Today's Outline - January 22, 2015

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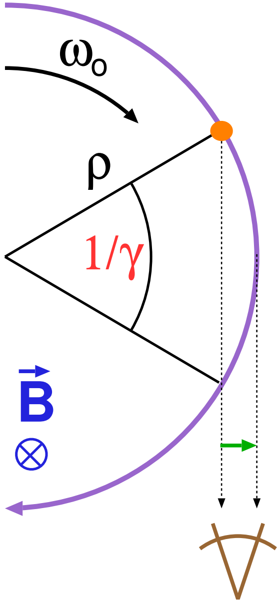
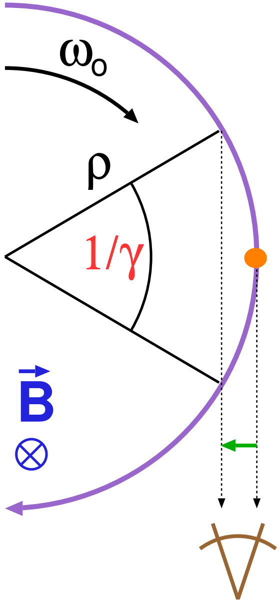
Homework Assignment #01:
Chapter Chapter 2: 2,3,5,6,8
due Thursday, January 29, 2015

Curved arc emission



The observer, looking in the plane of the circular trajectory,

Curved arc emission



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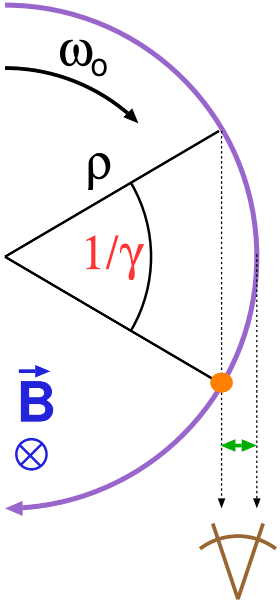

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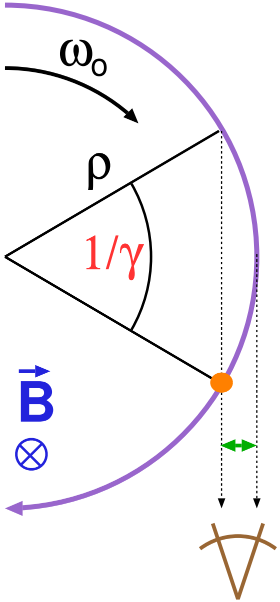

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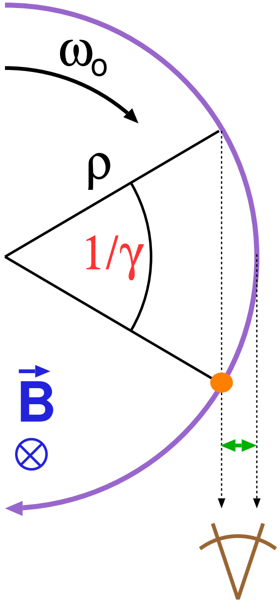

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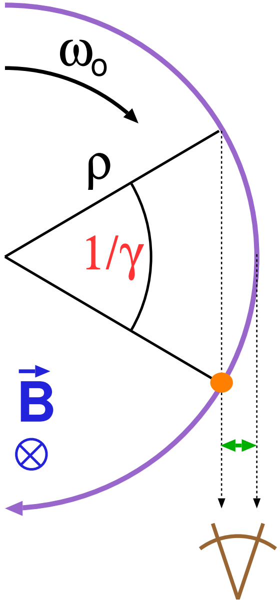

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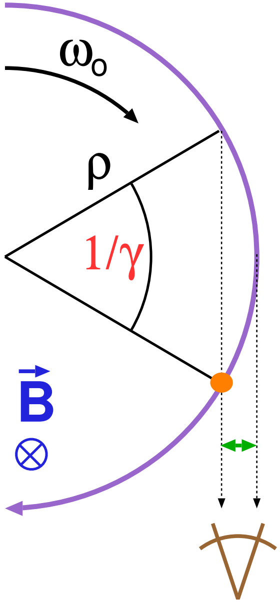

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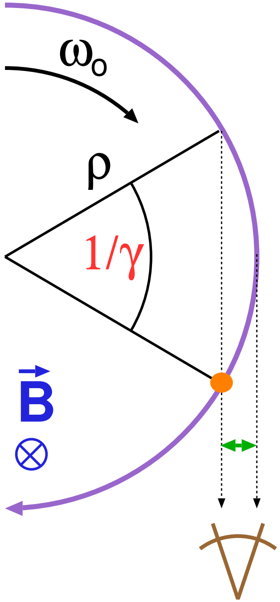

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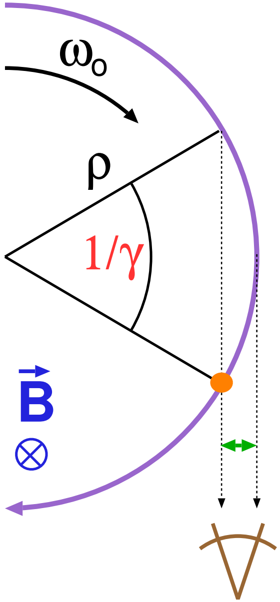

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The Fourier transform of this pulse is the spectrum of the radiation from the bending magnet.

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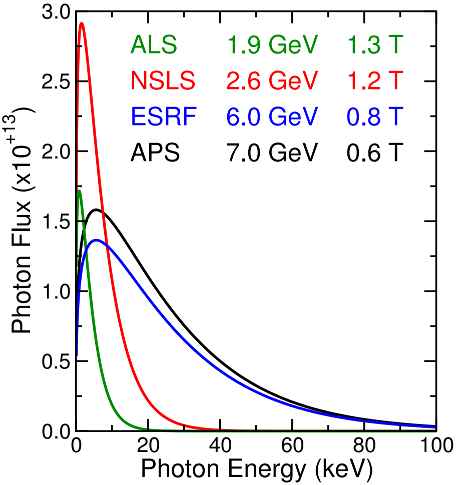
$$\mathcal{E}_c[\text{keV}] = 0.665\mathcal{E}^2[\text{GeV}]B[\text{T}]$$

Bending magnet spectrum

When the radiation pulse time is Fourier transformed, we obtain the spectrum of a bending magnet.



