Today's Outline - January 20, 2015

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- Coherence of x-ray sources

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Homework Assignment #01:
Chapter Chapter 2: 2,3,5,6,8
due Thursday, January 29, 2015

Coherence: what is it?

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Because of these imperfections the "coherence length" of an x-ray beam is finite and we can calculate it.

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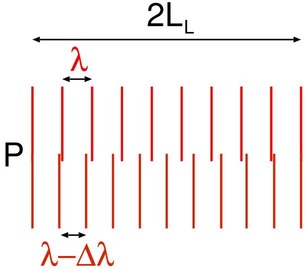

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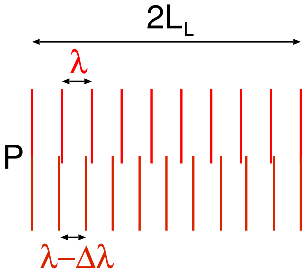

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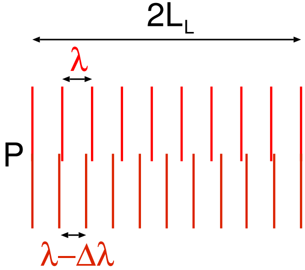

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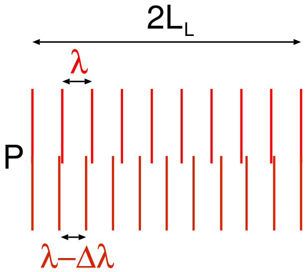

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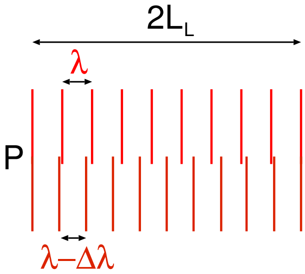

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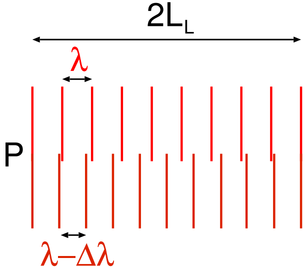

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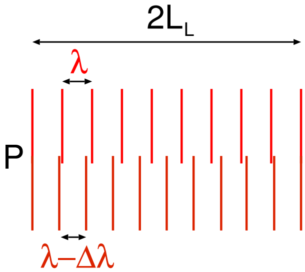

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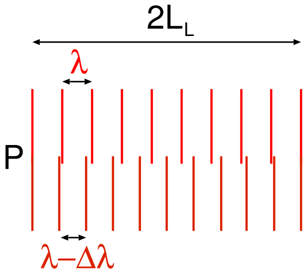

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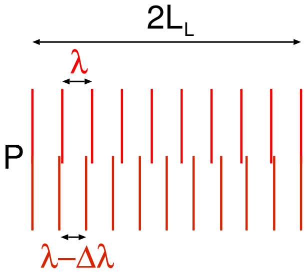

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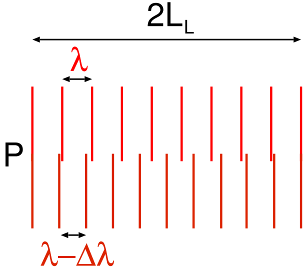

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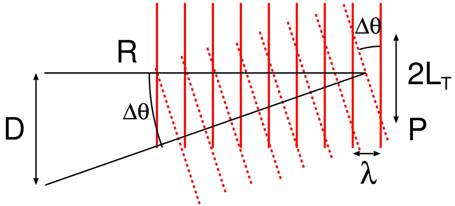

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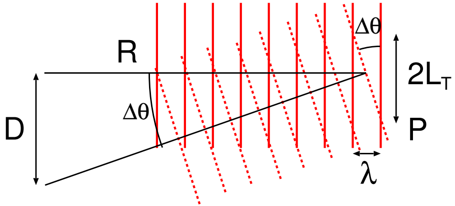

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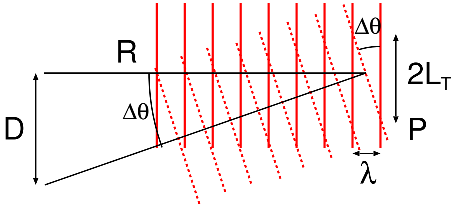

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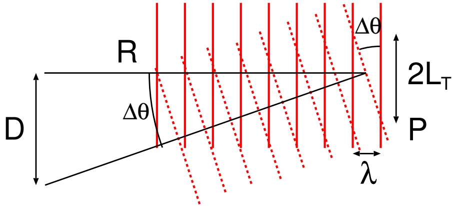

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Coherence lengths at the APS

For a typical 3^{rd} generation undulator source, such as at the Advanced Photon Source the vertical source size is $D = 100\mu$m and we are typically $R = 50$m away with our experiment. If we assume a typical wavelength of $\lambda = 1$Å, and a monochromator resolution of $\Delta\lambda/\lambda = 10^{-5}$ we have for the vertical direction:

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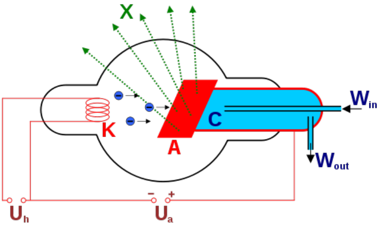
$$L_T = \frac{\lambda R}{2D} = \frac{(1 \times 10^{-10}) \cdot 50}{2 \cdot (100 \times 10^{-6})} = 25\mu\text{m}$$

X-ray tube schematics

Fixed anode tube



- low power
- low maintenance

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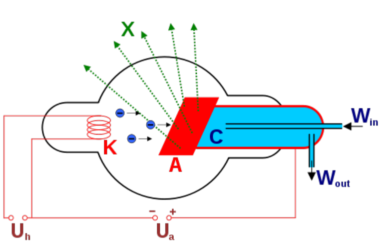

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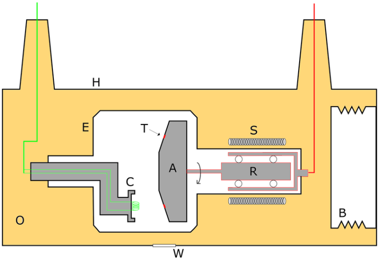
Rotating anode tube



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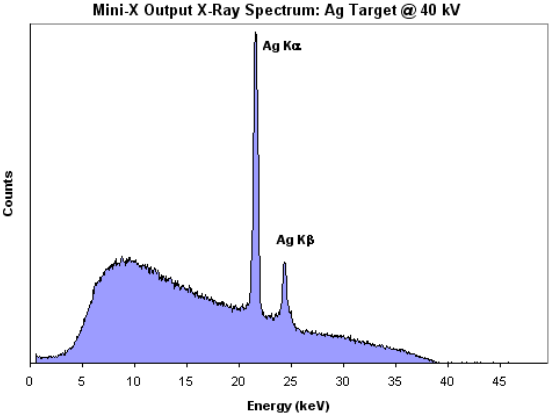
X-ray tube spectrum



