

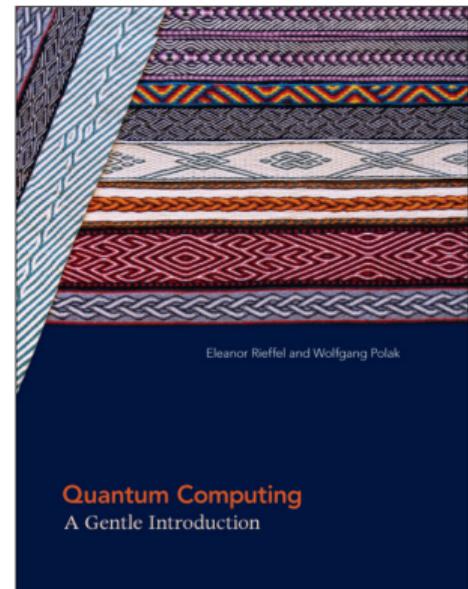
PHYS 407/507 & CS 495 - Introduction to Quantum Computing



Term: Spring 2026
Meetings: Tuesday & Thursday 17:00-18:15
Location: Room 117 Wishnick Hall
Video: All sessions recorded for online viewing
Instructor: Carlo Segre
Office: 166D/172 Pritzker Science
Phone: 312.567.3498
email: segre@illinoistech.edu
Resources: *Quantum Computing: A Gentle Introduction*,
E. Rieffel & W. Polak (MIT Univ Press, 2011)

Introduction to Classical and Quantum Computing,
T. Wong (Rooted Grove, 2022)

Quirk: A drag-and-drop quantum circuit simulator, Craig Gidney
Web Site: <http://phys.iit.edu/~segre/phys407/26S>





Course objectives for PHYS 407/507 & CS 495

1. Clearly describe the building blocks of quantum computing.



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2. Apply tools of quantum computing to manipulate qubits.



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5. Use the concept of quantum entanglement to develop quantum algorithms.



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7. Build quantum algorithms using Quirk.

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5. Use the concept of quantum entanglement to develop quantum algorithms.
6. Clearly describe the techniques of quantum error correction and fault tolerance.
7. Build quantum algorithms using Quirk.
8. Demonstrate the programming of a quantum computer using Python.

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5. Use the concept of quantum entanglement to develop quantum algorithms.
6. Clearly describe the techniques of quantum error correction and fault tolerance.
7. Build quantum algorithms using Quirk.
8. Demonstrate the programming of a quantum computer using Python.
9. Successfully program an agreed-upon quantum algorithm on an IBM quantum computer.



Course syllabus

- Homework Assignments



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- Qiskit Progress Reports



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- Midterm Exams – Online, handwritten uploads to Canvas



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- Final Exam – Presentations on research articles
 - Choose a recent research article which features a synchrotron technique
 - Get approval before starting
 - Timetable will be posted



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- Final project - programming a quantum algorithm
 - Start thinking about a suitable project right away
 - Make proposal and get approval before starting



Course grading

PHYS 407 & CS 495

PHYS 507



Course grading

PHYS 407 & CS 495

30% – Homework assignments

PHYS 507

20% – Homework assignments



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Grading scale

A – 88% to 100%

B – 75% to 88%

C – 62% to 75%

D – 50% to 62%

E – 0% to 50%

PHYS 507

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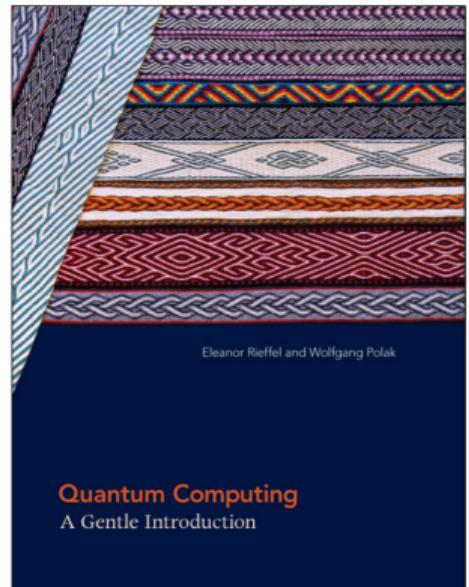
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Topics to be covered (chapter titles)

