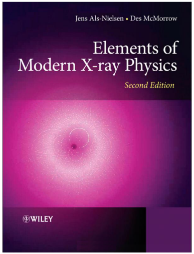
PHYS 570 - Introduction to Synchrotron Radiation



Term:	Spring 2015
Meetings:	Tuesday & Thursday 17:00-18:15
Location:	204 Stuart Building
Instructor:	Carlo Segre
Office:	166A Life Sciences
Phone:	312.567.3498
email:	segre@iit.edu
Book:	*Elements of Modern X-Ray Physics, 2nd ed.*, J. Als-Nielsen and D. McMorrow (Wiley, 2011)
Web Site:	http://csrri.iit.edu/~segre/phys570/15S

Course objectives

- Understand the means of production of synchrotron x-ray radiation

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- Understand the physics behind a variety of experimental techniques
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- Be able to write a General User Proposal in the format used by the Advanced Photon Source

Course syllabus

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- Visits to Advanced Photon Source (outside class, not required)
  - All students who plan to attend will need to request badges from APS
  - Go to the APS User Portal, https://www1.aps.anl.gov/Users-Information and register as a new user.
  - Use MRCAT (Sector 10) as location of experiment
  - Use Carlo Segre as local contact
  - State that your beamtime will be in the first week of March

Course grading

33% – Homework assignments

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Weekly or bi-weekly

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Grading scale

A	–	80%	to	100%
B	–	65%	to	80%
C	–	50%	to	65%
E	–	0%	to	50%

Topics to be covered (at a minimum)

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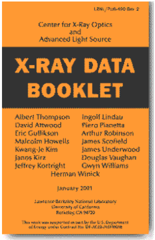
- Imaging

Resources for the course

- Orange x-ray data booklet:
  http://xdb.lbl.gov/xdb-new.pdf



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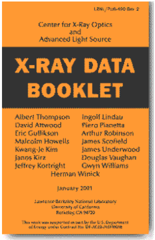

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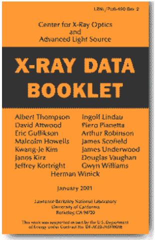
- Hephaestus from the Demeter suite:
  http://bruceravel.github.io/demeter/



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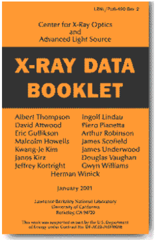

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- X-ray Oriented Programs:
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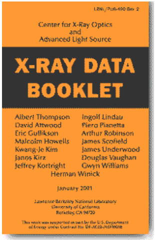

Today's outline - January 13, 2015

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Reading Assignment: Chapter 1.1–1.6; 2.1–2.2

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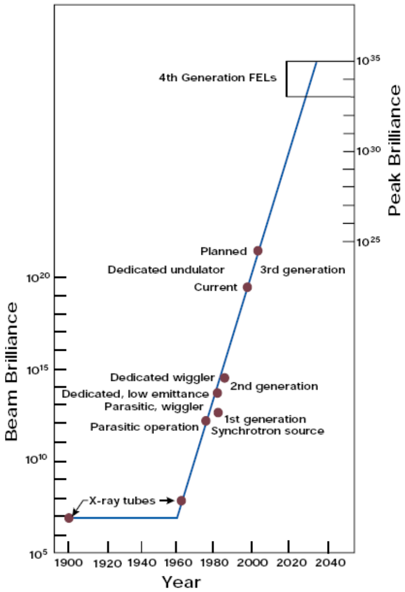

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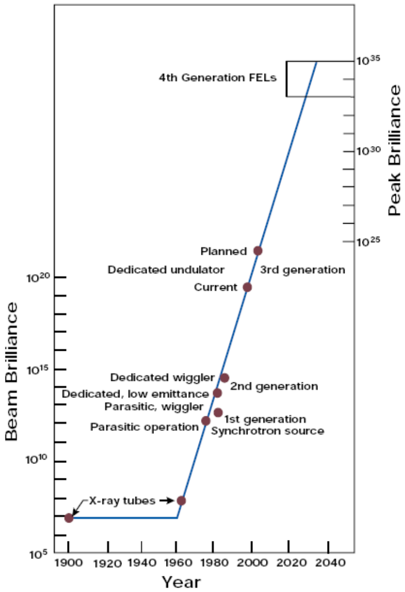

