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Reading Assignment: Chapter 7.7-7.8

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Homework Assignment #05:
Chapter 7:1,3,4
due Thursday, February 24, 2022

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For each vector $|i\rangle$ in the sum that makes up $|\psi\rangle$, the Walsh transform applies a sign change depending on the number of common 1 bits between $|i\rangle$ and $|j\rangle$

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This solves the Deutsch-Jozsa problem with a single call to U_f which is exponentially better than the classical solution

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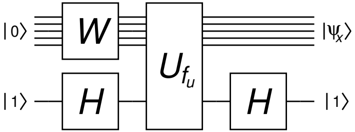

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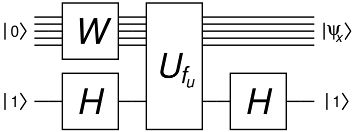

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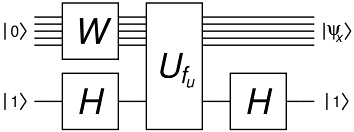

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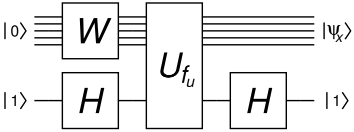

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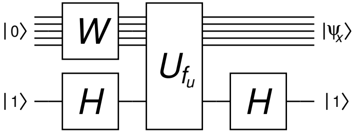

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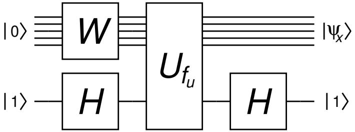

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But from the discussion of the Hanning distance, we have that

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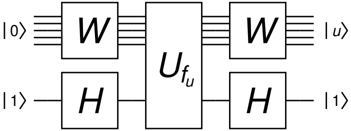

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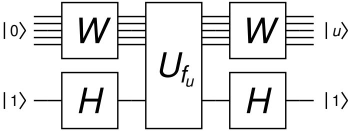

This illustrates a common interpretation of how quantum circuits work, that is using parallelism to perform a computation on all possible inputs then manipulate the resulting superposition to get the result

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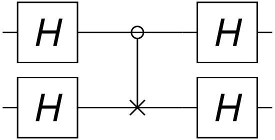

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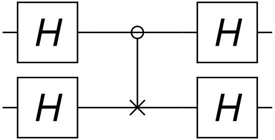

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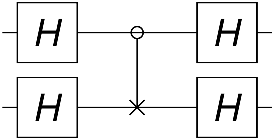

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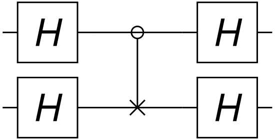

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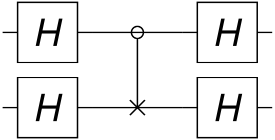

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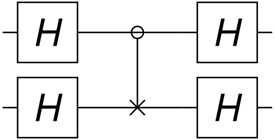

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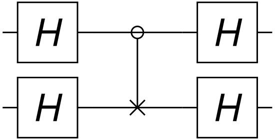

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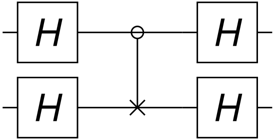

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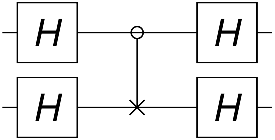

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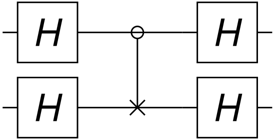

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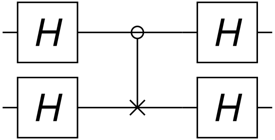

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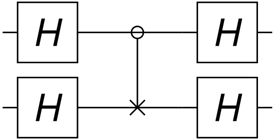

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Initial Final

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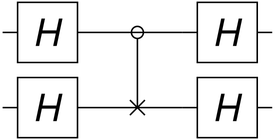

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Initial			Final	
0	0	$\longrightarrow$	0	0

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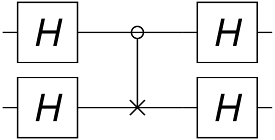

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0	0	$\longrightarrow$	0	0
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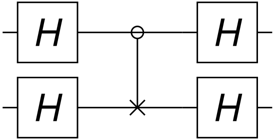

