Today's outline - January 27, 2022

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Reading Assignment: Chapter 5.3-5.4

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Homework Assignment #03:
Chapter 4:1,2,7,10,15,18
due Thursday, February 03, 2022

Quantum operators

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where arbitrary choices for the sign and phase factor of b have been made

Quantum gates

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Phase Shift	$\theta = 0$	ϕ	$\lambda = 0$	leaves $\|0\rangle$ unchanged and rotates $\|1\rangle$ on Bloch sphere by ϕ	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

No clone year: 1982

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LETTERS TO NATURE

A single quantum cannot be cloned

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If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6–8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \quad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\mathrm{vert}}\rangle|\updownarrow\updownarrow\rangle \quad (2)$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\mathrm{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \quad (3)$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha=\beta=2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle) \rightarrow \alpha|A_{\mathrm{vert}}\rangle|\updownarrow\updownarrow\rangle+\beta|A_{\mathrm{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \quad (4)$$

If the apparatus states $|A_{\mathrm{vert}}\rangle$ and $|A_{\mathrm{hor}}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle+\beta|\leftrightarrow\leftrightarrow\rangle \quad (5)$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^{+}_{\mathrm{vert}}+\beta a^{+}_{\mathrm{hor}})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle+2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle+\beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

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"A single quantum cannot be cloned," W.K. Wouters and W.H. Zurek, *Nature* **299**, 802 (1982).

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COMMUNICATION BY EPR DEVICES

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A recent proposal to achieve faster-than-light communication by means of an EPR-type experimental set-up is examined. We demonstrate that such superluminal communication is not possible. The crucial role of the linearity of the quantum mechanical evolution laws in preventing causal anomalies is stressed.

The existence, according to quantum theory, of correlations between spatially separated systems in EPR-like experiments has suggested to several authors the possibility of message transmission at speeds greater than that of light. The idea is that the correlations subsist between measurement results which do not – as in classical physics – correspond to properties possessed by the systems before the measurement. An experimenter A can therefore choose what kind of experiment to perform at system I and is thus able to influence the probability distribution of outcomes obtained by experimenter B who is measuring on system II. If B were able to recognize this change in the probability distribution he would know what kind of experiment A had decided to perform; and this transmission of information could be used to develop a code for sending messages from A to B (and vice versa). However, it can easily be proved [1] that, due to the fact that the operators representing two measurements at space-like separation commute, all expectation values of physical quantities measured by B remain the same irrespective of A's decisions. Repetition of the experiment therefore will not provide B with any means to discover what A has done. The idea of communication by superluminal velocity thereby seems to be refuted.

There is nevertheless a remaining possibility, recently pointed out by Herbert [2]. The central idea here is to use one single experiment (and not a series of repeated experiments) to transmit one unit of information. In order to ascertain whether or not a change in the probability distribution has taken place a "multiplying device" is included in the experimental set-up. We shall discuss this idea in the context of Bohm's familiar version of the EPR-experiment (see ref. [2] for an exposition in terms of photon polarizations). It will be shown that the laws of quantum theory by virtue of their linearity, prevent such a "quantum communicator" from working.

Suppose that a compound $S = 0$ state decays into two spin 1/2 particles (electrons, say). Experimenter A has the choice to measure either the x-component or the z-component of the spin of electron I. In the path of electron II a "multiplying device" is positioned, in such a way as to ensure that II enters the device *after* A has performed a measurement upon I. The function of the "multiplying device" is to produce a burst of electrons all in exactly the same spin state as the single input electron. The large number N of electrons coming from this device are then examined by B, by means of a Stern–Gerlach apparatus adjusted to measure the x-component of the spin. There are now two possibilities:

(i) A has decided to measure the x-component of the spin of I. Immediately after this measurement II can be described (as far as spin is concerned) with an eigenstate of s_x, and therefore all the electrons emerging from the multiplier will be in this state. The subsequent measurement by B will then have as a result *all* N electrons in either the $s_x = \frac{1}{2}$ or $s_x = -\frac{1}{2}$ channel.

(ii) A has chosen to measure the z-component of the spin of I. Then the electrons emerging from the multiplier will be in an eigenstate of s_z. For each of

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"Communication by EPR devices," D. Dieks, *Phys. Lett.* **92A**, 271 (1982).

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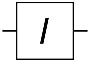

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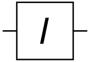
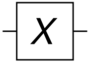


returns the same qubit

$$X = |1\rangle\langle 0| + |0\rangle\langle 1| \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



negates the qubit

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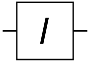

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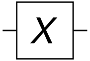
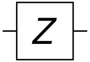
$$X = |1\rangle\langle 0| + |0\rangle\langle 1| \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



negates the qubit

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1| \qquad \begin{pmatrix} 1 & 0 \\ 0 & \bar{1} \end{pmatrix}$$



changes phase of qubit

Common single qubit gates

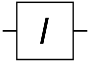
The most common single qubit transformations are the Pauli transformations

$I = \|0\rangle\langle 0\| + \|1\rangle\langle 1\|$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	I	returns the same qubit
$X = \|1\rangle\langle 0\| + \|0\rangle\langle 1\|$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	X	negates the qubit
$Z = \|0\rangle\langle 0\| - \|1\rangle\langle 1\|$	$\begin{pmatrix} 1 & 0 \\ 0 & \bar{1} \end{pmatrix}$	Z	changes phase of qubit
$Y = \|0\rangle\langle 1\| - \|1\rangle\langle 0\|$	$\begin{pmatrix} 0 & 1 \\ \bar{1} & 0 \end{pmatrix}$	Y	negate and change phase of qubit

