Today's Outline - February 03, 2015

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Reading Assignment: Chapter 3.4

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Reading Assignment: Chapter 3.4

Homework Assignment #02:
Problems to be provided
due Thursday, February 12, 2015

HW #02

1. Knowing that the photoelectric absorption of an element scales as the inverse of the energy cubed, calculate:

(a) the absorption coefficient at 10keV for copper when the value at 5keV is 1698.3 cm^{-1};

(b) The actual absorption coefficient of copper at 10keV is 1942.1 cm^{-1}, why is this so different than your calculated value?

2. A 30 cm long, ionization chamber, filled with 80% helium and 20% nitrogen gases at 1 atmosphere, is being used to measure the photon rate (photons/sec) in a synchrotron beamline at 12 keV. If a current of 10 nA is measured, what is the photon flux entering the ionization chamber?

3. A 5 cm deep ionization chamber is used to measure the fluorescence from a sample containing arsenic (As). Using any noble gases or nitrogen, determine a gas fill (at 1 atmosphere) for this chamber which absorbs at least 60% of the incident photons. How does this change if you are measuring the fluorescence from ruthenium (Ru)?

HW #02

4. Calculate the characteristic angle of reflection of 10 keV and 30 keV x-rays for:

(a) A slab of glass (SiO_2);

(b) A thick chromium mirror;

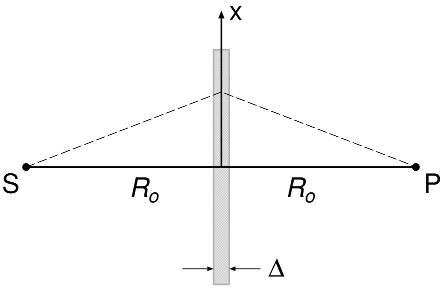
(c) A thick platinum mirror.

(d) If the incident x-ray beam is 2 mm high, what length of mirror is required to reflect the entire beam for each material?

5. Calculate the fraction of silver (Ag) fluorescence x-rays which are absorbed in a 1 mm thick silicon (Si) detector and the charge pulse expected for each absorbed photon. Repeat the calculation for a 1 mm thick germanium (Ge) detector.

Thin plate response - scattering approach

Consider a thin plate of thickness Δ onto which x-rays are incident from a point source S a perpendicular distance R_o away.



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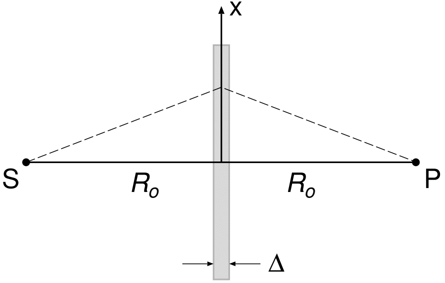

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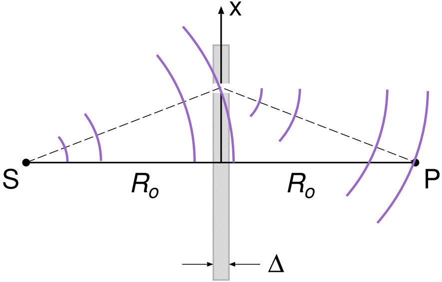

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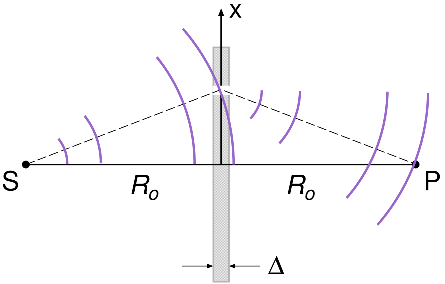

The plate has electron density ρ and the volume $\Delta dxdy$ contains $\rho\Delta dxdy$ electrons which scatter the x-rays.