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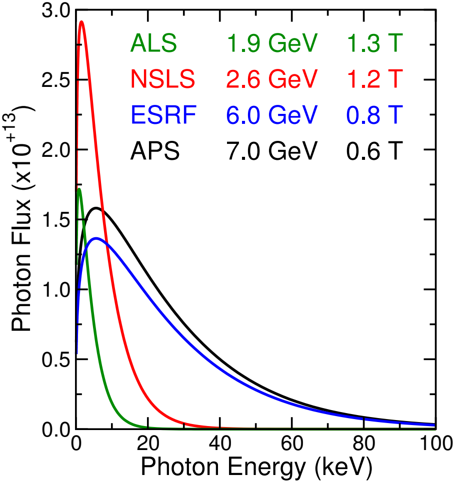

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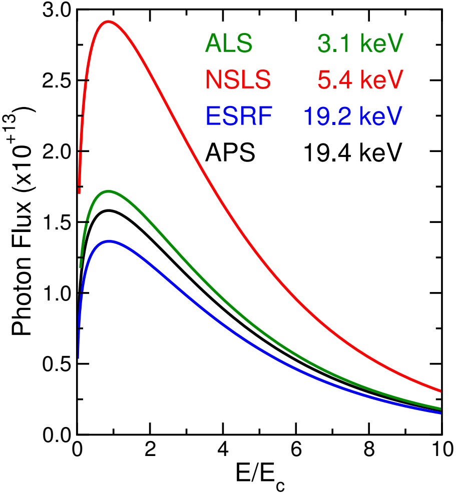

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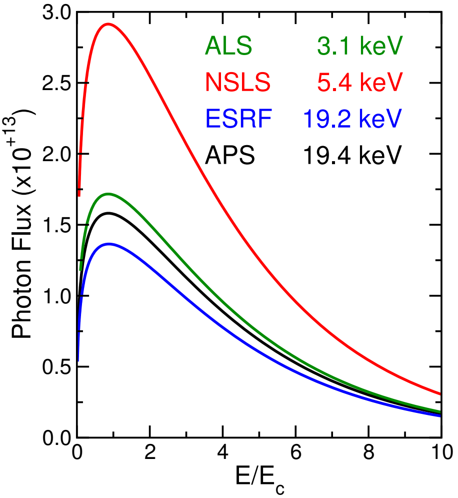
$$1.33\times10^{13}\mathcal{E}^2\, I \left(\frac{\omega}{\omega_c}\right)^2 K^2_{2/3}\left(\frac{\omega}{2\omega_c}\right)$$

where $K_{2/3}$ is a modified Bessel function of the second kind.



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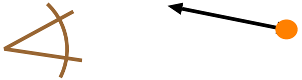
$$P = 1.266(6\text{GeV})^2(0.8\text{T})^2(1.24\times 10^{-3}\text{m})(0.2\text{A}) = 7.3\text{W}$$

Polarization

A bending magnet also produces circularly polarized radiation



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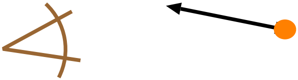

- If the observer is in the plane of the electron orbit, the electron motion looks like a half period of linear sinusoidal motion

Polarization

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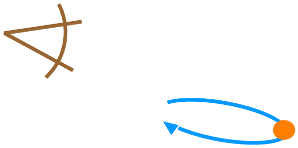

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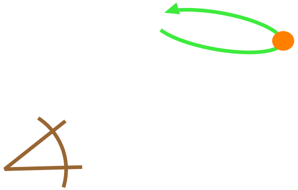

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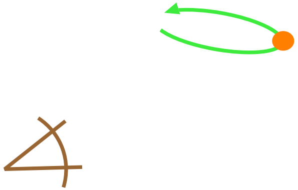

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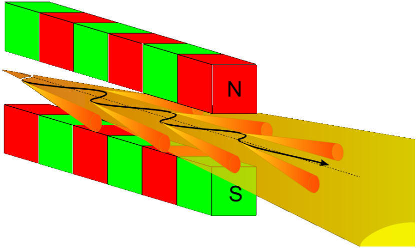
The result is circularly polarized radiation above and below the on-axis radiation.

Wigglers and undulators

Wiggler



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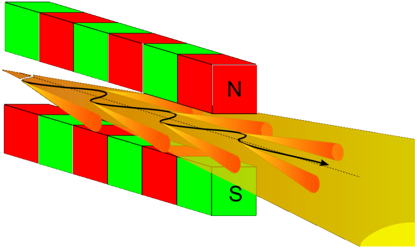

Like bending magnet except:

Wigglers and undulators

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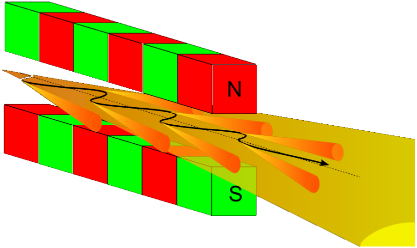

Like bending magnet except:

- larger $\vec{B} \rightarrow$ higher E_c

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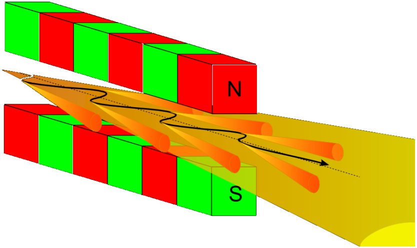

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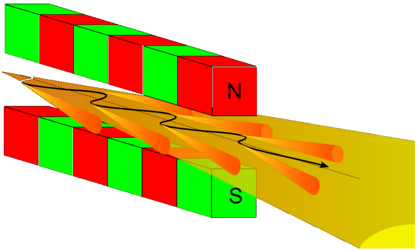
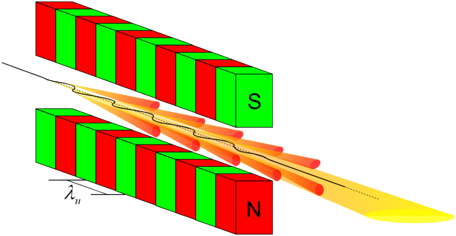


