

Today's outline - March 08, 2022





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- Exam #1 solutions



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- Exam #1 solutions
 - Average: 63.4%



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 - High: 94%



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- Introduction to Quirk

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Reading assignment: 9.1 – 9.2



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Homework Assignment #06:
TBA on Thursday, March 10



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- Example Quirk circuits

Reading assignment: 9.1 – 9.2

Homework Assignment #06:
TBA on Thursday, March 10

Quantum circuit simulator <https://algassert.com/quirk>

Introduction to Quirk



Menu Export Clear Circuit Clear ALL Undo Redo Make Gate Version 2.3

Toolbox

Probes		Displays		Half Turns		Quarter Turns		Eighth Turns		Spinning		Formulaic		Parametrized		Sampling		Parity
$ 0\rangle\langle 0 $	$ 1\rangle\langle 1 $	Density	Bloch	Z	Swap	S	S^{-1}	T	T^{-1}	z^t	z^{-t}	$f(t)$	$Rz(f(t))$	$z^{A/2^n}$	$z^{-A/2^n}$	Z	Y	$[Z]$
\circ	\bullet	Chance	Amps	Y		$Y^{1/2}$	$Y^{-1/2}$	$Y^{1/4}$	$Y^{-1/4}$	y^t	y^{-t}	$f(t)$	$Ry(f(t))$	$Y^{A/2^n}$	$Y^{-A/2^n}$	Y	$[Y]$	$[Z]$
\oplus				H		$X^{1/2}$	$X^{-1/2}$	$X^{1/4}$	$X^{-1/4}$	x^t	x^{-t}	$X(f(t))$	$Rx(f(t))$	$x^{A/2^n}$	$x^{-A/2^n}$	X	$[X]$	$[X]$

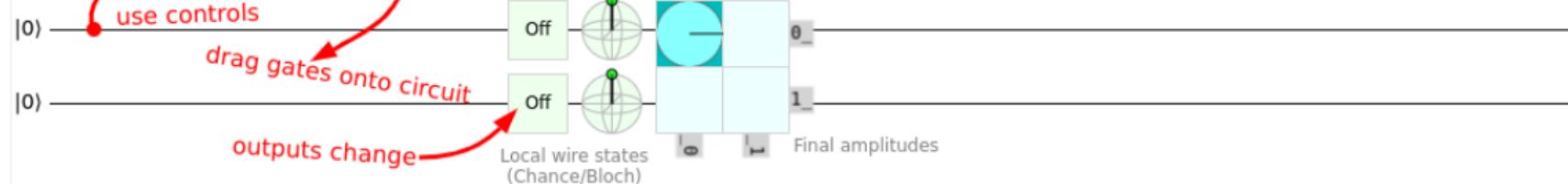
use controls

drag gates onto circuit

outputs change

Local wire states (Chance/Bloch)

Final amplitudes



Toolbox₂

\ominus		\oplus		$+[t]$		$-[t]$		QFT		QFT^\dagger		input A		$A = \#$ default		$+1$		-1		$\oplus A < B$		$\oplus A > B$		$+1$ mod R		-1 mod R		...		0		
\emptyset	\otimes											$A = \#$ default	$B = \#$ default	$R = \#$ default	$+A$	$-A$	$\oplus A \leq B$	$\oplus A \geq B$	$\oplus A = B$	$\oplus A \neq B$	$+A$	$-A$	$\times A$	$\times A^{-1}$	$\times A$ mod R	$\times A^{-1}$ mod R	$\times B$ mod R	$\times B^{-1}$ mod R	i	$-i$	\sqrt{i}	$\sqrt{-i}$
$ +\rangle\langle + $	$ -\rangle\langle - $																															
$ i\rangle\langle i $	$ i\rangle\langle -i $																															

X/Y Probes Order Frequency Inputs Arithmetic Compare Modular Scalar Custom Gates



Quirk circuit for teleportation

Given an entangled state $|\psi_0\rangle$ and a message qubit $|\phi\rangle$ we have a 3-qubit state $|\phi\rangle \otimes |\psi_0\rangle$



Quirk circuit for teleportation

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Alice applies $C_{not} \otimes I$ followed by $H \otimes I \otimes I$



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Alice then measures her two qubits and transmits the result to Bob



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Bob applies a transformation depending on the 2 bits received



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00 \longrightarrow /

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00 \longrightarrow I

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01 \longrightarrow X



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00	\longrightarrow	I
01	\longrightarrow	X
10	\longrightarrow	Z



Quirk circuit for teleportation

Given an entangled state $|\psi_0\rangle$ and a message qubit $|\phi\rangle$ we have a 3-qubit state $|\phi\rangle \otimes |\psi_0\rangle$

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Bob applies a transformation depending on the 2 bits received