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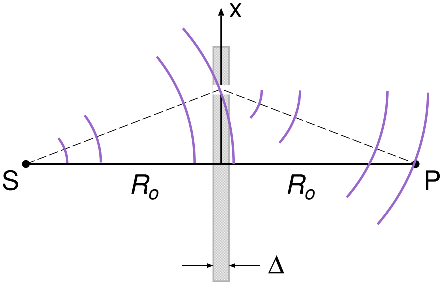

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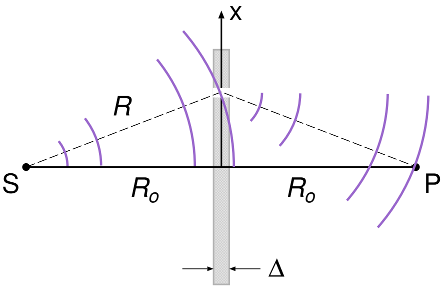

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$$R = \sqrt{R_o^2 + x^2 + y^2}$$

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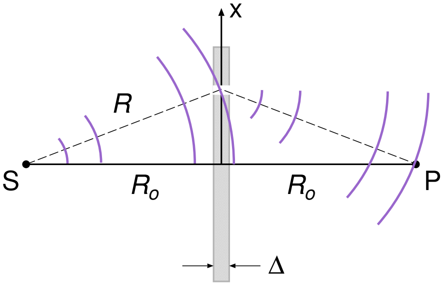

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$$R = R_o\sqrt{1 + \frac{x^2 + y^2}{R_o^2}}$$

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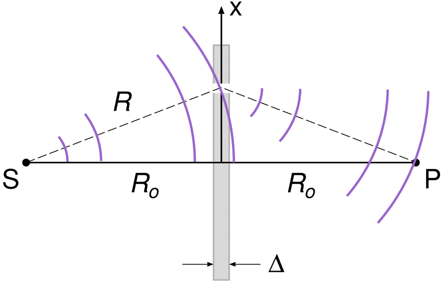

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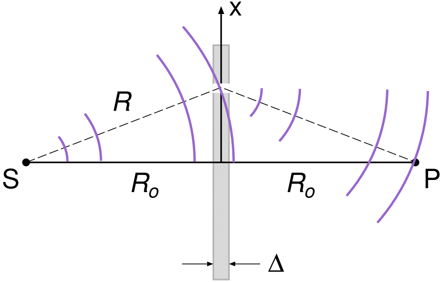
Thin plate response - scattering approach

R is also the distance between the scattering volume and P so, a wave (x-ray) which travels from $S \rightarrow P$ through the scattering volume will have an extra phase shift



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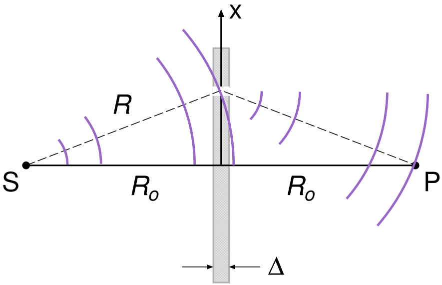

$$\phi(x, y) = 2k\frac{x^2 + y^2}{2R_o^2}$$

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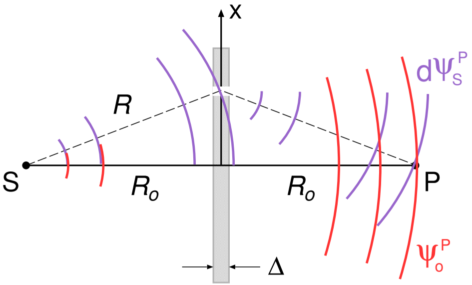

$$\phi(x,y) = 2k\frac{x^2+y^2}{2R_o^2} = \frac{x^2+y^2}{R_o^2}k$$

compared to a wave which travels directly along the z-axis.

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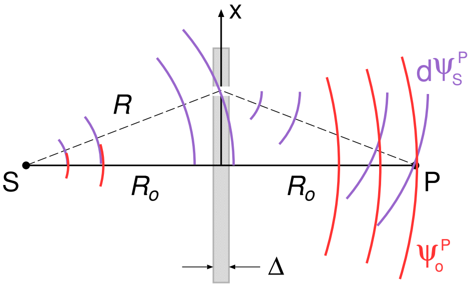

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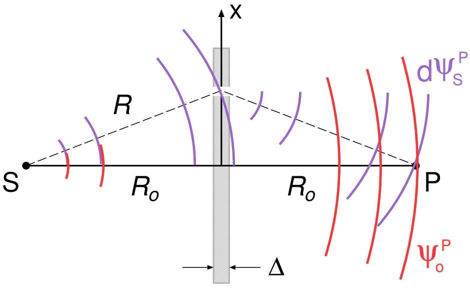