Undulator



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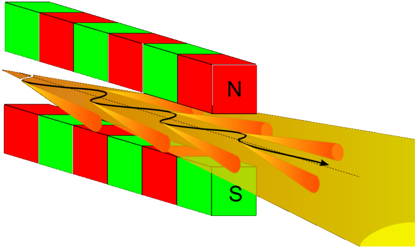

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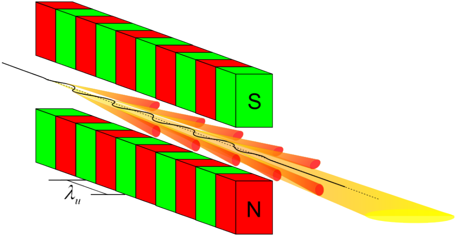

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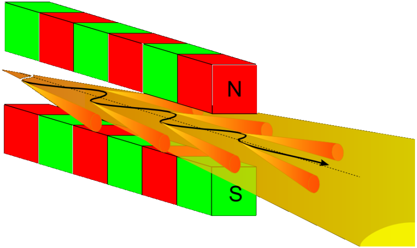
Different from bending magnet:

Wigglers and undulators

Wiggler



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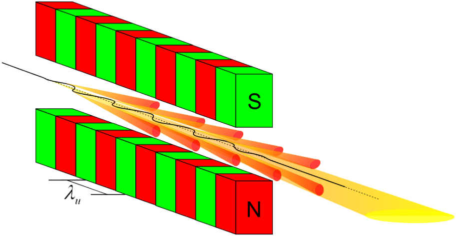

Like bending magnet except:

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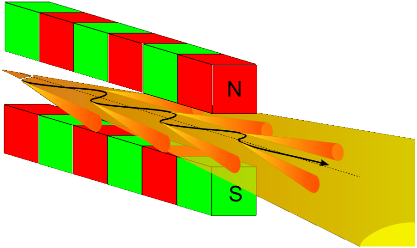
Different from bending magnet:

- shallow bends $\rightarrow$ smaller source

Wigglers and undulators

Wiggler



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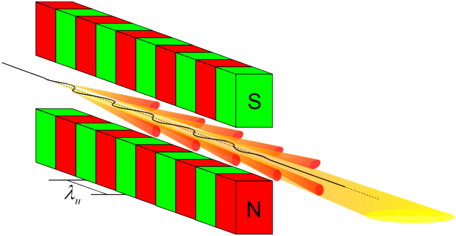

Like bending magnet except:

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Different from bending magnet:

- shallow bends $\rightarrow$ smaller source
- interference $\rightarrow$ peaked spectrum

Wiggler radiation

- The electron's trajectory through a wiggler can be considered as a series of short circular arcs, each like a bending magnet

$$Power[\mathrm{kW}] = 1.266\mathcal{E}_e^2[\mathrm{GeV}]B^2[\mathrm{T}]L[\mathrm{m}]I[\mathrm{A}]$$

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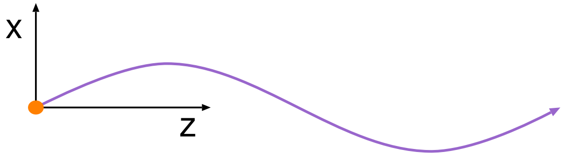
- The magnetic field varies along the length of the wiggler and is higher than that in a bending magnet, having an average value of $B_{rms} = B_o/\sqrt{2}$
- This results in a significantly higher power load on all downstream components

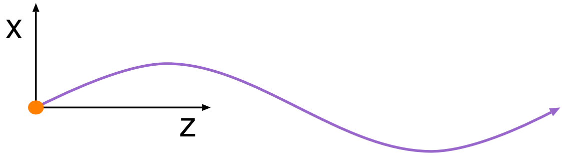
$$Power[\text{kW}] = 0.633\mathcal{E}_e^2[\text{GeV}]B_o^2[\text{T}]L[\text{m}]I[\text{A}]$$

Undulator characterization



Undulator radiation is characterized by three parameters:

Undulator characterization



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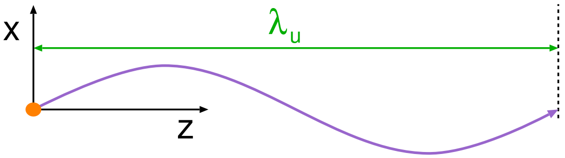

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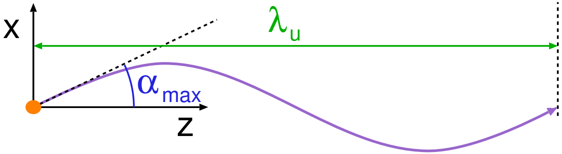

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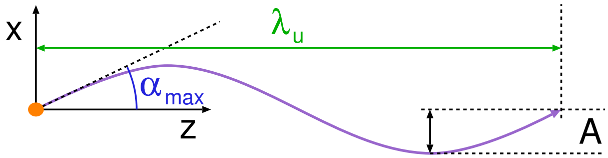

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$$x = A\sin(k_u z)$$

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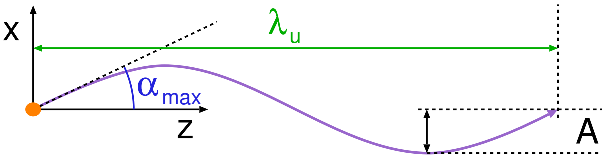

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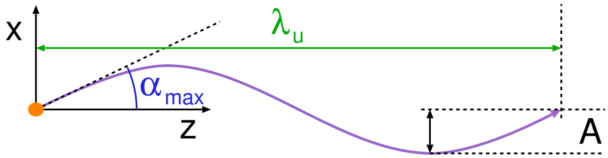

