\sin(\pi\Gamma\zeta)}{\pi\Gamma\zeta}\frac{1-e^{i2\pi\zeta N}e^{-\beta N}}{1-e^{i2\pi\zeta}e^{-\beta}}$$

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The incident x-ray has a path length $\Lambda/\sin\theta$ in a bilayer, a fraction Γ through n_A and a fraction $(1-\Gamma)$ through n_B.

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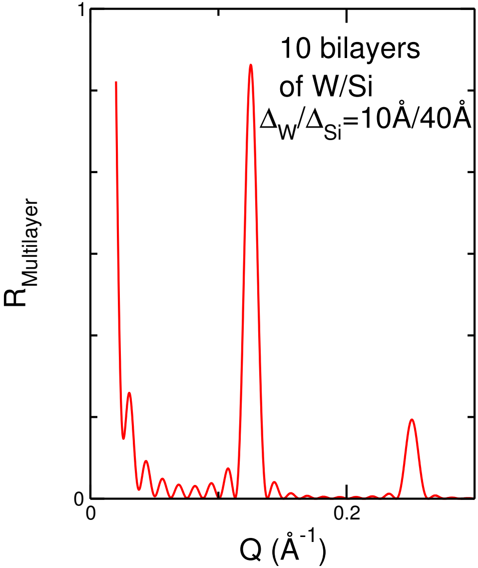
Reflectivity Calculation



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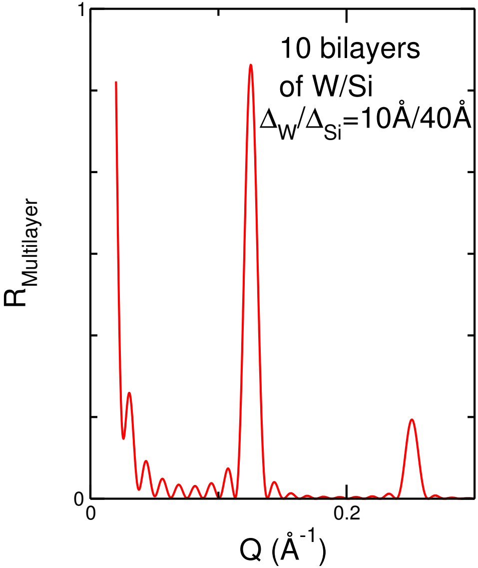

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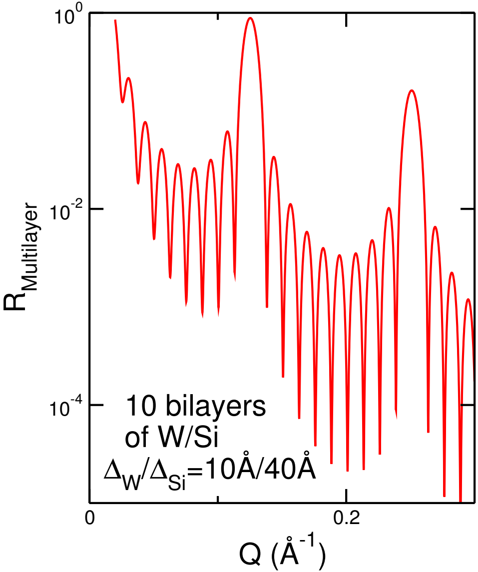

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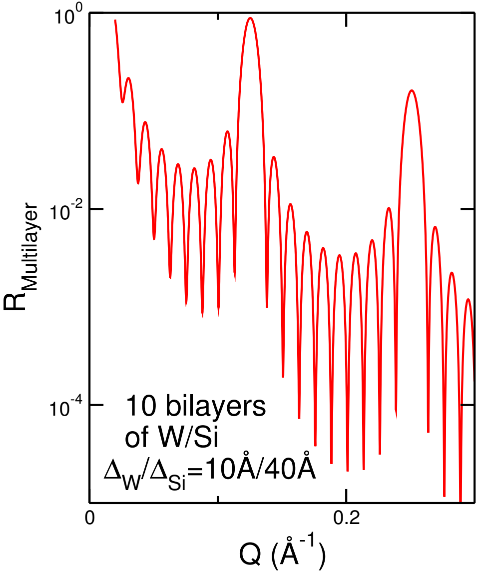

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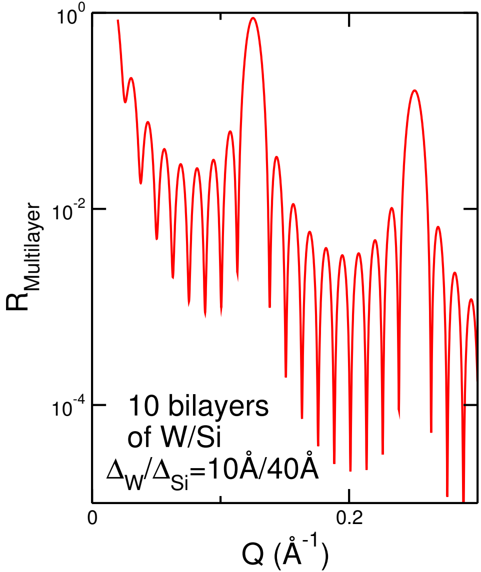