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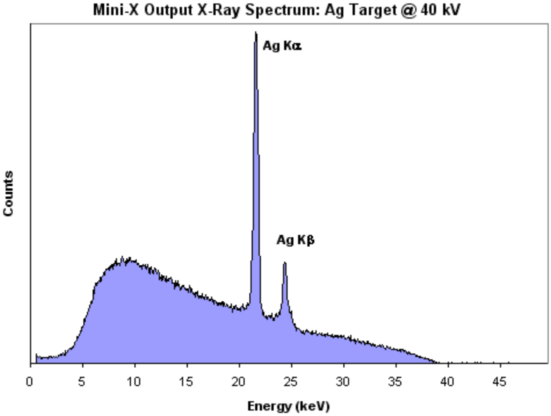
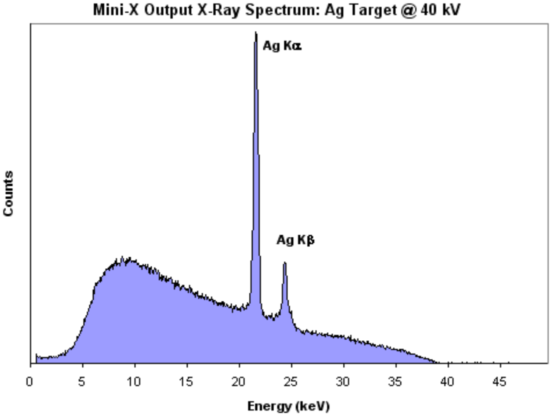


- Minimum wavelength (maximum energy) set by accelerating potential

X-ray tube spectrum



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- Bremßtrahlung radiation provides smooth background (charged particle deceleration)

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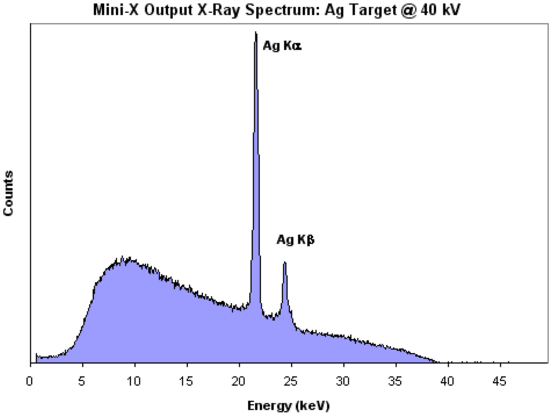

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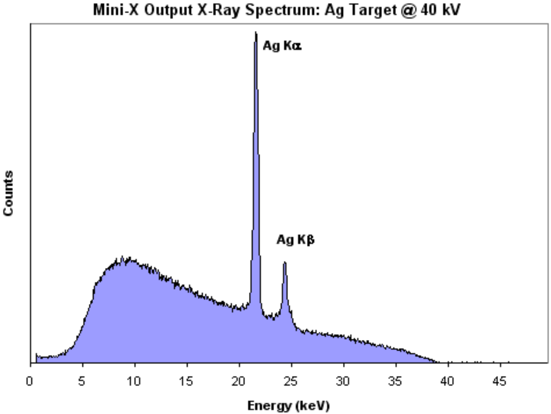

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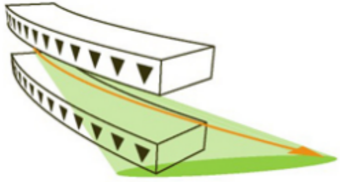
- Highest intensity at the characteristic fluorescence emission energy of the anode material
- Unpolarized, incoherent x-rays emitted in all directions from anode surface, must be collimated with slits

Synchrotron sources

Bending magnet



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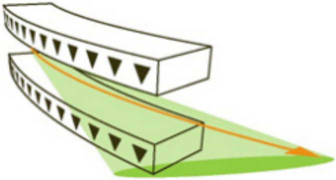

- Wide horizontal beam

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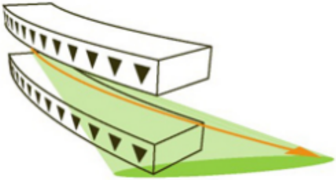

- Wide horizontal beam
- Broad spectrum to high energies

Synchrotron sources

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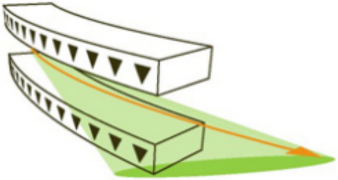

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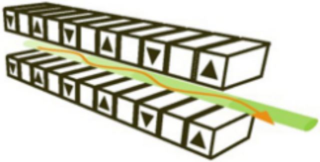

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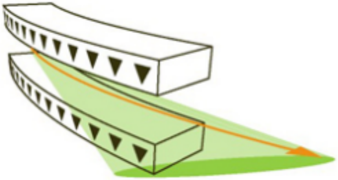

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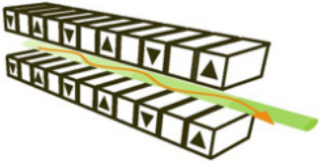

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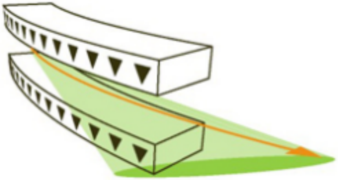

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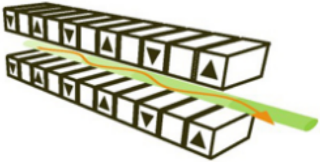
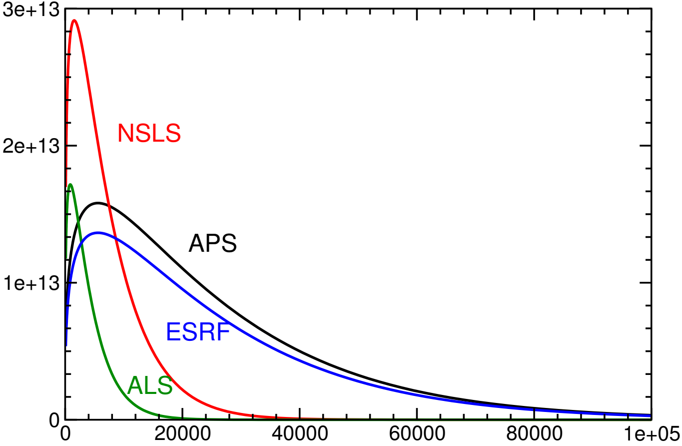


- Highly collimated beam
- Highly peaked spectrum with harmonics

Bending magnet spectra



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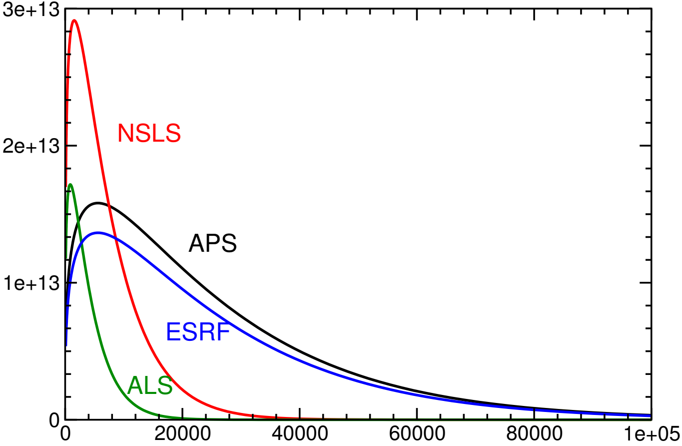

Lower energy sources, such as NSLS have lower peak energy and higher intensity at the peak.