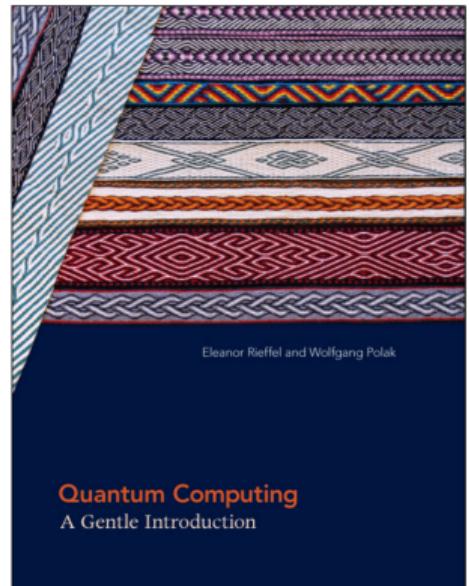
1. Quantum building blocks





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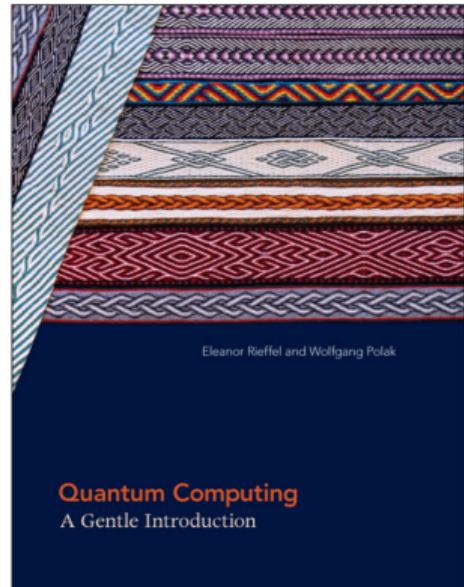
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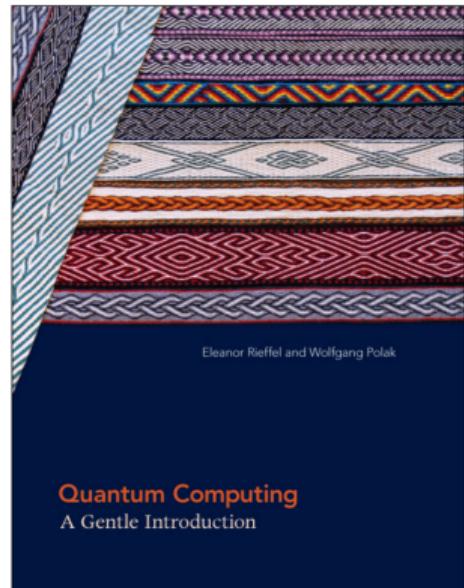
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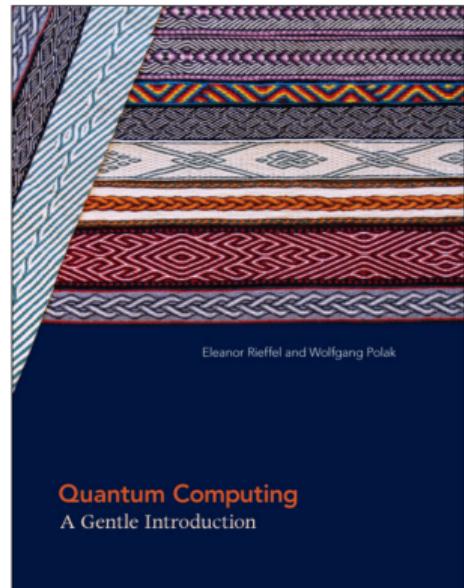
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4. Quantum computing hardware





Topics to be covered (chapter titles)

1. Quantum building blocks
2. Quantum algorithms
3. Entangled subsystems and robust computation
4. Quantum computing hardware
5. Other topics as appropriate





Why study quantum computing?