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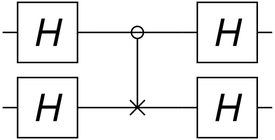

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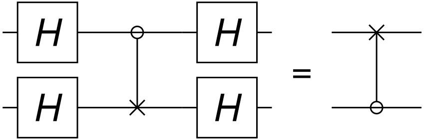

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Initial			Final	
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This is simply a C_{not} gate applied to the first qubit controlled by the second

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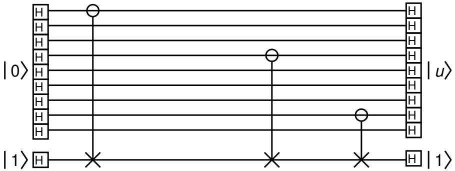

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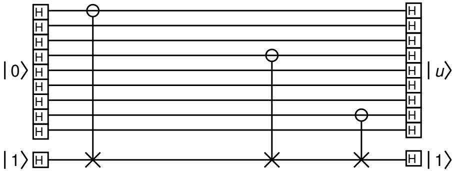

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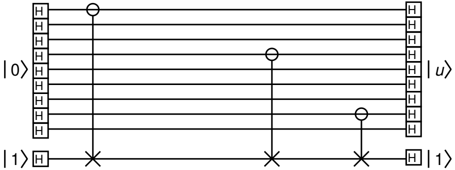

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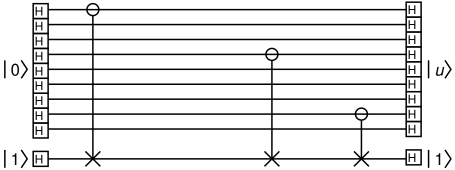

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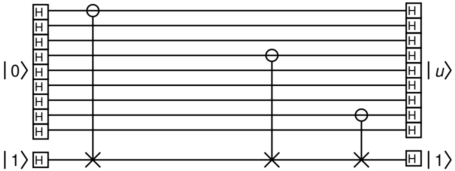

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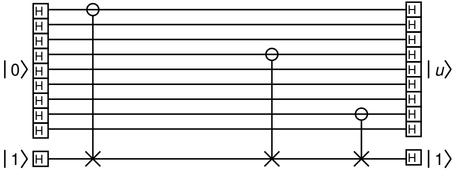

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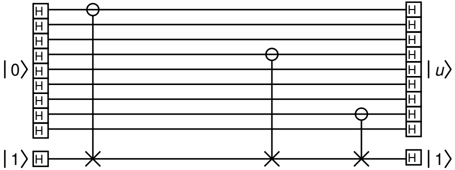

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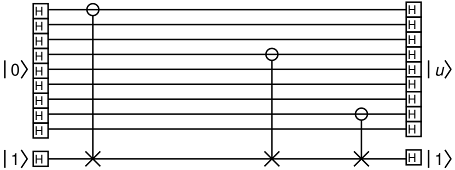

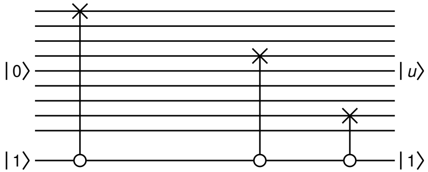

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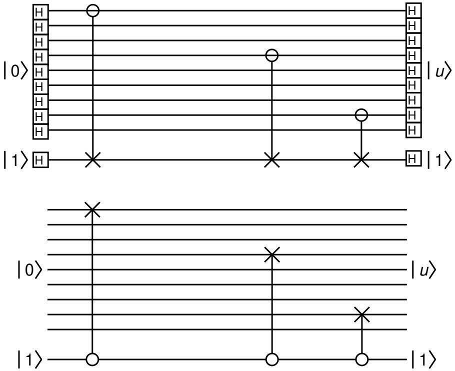

