Today's outline - January 25, 2022

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- The EPR paradox

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Reading Assignment: Chapter 5.1-5.2

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Homework Assignment #02:
Chapter 3:1,4,8,10,14,15
due Thursday, January 27, 2022

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- The EPR paradox
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Reading Assignment: Chapter 5.1-5.2

Homework Assignment #02:
Chapter 3:1,4,8,10,14,15
due Thursday, January 27, 2022

Homework Assignment #03:
Chapter 4:1,2,7,10,15,18
due Thursday, February 03, 2022

Einstein Podolsky Rosen paradox

of lanthanum is 7/2, hence the nuclear magnetic moment as determined by this analysis is 2.5 nuclear magnetons. This is in fair agreement with the value 2.8 nuclear magnetons determined from La III hyperfine structures by the writer and N. S. Grace.[9]

[9] M. F. Crawford and N. S. Grace, Phys. Rev. **47**, 536 (1935).

This investigation was carried out under the supervision of Professor G. Breit, and I wish to thank him for the invaluable advice and assistance so freely given. I also take this opportunity to acknowledge the award of a Fellowship by the Royal Society of Canada, and to thank the University of Wisconsin and the Department of Physics for the privilege of working here.

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Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

such way, whenever the conditions set down in it occur. Regarded not as a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.

To illustrate the ideas involved let us consider the quantum-mechanical description of the behavior of a particle having a single degree of freedom. The fundamental concept of the theory is the concept of *state*, which is supposed to be completely characterized by the wave function ψ, which is a function of the variables chosen to describe the particle's behavior. Corresponding to each physically observable quantity A there is an operator, which may be designated by the same letter.

If ψ is an eigenfunction of the operator A, that is, if

$$\psi' \equiv A\psi = a\psi, \tag{1}$$

where a is a number, then the physical quantity A has with certainty the value a whenever the particle is in the state given by ψ. In accordance with our criterion of reality, for a particle in the state given by ψ for which Eq. (1) holds, there is an element of physical reality corresponding to the physical quantity A. Let, for example,

$$\psi = e^{(2\pi i/h)p_0 x}, \tag{2}$$

where h is Planck's constant, p_0 is some constant number, and x the independent variable. Since the operator corresponding to the momentum of the particle is

$$p = (h/2\pi i)\partial/\partial x, \tag{3}$$

we obtain

$$\psi' = p\psi = (h/2\pi i)\partial\psi/\partial x = p_0\psi. \tag{4}$$

Thus, in the state given by Eq. (2), the momentum has certainly the value p_0. It thus has meaning to say that the momentum of the particle in the state given by Eq. (2) is real.

On the other hand if Eq. (1) does not hold, we can no longer speak of the physical quantity A having a particular value. This is the case, for example, with the coordinate of the particle. The operator corresponding to it, say q, is the operator of multiplication by the independent variable. Thus,

$$q\psi = x\psi \neq a\psi. \tag{5}$$

In accordance with quantum mechanics we can only say that the relative probability that a measurement of the coordinate will give a result lying between a and b is

$$P(a,b) = \int_a^b \bar{\psi}\psi dx = \int_a^b dx = b - a. \tag{6}$$

Since this probability is independent of a, but depends only upon the difference $b-a$, we see that all values of the coordinate are equally probable.

A definite value of the coordinate, for a particle in the state given by Eq. (2), is thus not predictable, but may be obtained only by a direct measurement. Such a measurement however disturbs the particle and thus alters its state. After the coordinate is determined, the particle will no longer be in the state given by Eq. (2). The usual conclusion from this in quantum mechanics is that *when the momentum of a particle is known, its coordinate has no physical reality*.

More generally, it is shown in quantum mechanics that, if the operators corresponding to two physical quantities, say A and B, do not commute, that is, if $AB \neq BA$, then the precise knowledge of one of them precludes such a knowledge of the other. Furthermore, any attempt to determine the latter experimentally will alter the state of the system in such a way as to destroy the knowledge of the first.

From this follows that either (1) *the quantum-mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality*. For if both of them had simultaneous reality—and thus definite values—these values would enter into the complete description, according to the condition of completeness. If then the wave function provided such a complete description of reality, it would contain these values; these would then be predictable. This not being the case, we are left with the alternatives stated.