Quantum computing is one part of a broader field called quantum information science which has revolutionized cryptography and secure communications



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Quantum computing is becoming interesting to a number of fields outside physics and could be even more relevant in the near future

Today's outline - January 13, 2026



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- Quantum fundamentals



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- Superposition



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Reading Assignment: Reiffel: 2.4-2.5; 3.1 Wong: 2.2.2-2.4.3; 4.2.1-4.2.2



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Homework Assignment #01:
due Thursday, January 22, 2026



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Homework Assignment #01:
due Thursday, January 22, 2026

Homework Assignment #02:
due Tuesday, February 03, 2026



Quantum mechanics fundamentals

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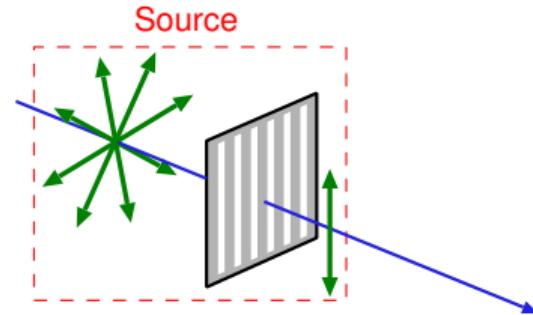
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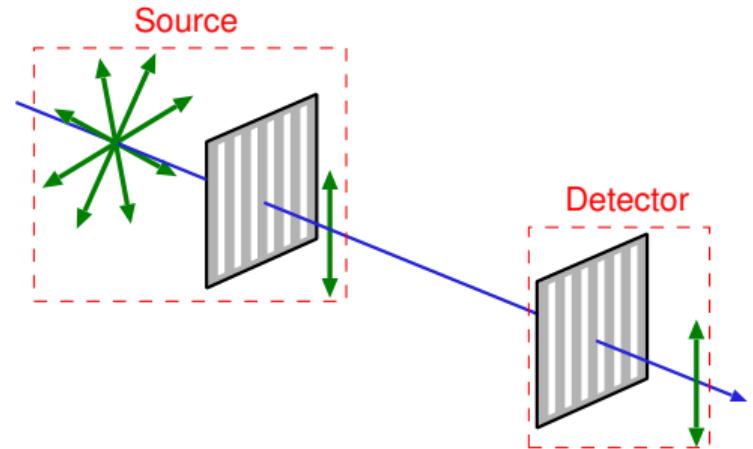
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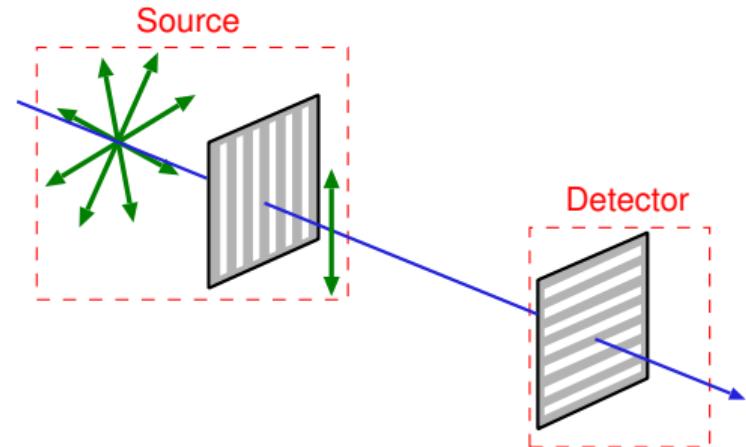
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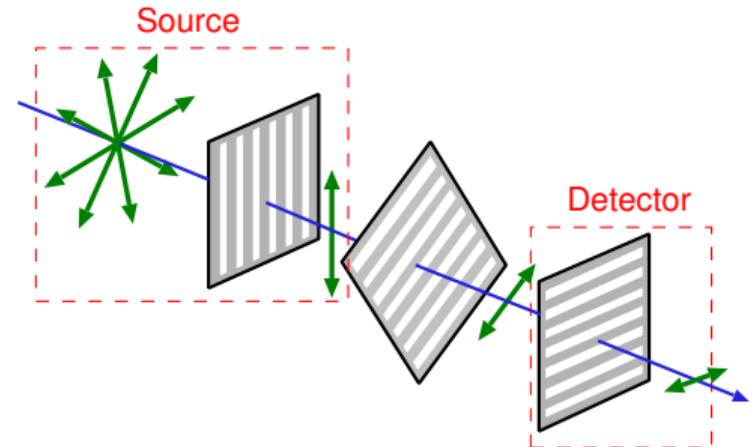
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If a tilted polarizer is placed in between, the horizontal detector now measures a smaller, but non-zero, value