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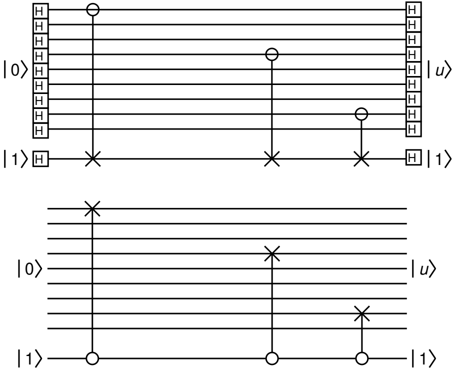

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Of course, this presupposes that one knows what $|u\rangle$ is so we are peering into the black box

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In this case, we can see that $a = 010 \oplus 111 = 101$ and this holds for all matched pairs in the table

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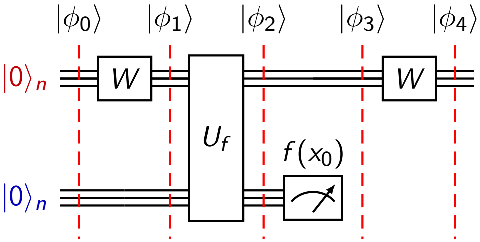

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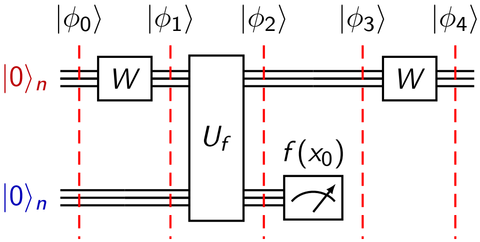

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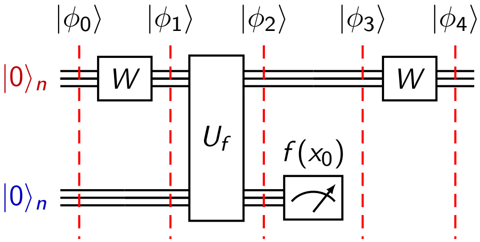

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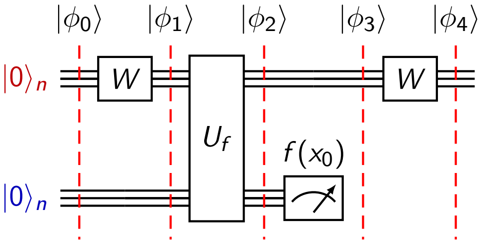

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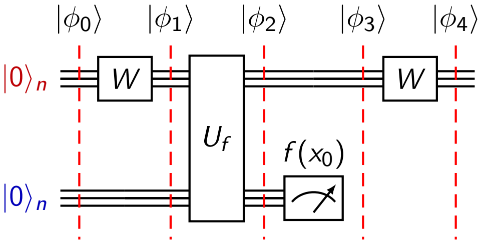

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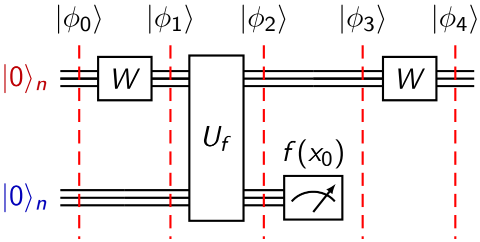

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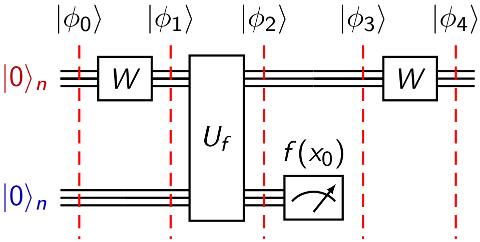

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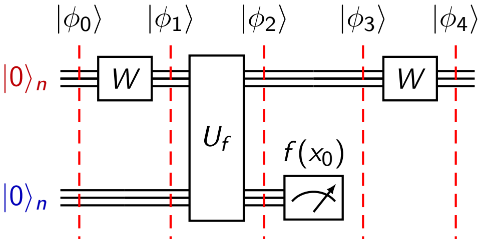

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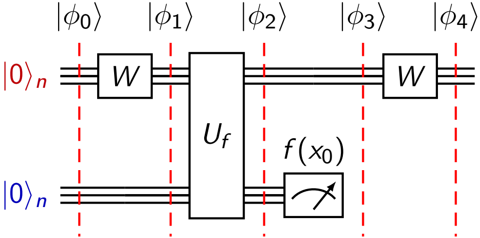