In quantum mechanics it is usually assumed that the wave function *does* contain a complete description of the physical reality of the system in the state to which it corresponds. At first

"Can quantum-mechanical description of physical reality be considered complete?," A. Einstein, B. Podolsky, and N. Rosen, *Physical Review* **47**, 777-779 (1935).

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Now Alice measures her photon and sees that it is the the $|0\rangle$ state which forces the original state to collapse: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \longrightarrow |00\rangle$

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There is no causality, just correlated random behavior

Einstein Podolsky Rosen paradox

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If such a theory is correct, then the result of the measurements is determined before the photons are separated and no possible violations of causality can occur

Bell's inequality

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received 4 November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

II. Formulation

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where $\vec{a}$ is some unit vector, yields the value $+1$ then, according to quantum mechanics, measurement of $\vec{\sigma}_2 \cdot \vec{a}$ must yield the value -1 and vice versa. Now we make the hypothesis [2], and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of $\vec{\sigma}_2$, by previously measuring the same component of $\vec{\sigma}_1$, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does *not* determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.

Let this more complete specification be effected by means of parameters λ. It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, we write as if λ were a single continuous parameter. The result A of measuring $\vec{\sigma}_1 \cdot \vec{a}$ is then determined by $\vec{a}$ and λ, and the result B of measuring $\vec{\sigma}_2 \cdot \vec{b}$ in the same instance is determined by $\vec{b}$ and λ, and

*Work supported in part by the U.S. Atomic Energy Commission
†On leave of absence from SLAC and CERN

$$A(\vec{a}, \lambda) = \pm 1, \; B(\vec{b}, \lambda) = \pm 1. \tag{1}$$

The vital assumption [2] is that the result B for particle 2 does not depend on the setting $\vec{a}$, of the magnet for particle 1, nor A on $\vec{b}$.

If $\rho(\lambda)$ is the probability distribution of λ then the expectation value of the product of the two components $\vec{\sigma}_1 . \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$ is

$$P(\vec{a}, \vec{b}) = \int d\lambda \, \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) \tag{2}$$

This should equal the quantum mechanical expectation value, which for the singlet state is

$$< \vec{\sigma}_1 \cdot \vec{a} \; \vec{\sigma}_2 \cdot \vec{b} > = -\vec{a} \cdot \vec{b} \, . \tag{3}$$

But it will be shown that this is not possible.

Some might prefer a formulation in which the hidden variables fall into two sets, with A dependent on one and B on the other; this possibility is contained in the above, since λ stands for any number of variables and the dependences thereon of A and B are unrestricted. In a complete physical theory of the type envisaged by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can then be thought of as initial values of these variables at some suitable instant.

III. Illustration

The proof of the main result is quite simple. Before giving it, however, a number of illustrations may serve to put it in perspective.

Firstly, there is no difficulty in giving a hidden variable account of spin measurements on a single particle. Suppose we have a spin half particle in a pure spin state with polarization denoted by a unit vector $\vec{p}$. Let the hidden variable be (for example) a unit vector $\vec{\lambda}$ with uniform probability distribution over the hemisphere $\vec{\lambda} \cdot \vec{p} > 0$. Specify that the result of measurement of a component $\vec{\sigma} \cdot \vec{a}$ is

$$\text{sign } \vec{\lambda} \cdot \vec{a}' \, , \tag{4}$$

where $\vec{a}'$ is a unit vector depending on $\vec{a}$ and $\vec{p}$ in a way to be specified, and the sign function is $+1$ or -1 according to the sign of its argument. Actually this leaves the result undetermined when $\lambda \cdot a' = 0$, but as the probability of this is zero we will not make special prescriptions for it. Averaging over $\vec{\lambda}$ the expectation value is

$$< \vec{\sigma} \cdot \vec{a} > = 1 - 2\theta'/\pi \, , \tag{5}$$

where θ' is the angle between $\vec{a}'$ and $\vec{p}$. Suppose then that $\vec{a}'$ is obtained from $\vec{a}$ by rotation towards $\vec{p}$ until

$$1 - \frac{2\theta'}{\pi} = \cos\theta \tag{6}$$

where θ is the angle between $\vec{a}$ and $\vec{p}$. Then we have the desired result

$$< \vec{\sigma} \cdot \vec{a} > = \cos\theta \tag{7}$$

So in this simple case there is no difficulty in the view that the result of every measurement is determined by the value of an extra variable, and that the statistical features of quantum mechanics arise because the value of this variable is unknown in individual instances.