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$$d\psi_S^P \approx \left(\frac{e^{ikR_o}}{R_o}\right)$$

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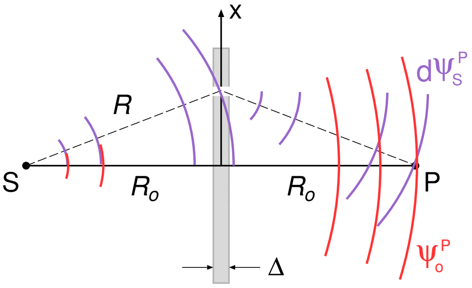

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$$d\psi_S^P \approx \left(\frac{e^{ikR_o}}{R_o}\right)(\rho\Delta dxdy)$$

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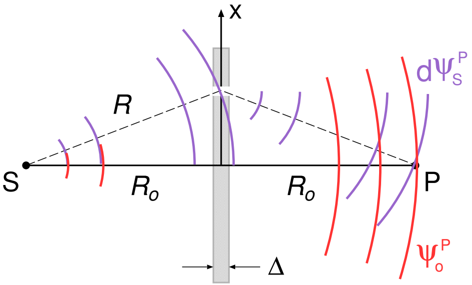

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$$d\psi_S^P \approx \left(\frac{e^{ikR_o}}{R_o}\right)(\rho\Delta dxdy)\left(-b\frac{e^{ikR_o}}{R_o}\right)$$

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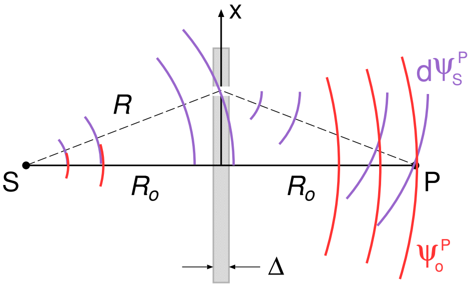

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Integrate the scattered wave over the entire plate

Thin plate response - scattering approach

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$$\psi_S^P = \int d\psi_S^P = -\rho b\Delta\frac{e^{i2kR_o}}{R_o^2}\int_{-\infty}^{\infty} e^{i\frac{x^2+y^2}{R_0^2}k}dxdy$$

Integrate the scattered wave over the entire plate This integral is basically a Gaussian integral with an imaginary (instead of real) constant in the exponent

$$I^2 = i\frac{\pi R_o}{k}$$

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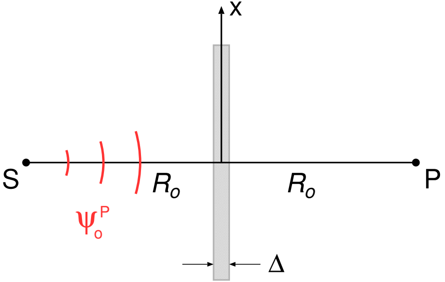
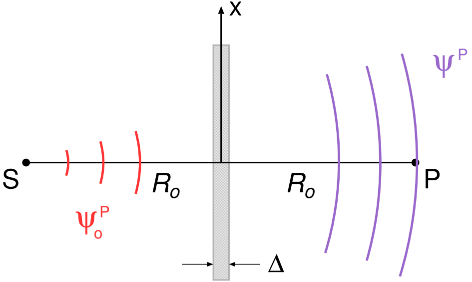
Now let's look at this phenomenon from a different point of view, that of refraction.



Thin plate response - refraction approach

Now let's look at this phenomenon from a different point of view, that of refraction. Assume that the wave passing through the plate simply gains a phase shift because it passes through a medium compared to a wave which does not have the plate present.