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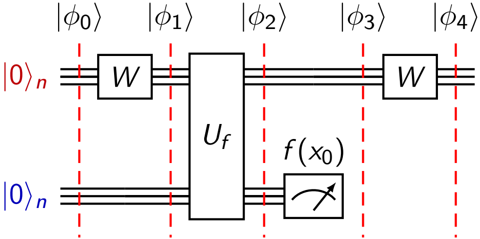

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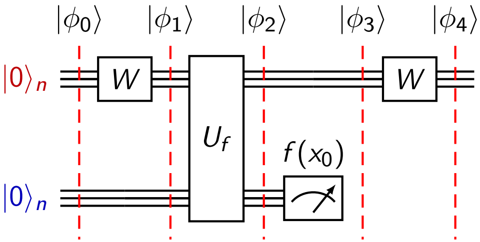

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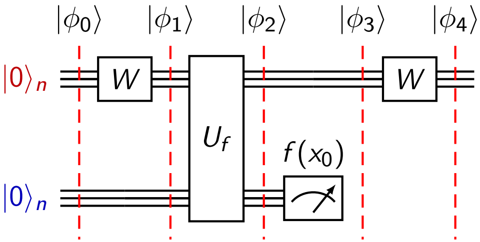

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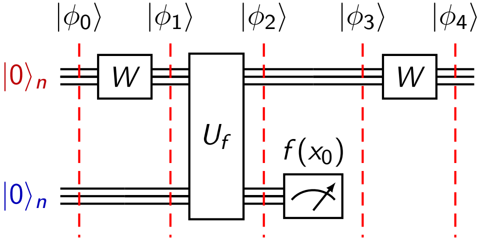

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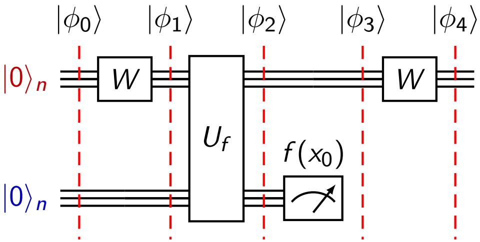

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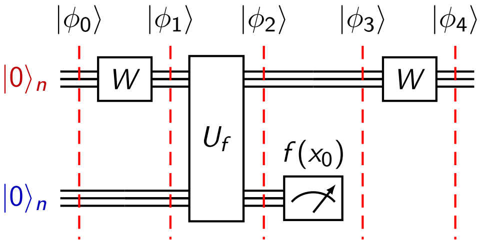

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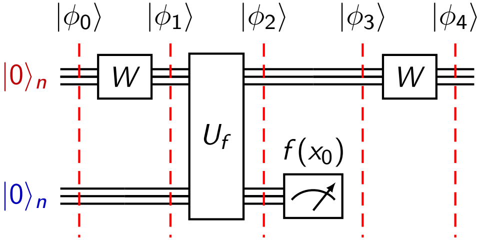

There are two cases to consider for the modulo 2 scalar product $a \cdot y$

$$y \cdot a \neq 0 \quad \longrightarrow \quad |\phi_4\rangle \equiv 0$$

The second case is for $a \cdot y = 0$, in which case

$$|\phi_4\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{y=0}^{2^n-1} (-1)^{x_0 \cdot y} \left[1 + 1\right] |y\rangle = \frac{1}{\sqrt{2^{n-1}}} \sum_{y=0}^{2^n-1} (-1)^{x_0 \cdot y} |y\rangle$$

Simon's algorithm – quantum circuit

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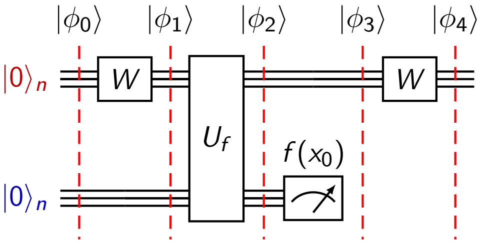

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This is a superposition of 2^n possible states, one of which will be observed when $|\phi_4\rangle$ is measured

