

Today's outline - March 08, 2022



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- Exam #1 solutions

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 - Average: 63.4%

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- Introduction to Quirk

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Reading assignment: 9.1 – 9.2

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Homework Assignment #06:
TBA on Thursday, March 10

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Reading assignment: 9.1 – 9.2

Homework Assignment #06:
TBA on Thursday, March 10

Quantum circuit simulator <https://algassert.com/quirk>

Introduction to Quirk



Menu

Export

Clear Circuit

Clear ALL

Undo

Redo

Make Gate

Version 2.3

	Probes	Displays	Half Turns	Quarter Turns	Eighth Turns	Spinning	Formulaic	Parametrized	Sampling	Parity
Toolbox			Z Swap	S S ⁻¹	T T ⁻¹	Z ^t Z ^{-t}	Z ^{f(t)} Rz(f(t))	Z ^{A/2ⁿ} Z ^{-A/2ⁿ}	Z	
	$ 0\rangle\langle 0 $ $ 1\rangle\langle 1 $	Density Bloch	Y	Y ^{1/2} Y ^{-1/2}	Y ^{1/4} Y ^{-1/4}	Y ^t Y ^{-t}	Y ^{f(t)} Ry(f(t))	Y ^{A/2ⁿ} Y ^{-A/2ⁿ}	Y	$\begin{bmatrix} Z \\ \text{par} \end{bmatrix}$
		Chance Amps		X ^{1/2} X ^{-1/2}	X ^{1/4} X ^{-1/4}	X ^t X ^{-t}	X ^{f(t)} Rx(f(t))	X ^{A/2ⁿ} X ^{-A/2ⁿ}	X	$\begin{bmatrix} Y \\ \text{par} \end{bmatrix}$ $\begin{bmatrix} X \\ \text{par} \end{bmatrix}$

$|0\rangle$ — use controls

$|0\rangle$ — drag gates onto circuit

— outputs change

Local wire states
(Chance/Bloch)

Final amplitudes

	X/Y Probes	Order	Frequency	Inputs	Arithmetic	Compare	Modular	Scalar	Custom Gates
Toolbox ₂	\ominus \oplus	$+ t $ $- t $	QFT QFT [†]	input A A=# default	+1 -1	$\oplus A < B$ $\oplus A > B$	+1 mod R -1 mod R	...	0
	\otimes \otimes	Reverse		input B B=# default	+A -A	$\oplus A \leq B$ $\oplus A \geq B$	+A mod R -A mod R	-	
	$ +\rangle\langle + $ $ -\rangle\langle - $		Grad ^{1/2} Grad ^{-1/2}	input R R=# default	+AB -AB	$\oplus A = B$ $\oplus A \neq B$	$\times A$ mod R $\times A^{-1}$ mod R	i -i	
	$ i\rangle\langle i $ $ -i\rangle\langle -i $		Grad ^t Grad ^{-t}		$\times A$ $\times A^{-1}$		$\times B^A$ mod R $\times B^{-A}$ mod R	\sqrt{i} $\sqrt{-i}$	

Quirk circuit for teleportation



Given an entangled state $|\psi_0\rangle$ and a message qubit $|\phi\rangle$ we have a 3-qubit state $|\phi\rangle \otimes |\psi_0\rangle$

Quirk circuit for teleportation



Given an entangled state $|\psi_0\rangle$ and a message qubit $|\phi\rangle$ we have a 3-qubit state $|\phi\rangle \otimes |\psi_0\rangle$

Alice applies $C_{not} \otimes I$ followed by $H \otimes I \otimes I$

Quirk circuit for teleportation



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Alice then measures her two qubits and transmits the result to Bob

Quirk circuit for teleportation



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Bob applies a transformation depending on the 2 bits received

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00 \longrightarrow I

Bob applies a transformation depending on the 2 bits received

01 \longrightarrow X

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Bob applies a transformation depending on the 2 bits received

00	→	I
01	→	X
10	→	Z

Quirk circuit for teleportation



Given an entangled state $|\psi_0\rangle$ and a message qubit $|\phi\rangle$ we have a 3-qubit state $|\phi\rangle \otimes |\psi_0\rangle$

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Bob applies a transformation depending on the 2 bits received