00	→	I
01	→	X
10	→	Z
11	→	Y

Quirk circuit for teleportation

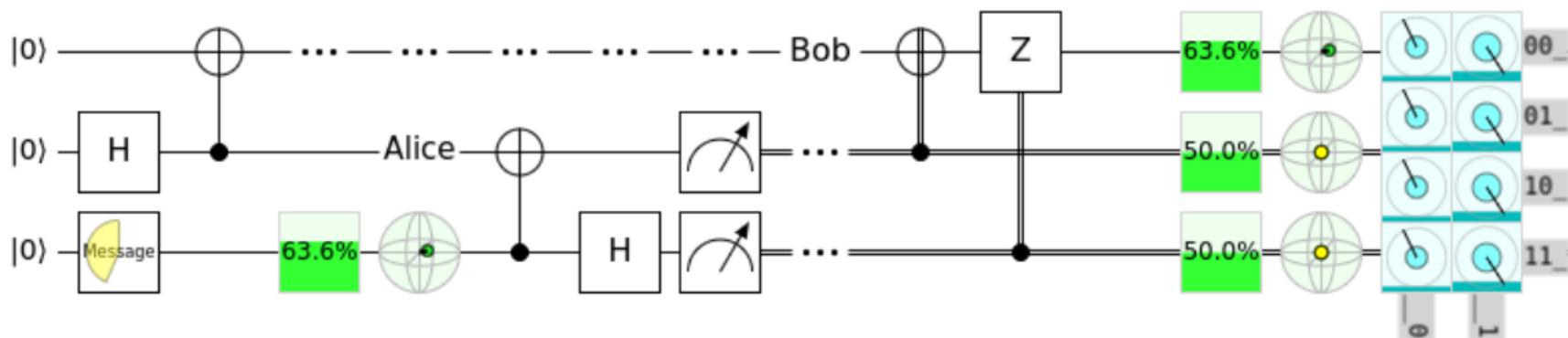
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00	\rightarrow	I
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Quirk circuit for teleportation

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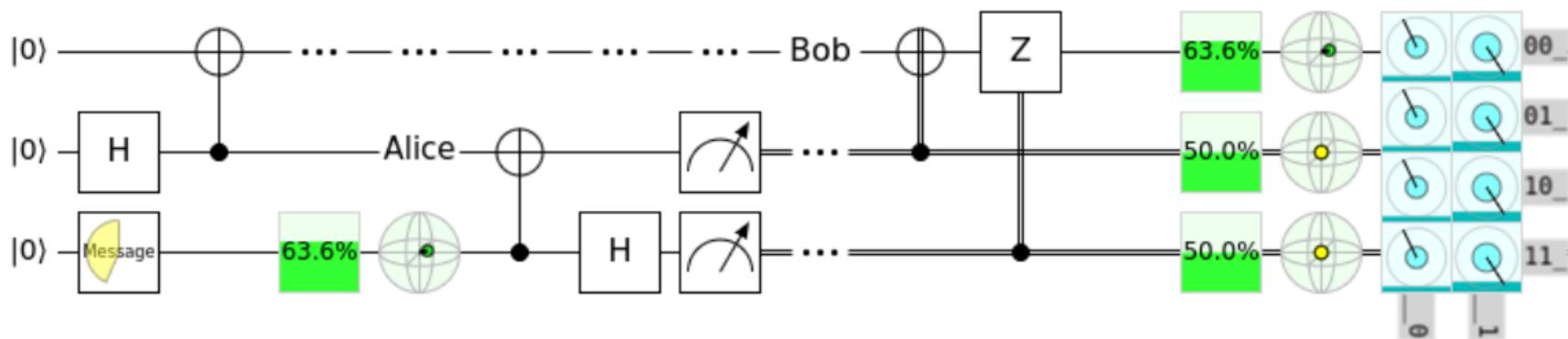
Alice applies $C_{not} \otimes I$ followed by $H \otimes I \otimes I$

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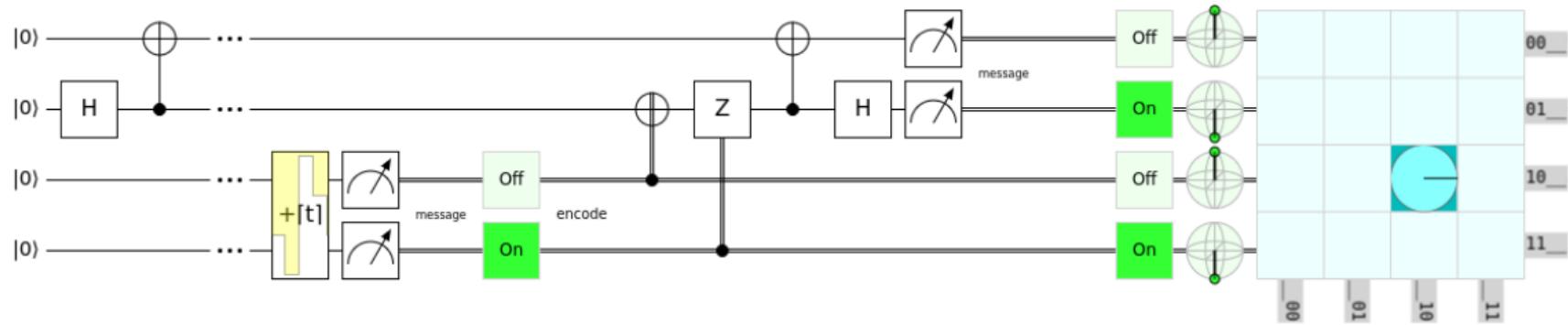
Bob applies a transformation depending on the 2 bits received