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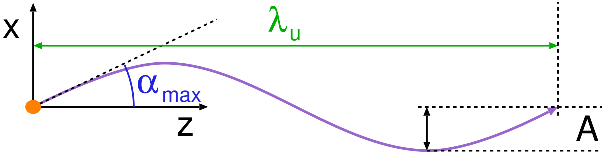

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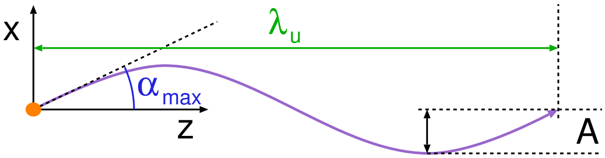

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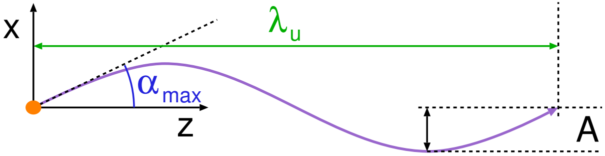

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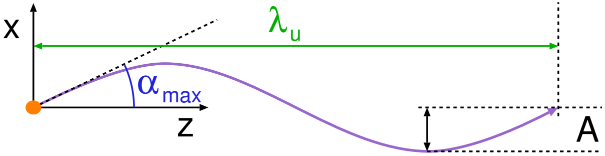
Define a dimensionless quantity, K which scales α_{max} to the natural opening angle of the radiation, $1/\gamma$

Undulator characterization



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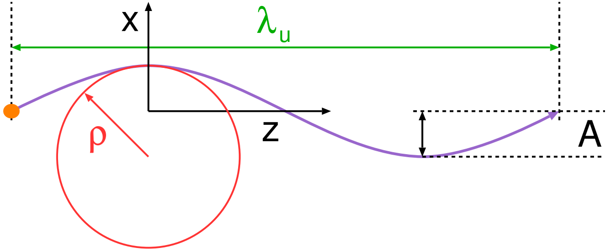
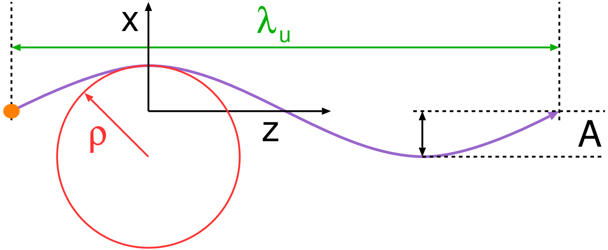
$$K = \alpha_{max}\gamma$$

Circular path approximation



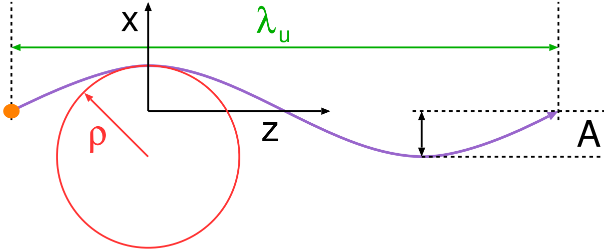
Consider the trajectory of the electron along one period of the undulator.

Circular path approximation



Consider the trajectory of the electron along one period of the undulator. Since the curvature is small, the path can be approximated by an arc or a circle of radius ρ whose origin lies at $x = -(\rho - A)$ and $z = 0$.

Circular path approximation



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The equation of the circle which approximates the arc is:

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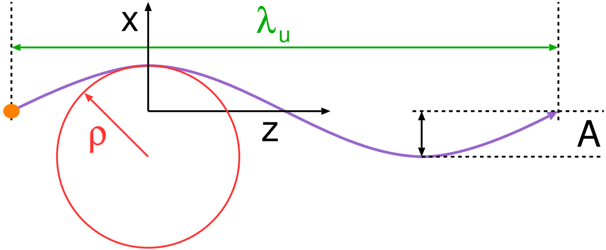

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$$\rho^2 = [x + (\rho - A)]^2 + z^2$$

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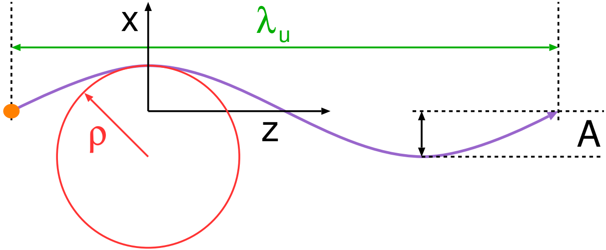

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The equation of the circle which approximates the arc is:

$$\rho^2 = [x + (\rho - A)]^2 + z^2$$
$$x + (\rho - A) = \sqrt{\rho^2 - z^2}$$

Radius of curvature

From the equation for a circle:

$$x = A - \rho + \sqrt{\rho^2 - z^2}$$

Radius of curvature

From the equation for a circle:

$$\begin{aligned} x &= A - \rho + \sqrt{\rho^2 - z^2} \\ &= A - \rho + \rho\sqrt{1 - \frac{z^2}{\rho^2}} \end{aligned}$$

Radius of curvature

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x &= A - \rho + \sqrt{\rho^2 - z^2} \\
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$$\frac{z^2}{2\rho} = \frac{A k_u^2 z^2}{2}$$

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Combining, we have

$$
\frac{z^2}{2\rho} = \frac{A k_u^2 z^2}{2} \quad \longrightarrow \quad \frac{1}{\rho} = A k_u^2
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Radius of curvature

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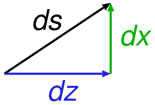
Combining, we have

$$\frac{z^2}{2\rho} = \frac{A k_u^2 z^2}{2} \quad \longrightarrow \quad \frac{1}{\rho} = A k_u^2 \quad \longrightarrow \quad \rho = \frac{1}{A k_u^2} = \frac{\lambda_u^2}{4\pi^2 A}$$

Electron path length

The displacement ds of the electron can be expressed in terms of the two coordinates, x and z as:



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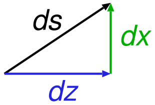