Common single qubit gates

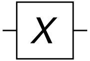
The most common single qubit transformations are the Pauli transformations and the Hadamard gate

$$I = |0\rangle\langle 0| + |1\rangle\langle 1| \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



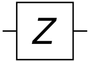
returns the same qubit

$$X = |1\rangle\langle 0| + |0\rangle\langle 1| \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



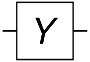
negates the qubit

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1| \qquad \begin{pmatrix} 1 & 0 \\ 0 & \bar{1} \end{pmatrix}$$



changes phase of qubit

$$Y = |0\rangle\langle 1| - |1\rangle\langle 0| \qquad \begin{pmatrix} 0 & 1 \\ \bar{1} & 0 \end{pmatrix}$$



negate and change phase of qubit

Common single qubit gates

The most common single qubit transformations are the Pauli transformations and the Hadamard gate

$I = \vert 0\rangle\langle 0\vert + \vert 1\rangle\langle 1\vert$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	I	returns the same qubit
$X = \vert 1\rangle\langle 0\vert + \vert 0\rangle\langle 1\vert$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	X	negates the qubit
$Z = \vert 0\rangle\langle 0\vert - \vert 1\rangle\langle 1\vert$	$\begin{pmatrix} 1 & 0 \\ 0 & \bar{1} \end{pmatrix}$	Z	changes phase of qubit
$Y = \vert 0\rangle\langle 1\vert - \vert 1\rangle\langle 0\vert$	$\begin{pmatrix} 0 & 1 \\ \bar{1} & 0 \end{pmatrix}$	Y	negate and change phase of qubit
$H = \frac{1}{\sqrt{2}}(\vert 0\rangle\langle 0\vert + \vert 1\rangle\langle 0\vert + \vert 0\rangle\langle 1\vert - \vert 1\rangle\langle 1\vert)$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & \bar{1} \end{pmatrix}$	H	Hadamard gate

The Hadamard gate

The Hadamard gate is particularly important as it transforms pure single qubit states into even superpositions

$$H = \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1| - |1\rangle\langle 1|\right)$$

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$$HH|0\rangle = H\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\left(H|0\rangle + H|1\rangle\right) = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right]$$

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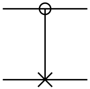

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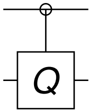

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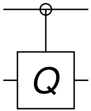

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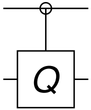

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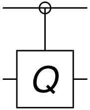

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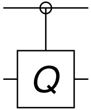

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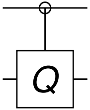

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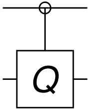

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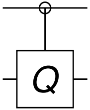

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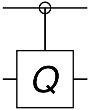

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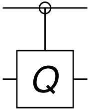

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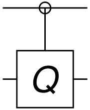

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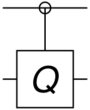

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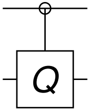

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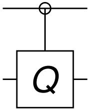

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Note the $\to$ and not $\mapsto$, the former being a transformation in a complex vector space while the latter works in the complex projective space where $e^{i\theta}|11\rangle \sim |11\rangle$

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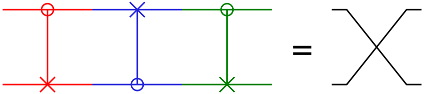

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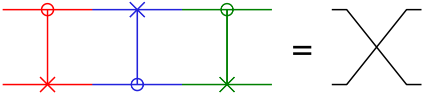

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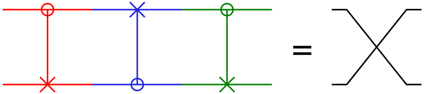

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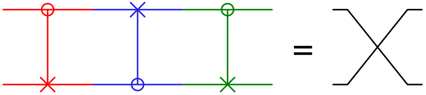

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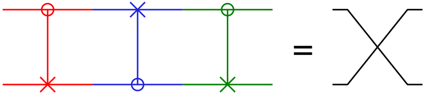

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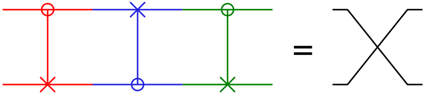

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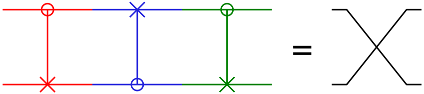

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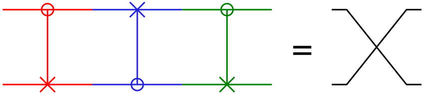

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Now apply the C_{not} gate: $C_{not} = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

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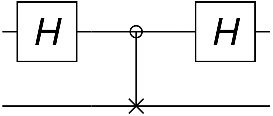

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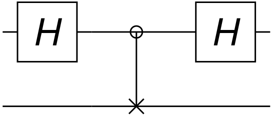

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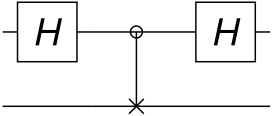

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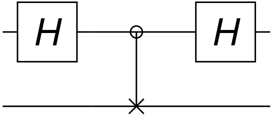

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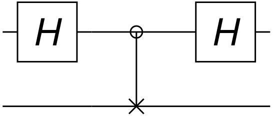

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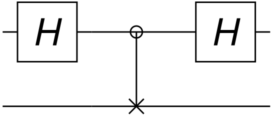

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