Today's outline - April 05, 2022

Today's outline - April 05, 2022

- LOCC

Today's outline - April 05, 2022

- LOCC
- Majorization

Today's outline - April 05, 2022

- LOCC
- Majorization
- Examples

Today's outline - April 05, 2022

- LOCC
- Majorization
- Examples
- Measurement of density operators

Today's outline - April 05, 2022

- LOCC
- Majorization
- Examples
- Measurement of density operators

Reading assignment: 10.4

Today's outline - April 05, 2022

- LOCC
- Majorization
- Examples
- Measurement of density operators

Reading assignment: 10.4

Exam #02 postponed, TBD

Quantum circuit simulator https://algassert.com/quirk

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

These individuals only have a restricted set of operations that they can perform on the system and these are called local operations with classical communications (LOCC)

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

These individuals only have a restricted set of operations that they can perform on the system and these are called local operations with classical communications (LOCC)

The measure of entanglement of a system cannot be increased by LOCC

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

These individuals only have a restricted set of operations that they can perform on the system and these are called local operations with classical communications (LOCC)

The measure of entanglement of a system cannot be increased by LOCC

Consider a system with a specific decomposition given by $X = X_1 \otimes \cdots \otimes X_n$

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

These individuals only have a restricted set of operations that they can perform on the system and these are called local operations with classical communications (LOCC)

The measure of entanglement of a system cannot be increased by LOCC

Consider a system with a specific decomposition given by $X = X_1 \otimes \cdots \otimes X_n$

If it is possible to convert $|\psi\rangle \in X$ into $|\phi\rangle \in X$ via a series of unitary transformations and measurements which guaranteed to succeed deterministically then the conversion is said to be done by LOCC with respect to the particular tensor decomposition

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

These individuals only have a restricted set of operations that they can perform on the system and these are called local operations with classical communications (LOCC)

The measure of entanglement of a system cannot be increased by LOCC

Consider a system with a specific decomposition given by $X = X_1 \otimes \cdots \otimes X_n$

If it is possible to convert $|\psi\rangle \in X$ into $|\phi\rangle \in X$ via a series of unitary transformations and measurements which guaranteed to succeed deterministically then the conversion is said to be done by LOCC with respect to the particular tensor decomposition

Two states are LOCC equivalent if they can be transformed into each other by LOCC

Local operations and classical communication (LOCC)

Suppose we have a multi-partite system where different subsystems are under the control of different people with only classical communications channels between them

These individuals only have a restricted set of operations that they can perform on the system and these are called local operations with classical communications (LOCC)

The measure of entanglement of a system cannot be increased by LOCC

Consider a system with a specific decomposition given by $X = X_1 \otimes \cdots \otimes X_n$

If it is possible to convert $|\psi\rangle \in X$ into $|\phi\rangle \in X$ via a series of unitary transformations and measurements which guaranteed to succeed deterministically then the conversion is said to be done by LOCC with respect to the particular tensor decomposition

Two states are LOCC equivalent if they can be transformed into each other by LOCC

The level of entanglement cannot be increased by LOCC so an unentangled state cannot be converted to an entangled state by LOCC alone

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \ldots, a_m), \quad b = (b_1, \ldots, b_m)$$

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \dots, a_m), \quad b = (b_1, \dots, b_m)$$

The vectors $a^{\downarrow}$ and $b^{\downarrow}$ are reordered in such a way that the coefficients are arranged by magnitude

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \dots, a_m), \quad b = (b_1, \dots, b_m)$$

The vectors $a^{\downarrow}$ and $b^{\downarrow}$ are reordered in such a way that the coefficients are arranged by magnitude

$$a^{\downarrow} = (a_1^{\downarrow}, \dots, a_m^{\downarrow}), \quad a_i^{\downarrow} \geq a_{i+1}^{\downarrow}$$

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \dots, a_m), \quad b = (b_1, \dots, b_m)$$

The vectors $a^{\downarrow}$ and $b^{\downarrow}$ are reordered in such a way that the coefficients are arranged by magnitude

$$a^{\downarrow} = (a_1^{\downarrow}, \dots, a_m^{\downarrow}), \quad a_i^{\downarrow} \geq a_{i+1}^{\downarrow}$$

$$b^{\downarrow} = (b_1^{\downarrow}, \dots, b_m^{\downarrow}), \quad b_i^{\downarrow} \geq b_{i+1}^{\downarrow}$$

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \ldots, a_m), \quad b = (b_1, \ldots, b_m)$$

The vectors $a^{\downarrow}$ and $b^{\downarrow}$ are reordered in such a way that the coefficients are arranged by magnitude

$$a^{\downarrow} = (a_1^{\downarrow}, \ldots, a_m^{\downarrow}), \quad a_i^{\downarrow} \geq a_{i+1}^{\downarrow}$$

$$b^{\downarrow} = (b_1^{\downarrow}, \ldots, b_m^{\downarrow}), \quad b_i^{\downarrow} \geq b_{i+1}^{\downarrow}$$

Note that $a^{\downarrow}$ and $b^{\downarrow}$ are not necessarily ordered in the same way!

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \dots, a_m), \quad b = (b_1, \dots, b_m)$$

The vectors $a^{\downarrow}$ and $b^{\downarrow}$ are reordered in such a way that the coefficients are arranged by magnitude

$$a^{\downarrow} = (a_1^{\downarrow}, \dots, a_m^{\downarrow}), \quad a_i^{\downarrow} \geq a_{i+1}^{\downarrow}$$

$$b^{\downarrow} = (b_1^{\downarrow}, \dots, b_m^{\downarrow}), \quad b_i^{\downarrow} \geq b_{i+1}^{\downarrow}$$

Note that $a^{\downarrow}$ and $b^{\downarrow}$ are not necessarily ordered in the same way!

b is said to majorize a ($b \succeq a$) if for each $k, 1 \leq k \leq m$

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \dots, a_m), \quad b = (b_1, \dots, b_m)$$

The vectors $a^{\downarrow}$ and $b^{\downarrow}$ are reordered in such a way that the coefficients are arranged by magnitude

$$a^{\downarrow} = (a_1^{\downarrow}, \dots, a_m^{\downarrow}), \quad a_i^{\downarrow} \geq a_{i+1}^{\downarrow}$$

$$b^{\downarrow} = (b_1^{\downarrow}, \dots, b_m^{\downarrow}), \quad b_i^{\downarrow} \geq b_{i+1}^{\downarrow}$$