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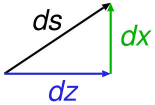

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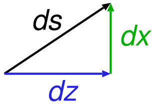

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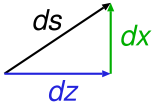

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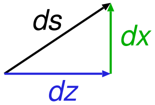

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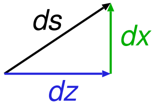

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$$S\lambda_u = \int_0^{\lambda_u} \sqrt{1 + \left(\frac{dx}{dz}\right)^2}\, dz$$

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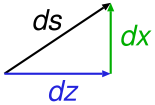

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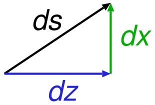

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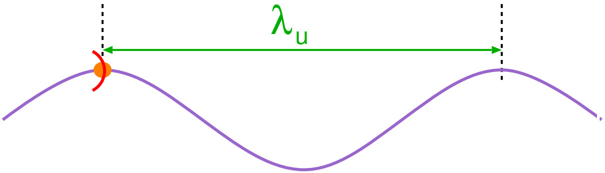
$$K = 0.934 \cdot 3.3[\mathrm{cm}] \cdot 0.6[\mathrm{T}] = 1.85$$

Undulator wavelength

Consider an electron traveling through the undulator and emitting radiation at the first maximum excursion from the center.



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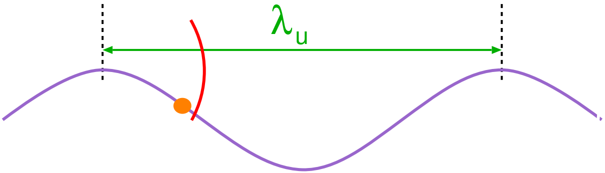

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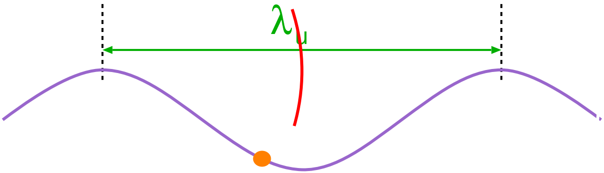

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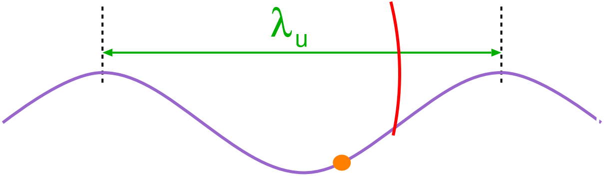

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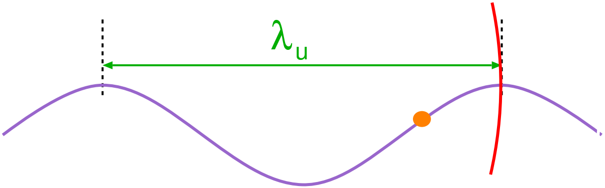

The emitted wave travels slightly faster than the electron.

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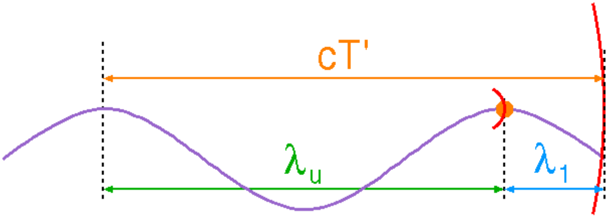

The emitted wave travels slightly faster than the electron.
It moves cT' in the time the electron travels a distance λ_u along the undulator.

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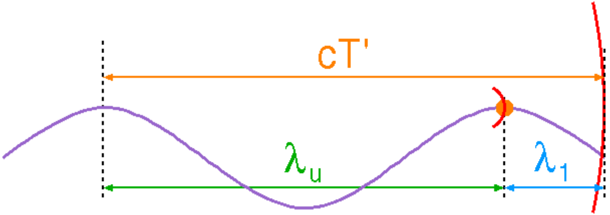

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The observer sees radiation with a compressed wavelength,

λ_1

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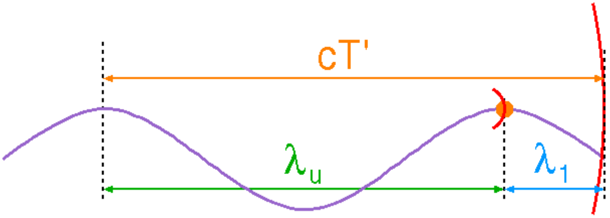

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$$\lambda_1 = cT' - \lambda_u$$

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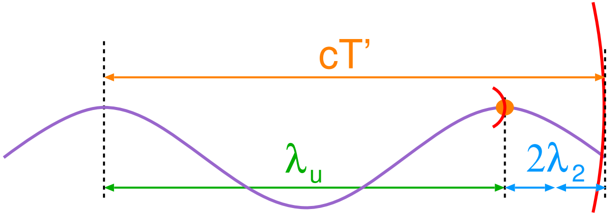

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The observer sees radiation with a compressed wavelength, along with harmonics which satisfy the same condition.

$$n\lambda_n = cT' - \lambda_u$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

Over the time T' the electron actually travels a distance $S\lambda_u$, so that

$$T' = \frac{S\lambda_u}{v}$$

The fundamental wavelength

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$$\lambda_1 = c T' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

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Since γ is large, the maximum observation angle θ is small so

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Since γ is large, the maximum observation angle θ is small so

$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right)$$

The fundamental wavelength

The fundamental wavelength must be corrected for the observer angle θ

$$\lambda_1 = cT' - \lambda_u \cos\theta$$

$$= \lambda_u \left(S\frac{c}{v} - \cos\theta \right)$$

$$= \lambda_u \left(\left[1 + \frac{K^2}{4\gamma^2} \right] \frac{1}{\beta} - \cos\theta \right)$$