"On the Einstein Podolsky Rosen paradox," J.S. Bell, *Physics* **1**, 195-200 (1964).

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$$P = \left(P^{v_1} \otimes I\right)\left(I \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes I\right)\left(I \otimes P^{v_2^\perp}\right)$$

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According to quantum mechanics, what is the probability of Alice and Bob obtaining the same value when they make their individual measurements, $O_{\theta_1} \otimes I$ and $I \otimes O_{\theta_2}$?

Start with the projectors for each of the measureable states $|v_i\rangle$ and $|v_i^\perp\rangle$

$$P^{v_i} = |v_i\rangle\langle v_i|, \qquad P^{v_i^\perp} = |v_i^\perp\rangle\langle v_i^\perp|$$

For the two measurements to result in $|v_1\rangle|v_2\rangle$ or $|v_1^\perp\rangle|v_2^\perp\rangle$, the projector must be

$$P = \left(P^{v_1} \otimes I\right)\left(I \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes I\right)\left(I \otimes P^{v_2^\perp}\right) = \left(P^{v_1} \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes P^{v_2^\perp}\right)$$

Bell's thought experiment

The pair of photons are emitted in an entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Alice and Bob have polarizers which can be set to vertical or $\pm 60^\circ$ from vertical

If O_θ is a 1-qubit observable with two basis vectors with results (eigenvalues) ± 1

$$|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle \longrightarrow +1$$
$$|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle \longrightarrow -1$$

According to quantum mechanics, what is the probability of Alice and Bob obtaining the same value when they make their individual measurements, $O_{\theta_1} \otimes I$ and $I \otimes O_{\theta_2}$?

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$$P = \left(P^{v_1} \otimes I\right)\left(I \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes I\right)\left(I \otimes P^{v_2^\perp}\right) = \left(P^{v_1} \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes P^{v_2^\perp}\right) = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Quantum mechanics prediction

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$$P^{v_1 v_2} = P^{v_1} \otimes P^{v_2}$$

Quantum mechanics prediction

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$$P^{v_1 v_2} = P^{v_1} \otimes P^{v_2} = \left(|v_1\rangle\langle v_1| \otimes |v_2\rangle\langle v_2|\right)$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

$$\begin{aligned} P^{v_1 v_2} &= P^{v_1} \otimes P^{v_2} = (|v_1\rangle\langle v_1| \otimes |v_2\rangle\langle v_2|) \\ &= |v_1\rangle|v_2\rangle(\qquad\qquad\qquad\qquad) \end{aligned}$$

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Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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$$P^{v_1 v_2} = P^{v_1} \otimes P^{v_2} = \left(|v_1\rangle\langle v_1| \otimes |v_2\rangle\langle v_2|\right)$$
$$= |v_1\rangle|v_2\rangle\left(\cos\theta_1\cos\theta_2\langle 00| + \cos\theta_1\sin\theta_2\langle 01| + \sin\theta_1\cos\theta_2\langle 10| \qquad\qquad \right)$$

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Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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$$P^{v_1^\perp v_2^\perp} = P^{v_1^\perp} \otimes P^{v_2^\perp}$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

$$
\begin{aligned}
P^{v_1 v_2} &= P^{v_1} \otimes P^{v_2} = \left(|v_1\rangle\langle v_1| \otimes |v_2\rangle\langle v_2|\right) \\
&= |v_1\rangle|v_2\rangle\left(\cos\theta_1 \cos\theta_2\langle 00| + \cos\theta_1 \sin\theta_2\langle 01| + \sin\theta_1 \cos\theta_2\langle 10| + \sin\theta_1 \sin\theta_2\langle 11|\right) \\
P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = \left(|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|\right)
\end{aligned}
$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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P^{v_1 v_2} &= P^{v_1} \otimes P^{v_2} = (|v_1\rangle\langle v_1| \otimes |v_2\rangle\langle v_2|) \\
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P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\qquad\qquad)
\end{aligned}$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1 \sin\theta_2 \langle 00| \qquad\qquad)
\end{aligned}
$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| \qquad)
\end{aligned}$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| - \cos\theta_1\sin\theta_2\langle 10| \qquad)
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&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| - \cos\theta_1\sin\theta_2\langle 10| + \cos\theta_1\cos\theta_2\langle 11|)
\end{aligned}$$

Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1 \sin\theta_2\langle 00| - \sin\theta_1 \cos\theta_2\langle 01| - \cos\theta_1 \sin\theta_2\langle 10| + \cos\theta_1 \cos\theta_2\langle 11|)
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$$P|\psi\rangle = \frac{1}{\sqrt{2}}|v_1\rangle|v_2\rangle(\qquad\qquad\qquad\qquad)$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| - \cos\theta_1\sin\theta_2\langle 10| + \cos\theta_1\cos\theta_2\langle 11|)
\end{aligned}$$

Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

$$P|\psi\rangle = \frac{1}{\sqrt{2}}|v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2 \qquad\qquad\qquad)$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| - \cos\theta_1\sin\theta_2\langle 10| + \cos\theta_1\cos\theta_2\langle 11|)
\end{aligned}$$

Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

$$P|\psi\rangle = \frac{1}{\sqrt{2}}|v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

$$
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&= |v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2\langle 00| + \cos\theta_1\sin\theta_2\langle 01| + \sin\theta_1\cos\theta_2\langle 10| + \sin\theta_1\sin\theta_2\langle 11|) \\
P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| - \cos\theta_1\sin\theta_2\langle 10| + \cos\theta_1\cos\theta_2\langle 11|)
\end{aligned}
$$

Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

$$
P|\psi\rangle = \frac{1}{\sqrt{2}}|v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + \frac{1}{\sqrt{2}}|v_1^\perp\rangle|v_2^\perp\rangle(\qquad\qquad)
$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

$$P|\psi\rangle = \frac{1}{\sqrt{2}}|v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + \frac{1}{\sqrt{2}}|v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2 \qquad)$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
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Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

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Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

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The probability of $|\psi\rangle$ being found in the $+1$ eigenspace generated by $\{|v_1\rangle|v_2\rangle, |v_1^\perp\rangle|v_2^\perp\rangle\}$

Photon polarization example

The three polarizations for each filter represent three different observables, M_{0°, M_{+60°, and M_{-60°

$$\langle\psi|O_{\theta_1} \otimes O_{v_2}|\psi\rangle = \cos^2(\theta_1 - \theta_2)$$

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If the polarizers are set randomly and independently, they will be the same $\frac{1}{3}$ of the time with 100% probability of the measurements agreeing and be different $\frac{2}{3}$ of the time with 25% probability of the measurements agreeing

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The overall probability of measurements agreeing is thus $\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{2}$

Consequences of a local hidden variable

Suppose there is a local hidden state associated with each photon which determines the result of the measurement in each of the three polarizer settings

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We know that when both filters are in the same position the two measurements of an EPR pair must coincide such that if Alice's measurements are to be *PAP*, then Bob's must also be *PAP* so we enumarate the 9 possible filter settings and see what the local hidden variables predict

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If the hidden state is h_0 or h_7 measurements agree for all possible filter settings but for the other 6 hidden states $\frac{5}{9}$ of the measurements will agree giving total probability $1 \cdot \frac{2}{8} + \frac{5}{9} \cdot \frac{6}{8} = \frac{8}{12}$

This does not match the quantum mechanics result of $\frac{1}{2}$

Bell's inequality

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Finally, let w_h be the probability with wich the EPR source emits photons of kind h

Bell's inequality (cont.)

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$$P_{ab} + P_{ac} + P_{bc} = \sum_h w_h \left(P_{ab}^h + P_{ac}^h + P_{bc}^h \right)$$

However, as we saw from the simple example, for every possible local hidden state h, the result of measuring the two photons will be the same for one or more of the three combinations, P_{ab}^h, P_{ac}^h, or P_{bc}^h and this forces the sum to be greater than 1 for any values of a, b, and c

$$P_{ab}^h + P_{ac}^h + P_{bc}^h \geq 1$$

Bell's inequality (cont.)