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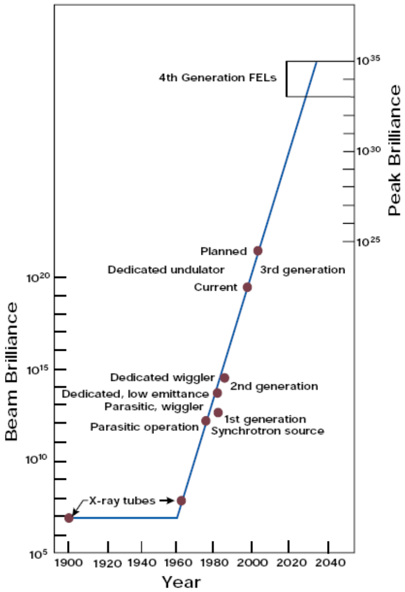

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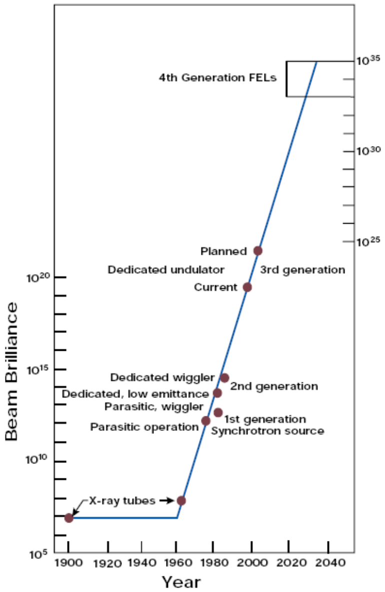

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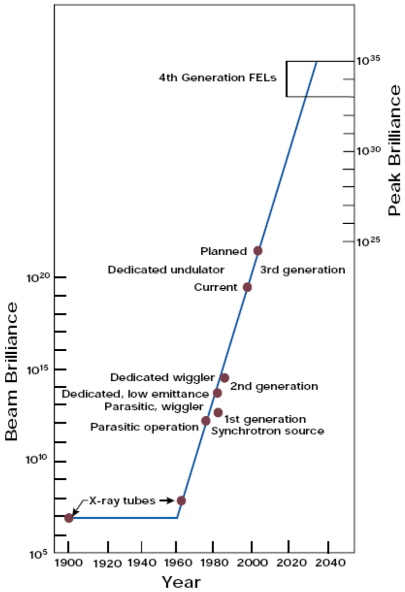

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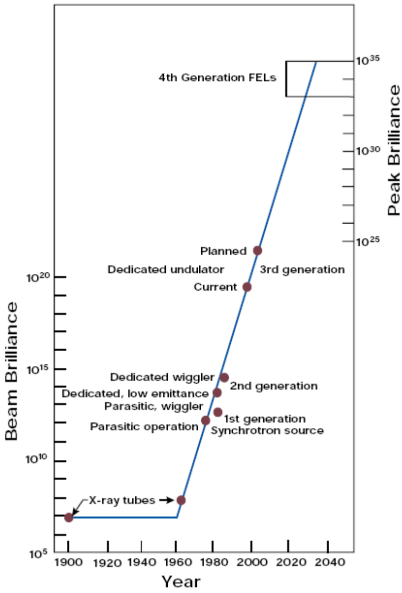

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- 4^{th} generation are free electron lasers (LCLS)

The classical x-ray

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$$\begin{aligned} \lambda &= hc/\mathcal{E} \\ &= (4.1357 \times 10^{-15}\,\text{eV}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})/\mathcal{E} \\ &= (4.1357 \times 10^{-18}\,\text{keV}\cdot\text{s})(2.9979 \times 10^{18}\,\text{Å/s})/\mathcal{E} \\ &= 12.398\,\text{Å}\cdot\text{keV}/\mathcal{E} \qquad \text{to give units of Å} \end{aligned}$$

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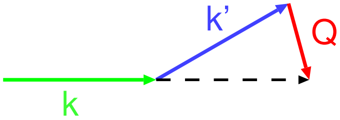
4. Pair production

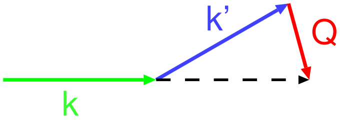
We will only discuss the first three.