Note that $a^{\downarrow}$ and $b^{\downarrow}$ are not necessarily ordered in the same way!

b is said to majorize a ($b \succeq a$) if for each $k, 1 \leq k \leq m$

$$\sum_{j=1}^{k} a_j^{\downarrow} \leq \sum_{j=1}^{k} b_j^{\downarrow}, \quad \sum_{j=1}^{m} a_j^{\downarrow} = \sum_{j=1}^{m} b_j^{\downarrow}$$

Classifying bipartite states

It is possible to classify pure states of bipartite systems using a metric called majorization of the eigenvalues of subsystem density operators

Let a and b be two vectors in the same m dimensional space

$$a = (a_1, \dots, a_m), \quad b = (b_1, \dots, b_m)$$

The vectors $a^\downarrow$ and $b^\downarrow$ are reordered in such a way that the coefficients are arranged by magnitude

$$a^\downarrow = (a_1^\downarrow, \dots, a_m^\downarrow), \quad a_i^\downarrow \geq a_{i+1}^\downarrow$$

$$b^\downarrow = (b_1^\downarrow, \dots, b_m^\downarrow), \quad b_i^\downarrow \geq b_{i+1}^\downarrow$$

Note that $a^\downarrow$ and $b^\downarrow$ are not necessarily ordered in the same way!

b is said to majorize a ($b \succeq a$) if for each $k, 1 \leq k \leq m$

$$\sum_{j=1}^{k} a_j^\downarrow \leq \sum_{j=1}^{k} b_j^\downarrow, \qquad \sum_{j=1}^{m} a_j^\downarrow = \sum_{j=1}^{m} b_j^\downarrow$$

This can be applied to density matrix eigenvalues to define LOCC equivalence and relative degrees of entanglement

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \mathrm{Tr}_B(|\phi\rangle\langle\phi|)$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and
$|\phi\rangle$ be states such that

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \mathrm{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and $|\phi\rangle$ be states such that

$$\lambda^\psi = (\lambda, 1 - \lambda),$$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and $|\phi\rangle$ be states such that

$$\lambda^\psi = (\lambda, 1-\lambda), \quad \lambda^\phi = (\mu, 1-\mu),$$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and $|\phi\rangle$ be states such that

$$\lambda^\psi = (\lambda, 1-\lambda), \quad \lambda^\phi = (\mu, 1-\mu), \quad \lambda, \mu \geq \tfrac{1}{2}$$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \text{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \text{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \ldots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \ldots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and $|\phi\rangle$ be states such that

$$\lambda^\psi = (\lambda, 1-\lambda), \quad \lambda^\phi = (\mu, 1-\mu), \quad \lambda, \mu \geq \tfrac{1}{2}$$

$|\psi\rangle$ can be converted to $|\phi\rangle$ only if $\mu \geq \lambda$ so that $\lambda^\phi \succeq \lambda^\psi$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \mathrm{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \dots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \dots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and $|\phi\rangle$ be states such that

$$\lambda^\psi = (\lambda, 1-\lambda), \quad \lambda^\phi = (\mu, 1-\mu), \quad \lambda, \mu \geq \tfrac{1}{2}$$

$|\psi\rangle$ can be converted to $|\phi\rangle$ only if $\mu \geq \lambda$ so that $\lambda^\phi \succeq \lambda^\psi$

Furthermore it is clear that $\lambda^\phi \succeq \lambda^\psi$ only when the von Neumann entropy satisfies $S[\mathrm{Tr}_2(|\psi\rangle\langle\psi|)] \geq S[\mathrm{Tr}_2(|\phi\rangle\langle\phi|)]$

Majorization & LOCC equivalence

For states $|\psi\rangle$ and $|\phi\rangle$ of a bipartite system $X = A \otimes B$, the partial density matrices with respect to subsystem A are $\rho_\psi = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ and $\rho_\phi = \mathrm{Tr}_B(|\phi\rangle\langle\phi|)$

The eigenvalues of ρ_ψ and ρ_ϕ are given by $\lambda^\psi = (\lambda_1^\psi, \dots, \lambda_m^\psi)$ and $\lambda^\phi = (\lambda_1^\phi, \dots, \lambda_m^\phi)$

It can be shown that $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ only if $\lambda^\phi \succeq \lambda^\psi$

An unambiguous definition of LOCC equivalence is thus $\lambda^\phi \succeq \lambda^\psi$ and $\lambda^\psi \succeq \lambda^\phi$

For a bipartite system of 2 qubits let $|\psi\rangle$ and $|\phi\rangle$ be states such that

$$\lambda^\psi = (\lambda, 1-\lambda), \quad \lambda^\phi = (\mu, 1-\mu), \quad \lambda, \mu \geq \tfrac{1}{2}$$

$|\psi\rangle$ can be converted to $|\phi\rangle$ only if $\mu \geq \lambda$ so that $\lambda^\phi \succeq \lambda^\psi$

Furthermore it is clear that $\lambda^\phi \succeq \lambda^\psi$ only when the von Neumann entropy satisfies $S[\mathrm{Tr}_2(|\psi\rangle\langle\psi|)] \geq S[\mathrm{Tr}_2(|\phi\rangle\langle\phi|)]$

Thus $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle$ is possible only when $|\psi\rangle$ is more, or equally as, entangled as $|\phi\rangle$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

$$|\psi\rangle = \frac{3}{4}|0\rangle|0\rangle + \frac{2}{4}|1\rangle|1\rangle + \frac{\sqrt{2}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

$$|\psi\rangle = \frac{3}{4}|0\rangle|0\rangle + \frac{2}{4}|1\rangle|1\rangle + \frac{\sqrt{2}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \frac{\sqrt{8}}{4}|0\rangle|0\rangle + \frac{\sqrt{6}}{4}|1\rangle|1\rangle + \frac{\sqrt{1}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