Over the time T' the electron actually travels a distance $S\lambda_u$, so that

$$T' = \frac{S\lambda_u}{v}$$

$$S \approx 1 + \frac{K^2}{4\gamma^2}$$

Since γ is large, the maximum observation angle θ is small so

$$\lambda_1 \approx \lambda_u \left(\frac{1}{\beta} + \frac{K^2}{4\gamma^2\beta} - 1 + \frac{\theta^2}{2} \right) = \frac{\lambda_u}{2\gamma^2} \left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2 \right)$$

The fundamental wavelength

$$\lambda_1 \approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2\gamma^2}{\beta} + \frac{K^2}{2\beta} - 2\gamma^2 + \gamma^2\theta^2\right)$$

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regrouping terms

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$$\approx \frac{\lambda_u}{2\gamma^2}\left(2\gamma^2\left[\frac{1}{\beta} - 1\right] + \frac{K^2}{2\beta} - (\gamma\theta)^2\right)$$

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$$1 - \beta^2 = (1+\beta)(1-\beta)$$

The fundamental wavelength

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$$1-\beta^2 = (1+\beta)(1-\beta)$$

$$\approx \frac{\lambda_u}{2\gamma^2}\left(\frac{2}{\beta(1+\beta)} + \frac{K^2}{2\beta} - [\gamma\theta]^2\right)$$

The fundamental wavelength

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The fundamental wavelength

If we assume that $\beta \sim 1$ for these highly relativistic electrons

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This corresponds to an energy $\mathcal{E}_1 \approx 8.2\text{keV}$ but as the undulator gap is widened

The fundamental wavelength

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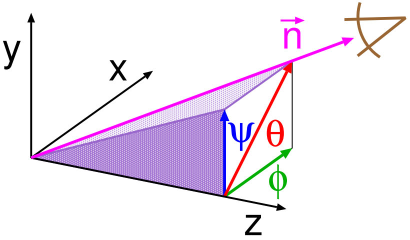
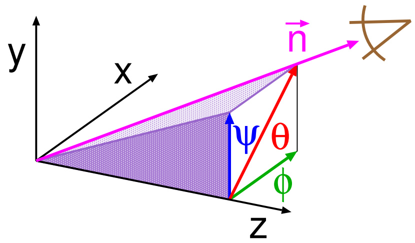
This corresponds to an energy $\mathcal{E}_1 \approx 8.2\text{keV}$ but as the undulator gap is widened, B_o decreases, K decreases, λ_1 decreases, and $\mathcal{E}_1$ increases.

Higher harmonics



Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

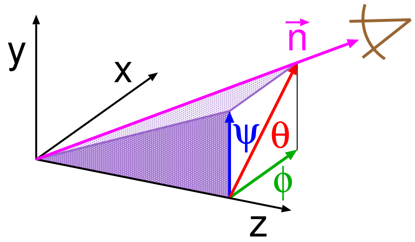
Higher harmonics



Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

$$\frac{dt}{dt'} = 1 - \vec{n} \cdot \vec{\beta}(t')$$

Higher harmonics



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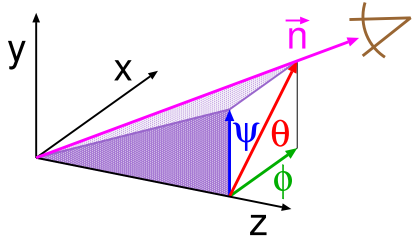
Recall that we developed an expression for the Doppler time compression of the emission from a moving electron as a function of the observer angle.

This can be rewritten in terms of the coordinates in the figure using the vector of unit length in the observer direction:

$$\vec{n} = \left\{ \phi, \psi, \sqrt{1-\theta^2} \right\}$$

$$\vec{\beta} = \beta \left\{ \alpha, 0, \sqrt{1-\alpha^2} \right\}$$

Higher harmonics



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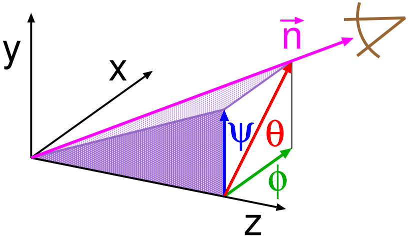
Higher harmonics



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$$\approx 1 - \beta \left[\alpha\phi + \left(1 - \frac{\theta^2}{2} - \frac{\alpha^2}{2} \right) \right]$$

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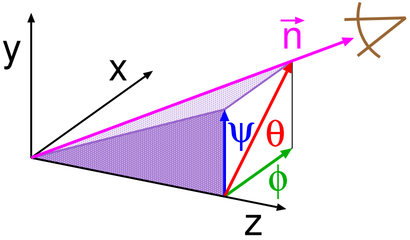
Higher harmonics



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Higher harmonics

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$$\approx 1 - 1 - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2}$$

Higher harmonics

$$\begin{aligned}\frac{dt}{dt'} &\approx 1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 + \alpha\phi - \frac{\theta^2}{2} - \frac{\alpha^2}{2}\right) \\ &\approx 1 - 1 - \alpha\phi + \frac{\theta^2}{2} + \frac{\alpha^2}{2} + \frac{1}{2\gamma^2} = \frac{1}{2}\left(\theta^2 + \alpha^2 + \frac{1}{\gamma^2}\right) - \alpha\phi\end{aligned}$$

Higher harmonics

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This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

Higher harmonics

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$\omega_1 t = \omega_u t'$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression

Higher harmonics

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This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2} \sin(2\omega_u t')$$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression , but they are not proportional because of the second

Higher harmonics

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This differential equation can be solved, realizing that ϕ and θ are constant while $\alpha(t')$ varies as the electron moves through the insertion device, and gives:

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1 + (\gamma\theta)^2 + K^2/2}\phi\sin(\omega_u t')$$

$\omega_1 \gg \omega_u$ as expected because of the Doppler compression , but they are not proportional because of the second and third terms.

Higher harmonics

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$\omega_1 \gg \omega_u$ as expected because of the Doppler compression , but they are not proportional because of the second and third terms.

The motion of the electron, $\sin\omega_u t'$, is always sinusoidal, but because of the additional terms, the motion as seen by the observer, $\sin\omega_1 t$, is not.