Elastic scattering geometry



an incident x-ray of wave number **k**

Elastic scattering geometry



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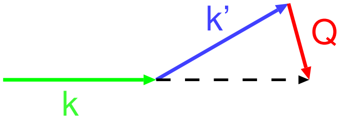

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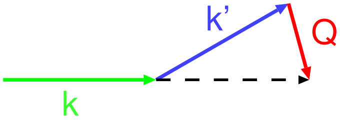

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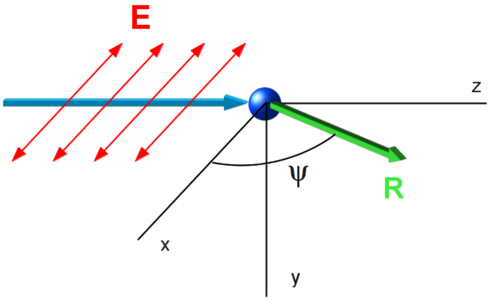
resulting in a scattering vector $\mathbf{Q}$
or in terms of momentum transfer: $\hbar\mathbf{Q} = \hbar\mathbf{k} - \hbar\mathbf{k}'$

Thomson scattering



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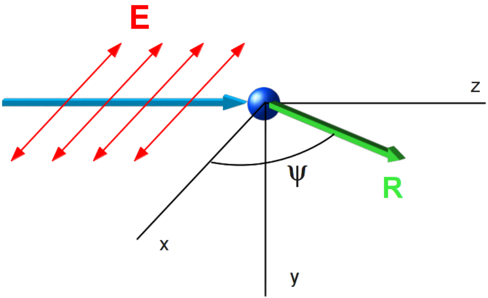

Assumptions:

plane wave of x-rays incident on a single electron

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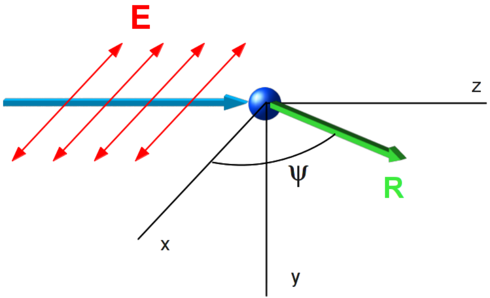

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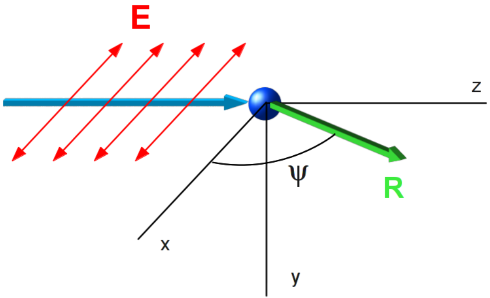

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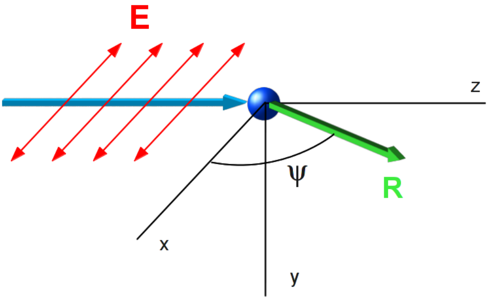

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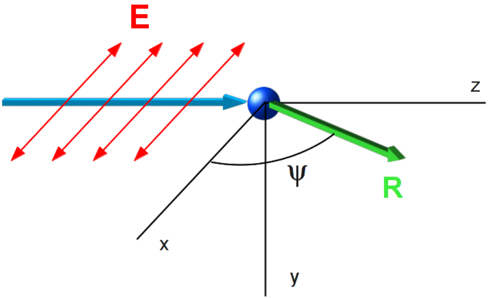
- scattered intensity $\propto 1/R^2$

Thomson scattering



$$E_{rad}(R,t) = -\frac{-e}{4\pi\epsilon_0 c^2 R} a_x(t') \sin\Psi$$

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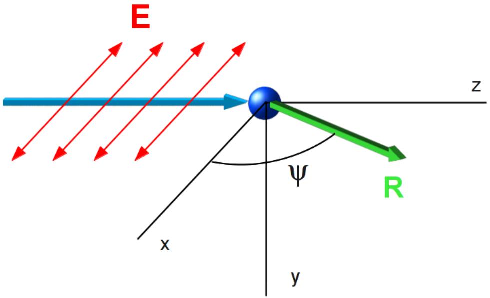