The sum of the observed probabilities the three combinations $P_{ab} + P_{ac} + P_{bc}$ is given by

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$$P^h_{ab} + P^h_{ac} + P^h_{bc} \geq 1 \quad \longrightarrow \quad P_{ab} + P_{ac} + P_{bc} > 1$$

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For the angle between a and b being θ and the angle between b and c being ϕ we have that

$$P_{ab} + P_{ac} + P_{bc} = \cos^2\theta + \cos^2(\theta + \phi) + \cos^2\phi > 1$$

Testing Bell's inequality

$$P_{ab} + P_{ac} + P_{bc} = \cos^2(\theta_a - \theta_b) + \cos^2(\theta_a - \theta_c) + \cos^2(\theta_b - \theta_c) > 1$$

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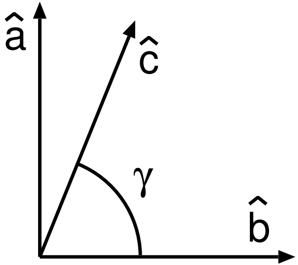
Take the worst case, that of $\theta_a = \frac{\pi}{2}$, $\theta_b = 0$, and $\theta_c = \gamma$



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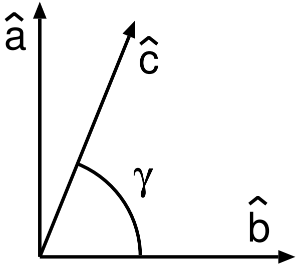

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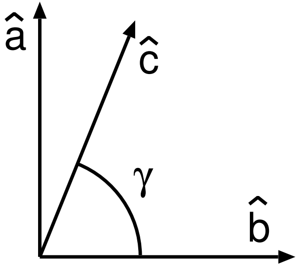

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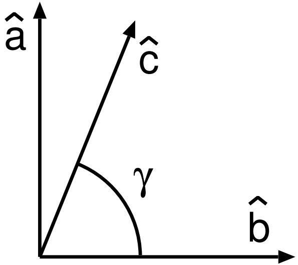

$$P_{ab} = \cos^2 \frac{\pi}{2} = 0$$

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Testing Bell's inequality

$$P_{ab} + P_{ac} + P_{bc} = \cos^2(\theta_a - \theta_b) + \cos^2(\theta_a - \theta_c) + \cos^2(\theta_b - \theta_c) > 1$$

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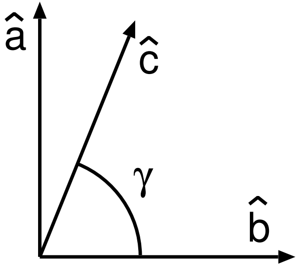

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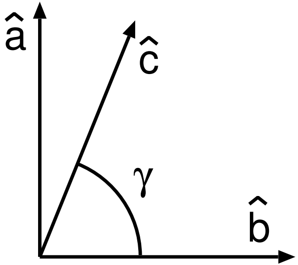

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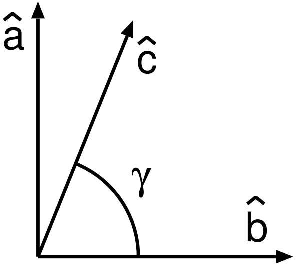
$$P_{ab} + P_{ac} + P_{bc} = 0 + \cos^2 \gamma + \sin^2 \gamma = 1 \not> 1$$



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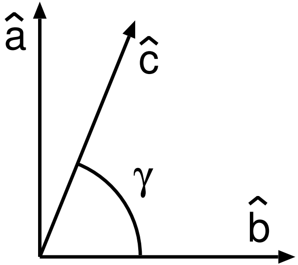

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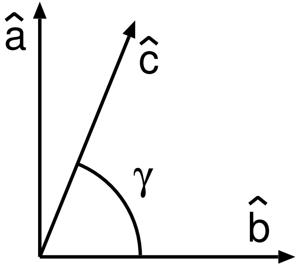

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Since Bell's paper, there have been many efforts to demonstrate the failure of this inequality

Bell's inequality tested

for trapping as b decreases is reflective of the incorporation of periodic components into the sequence of numbers generated.