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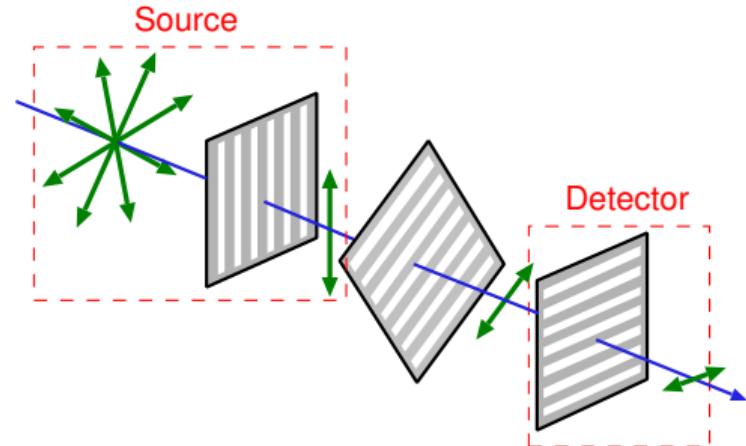
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Because photons are quantum particles, this effect works even for single photons with the measuring a fraction of the photons to be horizontal





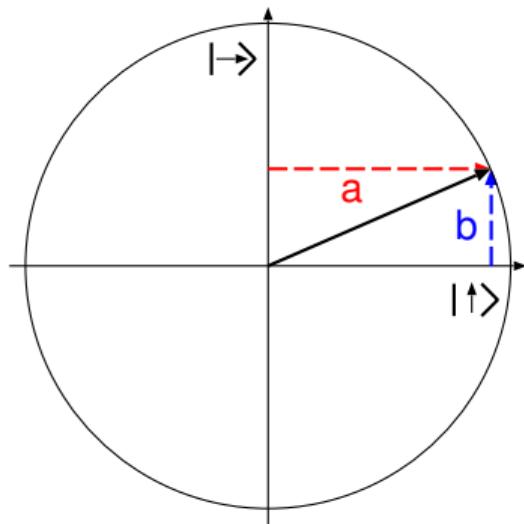
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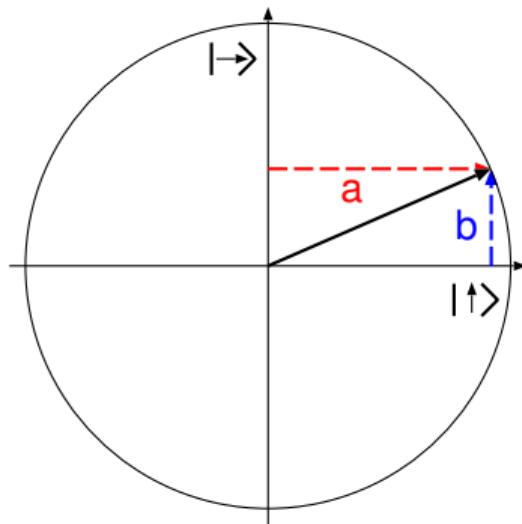
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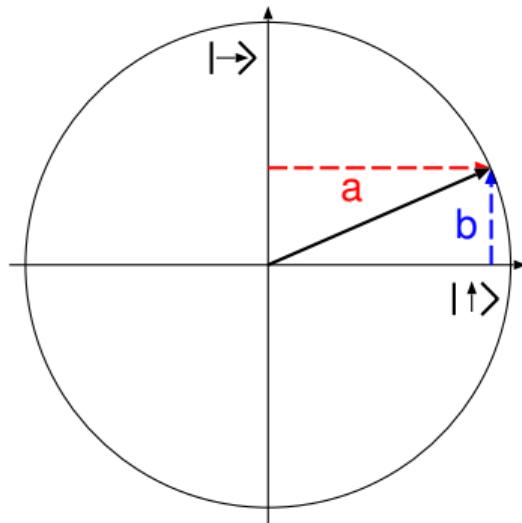