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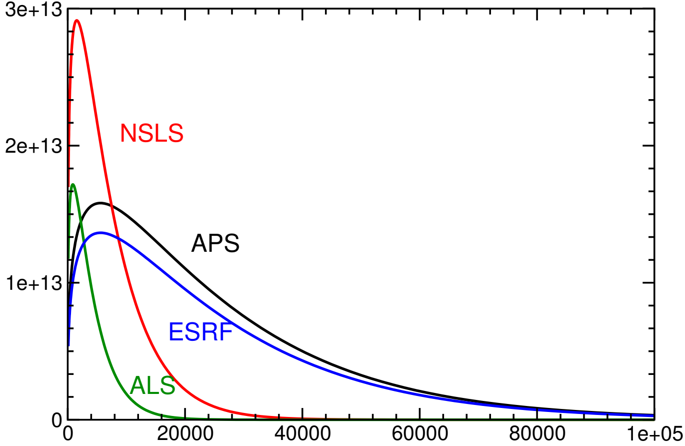
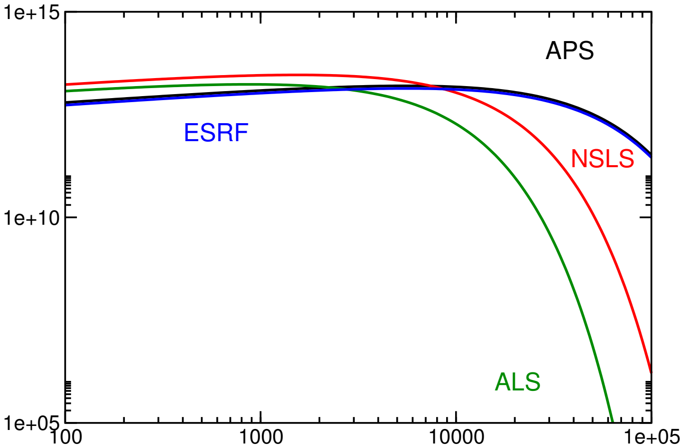


Lower energy sources, such as NSLS have lower peak energy and higher intensity at the peak.

Higher energy sources, such as APS have higher energy spectrum and are only off by a factor of 2 intensity at low energy.

Bending magnet spectra



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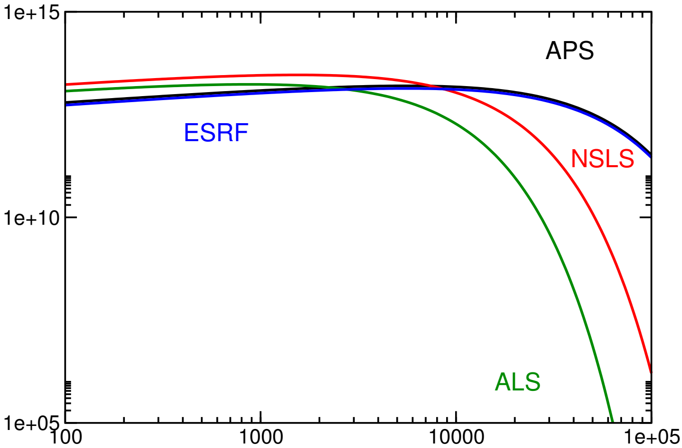
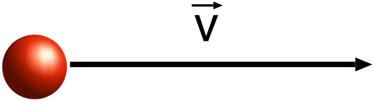


Logarithmic scale shows clearly how much more energetic and intense the bending magnet sources at the 6 GeV and 7 GeV sources are.

Review of special relativity



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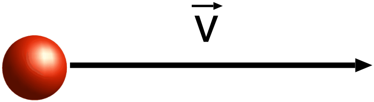
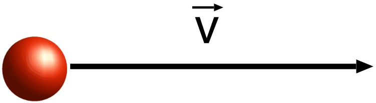


$$\beta = \frac{v}{c}$$

Review of special relativity



$$\beta = \frac{v}{c} \qquad \gamma = \sqrt{\frac{1}{1-\beta^2}}$$

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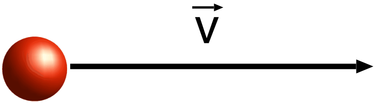

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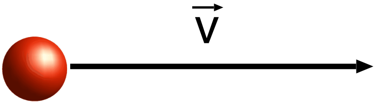

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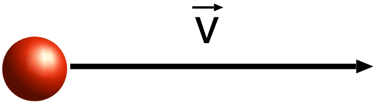

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use binomial expansion since $1/\gamma^2 << 1$

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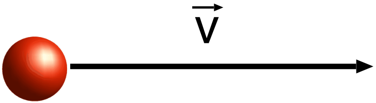

Let's calculate these quantities for an electron at NSLS and APS

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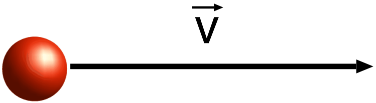

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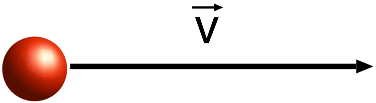

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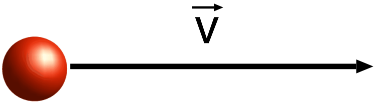

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APS: $E = 7.0$ GeV

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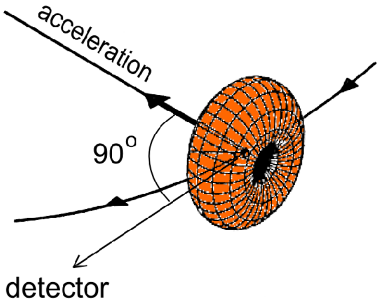
"Headlight" effect

In electron rest frame:



emission is symmetric about the axis of the acceleration vector

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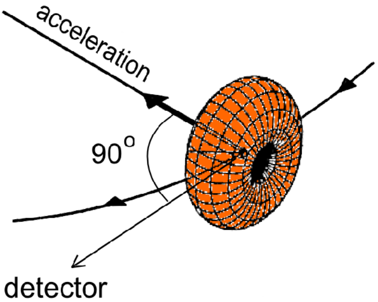

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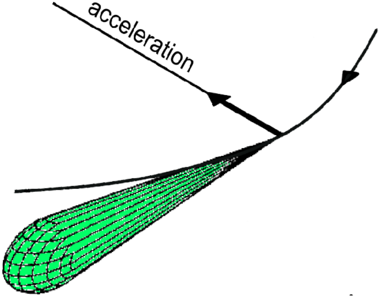
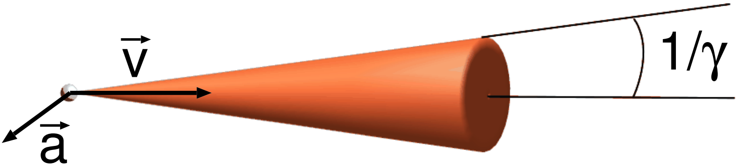
In lab frame:



emission is pushed into the direction of motion of the electron

Relativistic emission



the electron is in constant transverse acceleration due to the Lorentz force from the magnetic field of the bending magnet

$$\vec{F} = e\vec{v} \times \vec{B} = m_e\vec{a}$$

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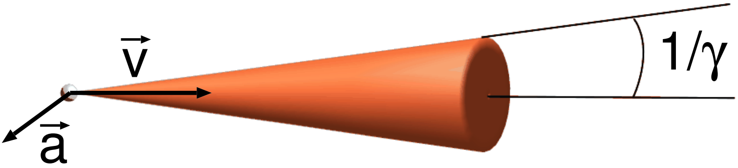

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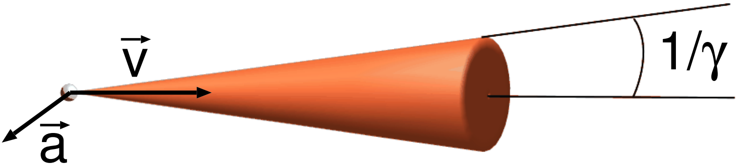