To summarize the motivation and principal conclusion of this Letter, we restate[1] that for values of b where numerically generated sequences *appear* to be chaotic, it has not been settled whether those sequences "are truly chaotic, or whether, in fact, they are really periodic, but with exceedingly large periods and very long transients required to settle down." On the one hand, Grossman and Thomae[5,6] have suggested that (only) the parameter value $b=1$ generates pure chaos [see the discussion following Eq. (31) of Ref. 5 and the correlations plotted in their Fig. 9]. On the other hand, for certain *other* values of b, numberical results of Lorenz (reported in Ref. 1) "strongly suggest that the sequences are truly chaotic." The purpose of this communication was to use an independent and exact result from the statistical-mechanical theory of $d=1$ random walks to test the randomness of the parabolic map for parameter values where the existence of "true chaos" is still an open question.

Our results strongly support the conclusions of Grossmann and Thomae.

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Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

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Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

PACS numbers: 03.65.Bz, 35.80.+s

Bell's inequalities apply to any correlated measurement on two correlated systems. For instance, in the optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*,[1] a source emits pairs of photons (Fig. 1). Measurements of the correlations of linear polarizations are performed on two photons belonging to the same pair. For pairs emitted in suitable states, the correlations are strong. To account for these correlations, Bell[2] considered theories which invoke common properties of both members of the

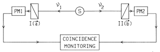


FIG. 1. Optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*. The pair of photons ν_1 and ν_2 is analyzed by linear polarizers I and II (in orientations $\vec{a}$ and $\vec{b}$) and photomultipliers. The coincidence rate is monitored.

pair. Such properties are referred to as supplementary parameters. This is very different from the quantum mechanical formalism, which does not involve such properties. With the addition of a reasonable locality assumption, Bell showed that such classical-looking theories are constrained by certain inequalities that are not always obeyed by quantum mechanical predictions.

Several experiments of increasing accuracy[3-5] have been performed and clearly favor quantum mechanics. Experiments using pairs of visible photons emitted in atomic radiative cascades seem to achieve a good realization of the ideal *Gedankenexperiment*.[5] However, all these experiments have been performed with static setups, in which polarizers are held fixed for the whole duration of a run. Then, one might question Bell's locality assumption, that states that the results of the measurement by polarizer II does not depend on the orientation $\vec{a}$ of polarizer I (and vice versa), nor does the way in which pairs are emitted depend on $\vec{a}$ or $\vec{b}$. Although highly reasonable, such a locality condition is not prescribed by any fundamental physical law. As pointed out by Bell,[2] it is possible, in such experiments, to reconcile supplementary-parameter theories and the experimentally verified predictions of quantum mechanics: "The settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light." If such interactions existed, Bell's locality condition would no longer hold for static experiments, nor would Bell's inequalities.

Bell thus insisted upon the importance of "experiments of the type proposed by Bohm and Aharonov,[6] in which the settings are changed during the flight of the particles." In such a "timing experiment," the locality condition would then become a consequence of Einstein's causality, preventing any faster-than-light influence.

In this Letter, we report the results of the first experiment using variable polarizers. Following our proposal,[7] we have used a modified scheme (Fig. 2). Each polarizer is replaced by a setup involving a switching device followed by two polarizers in two different orientations: $\vec{a}$ and $\vec{a}'$ on side I, and $\vec{b}$ and $\vec{b}'$ on side II. Such an optical switch is able to rapidly redirect the incident light from one polarizer to the other one. If the two switches work at random and are uncorrelated, it is possible to write generalized Bell's inequalities in a form similar to Clauser-Horne-

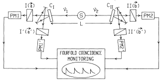


FIG. 2. Timing experiment with optical switches. Each switching device (C_I, C_{II}) is followed by two polarizers in two different orientations. Each combination is equivalent to a polarizer switched fast between two orientations.