The two controlled gates do exactly this since neither gate is applied for 00, only X for 01, only Z for 10 and both $ZX = Y$ for 11

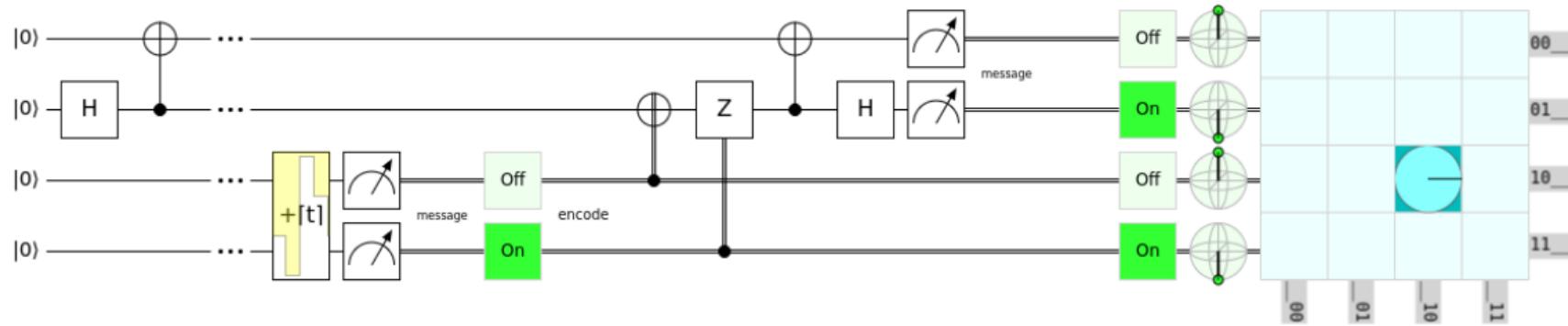
00	\rightarrow	I
01	\rightarrow	X
10	\rightarrow	Z
11	\rightarrow	Y