There is an inconsistency which prevents a a comparison

$$|\psi\rangle = \frac{3}{4}|0\rangle|0\rangle + \frac{2}{4}|1\rangle|1\rangle + \frac{\sqrt{2}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \frac{\sqrt{8}}{4}|0\rangle|0\rangle + \frac{\sqrt{6}}{4}|1\rangle|1\rangle + \frac{\sqrt{1}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

There is an inconsistency which prevents a a comparison

$$|\psi\rangle = \frac{3}{4}|0\rangle|0\rangle + \frac{2}{4}|1\rangle|1\rangle + \frac{\sqrt{2}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \frac{\sqrt{8}}{4}|0\rangle|0\rangle + \frac{\sqrt{6}}{4}|1\rangle|1\rangle + \frac{\sqrt{1}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

$$\lambda_1^{\psi} = \frac{9}{16} > \frac{1}{2} = \lambda_1^{\phi}$$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

There is an inconsistency which prevents a a comparison

$$|\psi\rangle = \frac{3}{4}|0\rangle|0\rangle + \frac{2}{4}|1\rangle|1\rangle + \frac{\sqrt{2}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \frac{\sqrt{8}}{4}|0\rangle|0\rangle + \frac{\sqrt{6}}{4}|1\rangle|1\rangle + \frac{\sqrt{1}}{4}|2\rangle|2\rangle + \frac{1}{4}|3\rangle|3\rangle$$

$$\lambda_1^{\psi} = \frac{9}{16} > \frac{1}{2} = \lambda_1^{\phi}$$

$$\lambda_1^{\psi} + \lambda_2^{\psi} = \frac{13}{16} < \frac{14}{16} = \lambda_1^{\phi} + \lambda_2^{\phi}$$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

There is an inconsistency which prevents a a comparison

$$|\psi\rangle = \tfrac{3}{4}|0\rangle|0\rangle + \tfrac{2}{4}|1\rangle|1\rangle + \tfrac{\sqrt{2}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \tfrac{\sqrt{8}}{4}|0\rangle|0\rangle + \tfrac{\sqrt{6}}{4}|1\rangle|1\rangle + \tfrac{\sqrt{1}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$\lambda_1^{\psi} = \tfrac{9}{16} > \tfrac{1}{2} = \lambda_1^{\phi}$$

$$\lambda_1^{\psi} + \lambda_2^{\psi} = \tfrac{13}{16} < \tfrac{14}{16} = \lambda_1^{\phi} + \lambda_2^{\phi}$$

However, it is unambiguous that in any bipartite system, the vector for an unentangled state majorizes all others and it can be shown that a maximally entangled state is majorized by all others

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

There is an inconsistency which prevents a a comparison

$$|\psi\rangle = \tfrac{3}{4}|0\rangle|0\rangle + \tfrac{2}{4}|1\rangle|1\rangle + \tfrac{\sqrt{2}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \tfrac{\sqrt{8}}{4}|0\rangle|0\rangle + \tfrac{\sqrt{6}}{4}|1\rangle|1\rangle + \tfrac{\sqrt{1}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$\lambda_1^\psi = \tfrac{9}{16} > \tfrac{1}{2} = \lambda_1^\phi$$

$$\lambda_1^\psi + \lambda_2^\psi = \tfrac{13}{16} < \tfrac{14}{16} = \lambda_1^\phi + \lambda_2^\phi$$

However, it is unambiguous that in any bipartite system, the vector for an unentangled state majorizes all others and it can be shown that a maximally entangled state is majorized by all others

Consider $|\psi\rangle$ in a bipartite system $X = A \otimes B$
where A and B have dimensions $N, M : N \geq M$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

$$|\psi\rangle = \tfrac{3}{4}|0\rangle|0\rangle + \tfrac{2}{4}|1\rangle|1\rangle + \tfrac{\sqrt{2}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \tfrac{\sqrt{8}}{4}|0\rangle|0\rangle + \tfrac{\sqrt{6}}{4}|1\rangle|1\rangle + \tfrac{\sqrt{1}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

There is an inconsistency which prevents a a comparison

$$\lambda_1^\psi = \tfrac{9}{16} > \tfrac{1}{2} = \lambda_1^\phi$$

$$\lambda_1^\psi + \lambda_2^\psi = \tfrac{13}{16} < \tfrac{14}{16} = \lambda_1^\phi + \lambda_2^\phi$$

However, it is unambiguous that in any bipartite system, the vector for an unentangled state majorizes all others and it can be shown that a maximally entangled state is majorized by all others

Consider $|\psi\rangle$ in a bipartite system $X = A \otimes B$ where A and B have dimensions $N, M : N \geq M$

$$|\psi\rangle = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} |\phi_i^A\rangle \otimes |\phi_i^B\rangle$$

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

$$|\psi\rangle = \tfrac{3}{4}|0\rangle|0\rangle + \tfrac{2}{4}|1\rangle|1\rangle + \tfrac{\sqrt{2}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \tfrac{\sqrt{8}}{4}|0\rangle|0\rangle + \tfrac{\sqrt{6}}{4}|1\rangle|1\rangle + \tfrac{\sqrt{1}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

There is an inconsistency which prevents a a comparison

$$\lambda_1^\psi = \tfrac{9}{16} > \tfrac{1}{2} = \lambda_1^\phi$$

$$\lambda_1^\psi + \lambda_2^\psi = \tfrac{13}{16} < \tfrac{14}{16} = \lambda_1^\phi + \lambda_2^\phi$$

However, it is unambiguous that in any bipartite system, the vector for an unentangled state majorizes all others and it can be shown that a maximally entangled state is majorized by all others

Consider $|\psi\rangle$ in a bipartite system $X = A \otimes B$ where A and B have dimensions $N, M : N \geq M$

$$|\psi\rangle = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} |\phi_i^A\rangle \otimes |\phi_i^B\rangle$$

$\{|\phi_i^A\rangle\}$ and $\{|\phi_i^B\rangle\}$ are orthonormal sets and the latter must be a basis for B