Shimony-Holt inequalities[8]:

$$-1 \leqslant S \leqslant 0,$$

with

$$S = \frac{N(\vec{a},\vec{b})}{N(\infty,\infty)} - \frac{N(\vec{a},\vec{b}')}{N(\infty,\infty')} + \frac{N(\vec{a}',\vec{b})}{N(\infty',\infty)} + \frac{N(\vec{a}',\vec{b}')}{N(\infty',\infty')} - \frac{N(\vec{a}',\infty)}{N(\infty',\infty)} - \frac{N(\infty,\vec{b})}{N(\infty,\infty)}.$$

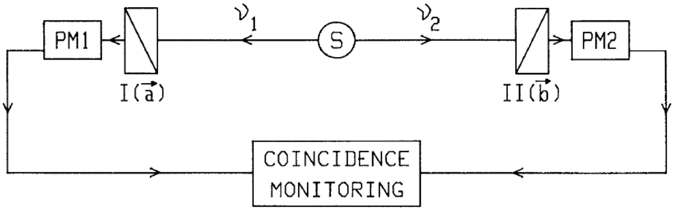
The quantity S involves (i) the four coincidence counting rates [$N(\vec{a},\vec{b})$, $N(\vec{a}',\vec{b})$, etc.] measured in a *single run*; (ii) the four corresponding coincidence rates [$N(\infty,\infty)$, $N(\infty',\infty)$, etc.] with all polarizers removed; and (iii) two coincidence rates [$N(\vec{a}',\infty)$, $N(\infty,\vec{b})$] with a polarizer removed on each side. The measurements (ii) and (iii) are performed in auxiliary runs.

In this experiment, switching between the two channels occurs about each 10 ns. Since this delay, as well as the lifetime of the intermediate level of the cascade (5 ns), is small compared to L/c (40 ns), a detection event on one side and the corresponding change of orientation on the other side are separated by a spacelike interval.

The switching of the light is effected by acousto-optical interaction with an ultrasonic standing wave in water.[9] As sketched in Fig. 3 the incidence angle is equal to the Bragg angle, $\theta_B = 5 \times 10^{-3}$ rad. It follows that light is either transmitted straight ahead or deflected at an angle $2\theta_B$. The light is completely transmitted when the amplitude of the standing wave is null, which occurs twice during an acoustical period. A quarter of a period later, the amplitude of the standing wave is maximum and, for a suitable value of the acoustical power, light is then fully

"Experimental test of Bell's inequalities using time-varying analyzers,", A. Aspect, J. Dalibard, and G. Roger, *Phys. Rev. Lett.* **49**, 1804-1807 (1982).

The EPR experiment



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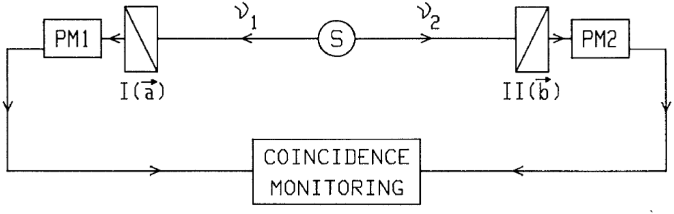

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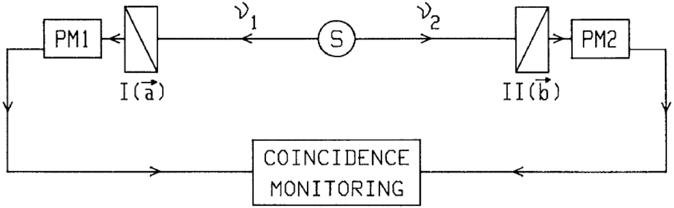

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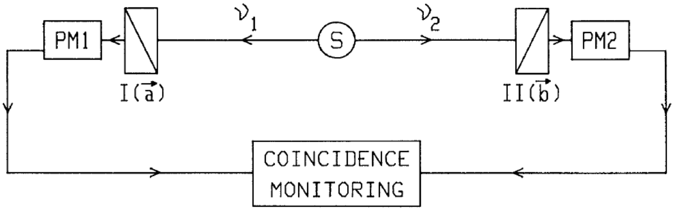

