Today's outline - March 08, 2022

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- Exam #1 solutions

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Reading assignment: 9.1 – 9.2

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Homework Assignment #06:
TBA on Thursday, March 10

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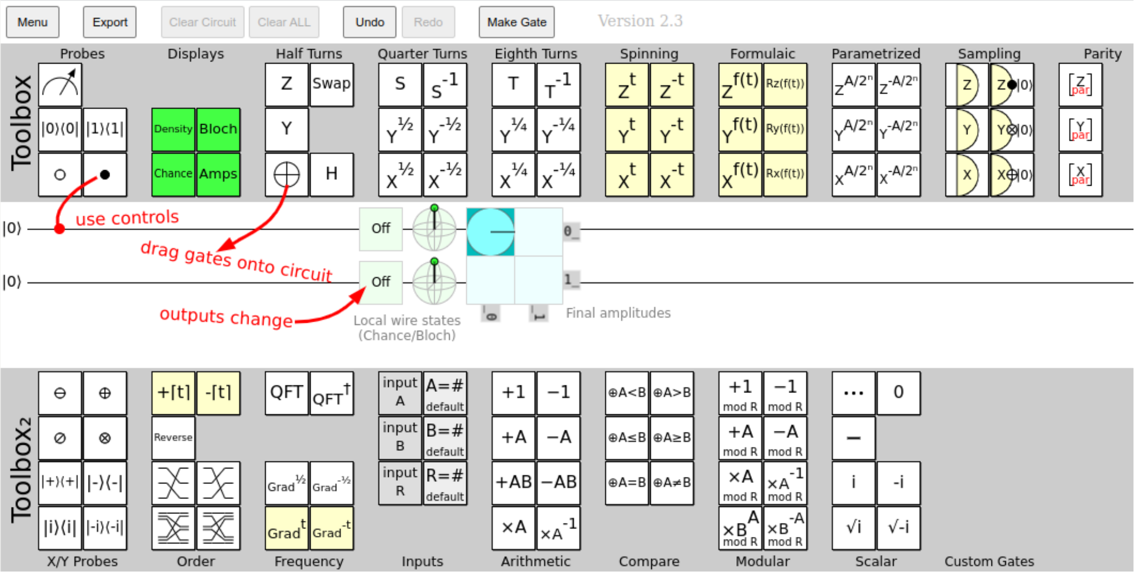
Homework Assignment #06:
TBA on Thursday, March 10

Quantum circuit simulator https://algassert.com/quirk

Introduction to Quirk



Quirk circuit for teleportation

Given an entangled state $|\psi_0\rangle$ and a message qubit $|\phi\rangle$ we have a 3-qubit state $|\phi\rangle \otimes |\psi_0\rangle$

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Alice applies $C_{not} \otimes I$ followed by $H \otimes I \otimes I$

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Bob applies a transformation depending on the 2 bits received

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$$00 \longrightarrow I$$

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$$\begin{array}{ccc} 00 & \longrightarrow & I \\ 01 & \longrightarrow & X \end{array}$$

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$$
\begin{aligned}
00 &\longrightarrow I \\
01 &\longrightarrow X \\
10 &\longrightarrow Z
\end{aligned}
$$

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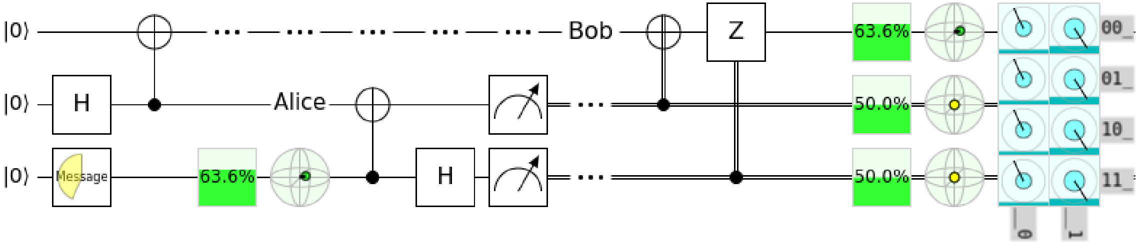

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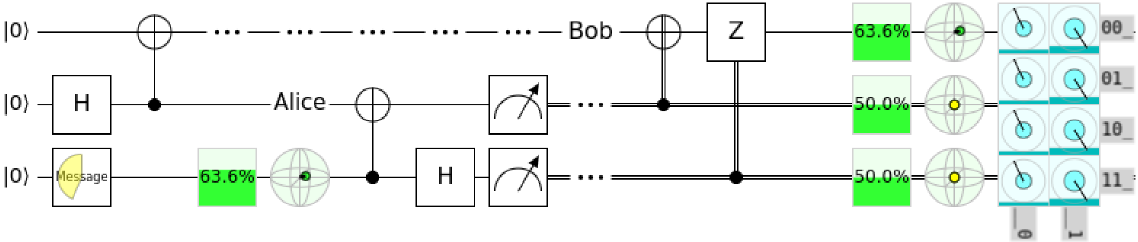
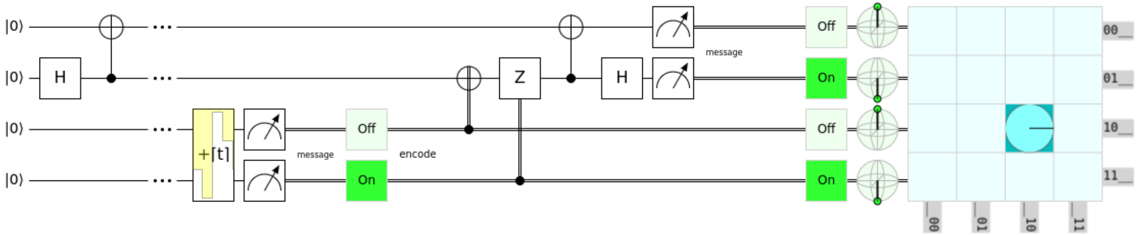
The two controlled gates do exactly this since neither gate is applied for 00, only X for 01, only Z for 10 and both $ZX = Y$ for 11