Larger bipartite systems

When bipartite systems have subsystems of more than one qubit, the situation is a bit more complex as there are states which cannot be compared for majorization

$$|\psi\rangle = \tfrac{3}{4}|0\rangle|0\rangle + \tfrac{2}{4}|1\rangle|1\rangle + \tfrac{\sqrt{2}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

$$|\phi\rangle = \tfrac{\sqrt{8}}{4}|0\rangle|0\rangle + \tfrac{\sqrt{6}}{4}|1\rangle|1\rangle + \tfrac{\sqrt{1}}{4}|2\rangle|2\rangle + \tfrac{1}{4}|3\rangle|3\rangle$$

There is an inconsistency which prevents a a comparison

$$\lambda_1^{\psi} = \tfrac{9}{16} > \tfrac{1}{2} = \lambda_1^{\phi}$$

$$\lambda_1^{\psi} + \lambda_2^{\psi} = \tfrac{13}{16} < \tfrac{14}{16} = \lambda_1^{\phi} + \lambda_2^{\phi}$$

However, it is unambiguous that in any bipartite system, the vector for an unentangled state majorizes all others and it can be shown that a maximally entangled state is majorized by all others

Consider $|\psi\rangle$ in a bipartite system $X = A \otimes B$ where A and B have dimensions $N, M : N \geq M$

$$|\psi\rangle = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} |\phi_i^A\rangle \otimes |\phi_i^B\rangle$$

$\{|\phi_i^A\rangle\}$ and $\{|\phi_i^B\rangle\}$ are orthonormal sets and the latter must be a basis for B so $\lambda^{\phi} \succeq \lambda^{\psi}$ must hold for all $|\phi\rangle \in A \otimes B$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix},$$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}),$$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

$$\lambda_1^{\Phi^+} = \lambda_1^{\Psi^+} = \frac{1}{2}$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \qquad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

$$\lambda_1^{\Phi^+} = \lambda_1^{\Psi^+} = \frac{1}{2}$$

$$\lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = \lambda_1^{\Psi^+} + \lambda_2^{\Psi^+} = 1$$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

$$\lambda_1^{\Phi^+} = \lambda_1^{\Psi^+} = \frac{1}{2}$$

$$\lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = \lambda_1^{\Psi^+} + \lambda_2^{\Psi^+} = 1$$

With von Neumann entropies of

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

$$\lambda_1^{\Phi^+} = \lambda_1^{\Psi^+} = \tfrac{1}{2}$$

$$\lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = \lambda_1^{\Psi^+} + \lambda_2^{\Psi^+} = 1$$

With von Neumann entropies of

$$S(\rho_{\Phi^+}) = -2(\tfrac{1}{2}\log_2 \tfrac{1}{2}) = 1 = S(\rho_{\Psi^+})$$

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

$$\lambda_1^{\Phi^+} = \lambda_1^{\Psi^+} = \frac{1}{2}$$

$$\lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = \lambda_1^{\Psi^+} + \lambda_2^{\Psi^+} = 1$$

With von Neumann entropies of

$$S(\rho_{\Phi^+}) = -2(\tfrac{1}{2}\log_2 \tfrac{1}{2}) = 1 = S(\rho_{\Psi^+})$$

These two states are both maximally entangled and thus are LOCC equivalent and can be transformed into each other using a local operation only

Example 10.2.5

Consider the two Bell states $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

The partial density matrices for each of these states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{\Psi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{\Psi^+} = (\tfrac{1}{2}, \tfrac{1}{2})$$

Therefore, it is easy to see that $\lambda^{\Phi^+} \succeq \lambda^{\Psi^+}$ and $\lambda^{\Psi^+} \succeq \lambda^{\Phi^+}$

$$\lambda_1^{\Phi^+} = \lambda_1^{\Psi^+} = \frac{1}{2}$$

$$\lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = \lambda_1^{\Psi^+} + \lambda_2^{\Psi^+} = 1$$

With von Neumann entropies of

$$S(\rho_{\Phi^+}) = -2(\tfrac{1}{2}\log_2 \tfrac{1}{2}) = 1 = S(\rho_{\Psi^+})$$

These two states are both maximally entangled and thus are LOCC equivalent and can be transformed into each other using a local operation only

In this case, applying an X transformation to the first qubit will transform $|\Phi^+\rangle \xleftrightarrow{X\otimes I} |\Psi^+\rangle$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix},$$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}),$$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

$$\lambda_1^{00} = 1 > \lambda_1^{\Phi^+} = \tfrac{1}{2}$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

$$\lambda_1^{00} = 1 > \lambda_1^{\Phi^+} = \frac{1}{2}$$

$$\lambda_1^{00} + \lambda_2^{00} = \lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = 1$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

$$\lambda_1^{00} = 1 > \lambda_1^{\Phi^+} = \frac{1}{2}$$

$$\lambda_1^{00} + \lambda_2^{00} = \lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = 1$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

With von Neumann entropies of

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

$$\lambda_1^{00} = 1 > \lambda_1^{\Phi^+} = \tfrac{1}{2}$$
$$\lambda_1^{00} + \lambda_2^{00} = \lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = 1$$

With von Neumann entropies of

$$S(\rho_{\Phi^+}) = 1, \quad S(\rho_{00}) = -1\log_2 1 - 0\log_2 0 = 0$$

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

$$\lambda_1^{00} = 1 > \lambda_1^{\Phi^+} = \frac{1}{2}$$

$$\lambda_1^{00} + \lambda_2^{00} = \lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = 1$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

With von Neumann entropies of

$$S(\rho_{\Phi^+}) = 1, \quad S(\rho_{00}) = -1\log_2 1 - 0\log_2 0 = 0$$

Thus it is possible to convert $|\Phi^+\rangle$ to $|00\rangle$ using LOCC

Example 10.2.6

Can the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be converted to $|00\rangle$ using LOCC?