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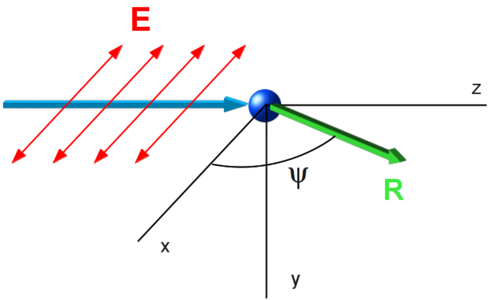

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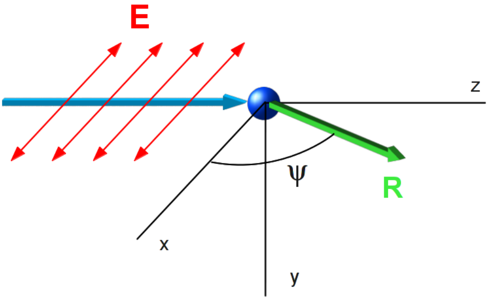

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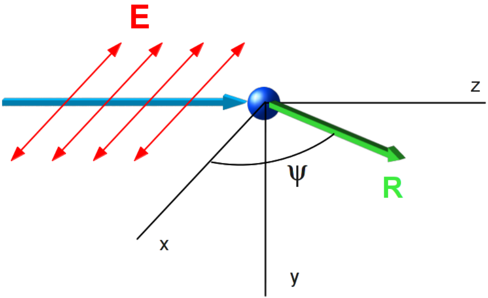

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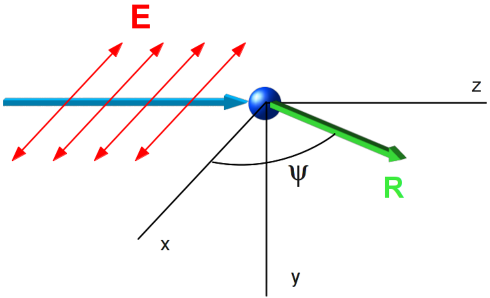

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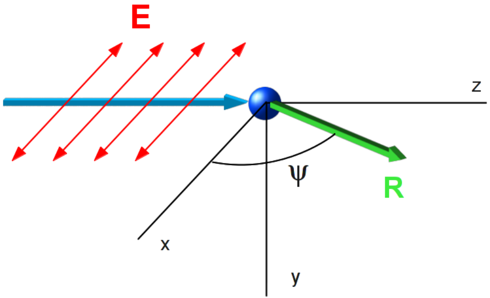

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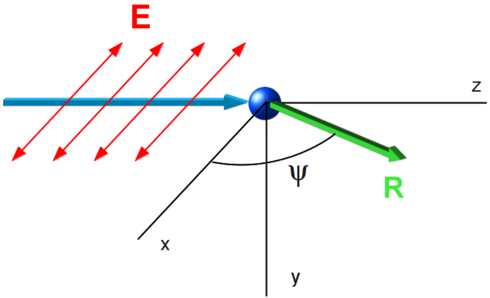

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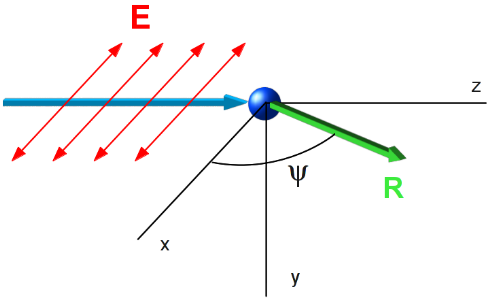
$$\frac{E_{rad}(R,t)}{E_{in}} = -\frac{e^2}{4\pi\epsilon_0 mc^2}\frac{e^{i\omega R/c}}{R}\sin\Psi \qquad but\ k = \omega/c$$

Thomson scattering



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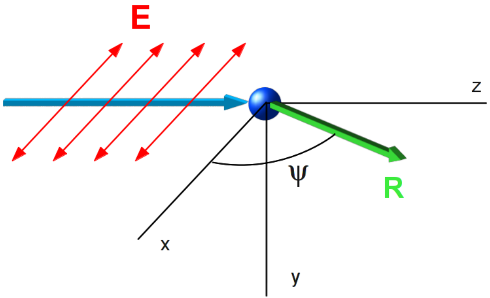

