Plasma based compact light sources

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Conventional undulators/wigglers

- Monochromatic & Tunable X-Rays
- High brilliance
- Good angular collimation

The fundamental wavelength is given by:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left[ 1 + \frac{K^2}{2} + (\gamma \theta)^2 \right]$$

$$\lambda_u \approx 1\text{cm}, \; K \leq 1$$

So in order to generate ‘hard’ x-rays we need $\gamma \approx 10^4$
Which corresponds to electrons with $>5\text{GeV}$

Can we make it smaller?
Conventional accelerators

Charged particles are accelerated using electric fields

Electrostatic accelerators are limited by breakdowns (electric discharge)

If AC fields are used we need something to transform the TEM mode propagating in vacuum to a mode with longitudinal electric field

But these are also limited by breakdowns to energy gains of a few tens MeV/m

Up: paschen’s curve describing the DC breakdown voltage between two electrodes as a function of pressure*distance
Plasma oscillations

1. We start with a neutral plasma - a ‘fluid’ composed of massive positive ions and electrons with zero net charge.

2. Imagine that the electrons are displaced slightly from their equilibrium positions by $\delta x$, while the ions are fixed.

3. Then we get a net negative charge (per unit area) $-n_e e \delta x$ and a net positive charge (per unit area) $+n_e e \delta x$, separated by $\delta x$ (polarization).

4. The resulting electric field is:

$$E = \frac{n_e e \delta x}{\varepsilon_0}$$

And the restoring force will cause the electrons to oscillate with a characteristic frequency:

$$\omega_p = \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \approx 9\sqrt{n_e} \frac{kHz}{cm^{1.5}}$$
Plasma Waves

But how can we displace the electrons?

A propagating EM wave has an associated momentum:
\[ \hat{S} \propto \hat{E} \times \hat{H} \]

When this wave interacts with the plasma it will ‘push’ the light electrons away, while the heavier ions remain in place.

Electrons are pushed away in the leading edge, while in the trailing edge electrons are attracted back to the positive ions which remained behind.

So we are left with an effective charge separation:
Plasma Waves

The wave-fronts generated by the pulse will be synchronized if the condition for resonant excitation is met:

\[ \tau_L = \tau_P = \frac{2\pi}{\omega_p} \]

In that case a plasma wave is created, with a phase velocity equal to the velocity of the generating pulse (the group velocity of the laser, close to the speed of light)

Another scheme to excite plasma waves is by using two lasers with frequencies:

\[ \omega_1, \omega_2 > \omega_p \text{ (Why?)} \]
\[ \omega_1 - \omega_2 = \omega_p \text{ (Beatwave excitation)} \]

The plasma had converted our TEM mode into a mode with a longitudinal electric field!
Plasma Acceleration

Because the plasma is not limited by breakdowns it can sustain high electric fields (up to $100\text{GeV/m}$) in the ‘ion channel’ created in the wake of the laser pulse.

If a bunch (pulse) of already relativistic electrons is injected into that channel, they can ‘surf’ the plasma wave and get accelerated to higher and higher energies.

State of affairs:

- LBNL: 1GeV over 3.3cm ($\sim 30\frac{\text{GeV}}{\text{m}}$)
- SLAC SLC: 42GeV over 85cm ($\sim 50\frac{\text{GeV}}{\text{m}}$)
- University of Texas: 2GeV over 2cm ($\sim 100\frac{\text{GeV}}{\text{m}}$)

“With current laser technology electron beams in the 100MeV range have been produced over millimeter distances”

[7,8]
Plasma Undulators
Principle of operation

The electric field pattern generated by the plasma wave can also be used to wiggle the electrons and produce x-rays.

In the plasma undulator the plasma wave propagates perpendicular to the electron beam direction.

The unulator period is equal to the plasma wavelength:

\[ \lambda_u = \lambda_p \]

because \( v_e = v_\phi \sim c \)
Plasma undulators

The strength parameter

The strength parameter is defined as:

\[ K = \gamma \varphi_{\text{max}} \sim \gamma \left( \frac{p_z}{p_x} \right)_{\text{max}} \sim \gamma \frac{(p_z)_{\text{max}}}{\gamma mc} = \frac{(p_z)_{\text{max}}}{mc} \]

The Hamiltonian is given by (for \( \beta \gamma \gg 1 \)):

\[ \mathcal{H} = T + V = m_e c^2 (\gamma - 1) + e\varphi \sim m_e c^2 \sqrt{1 + \left( \frac{p}{m_e c} \right)^2} + e\varphi (z - ct) \]

\( \varphi \) — plasma wave potential, \( p = \gamma mv \)

Assuming that the electron starts on axis with \( \vec{p} = m_e c \sqrt{(\gamma_i)^2 - 1} \hat{x} \), when \( \varphi = 0 \)

Then we get following constants of motion:

\[ y = p_y(0) = 0, p_x = p_x(0) = m_e c \sqrt{(\gamma_i)^2 - 1} \]

\[ \frac{\gamma m_e c^2 + e\varphi - cp_z = \text{const} = \gamma_i m_e c^2} \]

For realistic plasma waves \( \frac{e\varphi}{\gamma_i m_e c^2} \ll 1 \)

so \( p_z \sim \frac{e\varphi}{c} \left( 1 + \frac{e\varphi}{2\gamma_i m_e c^2} \right) \rightarrow K \sim \frac{e\varphi_{\text{max}}}{m_e c^2} \)

So \( K \) is determined by the intensity of the laser pulse and can approach unity (wiggler regime) for intense beams
Plasma undulator
typical figures of merit

For a typical plasma density $n_e \sim 10^{19} \frac{electrons}{cm^3}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_p (&lt; f_L)$</td>
<td>30THz</td>
</tr>
<tr>
<td>$\tau_p = \tau_L &gt; \tau_{bunch}$</td>
<td>35fsec</td>
</tr>
<tr>
<td>$\lambda_u = \lambda_p$</td>
<td>10um</td>
</tr>
<tr>
<td>$\gamma_e^-$</td>
<td>200 ($\sim 100MeV$)</td>
</tr>
<tr>
<td>$\epsilon_x$ rays</td>
<td>10keV (on axis)</td>
</tr>
<tr>
<td>$K$</td>
<td>0.01</td>
</tr>
<tr>
<td>$L$</td>
<td>1mm for 100 undulations</td>
</tr>
</tbody>
</table>

$\tau_{bunch}$ is the temporal width of the electron bunch, and would also be the width of the generated x-rays pulse.

If I use $\phi_{max} \sim E \frac{\lambda_p}{2}$

With a modest electric field gradient in the plasma of ‘only’ $E = 1 \frac{Gev}{m}$.
This corresponds to an effective magnetic field:

$$B_0 = \frac{2\pi mc}{e\lambda_u} K = 10.7T$$
Summary

• Plasma based technology looks like an exciting and promising route towards compact light sources

• Tunable high brilliance hard x-rays seem to be feasible from cm scale devices

• Simulations indicate that the good properties of undulator radiation should be preserved. However, that strongly depends on the quality of the input electron beam
References

2. www.aps.anl.gov
5. tesla.desy.de
6. www.aist.go.jp

Thank you & Enjoy the break!!