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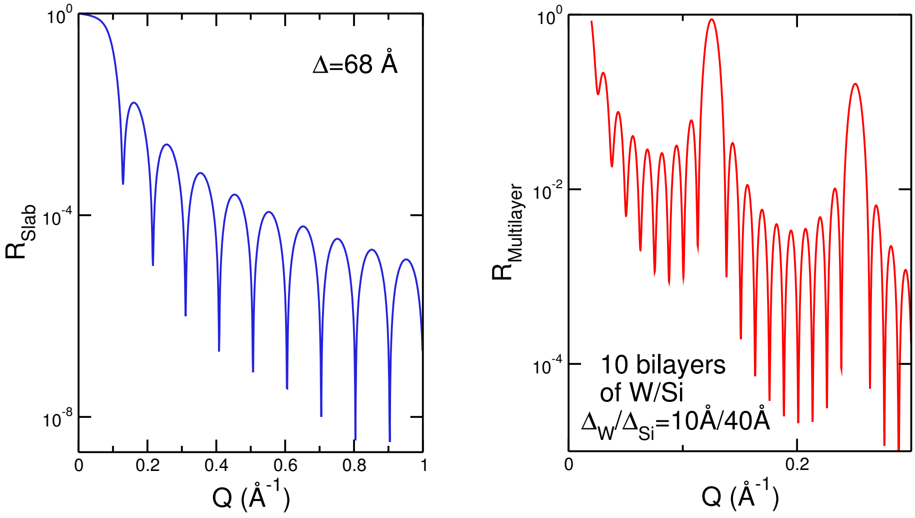
- Multilayers are used commonly on laboratory sources as well as at synchrotrons as mirrors

Slab - Multilayer Comparison



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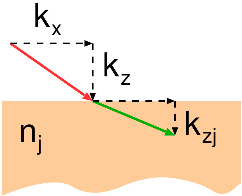

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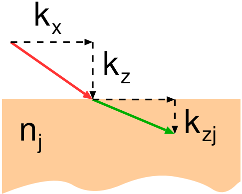

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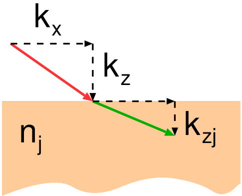

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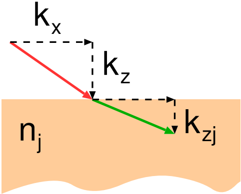

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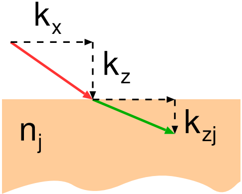

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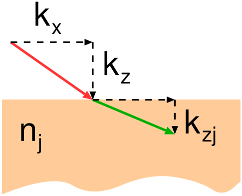

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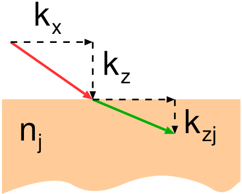

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The recursive relation can be seen from the calculation of reflectivity of the next layer up

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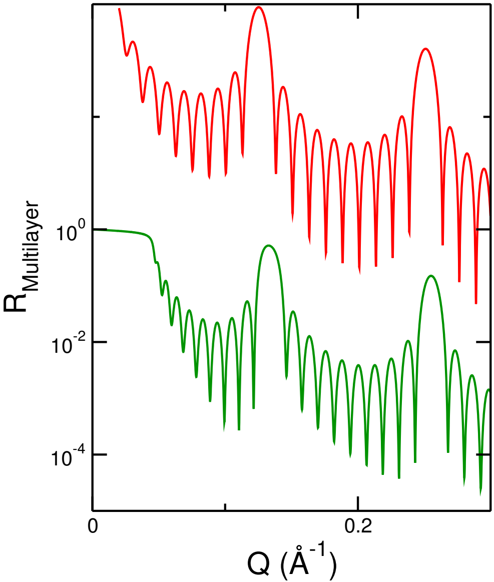
Kinematical - Parratt Comparison



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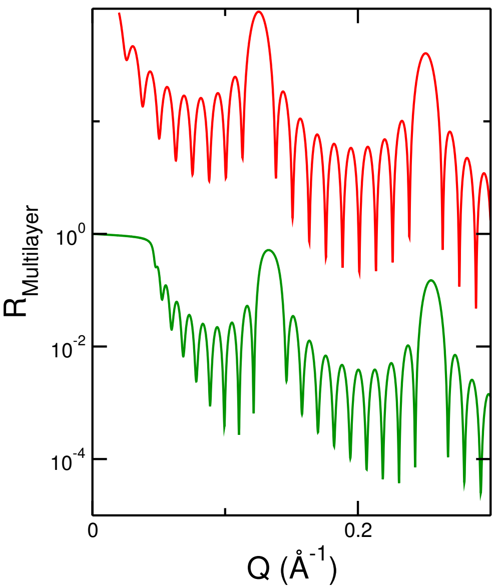