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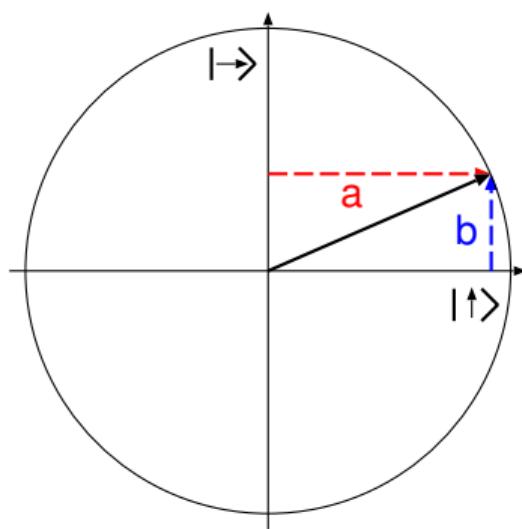
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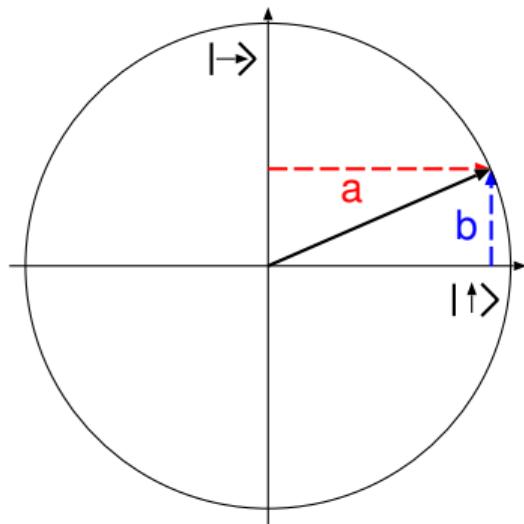
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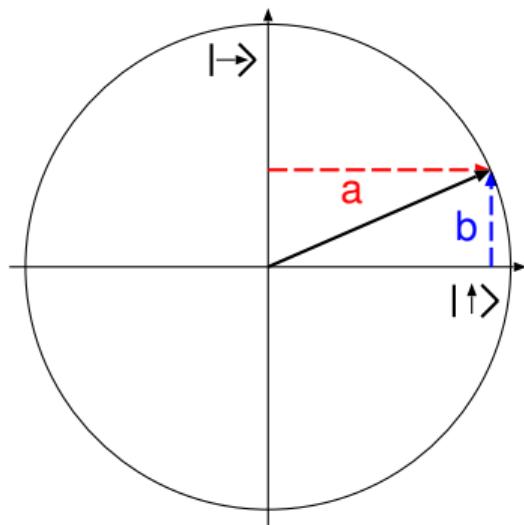
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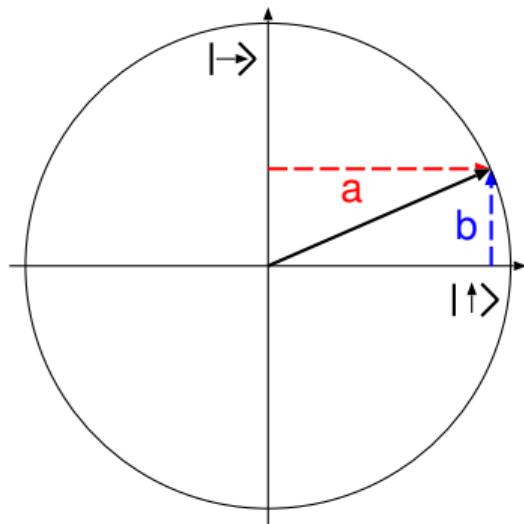
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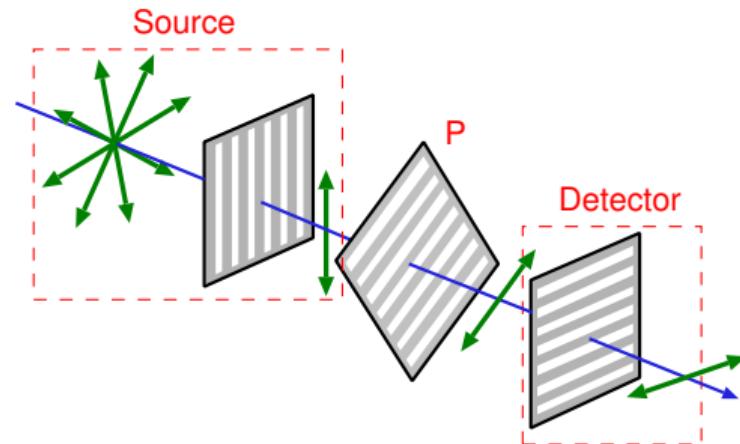
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This formalism allows us to describe the polarization experiment