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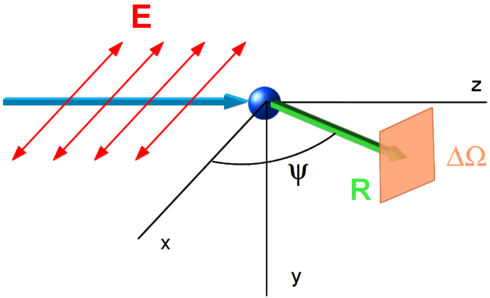
$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} = 2.82\times 10^{-5}\text{Å}$$

Scattering cross-section



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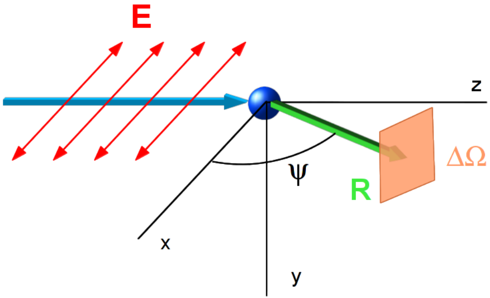

Detector of solid angle $\Delta\Omega$ at a distance R from electron

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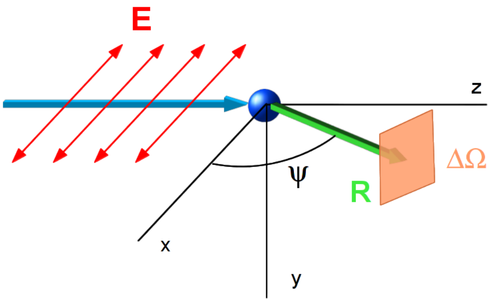

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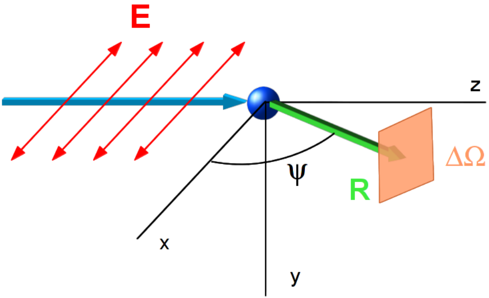

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Cross section of scattered beam (into detector) $= R^2\Delta\Omega$

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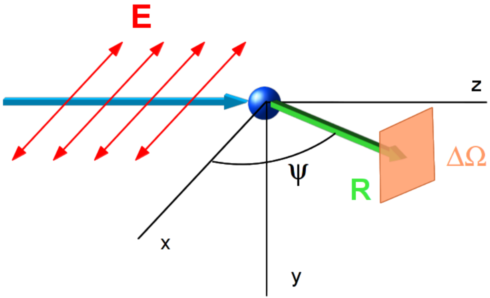

Detector of solid angle $\Delta\Omega$ at a distance R from electron
Cross-section of incoming beam $= A_o$
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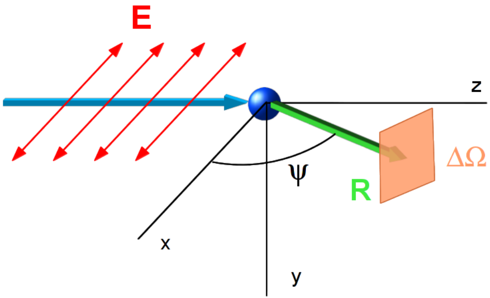
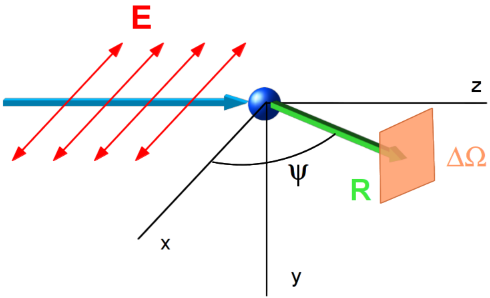


Differential cross-section is obtained by normalizing

Scattering cross-section



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$$\frac{d\sigma}{d\Omega} =$$

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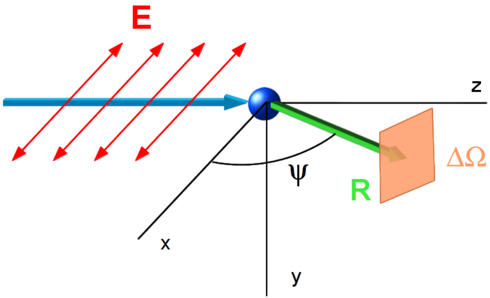