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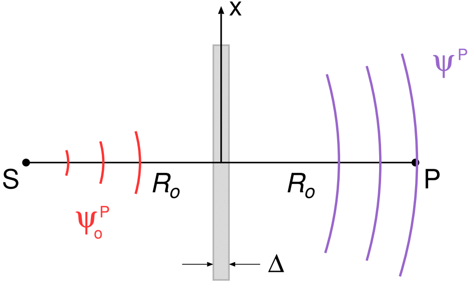

The phase shift depends on the thickness and the difference between the index of refraction of the medium and that of vacuum

$$\phi = (n-1)k\Delta$$

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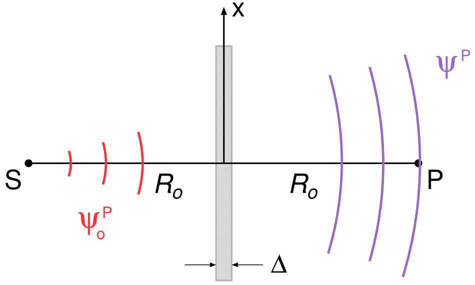

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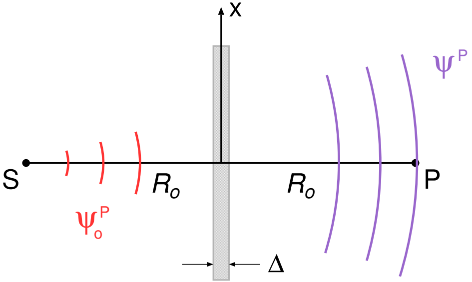

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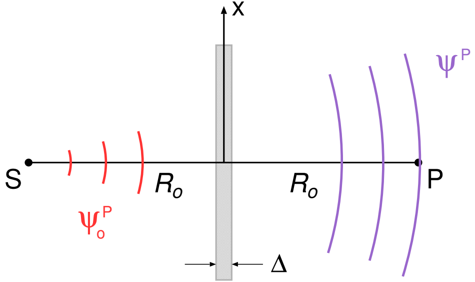

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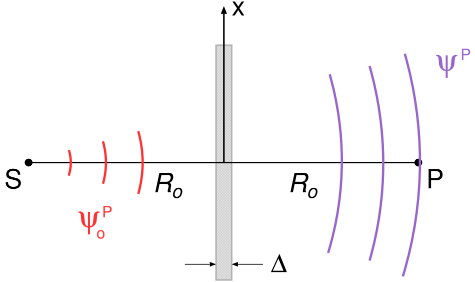

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Calculating n

We can now compare the expressions obtained by the scattering and refraction approaches.

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Scattering

Refraction

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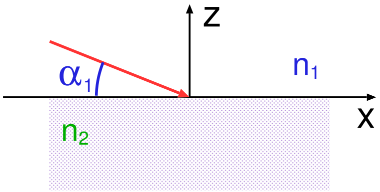
Index of refraction & critical angle

Now that we have an expression for the index of refraction, we can examine the consequences and estimate it's magnitude.



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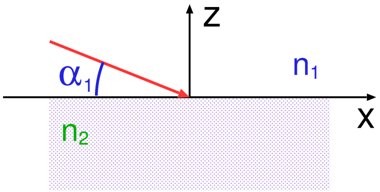

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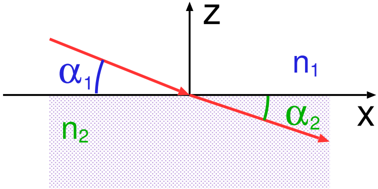

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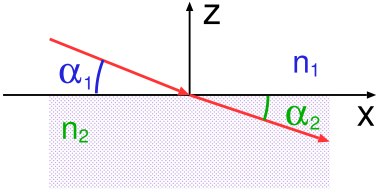

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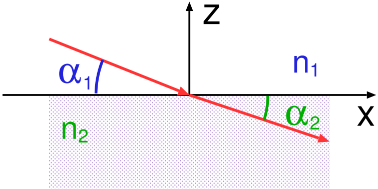

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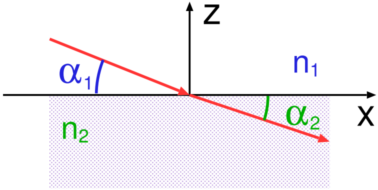

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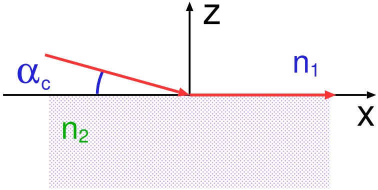

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\cos\alpha_c &= 1-\delta
\end{aligned}
$$

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When the incident angle becomes small enough, there will be total external reflection

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$$1-\delta = \cos\alpha_c$$

$$1-\delta = 1-\frac{{\alpha_c}^2}{2}+\cdots$$

$$1-\delta \approx 1-\frac{{\alpha_c}^2}{2}$$

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For small angles, the cosine function can expanded to give a simple relation for the critical angle