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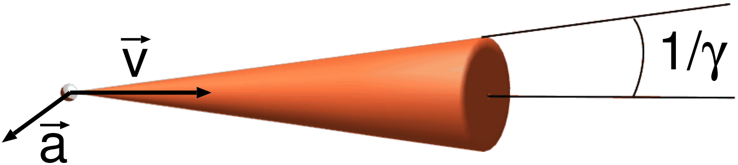

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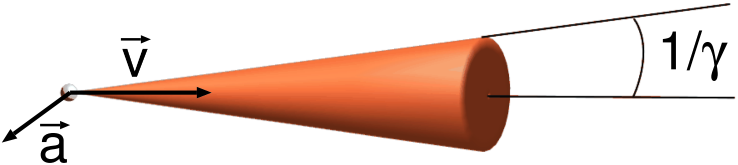

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for the APS, with $\gamma \approx 10^4$ we have

$$E_{max} \approx (10^4)^3 \cdot 10^6 = 10^{18}$$

Flux and brilliance

There are a number of important quantities which are relevant to the quality of an x-ray source:

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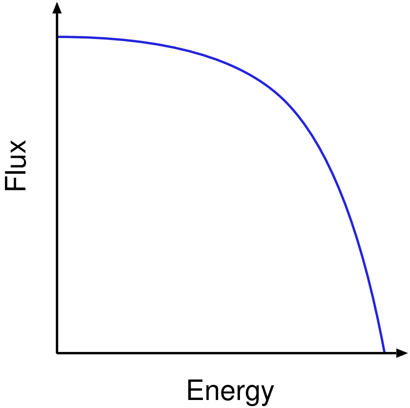

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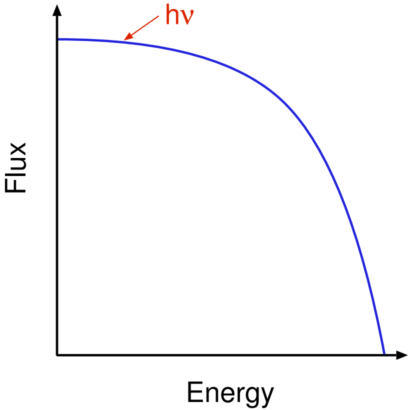

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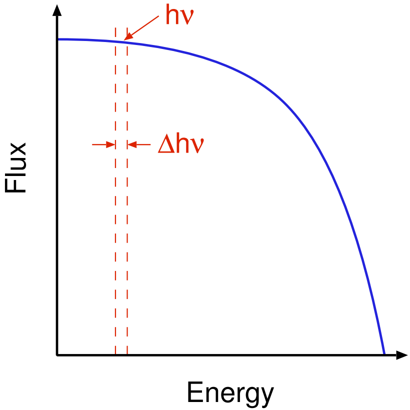

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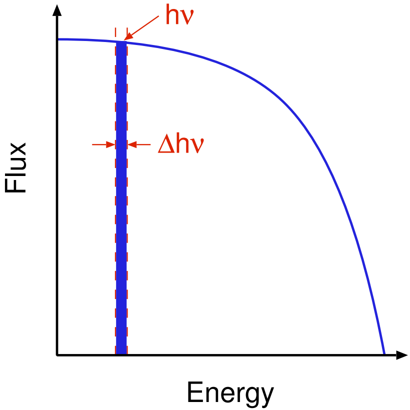

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Compute the integrated photon flux in that bandwidth.

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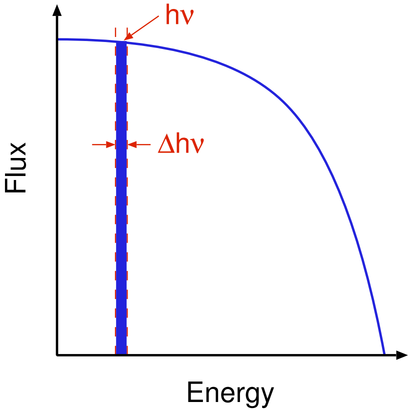

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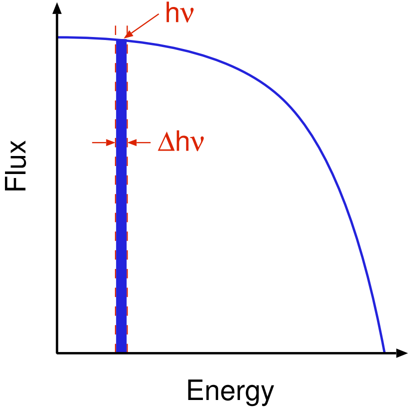

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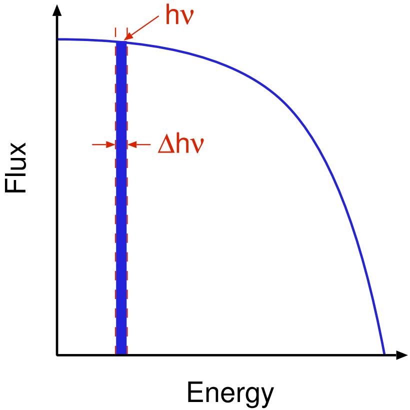

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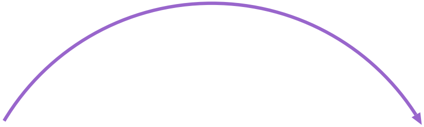
$$\alpha \approx x/z \qquad \beta \approx y/z,$$

where z is the distance from the source over which there is a lateral spread x and y in each direction

Segmented arc approximation



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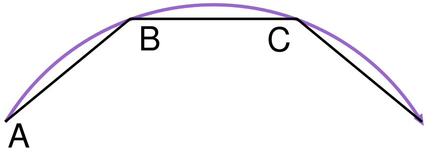
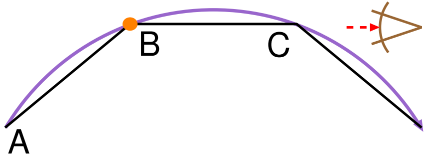


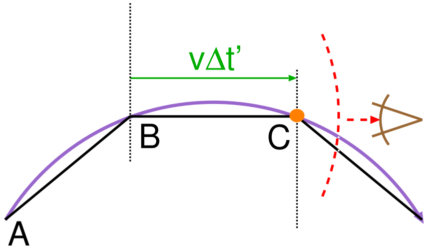
- Approximate the electron's path as a series of segments

Segmented arc approximation



- Approximate the electron's path as a series of segments
- At each corner the electron is accelerated and emits radiation

Segmented arc approximation



- Approximate the electron's path as a series of segments
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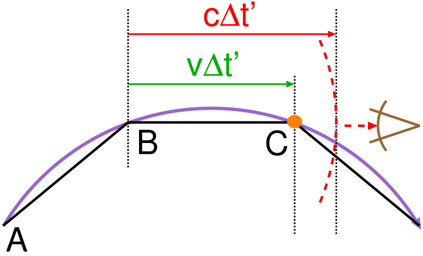

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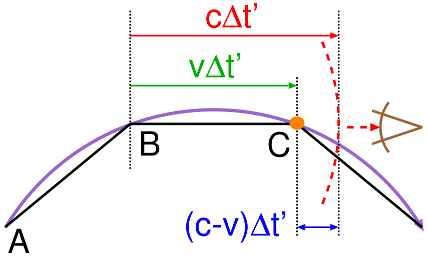

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The electron travels the distance from B to C in $\Delta t'$ while the light pulse emitted at B travels further, $c\Delta t'$, in the same time.