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$$\frac{d\sigma}{d\Omega} = \frac{I_{scatt}}{(I_o/A_o)\,\Delta\Omega}$$

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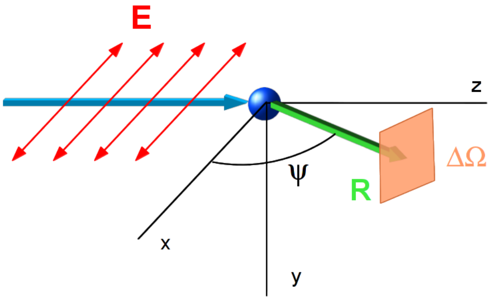

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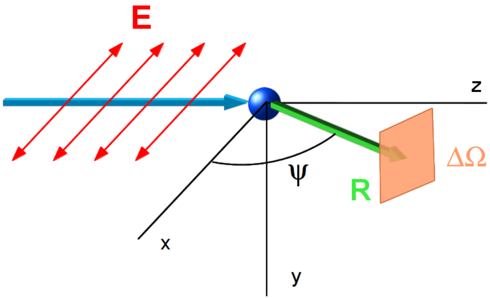

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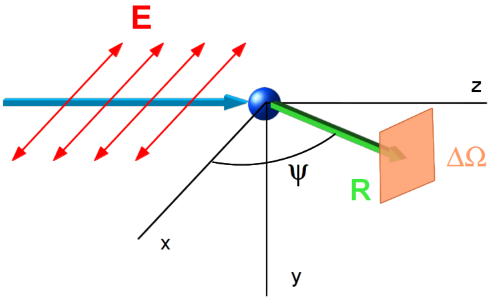

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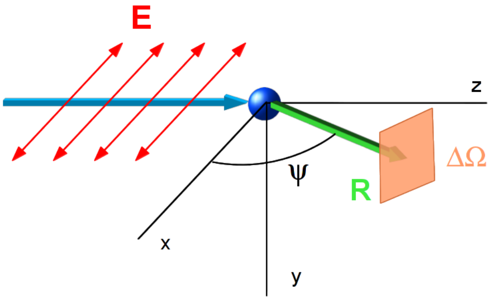

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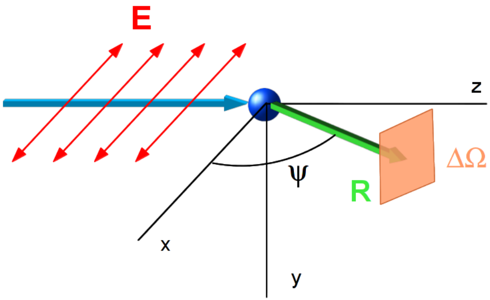

Integrate to obtain the total Thomson scattering cross-section from an electron.

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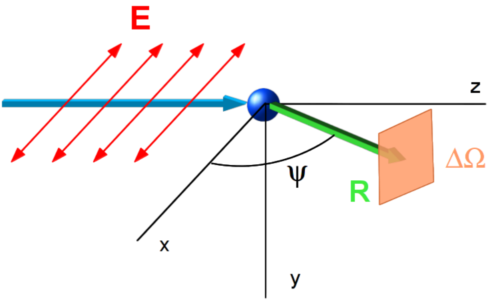

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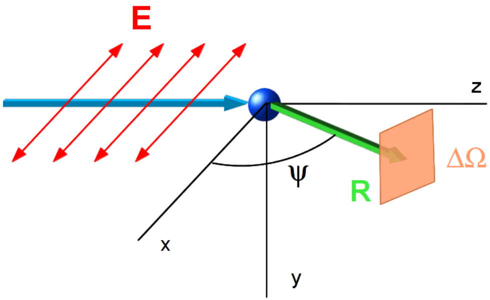

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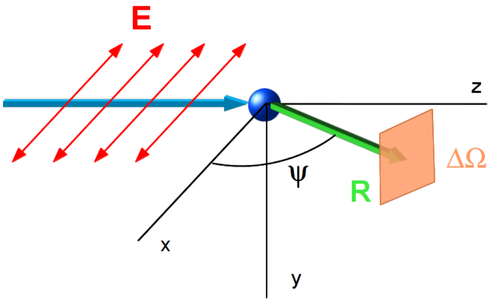