We know that the partial density matrices for the two states with respect to the first qubit are

$$\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The ordered eigenvalues are thus

$$\lambda^{\Phi^+} = (\tfrac{1}{2}, \tfrac{1}{2}), \qquad \lambda^{00} = (1, 0)$$

Therefore since $\lambda^{00} \succeq \lambda^{\Phi^+}$ we see that $|00\rangle$ majorizes $|\Phi^+\rangle$

$$\lambda_1^{00} = 1 > \lambda_1^{\Phi^+} = \tfrac{1}{2}$$

$$\lambda_1^{00} + \lambda_2^{00} = \lambda_1^{\Phi^+} + \lambda_2^{\Phi^+} = 1$$

With von Neumann entropies of

$$S(\rho_{\Phi^+}) = 1, \quad S(\rho_{00}) = -1\log_2 1 - 0\log_2 0 = 0$$

Thus it is possible to convert $|\Phi^+\rangle$ to $|00\rangle$ using LOCC

This can be done by measuring the first qubit and and then applying X to each qubit if the result is $|1\rangle$ and doing nothing if the result is $|0\rangle$

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

$$\rho = \frac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{2}|\Phi^-\rangle\langle\Phi^-|$$

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

$$\rho = \tfrac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \tfrac{1}{2}|\Phi^-\rangle\langle\Phi^-| = \tfrac{1}{2}\left[(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)\right] + \tfrac{1}{2}\left[(|00\rangle - |11\rangle)(\langle 00| - \langle 11|)\right]$$

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

$$\rho = \tfrac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \tfrac{1}{2}|\Phi^-\rangle\langle\Phi^-| = \tfrac{1}{2}\left[(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)\right] + \tfrac{1}{2}\left[(|00\rangle - |11\rangle)(\langle 00| - \langle 11|)\right]$$

$$= \tfrac{1}{2}\left[2|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + 2|11\rangle\langle 11| - |00\rangle\langle 11| - |11\rangle\langle 00|\right]$$

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

$$\rho = \frac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{2}|\Phi^-\rangle\langle\Phi^-| = \frac{1}{2}\left[(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)\right] + \frac{1}{2}\left[(|00\rangle - |11\rangle)(\langle 00| - \langle 11|)\right]$$

$$= \frac{1}{2}\left[2|00\rangle\langle 00| + \cancel{|00\rangle\langle 11|} + \cancel{|11\rangle\langle 00|} + 2|11\rangle\langle 11| - \cancel{|00\rangle\langle 11|} - \cancel{|11\rangle\langle 00|}\right] = \frac{1}{2}\left[|00\rangle\langle 00| + |11\rangle\langle 11|\right]$$

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

$$\rho = \tfrac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \tfrac{1}{2}|\Phi^-\rangle\langle\Phi^-| = \tfrac{1}{2}\left[(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)\right] + \tfrac{1}{2}\left[(|00\rangle - |11\rangle)(\langle 00| - \langle 11|)\right]$$

$$= \tfrac{1}{2}\left[2|00\rangle\langle 00| + \cancel{|00\rangle\langle 11|} + \cancel{|11\rangle\langle 00|} + 2|11\rangle\langle 11| - \cancel{|00\rangle\langle 11|} - \cancel{|11\rangle\langle 00|}\right] = \tfrac{1}{2}\left[|00\rangle\langle 00| + |11\rangle\langle 11|\right]$$

Which is separable and really just a probabilistic mixture of product states

Mixed bipartite systems

A mixed state of a quantum system is separable with respect to a particular tensor decomposition $V_0 \otimes \cdots \otimes V_{N-1}$ if it can be written as a probabilistic mixture of unentangled states

$$\rho = \sum_{j=1}^{m} p_j |\phi_j^{(0)}\rangle\langle\phi_j^{(0)}| \otimes \cdots \otimes |\phi_j^{(N-1)}\rangle\langle\phi_j^{(N-1)}|, \quad |\phi_j^{(i)}\rangle \in V_i, \quad p_i \geq 0, \quad \sum_{i=0}^{N-1} p_i = 1$$

For any given value of i, the $|\phi_j^{(i)}\rangle$ do not need to be orthogonal

If an mixed state cannot be written in this way it is said to be entangled

A mixed state that can be written as a probabilistic mixture of entangled states can still be separable

$$\rho = \tfrac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \tfrac{1}{2}|\Phi^-\rangle\langle\Phi^-| = \tfrac{1}{2}\left[(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)\right] + \tfrac{1}{2}\left[(|00\rangle - |11\rangle)(\langle 00| - \langle 11|)\right]$$
$$= \tfrac{1}{2}\left[2|00\rangle\langle 00| + \cancel{|00\rangle\langle 11|} + \cancel{|11\rangle\langle 00|} + 2|11\rangle\langle 11| - \cancel{|00\rangle\langle 11|} - \cancel{|11\rangle\langle 00|}\right] = \tfrac{1}{2}\left[|00\rangle\langle 00| + |11\rangle\langle 11|\right]$$

Which is separable and really just a probabilistic mixture of product states

Measuring the entropy of a bipartite mixed state is complicated and can be done in a number of different ways

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}}P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

$$\rho_x^O = \sum_{j=0}^{K-1} p_j \rho_x^j$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}}P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j}P_j|x\rangle\langle x|P_j = \frac{1}{p_j}P_j\rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

$$\rho_x^O = \sum_{j=0}^{K-1} p_j\rho_x^j = \sum_{j=0}^{K-1} P_j\rho_x P_j^\dagger$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

$$\rho_x^O = \sum_{j=0}^{K-1} p_j \rho_x^j = \sum_{j=0}^{K-1} P_j \rho_x P_j^\dagger$$

Let $\{|\alpha_i\rangle\}$ be an eigenbasis of the operator O that contains the result of the measurement of $|x\rangle$ by P_j as the first K elements of the N-element basis

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

$$\rho_x^O = \sum_{j=0}^{K-1} p_j \rho_x^j = \sum_{j=0}^{K-1} P_j \rho_x P_j^\dagger$$

Let $\{|\alpha_i\rangle\}$ be an eigenbasis of the operator O that contains the result of the measurement of $|x\rangle$ by P_j as the first K elements of the N-element basis

$$|x\rangle = \sum_{j=0}^{K-1} \frac{P_j|x\rangle}{|P_j|x\rangle|}$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

$$\rho_x^O = \sum_{j=0}^{K-1} p_j \rho_x^j = \sum_{j=0}^{K-1} P_j \rho_x P_j^\dagger$$

Let $\{|\alpha_i\rangle\}$ be an eigenbasis of the operator O that contains the result of the measurement of $|x\rangle$ by P_j as the first K elements of the N-element basis

$$|x\rangle = \sum_{j=0}^{K-1} \frac{P_j|x\rangle}{|P_j|x\rangle|} = \sum_{i=0}^{N-1} x_i|\alpha_i\rangle$$