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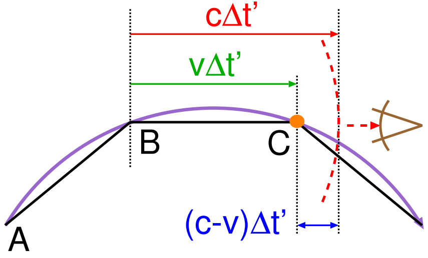

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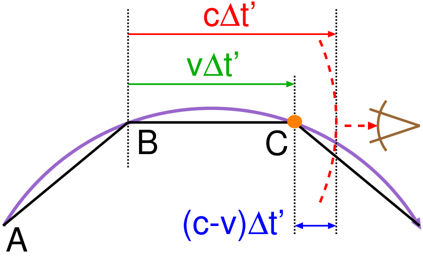

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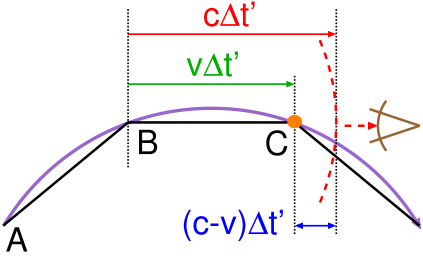
The observer will measure a time between the two pulses:

Segmented arc approximation



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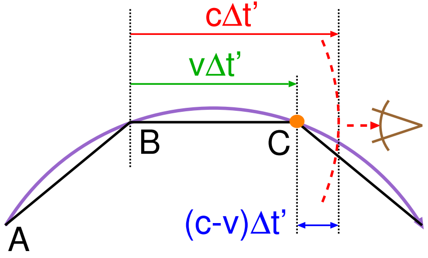

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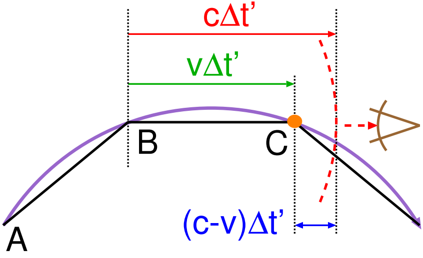

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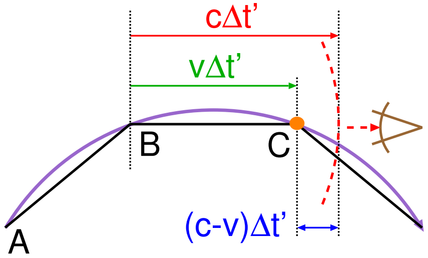

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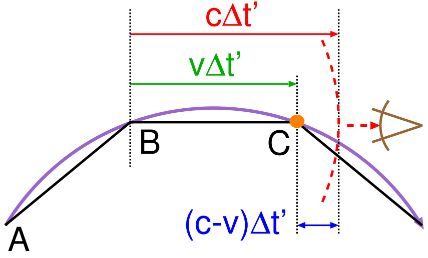

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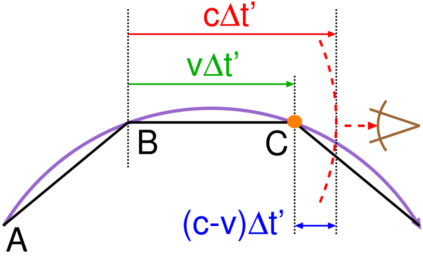

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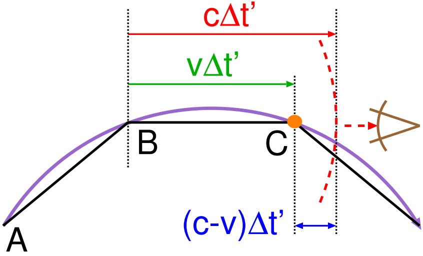

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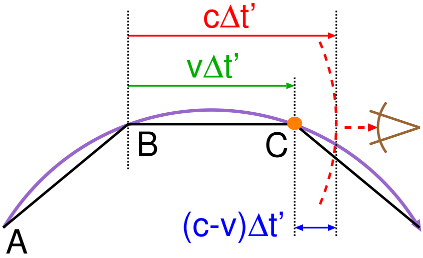

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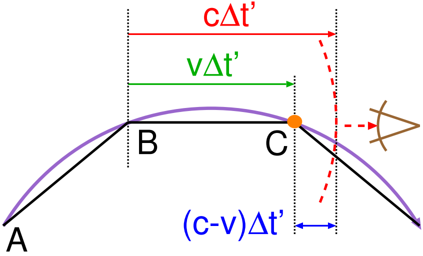

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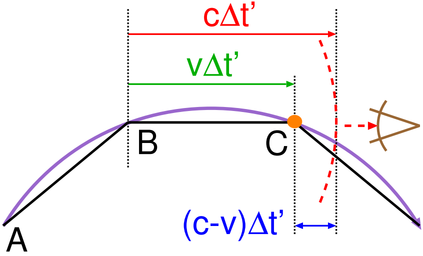

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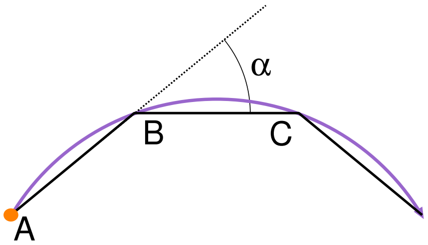
Off-axis emission



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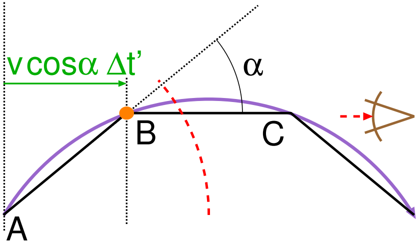

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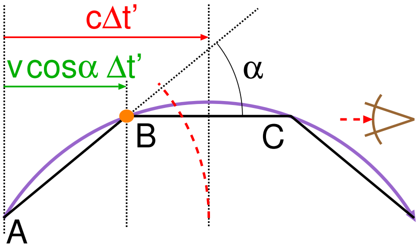

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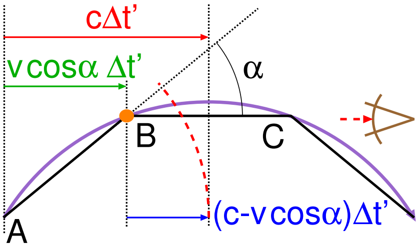

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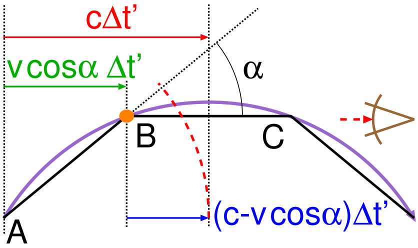

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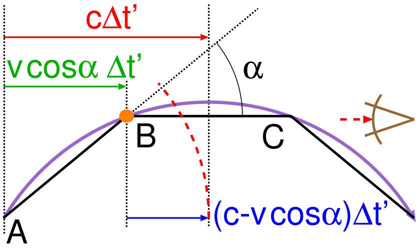

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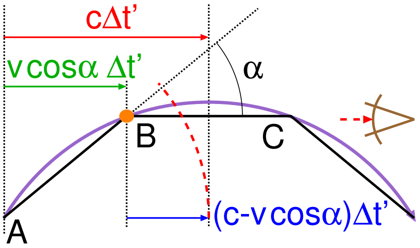

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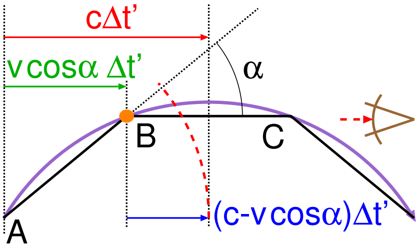