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2} \sin(2\omega_u t')$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1 + (\gamma\theta)^2 + K^2/2} \sin\left(2\omega_u t'\right)$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6} \sin\left(2\omega_u t'\right)$$

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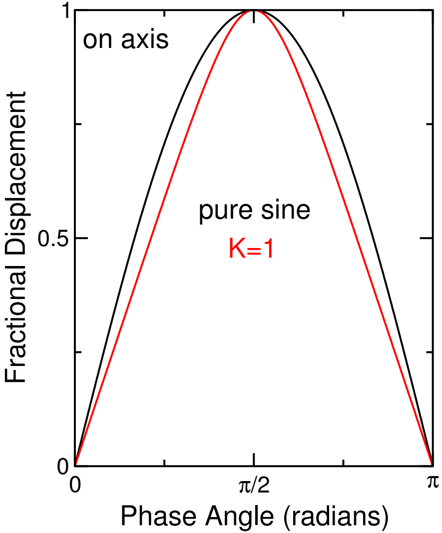
Plotting $sin\omega_u t'$ and $sin\omega_1 t$ shows the deviation from sinusoidal.



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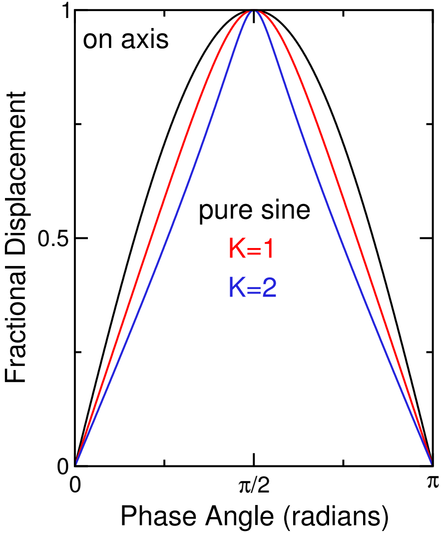
Similarly, for $K = 2$



On-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t')$$

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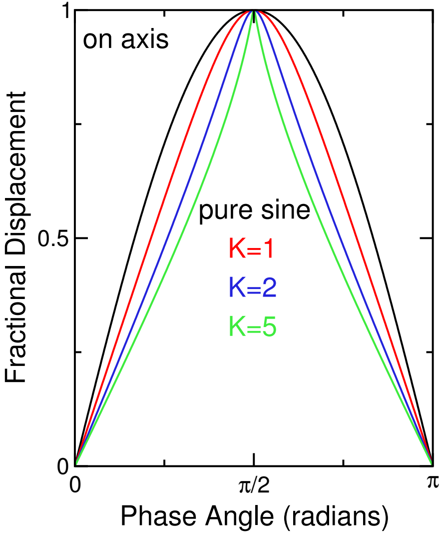
Similarly, for $K = 2$ and $K = 5$, the deviation becomes more pronounced.



On-axis undulator characteristics

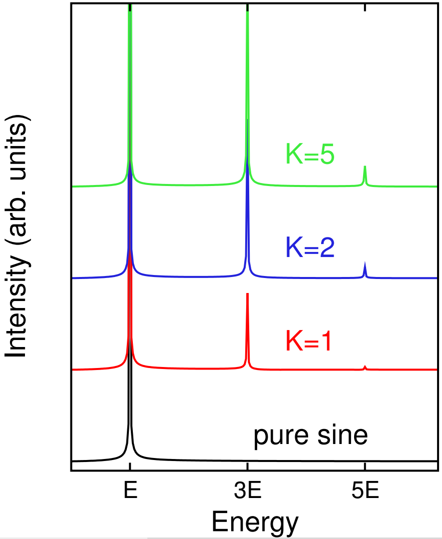
$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t')$$

Suppose we have $K = 1$ and $\theta = 0$ (on axis), then

$$\omega_1 t = \omega_u t' + \frac{1}{6}\sin(2\omega_u t')$$

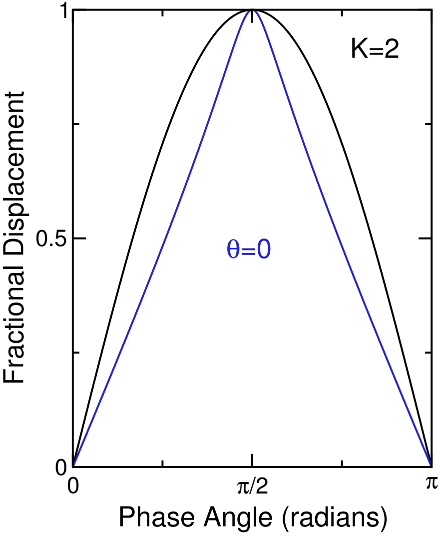
Plotting $sin\omega_u t'$ and $sin\omega_1 t$ shows the deviation from sinusoidal.

Similarly, for $K = 2$ and $K = 5$, the deviation becomes more pronounced. This shows how higher harmonics must be present in the radiation as seen by the observer.



Off-axis undulator characteristics

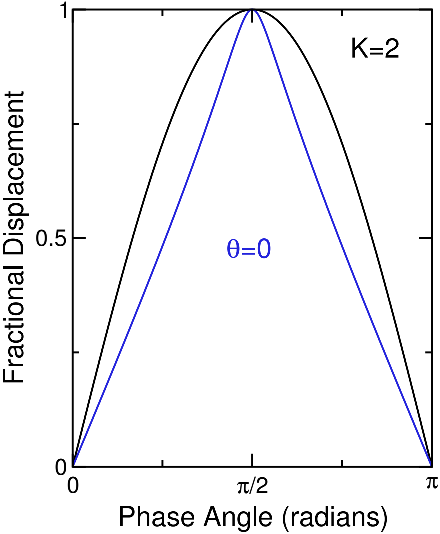
$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$