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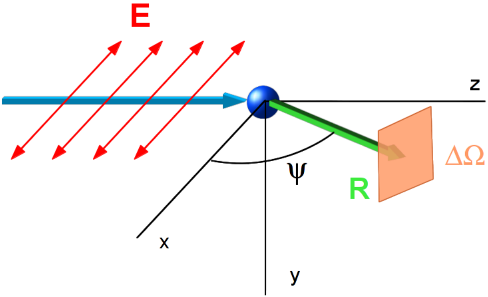

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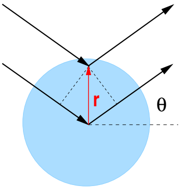
$$\text{Polarization factor} = \begin{cases} 1 \\ \sin^2\Psi \\ \frac{1}{2}\left(1+\sin^2\Psi\right)\end{cases}$$

Atomic scattering

phase shift arises from scattering off different portions of extended electron distribution

$$\Delta\phi(\mathbf{r}) = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{r} = \mathbf{Q} \cdot \mathbf{r}$$



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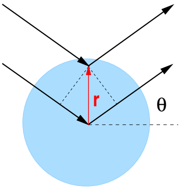

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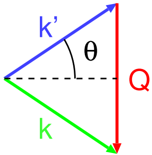

$$|\mathbf{Q}| = 2\,|\mathbf{k}|\,sin\theta = \frac{4\pi}{\lambda} sin\theta$$

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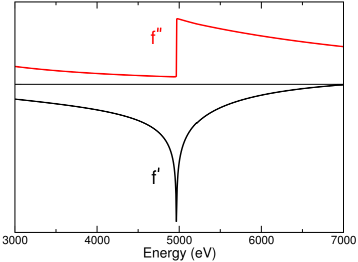

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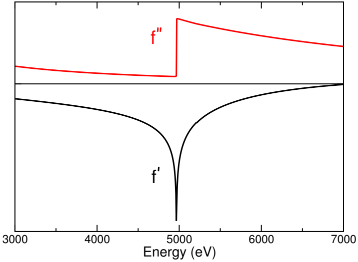

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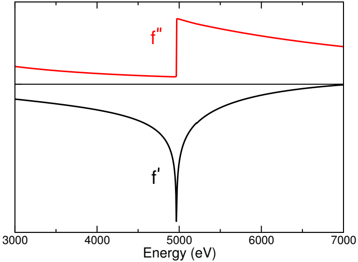

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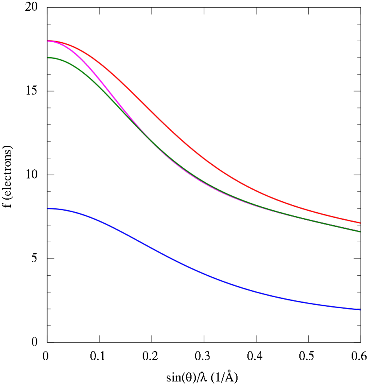
$$f(\mathbf{Q},\hbar\omega) = f^o(\mathbf{Q}) + f'(\hbar\omega) + if''(\hbar\omega)$$

Atomic form factor



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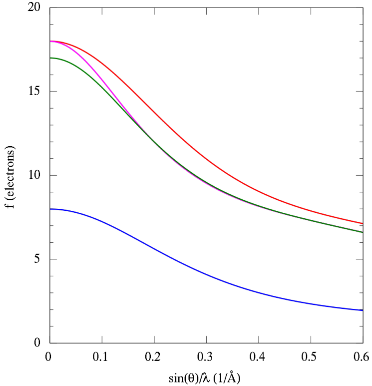

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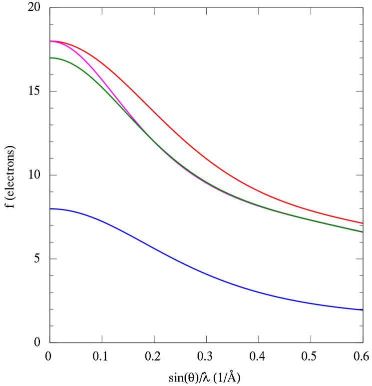

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Lighter atoms (blue is oxygen) have wider form factor

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Scattering from an atom is built up from component quantities:

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