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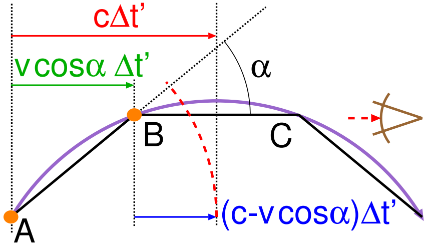
Corrected Doppler shift



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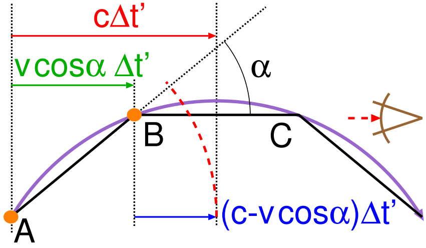

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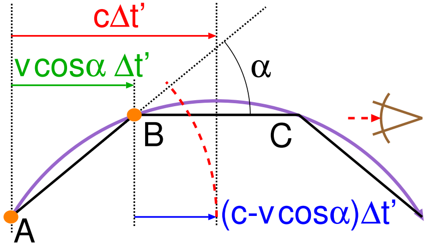

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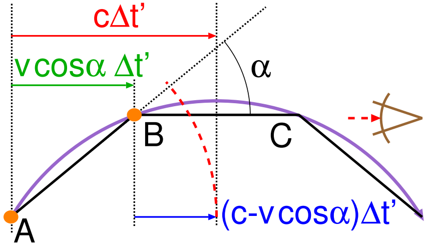

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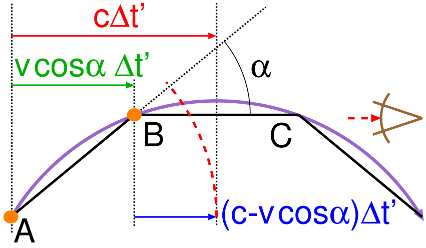

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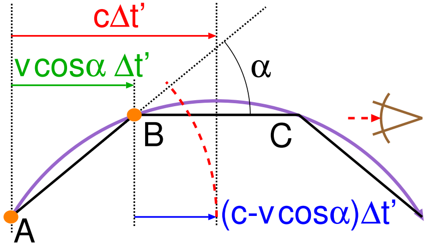

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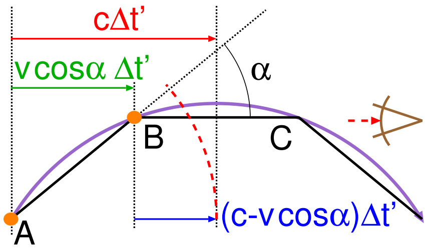

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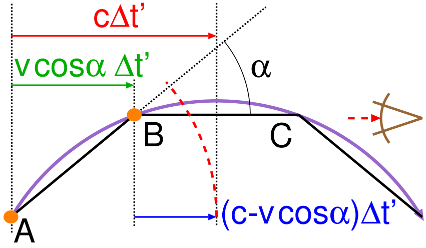

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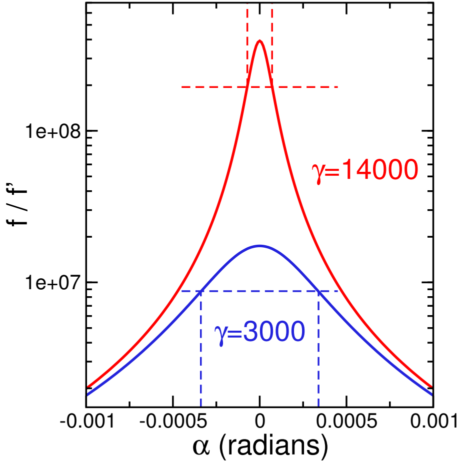
called the time compression ratio.

Radiation opening angle

The Doppler shift is defined in terms of the time compression ratio

$$\frac{f}{f'} = \frac{\Delta t'}{\Delta t} = \frac{2\gamma^2}{1+\alpha^2\gamma^2}$$



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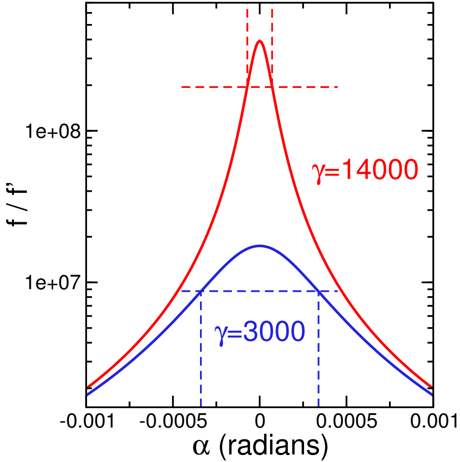

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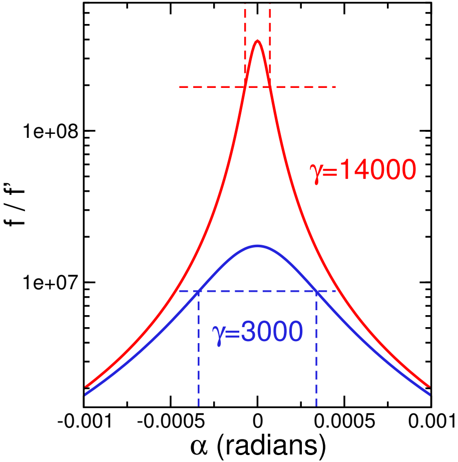

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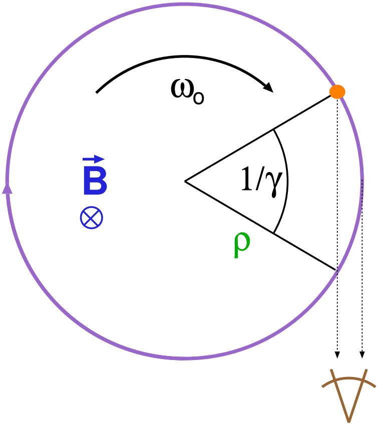
- The intesection of the horizontal and vertical dashed lines indicate where $\alpha = \pm 1/\gamma$ and f/f' is one half of it's maximum value
- The highest energy emitted radiation appears within a cone of half angle $1/\gamma$

Curved arc emission



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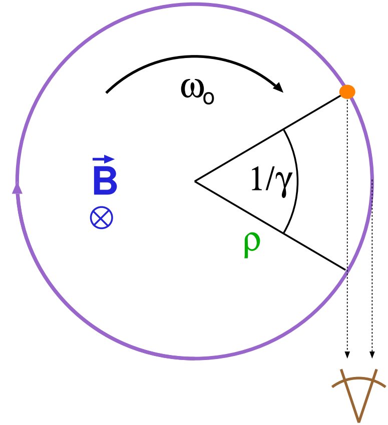

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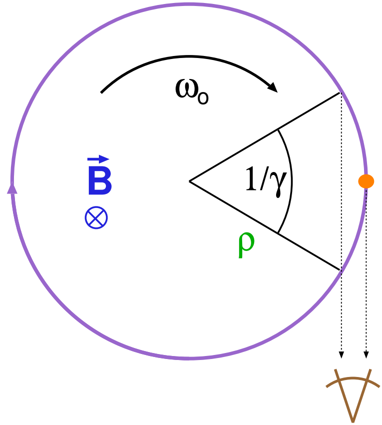