00	→	I
01	→	X
10	→	Z
11	→	Y

Quirk circuit for teleportation



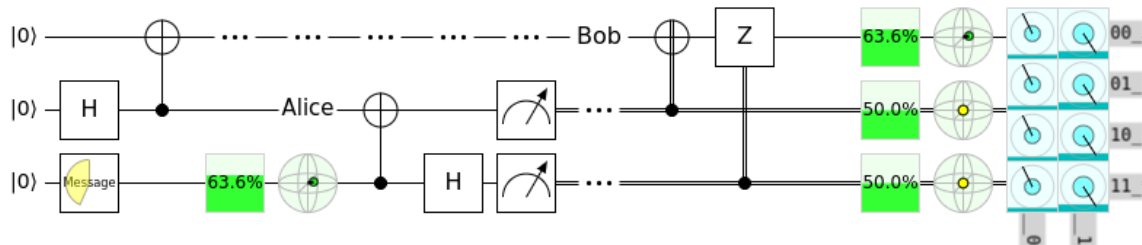
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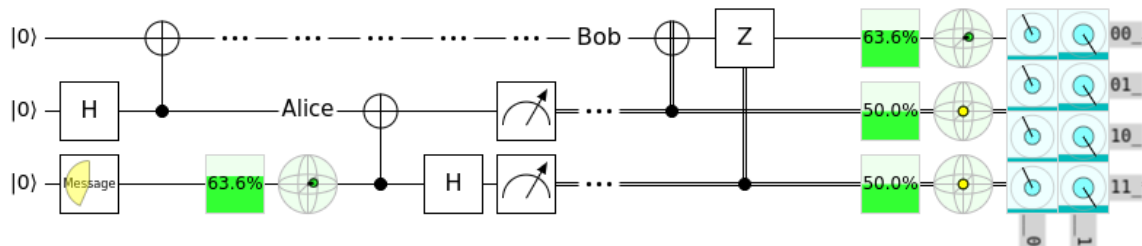
Alice applies $C_{not} \otimes I$ followed by $H \otimes I \otimes I$

Alice then measures her two qubits and transmits the result to Bob

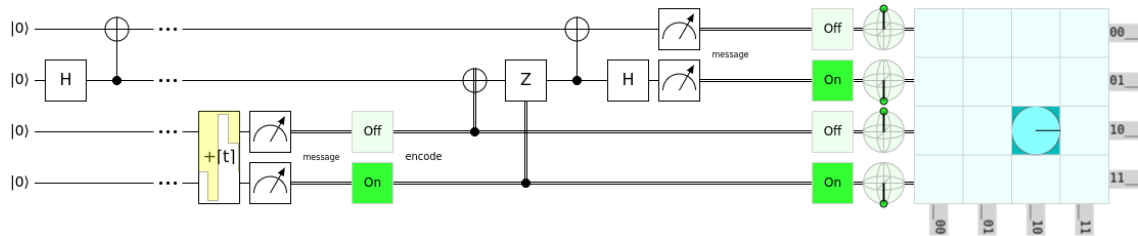
Bob applies a transformation depending on the 2 bits received

The two controlled gates do exactly this since neither gate is applied for 00, only X for 01, only Z for 10 and both $ZX = Y$ for 11