Measurement of density operators

Let $|x\rangle$ be an element of an $N = 2^n$ dimensional vector space X with density operator $\rho_x = |x\rangle\langle x|$

If $|x\rangle$ is measured with an operator O that has K associated projectors P_j the result, with probability p_j is

$$\frac{P_j|x\rangle}{|P_j|x\rangle|} = \frac{1}{\sqrt{p_j}} P_j|x\rangle, \qquad p_j = \langle x|P_j|x\rangle$$

The density operator for each of these states is

$$\rho_x^j = \frac{1}{p_j} P_j|x\rangle\langle x|P_j = \frac{1}{p_j} P_j \rho_x P_j$$

The density operator for O which summarizes the possible outcomes of the measurement is

$$\rho_x^O = \sum_{j=0}^{K-1} p_j \rho_x^j = \sum_{j=0}^{K-1} P_j \rho_x P_j^\dagger$$

Let $\{|\alpha_i\rangle\}$ be an eigenbasis of the operator O that contains the result of the measurement of $|x\rangle$ by P_j as the first K elements of the N-element basis

$$|x\rangle = \sum_{j=0}^{K-1} \frac{P_j|x\rangle}{|P_j|x\rangle|} = \sum_{i=0}^{N-1} x_i|\alpha_i\rangle$$

$$x_i = \begin{cases} \sqrt{p_i} & i < K \\ 0 & i \geq K \end{cases}$$

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i |\alpha_i\rangle \right) \left(\sum_{j=0}^{N-1} x_j |\alpha_j\rangle \right)^{\dagger}$$

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i|\alpha_i\rangle\right)\left(\sum_{j=0}^{N-1} x_j|\alpha_j\rangle\right)^\dagger = \sum_{j=0}^{N-1}\sum_{i=0}^{N-1} x_i\overline{x_j}|\alpha_i\rangle\langle\alpha_j|$$

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i|\alpha_i\rangle\right)\left(\sum_{j=0}^{N-1} x_j|\alpha_j\rangle\right)^\dagger = \sum_{j=0}^{N-1}\sum_{i=0}^{N-1} x_i\overline{x_j}|\alpha_i\rangle\langle\alpha_j|$$

The ij^{th} entry of the matrix ρ_x in the $\{|\alpha_k\rangle\}$ basis is just $\overline{x_i}x_j$

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i|\alpha_i\rangle\right)\left(\sum_{j=0}^{N-1} x_j|\alpha_j\rangle\right)^\dagger = \sum_{j=0}^{N-1}\sum_{i=0}^{N-1} x_i\overline{x_j}|\alpha_i\rangle\langle\alpha_j|$$

The ij^{th} entry of the matrix ρ_x in the $\{|\alpha_k\rangle\}$ basis is just $\overline{x_i}x_j$

The density operator ρ_x^O is thus

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i |\alpha_i\rangle \right) \left(\sum_{j=0}^{N-1} x_j |\alpha_j\rangle \right)^{\dagger} = \sum_{j=0}^{N-1} \sum_{i=0}^{N-1} x_i \overline{x_j} |\alpha_i\rangle\langle\alpha_j|$$

The ij^{th} entry of the matrix ρ_x in the $\{|\alpha_k\rangle\}$ basis is just $\overline{x_i} x_j$

The density operator ρ_x^O is thus

$$\rho_x^O = \sum_{j=0}^{N-1} x_j \overline{x_j} |\alpha_j\rangle\langle\alpha_j|$$

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i|\alpha_i\rangle\right)\left(\sum_{j=0}^{N-1} x_j|\alpha_j\rangle\right)^\dagger = \sum_{j=0}^{N-1}\sum_{i=0}^{N-1} x_i\overline{x_j}|\alpha_i\rangle\langle\alpha_j|$$

The ij^{th} entry of the matrix ρ_x in the $\{|\alpha_k\rangle\}$ basis is just $\overline{x_i}x_j$

The density operator ρ_x^O is thus

$$\rho_x^O = \sum_{j=0}^{N-1} x_j\overline{x_j}|\alpha_j\rangle\langle\alpha_j| = \sum_{j=0}^{N-1} P_j|x\rangle\langle x|P_j^\dagger$$

Measurement of density operators

We can now write the density operator ρ_x in terms of this eigenbasis

$$\rho_x = |x\rangle\langle x| = \left(\sum_{i=0}^{N-1} x_i|\alpha_i\rangle\right)\left(\sum_{j=0}^{N-1} x_j|\alpha_j\rangle\right)^\dagger = \sum_{j=0}^{N-1}\sum_{i=0}^{N-1} x_i\overline{x_j}|\alpha_i\rangle\langle\alpha_j|$$

The ij^{th} entry of the matrix ρ_x in the $\{|\alpha_k\rangle\}$ basis is just $\overline{x_i}x_j$

The density operator ρ_x^O is thus

$$\rho_x^O = \sum_{j=0}^{N-1} x_j\overline{x_j}|\alpha_j\rangle\langle\alpha_j| = \sum_{j=0}^{N-1} P_j|x\rangle\langle x|P_j^\dagger$$

So ρ_x^O in the $\{|\alpha_k\rangle\}$ basis is simply ρ_x with all the off-diagonal elements set to zero

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho'_i = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho_i' = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho'_i = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho_i' = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger = \sum_j P_j \left(\sum_i q_i |\psi_i\rangle\langle\psi_i| \right) P_j^\dagger$$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho'_i = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger = \sum_j P_j \left(\sum_i q_i |\psi_i\rangle\langle\psi_i| \right) P_j^\dagger = \sum_j P_j \rho P_j^\dagger$$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho'_i = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger = \sum_j P_j \left(\sum_i q_i |\psi_i\rangle\langle\psi_i| \right) P_j^\dagger = \sum_j P_j \rho P_j^\dagger$$

The term $P_j \rho P_j^\dagger$ is not necessarily a density operator but is positive and Hermitian with a trace $\mathrm{Tr}(P_j \rho P_j^\dagger) \leq 1$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho_i' = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger = \sum_j P_j \left(\sum_i q_i |\psi_i\rangle\langle\psi_i| \right) P_j^\dagger = \sum_j P_j \rho P_j^\dagger$$