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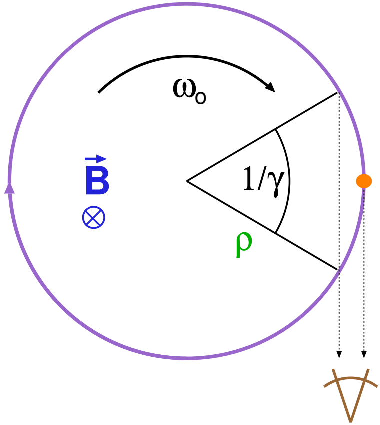

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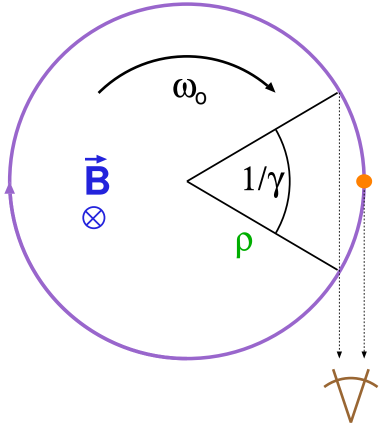

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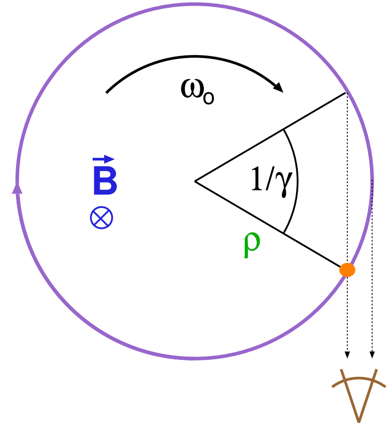

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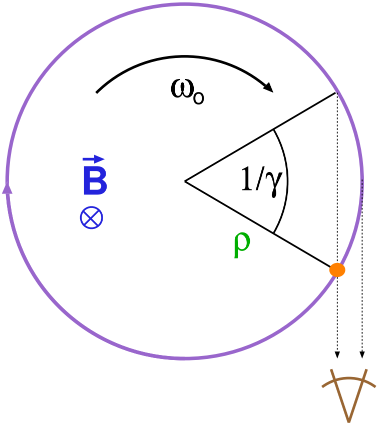

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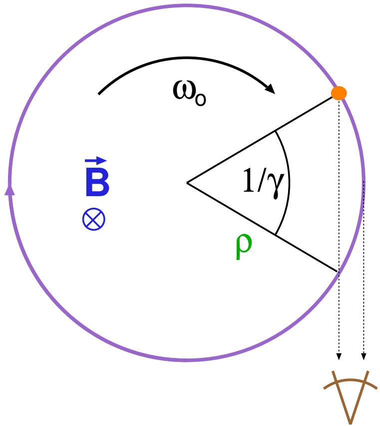
Electron bending radius



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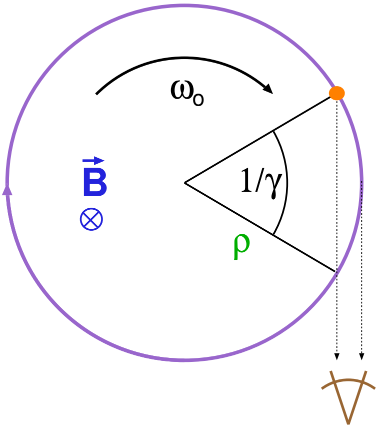

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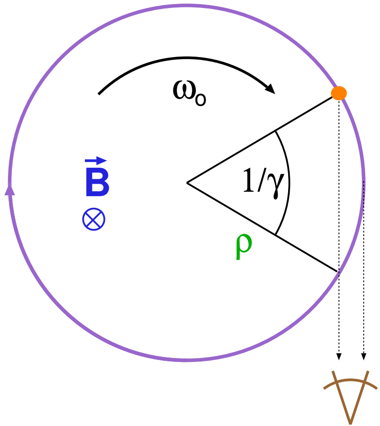
Electron bending radius



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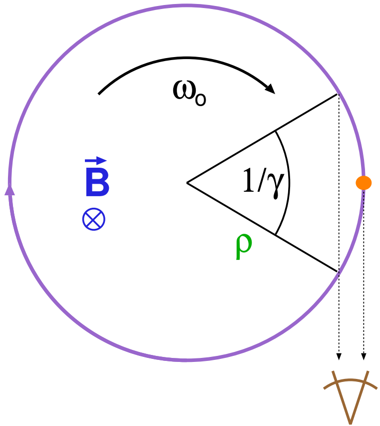

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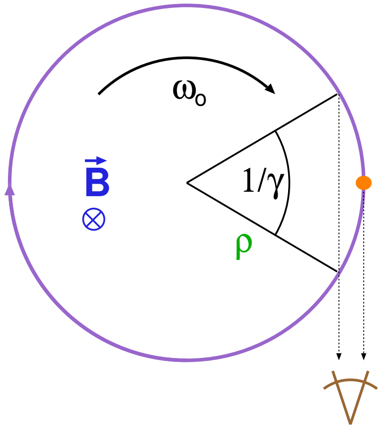

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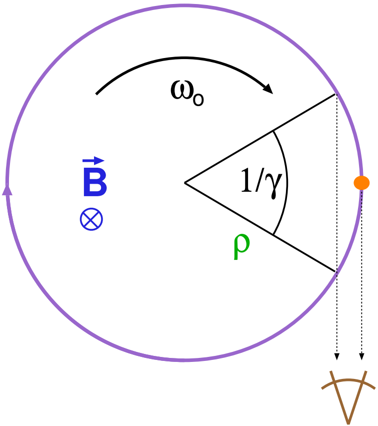

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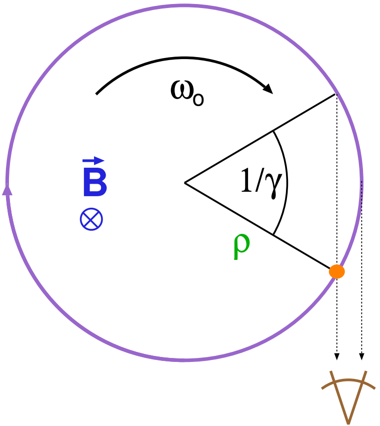

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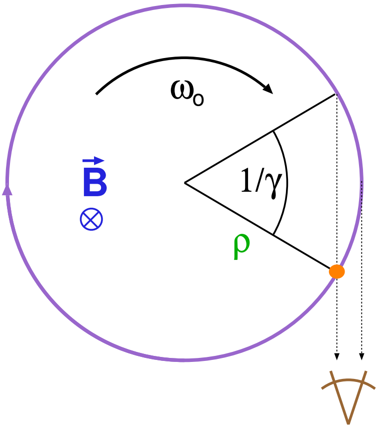

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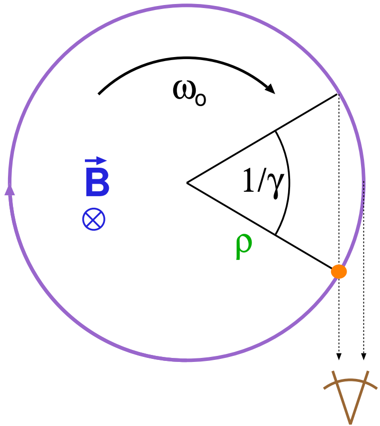