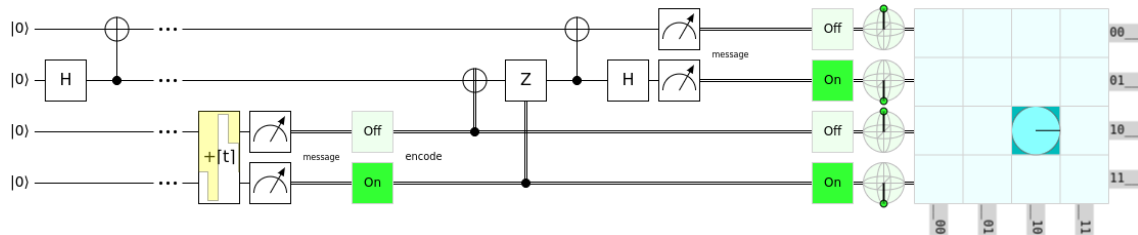
00	→	I
01	→	X
10	→	Z
11	→	Y



Quirk circuit for dense-coding

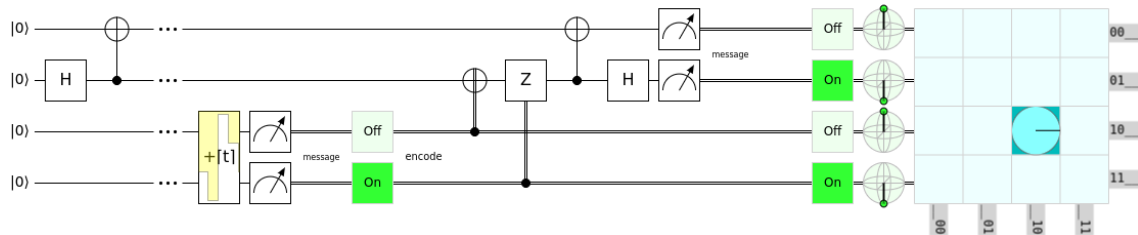


Quirk circuit for dense-coding



The the two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

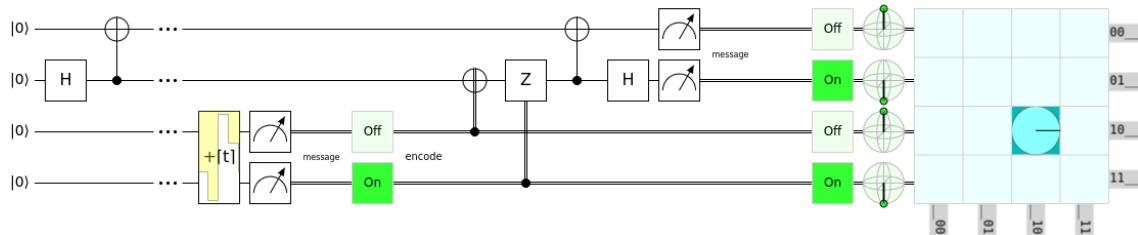
Quirk circuit for dense-coding



The the two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Alice (2^{nd} row) encodes the message into her entangled qubit and then sends it to Bob

Quirk circuit for dense-coding

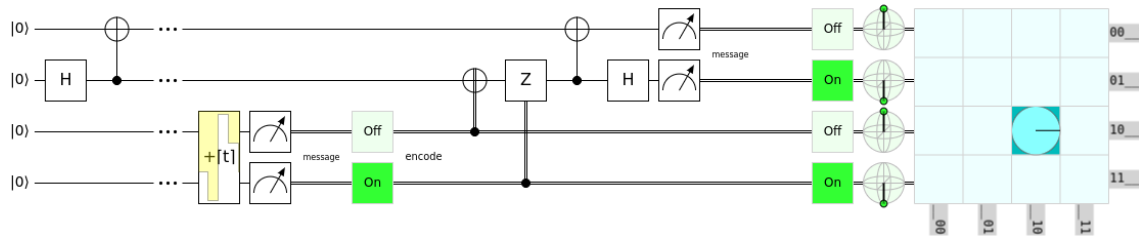


The the two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Alice (2^{nd} row) encodes the message into her entangled qubit and then sends it to Bob

Bob (1^{st} row) applies a $C_{not}^{10}(h \otimes I)$ to the entangled qubits to recover the message

Quirk circuit for dense-coding



The the two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Alice (2^{nd} row) encodes the message into her entangled qubit and then sends it to Bob

Bob (1^{st} row) applies a $C_{not}^{10}(h \otimes I)$ to the entangled qubits to recover the message

Bob and Alice have swapped transformations from the ones used in teleportation

Concluding remarks on Shor's algorithm



The measurement of the second register to obtain $|u\rangle$ can be omitted in the implementation of Shor's algorithm

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Consider the state after the U_f transformation has been applied

Concluding remarks on Shor's algorithm



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Consider the state after the U_f transformation has been applied

$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle$$

Concluding remarks on Shor's algorithm



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Consider the state after the U_f transformation has been applied

$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle$$

This can be separated into sums over all the periodicities u and all the states $x \in X_u$ where $x_u = \{x | f(x) = u\}$

Concluding remarks on Shor's algorithm



The measurement of the second register to obtain $|u\rangle$ can be omitted in the implementation of Shor's algorithm

Consider the state after the U_f transformation has been applied

$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle = \frac{1}{\sqrt{N}} \sum_{u \in R} \sum_{x \in X_u} |x\rangle |u\rangle$$

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Concluding remarks on Shor's algorithm



The measurement of the second register to obtain $|u\rangle$ can be omitted in the implementation of Shor's algorithm

Consider the state after the U_f transformation has been applied

This can be separated into sums over all the periodicities u and all the states $x \in X_u$ where $x_u = \{x | f(x) = u\}$

The amplitudes in states with different u can never interfere with each other so the measurement of $|u\rangle$ is unnecessary

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$$U_F \otimes I \left[\frac{1}{\sqrt{N}} \sum_{u \in R} \left(\sum_{x=0}^{N-1} g_u(x) |x\rangle \right) |u\rangle \right]$$

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$$U_F \otimes I \left[\frac{1}{\sqrt{N}} \sum_{u \in R} \left(\sum_{x=0}^{N-1} g_u(x) |x\rangle \right) |u\rangle \right] = \frac{1}{\sqrt{N}} \sum_{u \in R} \left(U_F \sum_{x=0}^{N-1} g_u(x) |x\rangle \right) |u\rangle$$

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Measuring the first part of this state will return a c close to a multiple of $2^n/r$