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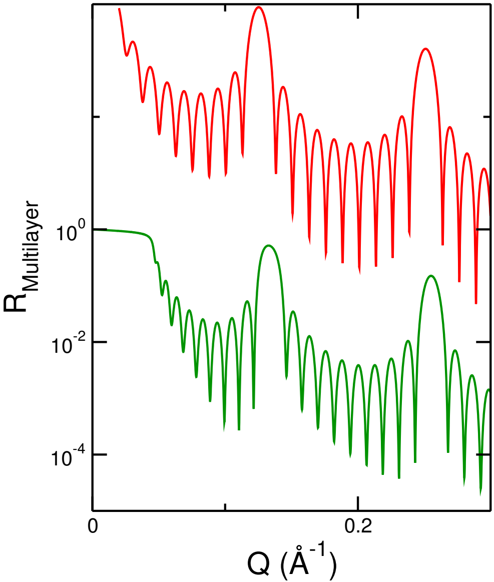

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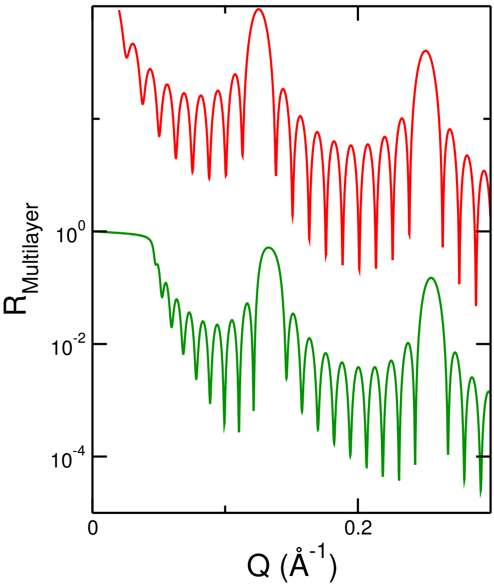

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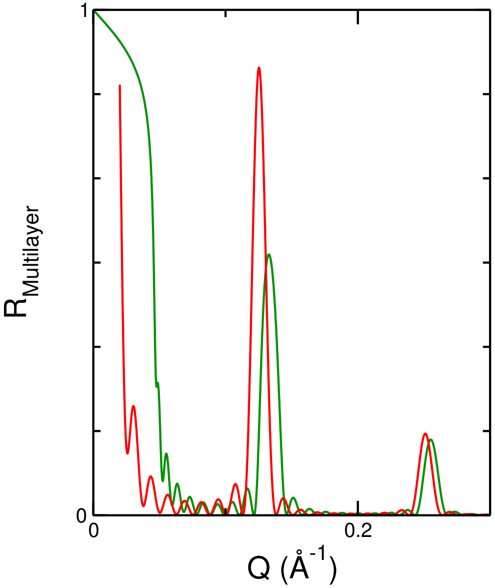

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Peaks in kinematical calculation are somewhat higher reflectivity than true value.

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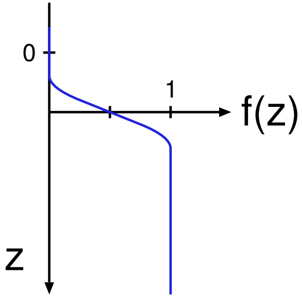

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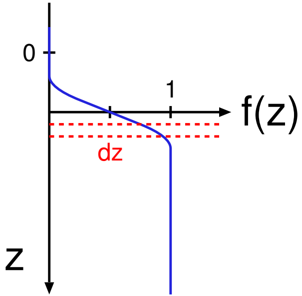

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The reflectivity can be computed as the superposition of the reflectivity of a series of infinitesmal slabs of thickness dz at a depth z.

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Calculating the full reflection coefficient relative to the Fresnel reflection coefficient

$$\frac{R(Q)}{R_F(Q)} = \left| \int_{-\infty}^{\infty} \left(\frac{df}{dz}\right) e^{iQz} dz \right|^2$$

The Error Function - a Specific Case

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$$f(z) = erf(\frac{z}{\sqrt{2}\sigma}) = \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{2}\sigma} e^{-t^2} dt$$

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$$f(z) = erf(\frac{z}{\sqrt{2}\sigma}) = \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{2}\sigma} e^{-t^2} dt$$

the gradient of the error function is simply a Gaussian

$$\frac{df(z)}{dz} = \frac{d}{dz} erf(\frac{z}{\sqrt{2}\sigma}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{z^2}{\sigma^2}}$$

whose Fourier transform is also a Gaussian, which when squared to obtain the reflection coefficient, gives. Or more accurately.

$$R(Q) = R_F(Q) e^{-Q^2\sigma^2} = R_F(Q) e^{-QQ'\sigma^2}$$

$$Q = k \sin\theta, \qquad Q' = k' \sin\theta'$$