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\begin{aligned}
\alpha_c &= \sqrt{2 \times 10^{-5}} \\
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&= 0.26^\circ
\end{aligned}
$$

Connection to atomic scattering

So far, we have made the assumption that the charge distribution is uniform. We know that this is not correct, and that usually electron charge distributions are those of the atoms making up the solid.

$$\psi^P = \psi_o^P \left[1 - i \frac{2\pi\rho b\Delta}{k} \right]$$

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So far, we have made the assumption that the charge distribution is uniform. We know that this is not correct, and that usually electron charge distributions are those of the atoms making up the solid. Therefore, it is useful to replace the uniform charge distribution, ρ, with a more realistic one, including the atom distribution ρ_a:

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In terms of the absorption coefficient, μ

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$$
\begin{aligned}
f'' &= -\frac{k^2}{2\pi\rho_a r_o}\frac{\mu}{2k} \\
&= -\frac{k}{4\pi r_o}\sigma_a
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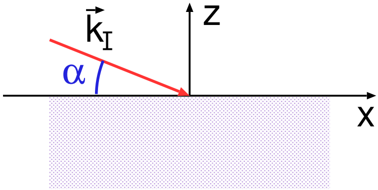
$$

Electromagnetic boundary conditions

Maxwell's equations require that an electromagnetic wave and its derivative be continuous in all directions at any interface. This condition places restrictions on the waves which exist at any interface:



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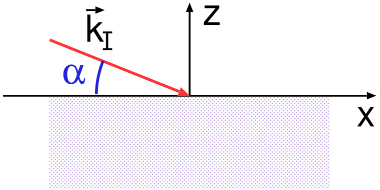

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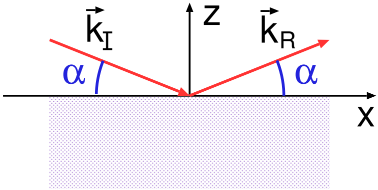

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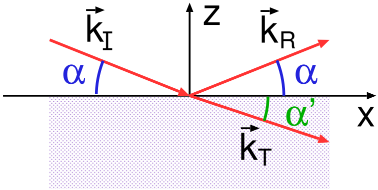

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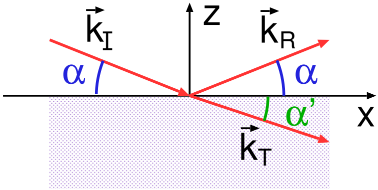

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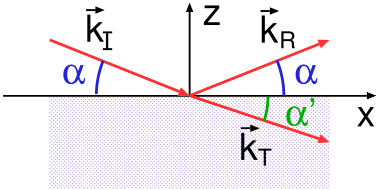

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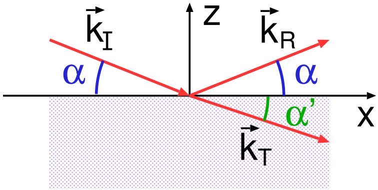

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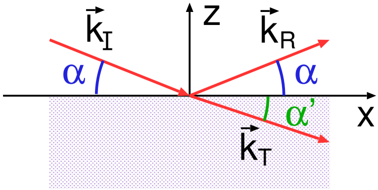

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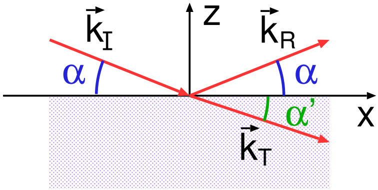

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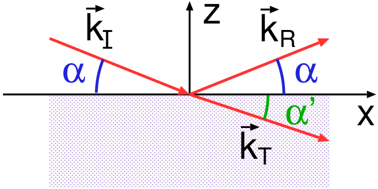

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Recalling that

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q is a convenient parameter to use because it is a combination of two parameters which are often varied in experiments, the angle of incidence α and the wavenumber (energy) of the x-ray, k.

Defining Equations in q

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reflected wave in phase with incident, almost total transmission

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The reflected wave is out of phase with the incident wave, there is only small transmission in the form of an evanescent wave, and the penetration depth is very short.

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The reflected wave is in phase with the incident, there is significant (larger amplitude than the reflection) transmission with a large penetration depth.