Quirk circuit for dense-coding

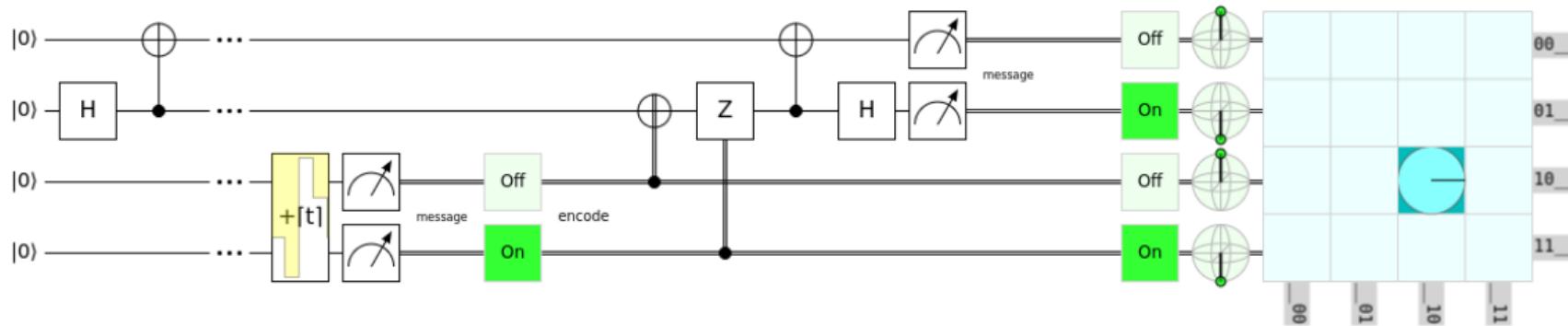


Quirk circuit for dense-coding



The two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

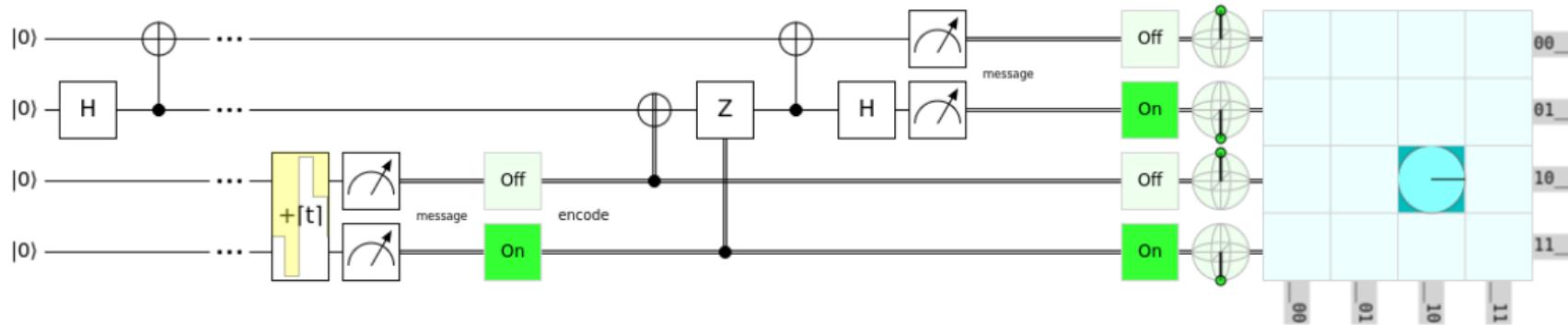
Quirk circuit for dense-coding



The two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Alice (2nd row) encodes the message into her entangled qubit and then sends it to Bob

Quirk circuit for dense-coding

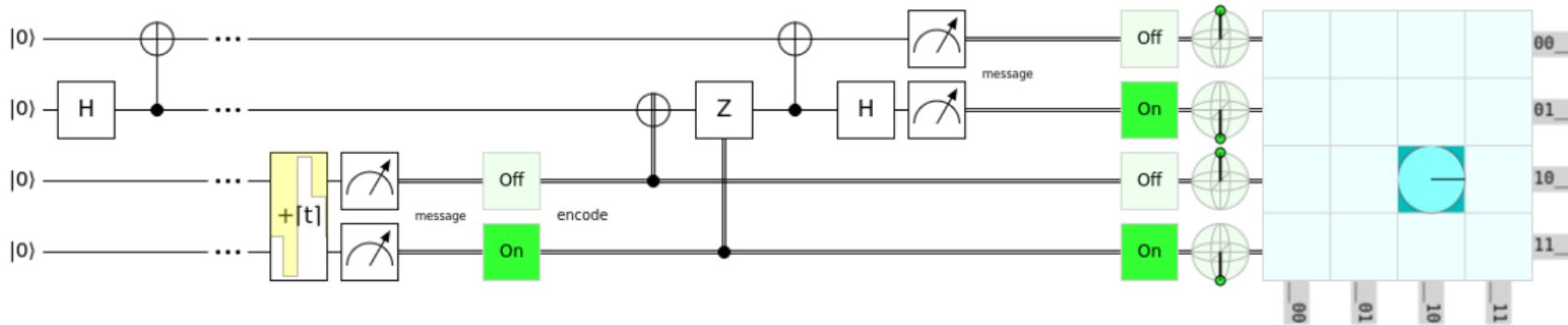


The two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Alice (2^{nd} row) encodes the message into her entangled qubit and then sends it to Bob

Bob (1^{st} row) applies a $C_{not}^{10}(h \otimes I)$ to the entangled qubits to recover the message

Quirk circuit for dense-coding



The two high order qubits are continuously incremented through its 4 states and is measured to make the classical message

Alice (2^{nd} row) encodes the message into her entangled qubit and then sends it to Bob

Bob (1^{st} row) applies a $C_{not}^{10}(h \otimes I)$ to the entangled qubits to recover the message

Bob and Alice have swapped transformations from the ones used in teleportation



Concluding remarks on Shor's algorithm

The measurement of the second register to obtain $|u\rangle$ can be omitted in the implementation of Shor's algorithm



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Consider the state after the U_f transformation has been applied

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Consider the state after the U_f transformation has been applied

$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle$$

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Consider the state after the U_f transformation has been applied

$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle$$

This can be separated into sums over all the periodicities u and all the states $x \in X_u$ where $X_u = \{x | f(x) = u\}$

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$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle |f(x)\rangle = \frac{1}{\sqrt{N}} \sum_{u \in R} \sum_{x \in X_u} |x\rangle |u\rangle$$



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Consider the state after the U_f transformation has been applied

This can be separated into sums over all the periodicities u and all the states $x \in X_u$ where $X_u = \{x | f(x) = u\}$

The amplitudes in states with different u can never interfere with each other so the measurement of $|u\rangle$ is unnecessary

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Measuring the first part of this state will return a c close to a multiple of $2^n/r$