$$\begin{aligned} 00 &\longrightarrow I \\ 01 &\longrightarrow X \\ 10 &\longrightarrow Z \\ 11 &\longrightarrow Y \end{aligned}$$



Quirk circuit for dense-coding



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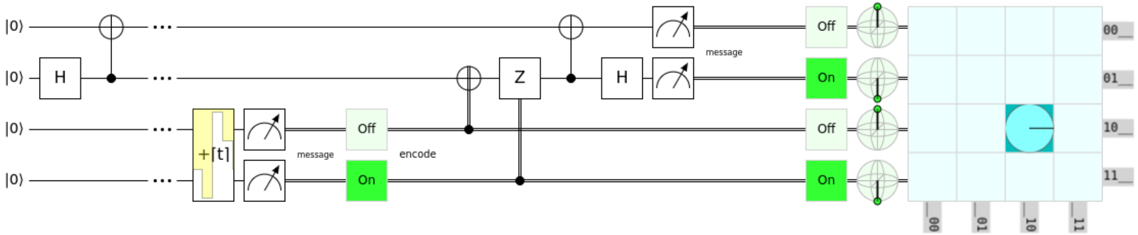
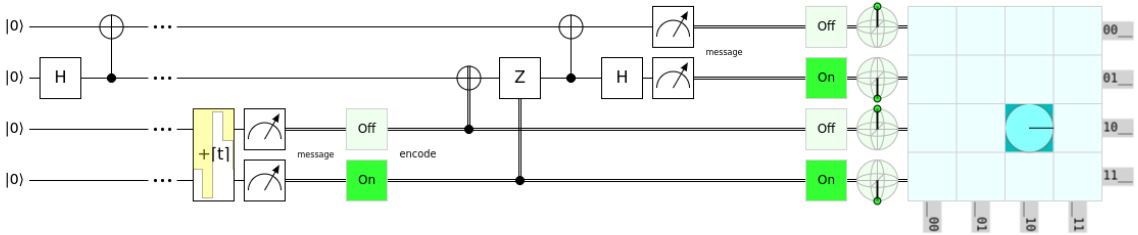


The the two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Quirk circuit for dense-coding



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Alice (2nd row) encodes the message into her entangled qubit and then sends it to Bob

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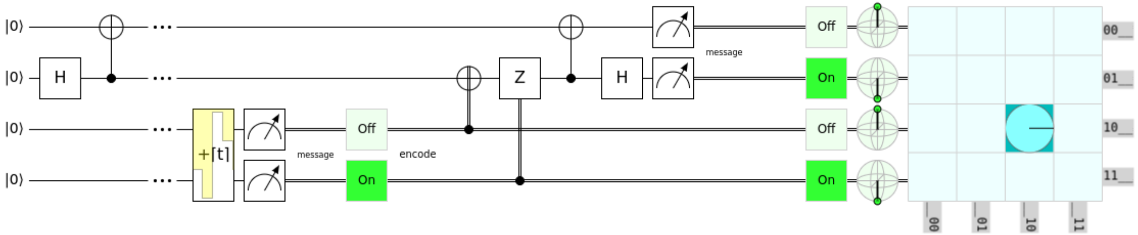

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Alice (2^{nd} row) encodes the message into her entangled qubit and then sends it to Bob

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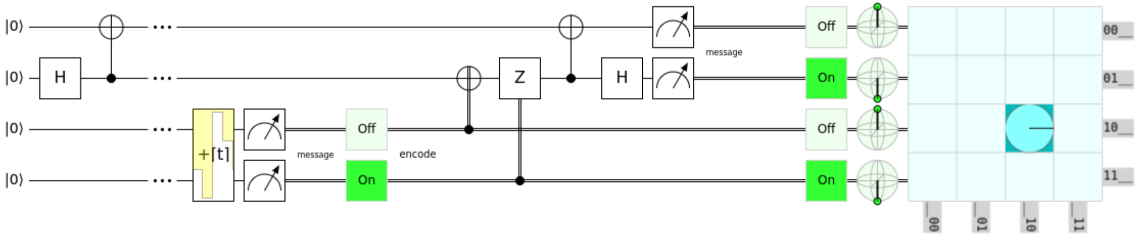

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Bob and Alice have swapped transformations from the ones used in teleportation

Concluding remarks on Shor's algorithm

The measurement of the second register to obtain $|u\rangle$ can be omitted in the implementation of Shor's algorithm

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$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle$$

This can be separated into sums over all the periodicities u and all the states $x \in X_u$ where $x_u = \{x|f(x) = u\}$

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$$U_F \otimes I \left[\frac{1}{\sqrt{N}}\sum_{u\in R}\left(\sum_{x=0}^{N-1} g_u(x)|x\rangle\right)|u\rangle\right]$$

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Measuring the first part of this state will return a c close to a multiple of $2^n/r$