When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

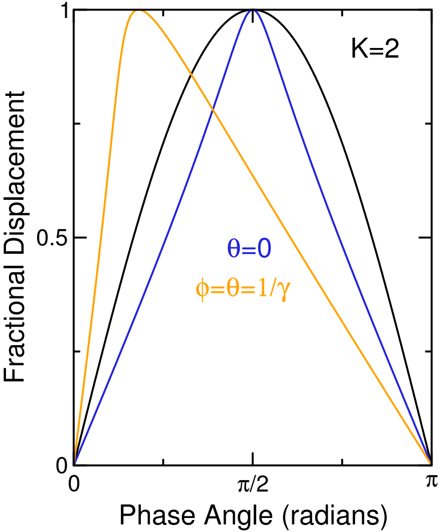


When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

$$\omega_1 t = \omega_u t' + \frac{1}{4}\sin(2\omega_u t') + \sin\omega_u t'$$

Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$



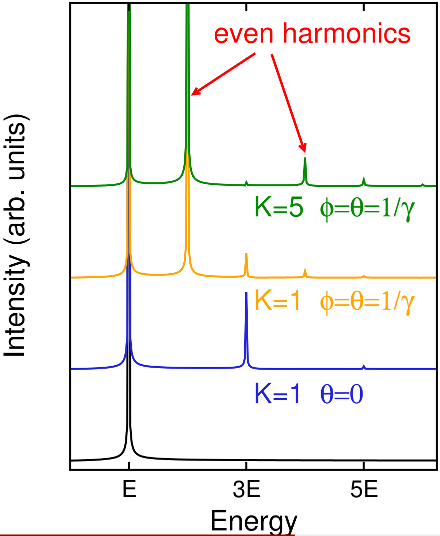
When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

$$\omega_1 t = \omega_u t' + \frac{1}{4}\sin(2\omega_u t') + \sin\omega_u t'$$

The last term introduces an antisymmetric term which skews the function

Off-axis undulator characteristics

$$\omega_1 t = \omega_u t' - \frac{K^2/4}{1+(\gamma\theta)^2+K^2/2}\sin(2\omega_u t') - \frac{2K\gamma}{1+(\gamma\theta)^2+K^2/2}\phi\sin(\omega_u t')$$

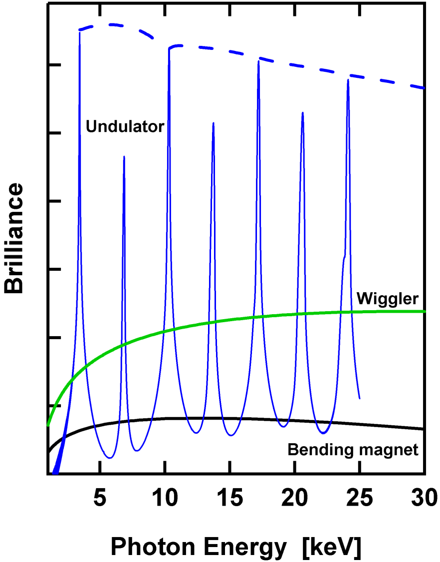


When $K = 2$ and $\theta = \phi = 1/\gamma$, we have

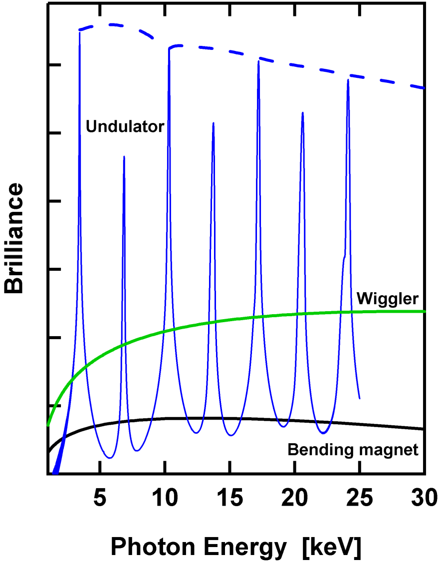
$$\omega_1 t = \omega_u t' + \frac{1}{4}\sin(2\omega_u t') + \sin\omega_u t'$$

The last term introduces an antisymmetric term which skews the function and leads to the presence of forbidden harmonics (2^{nd}, 4^{th}, etc) in the radiation from the undulator compared to the on-axis radiation.

Spectral comparison

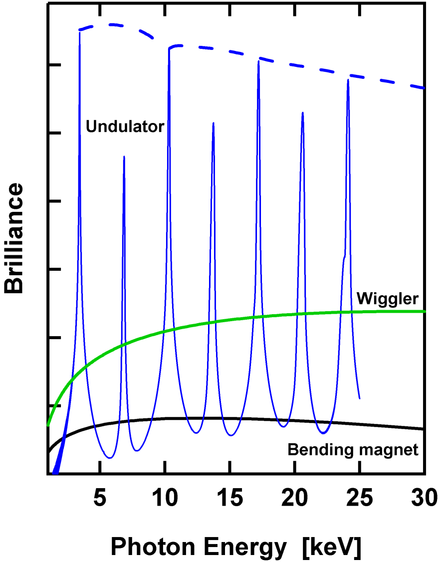


Spectral comparison



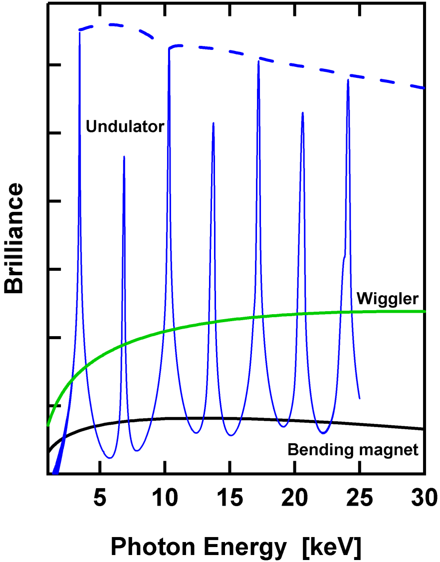
- Brilliance is 6 orders larger than a bending magnet

Spectral comparison



- Brilliance is 6 orders larger than a bending magnet
- Both odd and even harmonics appear

Spectral comparison



- Brilliance is 6 orders larger than a bending magnet
- Both odd and even harmonics appear
- Harmonics can be tuned in energy (dashed lines)