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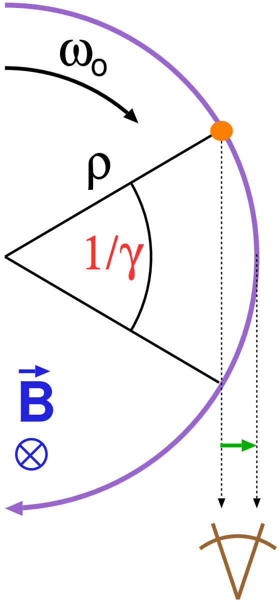
Curved arc emission



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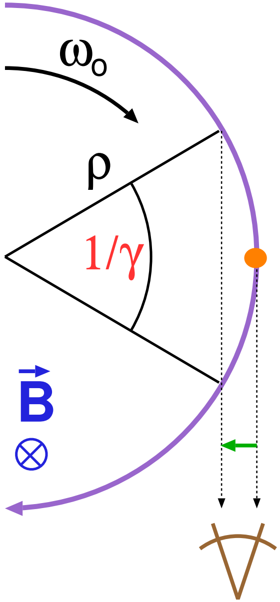

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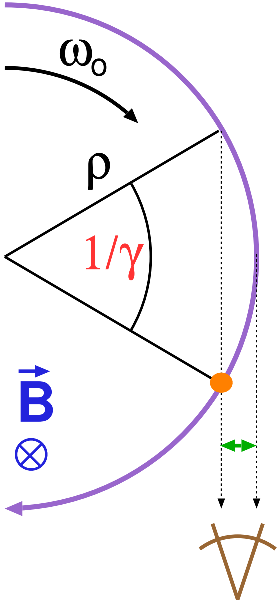

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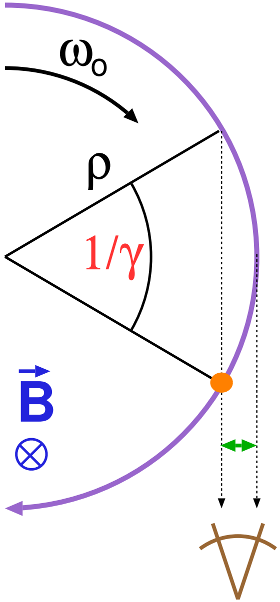

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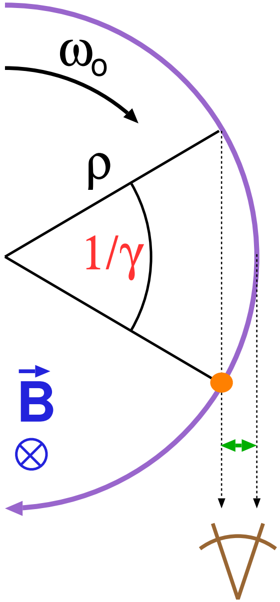

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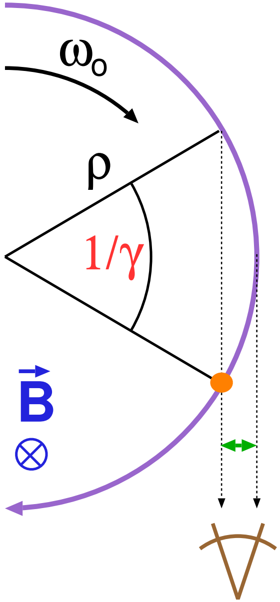

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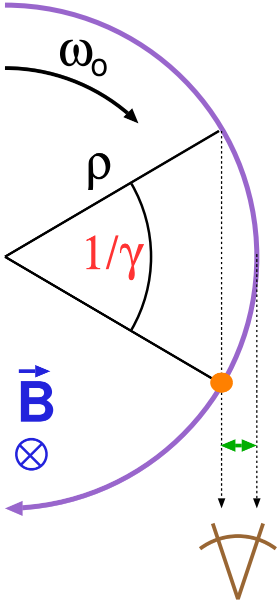

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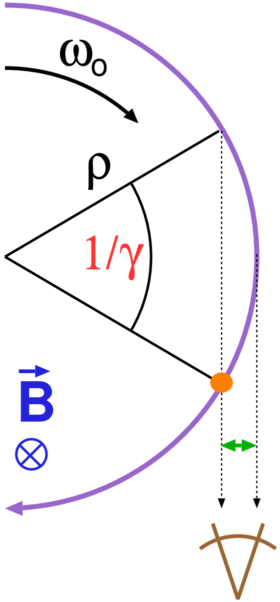

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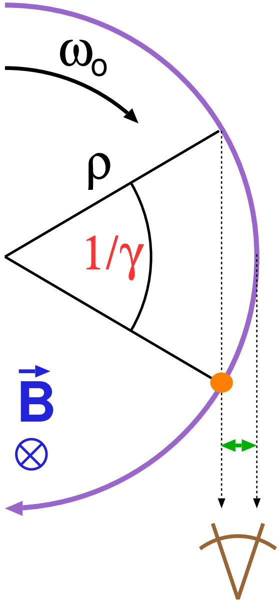

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The Fourier transform of this pulse is the spectrum of the radiation from the bending magnet.

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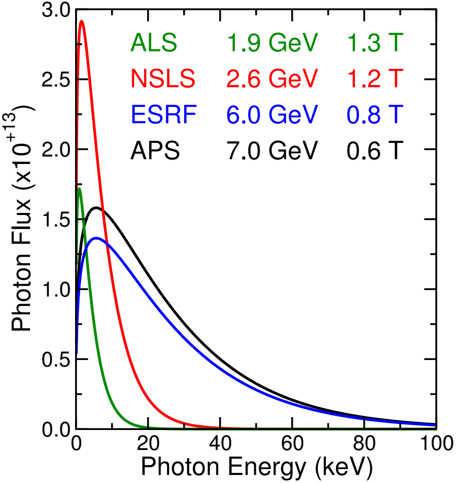
$$\mathcal{E}_c[\text{keV}] = 0.665\mathcal{E}^2[\text{GeV}]B[\text{T}]$$

Bending magnet spectrum

When the radiation pulse time is Fourier transformed, we obtain the spectrum of a bending magnet.



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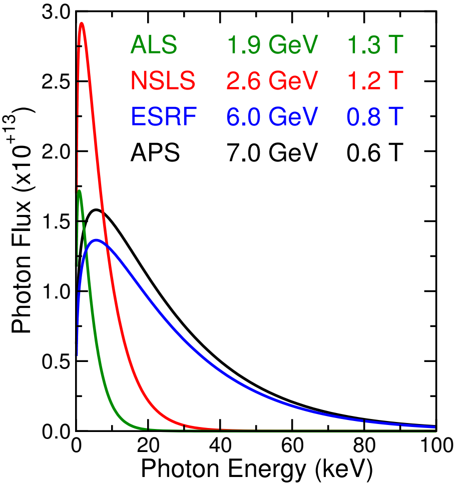

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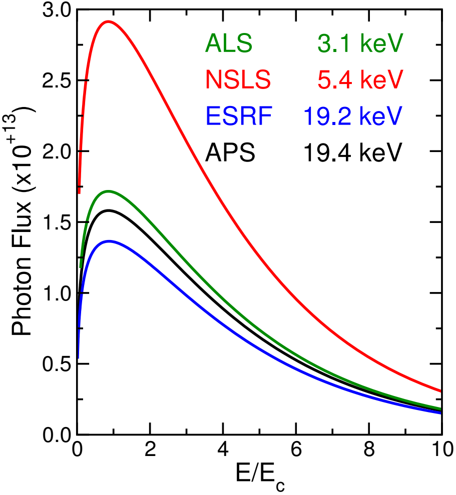

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$$1.33\times 10^{13}\mathcal{E}^2 \, I \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\right)$$

where $K_{2/3}$ is a modified Bessel function of the second kind.