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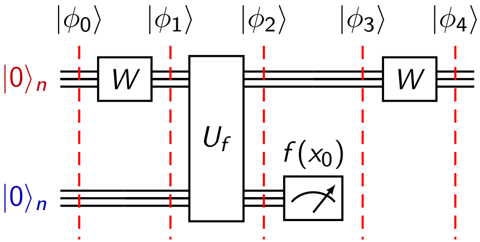

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Simon's algorithm – example

Suppose a system with $n = 4$ and $a = 1001$, $f(x)$ has the truth table

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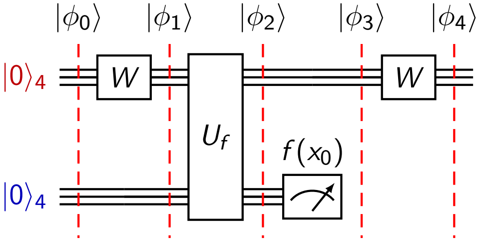

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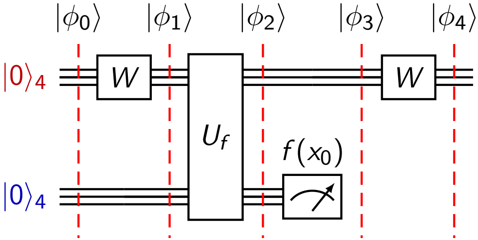

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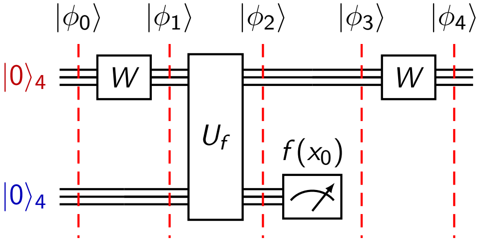

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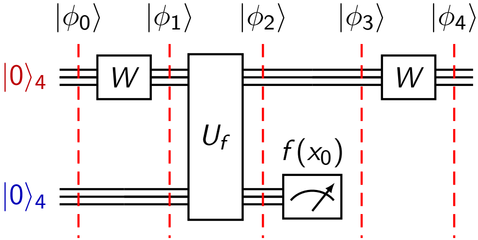

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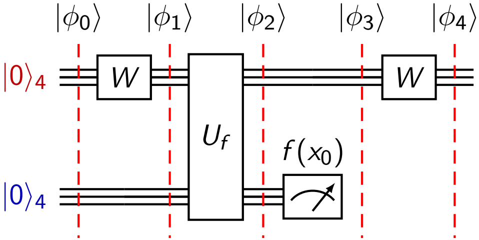

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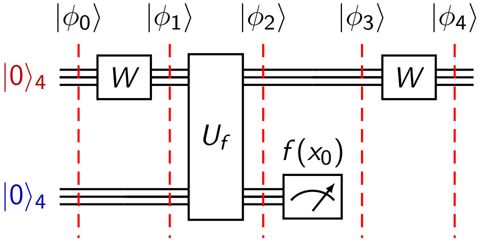

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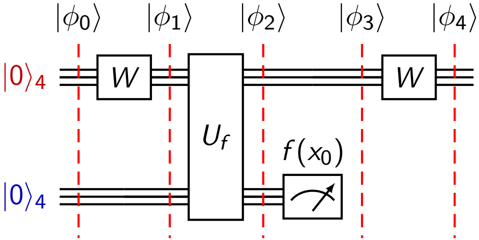

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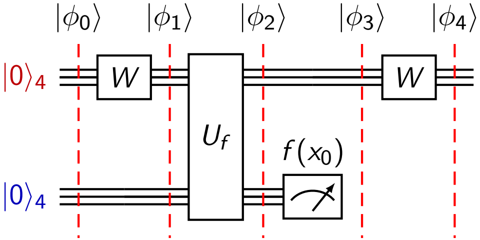

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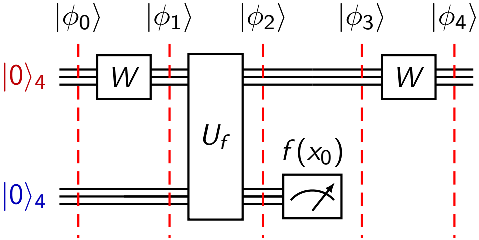

