Today's outline - April 19, 2022

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- Mixtures of correctable errors

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Homework Assignment #07:
Chapter 9:2,3,4; Chapter 10:3,4,11
Due Thursday, April 21, 2022

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Final Exam will be 10 minute presentations on a quantum computing journal article

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Measurement with O will give the state $P_i \rho' P_i^\dagger$ with probability $|\alpha_i|^2$ and will leave the system in the pure state $E_i|c\rangle$

$$P_i \rho' P_i^\dagger = E_i|c\rangle\langle c|E_i^\dagger$$

Knowing the result of the measurement, λ_i, means that the error can be corrected by application of $E_i^\dagger$ thus recovering the corrected state $|c\rangle$

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Quantum independent error models

$$\langle c_1|X_i^\dagger Z_i|c_2\rangle = \langle c_1|Z_i^\dagger X_i|c_2\rangle = \langle c_1|I\, Z_i|c_2\rangle = \langle c_1|I\, X_i|c_2\rangle = 0$$

In addition, it is possible to show that Y_i are also correctable errors using multiplication in the Pauli group and the commutator relations

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So any code that corrects all bit-flip (X) and phase-flip (Z) errors also will correct all Y errors

Requirements for quantum computers

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These hold for the standard circuit model, alternative models require more general criteria

NMR-based quantum computers

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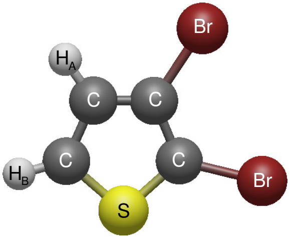

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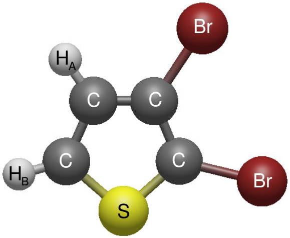
In a 4.7 T magnetic field, the proton spin states are separated by 200 MHz but are slightly different because of the differing local environment



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where $\alpha_i = \hbar\omega_i / 2kT \approx 4 \times 10^{-6} B_0$

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The deviation from equilibrium is what is being measured as the qubit state

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The dynamics of the ensemble can be approximated by just the deviation density matrix whose macroscopic signal has a relatively long decoherence time

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Single spin operations are possible because each spin has a different resonant frequency and can be manipulated individually by applying an appropriately timed RF pulse to rotate the spin

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2-qubit NMR computer

At equilibrium, suppose that the deviation density matrix is given by

$$\rho_\Delta = \alpha \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

This means that the distribution of the 10^{23} molecules among the four states is

$$p_{|\downarrow\downarrow\rangle} = (\tfrac{1}{4} + 3\alpha) \times 10^{23}$$

$$p_{|\downarrow\uparrow\rangle} = p_{|\uparrow\downarrow\rangle} = p_{|\uparrow\uparrow\rangle} = (\tfrac{1}{4} - \alpha) \times 10^{23}$$

This is a pseudo-pure state, a mixed state that behaves like a pure state

For example if an RF pulse is provided that sends one of the spins in all of the molecules into the transverse plane, the excess population in the $|\downarrow\downarrow\rangle$ state will produce a signal

Single spin operations are possible because each spin has a different resonant frequency and can be manipulated individually by applying an appropriately timed RF pulse to rotate the spin

The C_{not} gate can be implemented due to the nonlinear interaction between spins on the same molecule

"Bulk spin-resonance quantum computation," N.A. Gershenfeld and I.L. Chuang, *Science* **275**, 350-356 (1997).

5-qubit NMR computer

In 2000, this was extended to a 5-qubit system using the same principles

"Approaching five-bit NMR quantum computing," R. Marx, A.F. Fahmy, J.M. Myers, W. Bermel, and S.J. Glaser, *Phys. Rev A* **62**, 012310 (2000).

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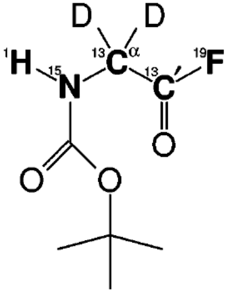

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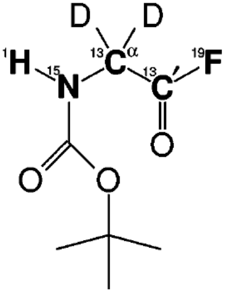

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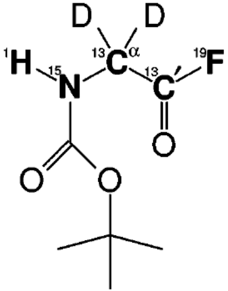

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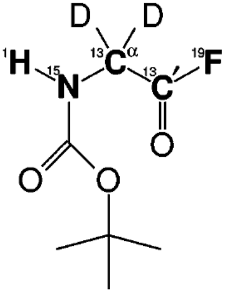

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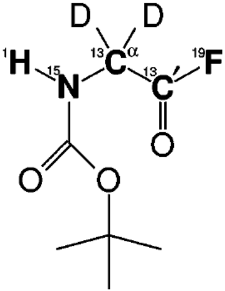

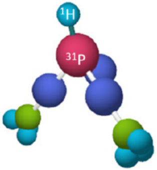

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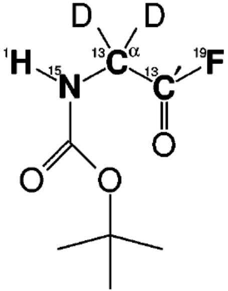

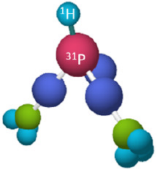

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The authors claim that this computer can demonstrate at least 10 quantum algorithms

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