The term $P_j \rho P_j^\dagger$ is not necessarily a density operator but is positive and Hermitian with a trace $\mathrm{Tr}(P_j \rho P_j^\dagger) \leq 1$

ρ' may be viewed as a probabilistic mixture of density operators $\rho_j = P_j \rho P_j^\dagger / \mathrm{Tr}(P_j \rho P_j^\dagger)$ with weighting $p_j = \mathrm{Tr}(P_j \rho P_j^\dagger)$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho_i' = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger = \sum_j P_j \left(\sum_i q_i |\psi_i\rangle\langle\psi_i| \right) P_j^\dagger = \sum_j P_j \rho P_j^\dagger$$

The term $P_j \rho P_j^\dagger$ is not necessarily a density operator but is positive and Hermitian with a trace $\mathrm{Tr}(P_j \rho P_j^\dagger) \leq 1$

ρ' may be viewed as a probabilistic mixture of density operators $\rho_j = P_j \rho P_j^\dagger / \mathrm{Tr}(P_j \rho P_j^\dagger)$ with weighting $p_j = \mathrm{Tr}(P_j \rho P_j^\dagger)$

$$\rho' = \sum_j p_j \rho_j$$

Measurement of mixed states

Let ρ be a density operator representing a mixed state which can be written as a probabilstic mixture of pure states $|\psi_i\rangle$

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

The outcomes of measuring the mixed state can be written as a probabilistic mixture of the density operators

$$\rho_i' = \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger$$

The density operator for the possible outcomes of measuring the mixed state ρ is

$$\rho' = \sum_i q_i \sum_j P_j |\psi_i\rangle\langle\psi_i| P_j^\dagger = \sum_j P_j \left(\sum_i q_i |\psi_i\rangle\langle\psi_i| \right) P_j^\dagger = \sum_j P_j \rho P_j^\dagger$$

The term $P_j \rho P_j^\dagger$ is not necessarily a density operator but is positive and Hermitian with a trace $\mathrm{Tr}(P_j \rho P_j^\dagger) \leq 1$

ρ' may be viewed as a probabilistic mixture of density operators $\rho_j = P_j \rho P_j^\dagger / \mathrm{Tr}(P_j \rho P_j^\dagger)$ with weighting $p_j = \mathrm{Tr}(P_j \rho P_j^\dagger)$

$$\rho' = \sum_j p_j \rho_j = \sum_j p_j \frac{P_j \rho P_j^\dagger}{\mathrm{Tr}(P_j \rho P_j^\dagger)}$$

A bit about me...

B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)

A bit about me...

B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)

Ph.D. in Physics, University of California, San Diego (1981)

A bit about me...

B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)

Ph.D. in Physics, University of California, San Diego (1981)

Illinois Tech Faculty member since 1983

A bit about me...

B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)

Ph.D. in Physics, University of California, San Diego (1981)

Illinois Tech Faculty member since 1983

Have held administrative positions as Associate Chair, Associate Dean, and most recently Interim Chair of Chemistry Dept.

A bit about me...

B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)

Ph.D. in Physics, University of California, San Diego (1981)

Illinois Tech Faculty member since 1983

Have held administrative positions as Associate Chair, Associate Dean, and most recently Interim Chair of Chemistry Dept.

Currently Academic Advisor for Applied Physics Program, Society of Physics Students, and CAURS-IIT

A bit about me...

B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)

Ph.D. in Physics, University of California, San Diego (1981)

Illinois Tech Faculty member since 1983

Have held administrative positions as Associate Chair, Associate Dean, and most recently Interim Chair of Chemistry Dept.

Currently Academic Advisor for Applied Physics Program, Society of Physics Students, and CAURS-IIT

Director of Center for Synchrotron Radiation Research & Instrumentation, Director of Materials Research Collaborative Access Team, Board Member, Sigma Xi, The Scientific Research Society & International Center for Diffraction Data

A bit about me...

- B.S. in Physics & Chemistry, University of Illinois at Urbana-Champaign (1976)
- Ph.D. in Physics, University of California, San Diego (1981)
- Illinois Tech Faculty member since 1983
- Have held administrative positions as Associate Chair, Associate Dean, and most recently Interim Chair of Chemistry Dept.
- Currently Academic Advisor for Applied Physics Program, Society of Physics Students, and CAURS-IIT
- Director of Center for Synchrotron Radiation Research & Instrumentation, Director of Materials Research Collaborative Access Team, Board Member, Sigma Xi, The Scientific Research Society & International Center for Diffraction Data
- For fun: Debian GNU/Linux Developer, Illinois Tech Representative & Super Moderator on CollegeConfidential

MRCAT

- Materials Research Collaborative Access Team
- Specializing in x-ray absorption spectroscopy for local structure & electronic measurements
- Focus on *in situ* experiments at time scales from 10 s to 2 min

Current active membership

Illinois Tech

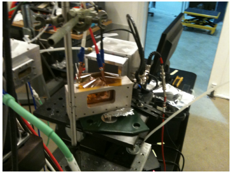
Argonne Chem. Sci. & Eng.

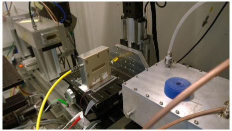
Argonne Biosciences

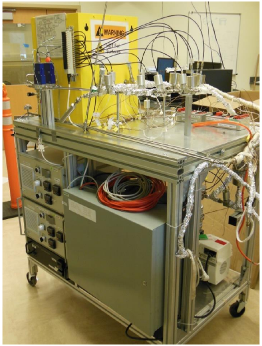
EPA Cincinnati

UOP Honeywell

BP Ventures

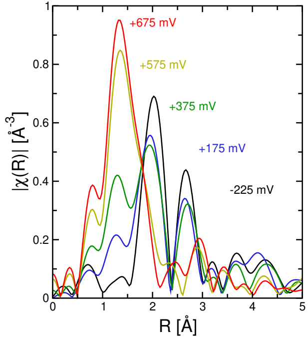






A bit about my research. . .