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now apply the Walsh transformation

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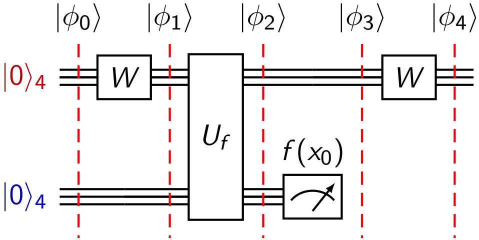

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$$|\phi_4\rangle = \frac{[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle]}{\sqrt{8}}$$

Simon's algorithm – example

Suppose a system with $n=4$ and $a=1001$, $f(x)$ has the truth table

x	$f(x)$
0000	1111
0001	0001
0010	1110
0011	1101
0100	0000
0101	0101
0110	1010
0111	1001
1000	0001
1001	1111
1010	1101
1011	1110
1100	0101
1101	0000
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1111	1010

$$|\phi_0\rangle = |0\rangle|0\rangle = |0000\rangle|0000\rangle$$

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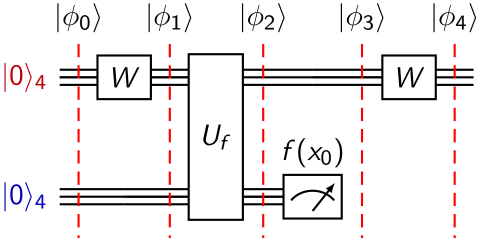

For example, suppose $f(x_0) = 1010$

$$|\phi_3\rangle = \frac{[|0110\rangle + |1111\rangle]}{\sqrt{2}}\cancel{|f(x_0)\rangle}$$ now apply the Walsh transformation

$$|\phi_4\rangle = \frac{[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle]}{\sqrt{8}}$$

Note that any value of $|f(x_0)\rangle$ measured will result in these 8 $|x_0\rangle$

Simon's algorithm – example

$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

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$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

The result of the final measurement, $|y\rangle$ will be one of these eight values and each of them should satisfy the linear equation $a \cdot y = 0$

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Since we know that $a = |1001\rangle$ for this example, we can check this identity

Simon's algorithm – example

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$$|1001\rangle \cdot |0000\rangle = 1{\cdot}0 + 0{\cdot}0 + 0{\cdot}0 + 1{\cdot}0 = 0$$

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$$|1001\rangle \cdot |0000\rangle = 1{\cdot}0 + 0{\cdot}0 + 0{\cdot}0 + 1{\cdot}0 = 0$$

$$|1001\rangle \cdot |1001\rangle = 1{\cdot}1 + 0{\cdot}0 + 0{\cdot}0 + 1{\cdot}1 = 2$$

Simon's algorithm – example

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and the other 6 have the same properties

It is now necessary to collect $n - 1 = 3$ independent values of $|y\rangle$ to solve for a

Simon's algorithm – example

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Trial	$\lvert y\rangle$	Indep.?

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$$|1001\rangle \cdot |0000\rangle = 1\cdot 0 + 0\cdot 0 + 0\cdot 0 + 1\cdot 0 = 0$$

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Trial	$\vert y\rangle$	Indep.?
1	$\vert 0000\rangle$	No

Simon's algorithm – example

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$$|1001\rangle \cdot |1001\rangle = 1\cdot 1 + 0\cdot 0 + 0\cdot 0 + 1\cdot 1 = 2 = 0$$

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Trial	$\|y\rangle$	Indep.?
1	$\|0000\rangle$	No
1	$\|0010\rangle$	Yes

Simon's algorithm – example

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$$|1001\rangle \cdot |1001\rangle = 1\cdot 1 + 0\cdot 0 + 0\cdot 0 + 1\cdot 1 = 2 = 0$$

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Trial	$\|y\rangle$	Indep.?
1	$\|0000\rangle$	No
1	$\|0010\rangle$	Yes
1	$\|0100\rangle$	Yes

Simon's algorithm – example

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$$|1001\rangle \cdot |1001\rangle = 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 = 2 = 0$$