Polarizer experiment

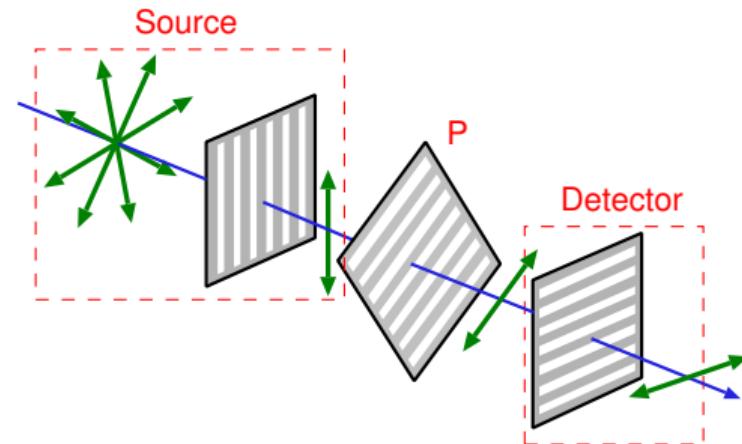
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In the axes of the polarizer P there are two possible states $|\nearrow\rangle$ and $|\nwarrow\rangle$ and the vertically polarized photon can be written as

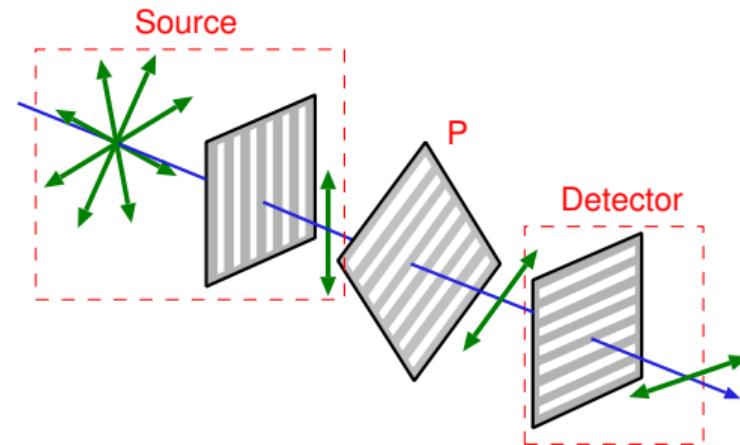


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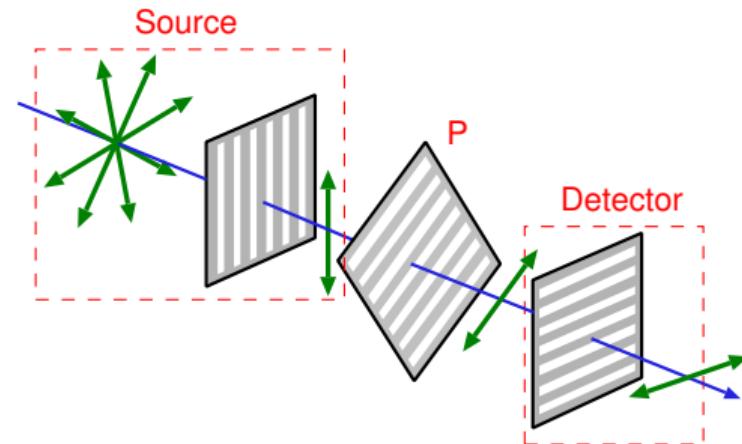
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The photon thus has an 0.5 probability of passing through the polarizer and will then be in a state