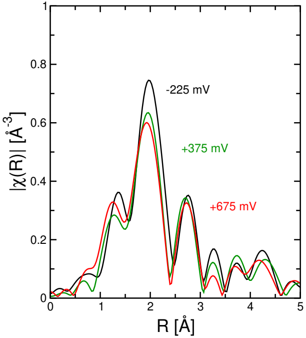
Mechanistic studies of catalysts by EXAFS



"In situ Ru K-edge x-ray absorption spectroscopy study of methanol oxidation mechanisms on model submonolayer Ru on Pt nanoparticle electrocatalyst," C.J. Pelliccione, E.V. Timofeeva, J.P. Katsoudas, and C.U. Segre, J. Phys. Chem. C 117, 18904-18912 (2013).

A bit about my research...

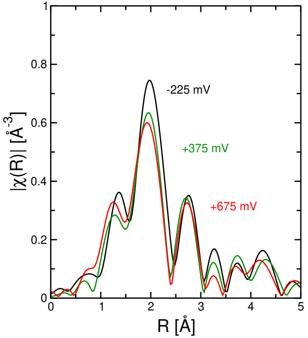
Mechanistic studies of catalysts
by EXAFS

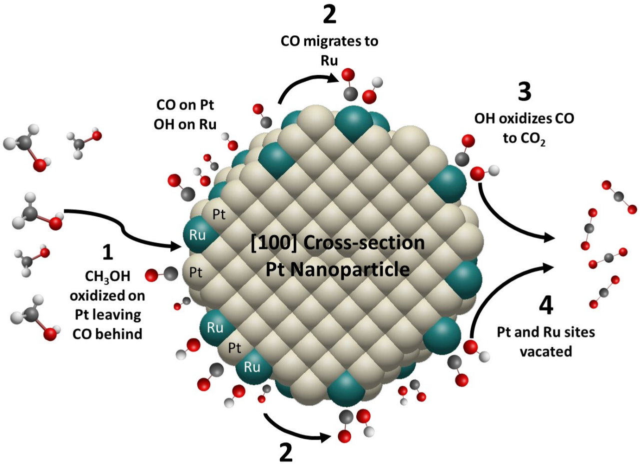


"In situ Ru K-edge x-ray absorption spectroscopy study of methanol oxidation mechanisms on model submonolayer Ru on Pt nanoparticle electrocatalyst," C.J. Pelliccione, E.V. Timofeeva, J.P. Katsoudas, and C.U. Segre, J. Phys. Chem. C 117, 18904-18912 (2013).

A bit about my research...

Mechanistic studies of catalysts by EXAFS

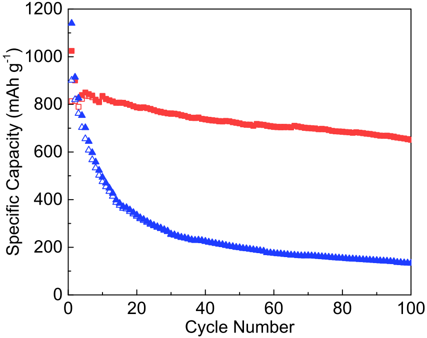




"In situ Ru K-edge x-ray absorption spectroscopy study of methanol oxidation mechanisms on model submonolayer Ru on Pt nanoparticle electrocatalyst," C.J. Pelliccione, E.V. Timofeeva, J.P. Katsoudas, and C.U. Segre, J. Phys. Chem. C 117, 18904-18912 (2013).

. . . and a little more

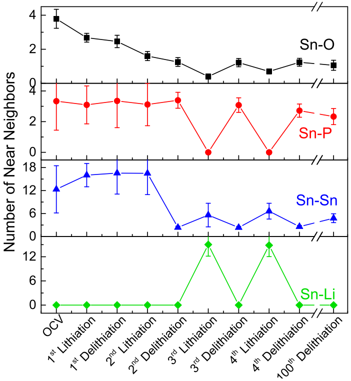
EXAFS studies of battery materials



"In situ EXAFS-derived mechanism of highly reversible tin phosphide/graphite composite anode for Li-ion batteries," Y. Ding, Z. Li, E.V. Timofeeva, and C.U. Segre, *Adv. Energy Mater.* 1702134 (2018).

. . . and a little more

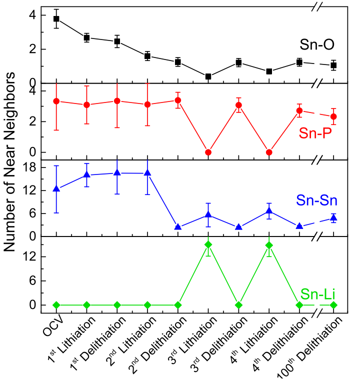
EXAFS studies of battery materials

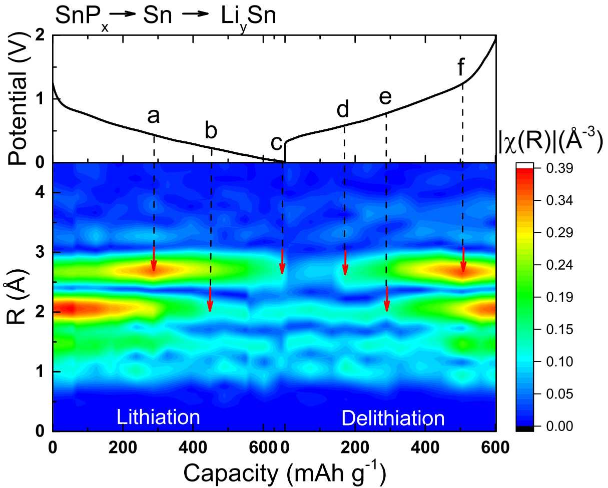


"In situ EXAFS-derived mechanism of highly reversible tin phosphide/graphite composite anode for Li-ion batteries," Y. Ding, Z. Li, E.V. Timofeeva, and C.U. Segre, *Adv. Energy Mater.* 1702134 (2018).

...and a little more

EXAFS studies of battery materials





"In situ EXAFS-derived mechanism of highly reversible tin phosphide/graphite composite anode for Li-ion batteries," Y. Ding, Z. Li, E.V. Timofeeva, and C.U. Segre, *Adv. Energy Mater.* 1702134 (2018).

The company . . . Influit Energy, LLC