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Trial	$\lvert y\rangle$	Indep.?
1	$\lvert 0000\rangle$	No
1	$\lvert 0010\rangle$	Yes
1	$\lvert 0100\rangle$	Yes
1	$\lvert 0110\rangle$	No

Simon's algorithm – example

$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

The result of the final measurement, $|y\rangle$ will be one of these eight values and each of them should satisfy the linear equation $a \cdot y = 0$

Since we know that $a = |1001\rangle$ for this example, we can check this identity

$$|1001\rangle \cdot |0000\rangle = 1\cdot 0 + 0\cdot 0 + 0\cdot 0 + 1\cdot 0 = 0$$

$$|1001\rangle \cdot |1001\rangle = 1\cdot 1 + 0\cdot 0 + 0\cdot 0 + 1\cdot 1 = 2 = 0$$

and the other 6 have the same properties

It is now necessary to collect $n - 1 = 3$ independent values of $|y\rangle$ to solve for a

Trial	$\|y\rangle$	Indep.?
1	$\|0000\rangle$	No
1	$\|0010\rangle$	Yes
1	$\|0100\rangle$	Yes
1	$\|0110\rangle$	No
1	$\|1001\rangle$	Yes

Simon's algorithm – example

$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

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1	$\lvert 0100\rangle$	Yes
1	$\lvert 0110\rangle$	No
1	$\lvert 1001\rangle$	Yes

Create a matrix from the $y \cdot a = 0$ equation and the three independent values obtained

Simon's algorithm – example

$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

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Create a matrix from the $y \cdot a = 0$ equation and the three independent values obtained

$$\begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Simon's algorithm – example

$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

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Create a matrix from the $y \cdot a = 0$ equation and the three independent values obtained

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} a_3 \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ \end{bmatrix}$$

Simon's algorithm – example

$$|\phi_4\rangle = \frac{1}{\sqrt{8}}\left[|0000\rangle - |0010\rangle - |0100\rangle + |0110\rangle + |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle\right]$$

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Simon's algorithm – example

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Create a matrix from the $y \cdot a = 0$ equation and the three independent values obtained

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ & & & \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \\ \end{bmatrix}$$

Simon's algorithm – example

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Create a matrix from the $y \cdot a = 0$ equation and the three independent values obtained

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Convert the matrix to an upper triangular form by swapping rows 1 and 3

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Convert the matrix to an upper triangular form by swapping rows 1 and 3

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$a_0 = 0$$

$$a_0 = 1$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$

$$a_1 = 0,$$

$$a_0 = 1$$

$$a_1 = 0,$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$
$$a_1 = 0, \quad a_2 = 0,$$

$$a_0 = 1$$
$$a_1 = 0, \quad a_2 = 0,$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_0 = 1$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$a_0 = 0$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_3 = 0 \quad \longrightarrow \quad a = |0000\rangle$$

$$a_0 = 1$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_3 = 0 \quad \longrightarrow \quad a = |0000\rangle$$

trivial, incorrect solution

$$a_0 = 1$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_3 = 0 \quad \longrightarrow \quad a = |0000\rangle$$

trivial, incorrect solution

$$a_0 = 1$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_3 = -1 = 1$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Convert the matrix to an upper triangular form by swapping rows 1 and 3

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$
$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$
$$a_3 = 0 \quad \longrightarrow \quad a = |0000\rangle$$

trivial, incorrect solution

$$a_0 = 1$$
$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$
$$a_3 = -1 = 1 \quad \longrightarrow \quad a = |1001\rangle$$

Simon's algorithm – example

Solve this matrix equation by Gaussian elimination

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Convert the matrix to an upper triangular form by swapping rows 1 and 3

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Since the bottom row of the matrix is all zeros, a_0 can be either 0 or 1

$$a_0 = 0$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_3 = 0 \quad \longrightarrow \quad a = |0000\rangle$$

trivial, incorrect solution

$$a_0 = 1$$

$$a_1 = 0, \quad a_2 = 0, \quad a_3 + a_0 = 0$$

$$a_3 = -1 = 1 \quad \longrightarrow \quad a = |1001\rangle$$

correct solution