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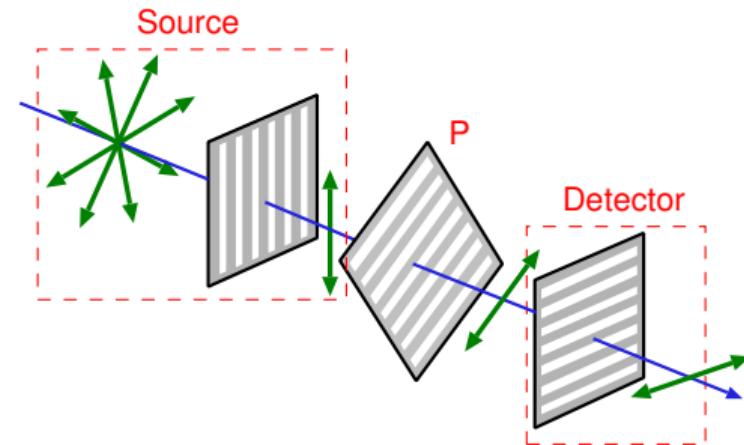
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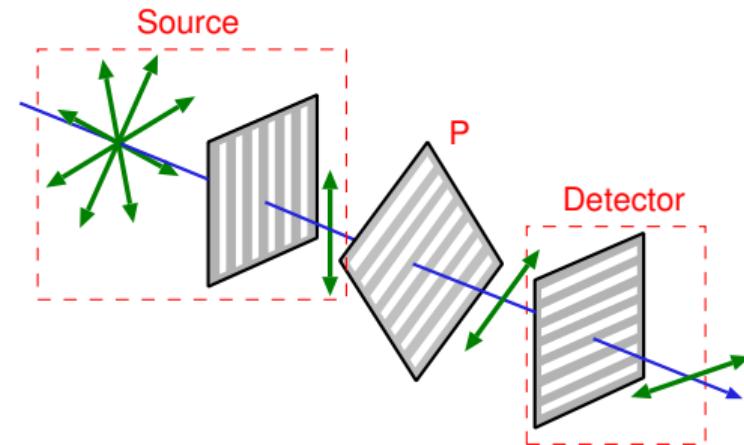
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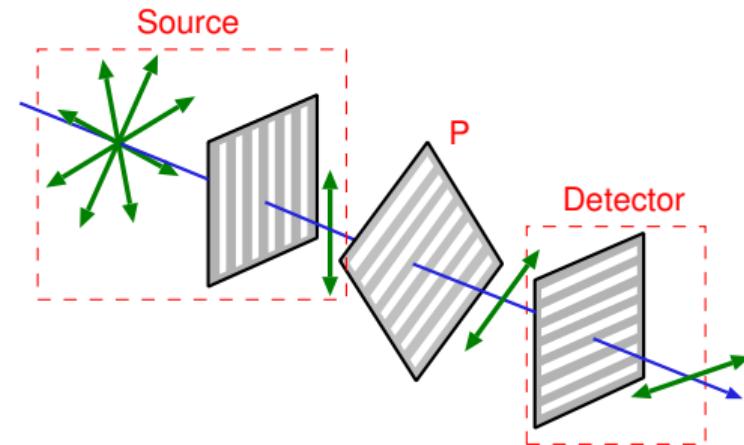
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Quantum particles (and qubits) behave probabilistically





Dirac notation

Any two-state quantum system can be considered a qubit and can be modeled as a superposition of the two linearly independent states



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Examples include photon polarization, electron spin, and ground/excited states of atoms

The infinite number of possible states in this system can all be described by the linear superposition $|q\rangle$

Dirac, or bra-ket, notation is used to describe quantum systems. The **ket** ($|x\rangle$) and **bra** ($\langle x|$) are used to represent a vector and its conjugate transpose respectively

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Dirac notation (cont.)

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After the measurement any photon that passed through the polarizer is now in the $|\uparrow\rangle$ state



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A superposition is not just a probabilistic mixture of two states, it is a definite state which consists of **both** its constituent states



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$$|\alpha\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle, \quad \{|+\rangle, |-\rangle\} \equiv \left\{ \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \right\}$$

A superposition is not just a probabilistic mixture of two states, it is a definite state which consists of **both** its constituent states

Qubits can exist in an infinite number of superposition states yet do not contain more information than classical bits since a single measurement produces only one of two answers depending on the basis



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Realizing an actual quantum computer requires a deep knowledge of quantum mechanics and experimental quantum systems