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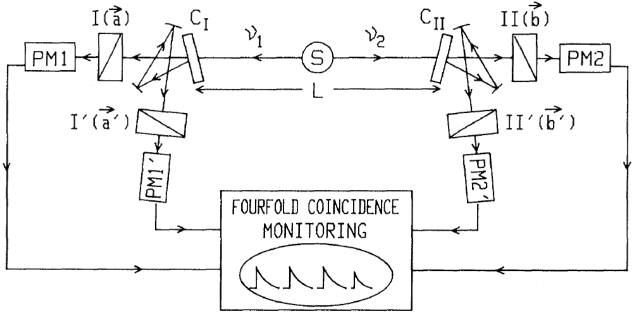
while a number of experiments of this kind yielded the expected result, the fact that the polarizers are static is problematic and could be argued to violate Bell's conditions

what is needed is a system where the relative orientation of $\hat{a}$ and $\hat{b}$ is randomized and unknown at the time of photon emission

“Randomized” EPR experiment



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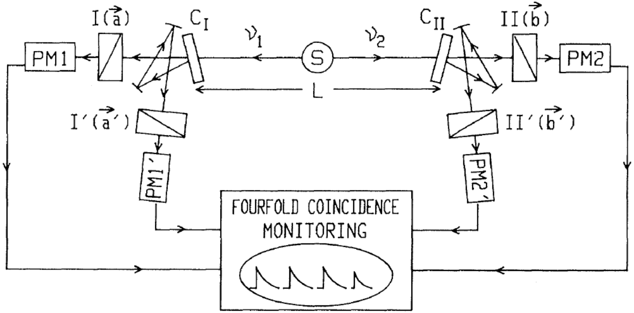

this experiment used two different orientations of the polarizers on each side and used fast switches to randomly and in an uncorrelated manner switch between the two possibilities

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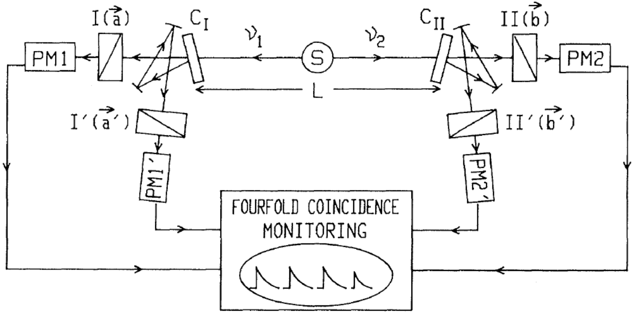

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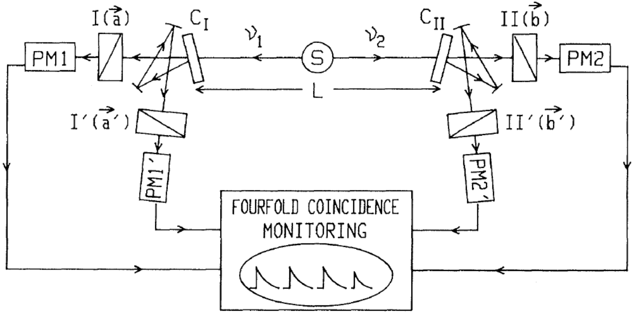

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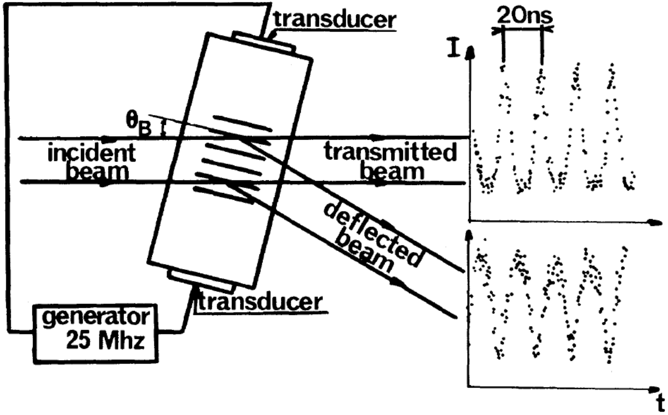
measurements are taken with all 4 polarizers in place, only two in place and none in place

Acoustical Bragg switching



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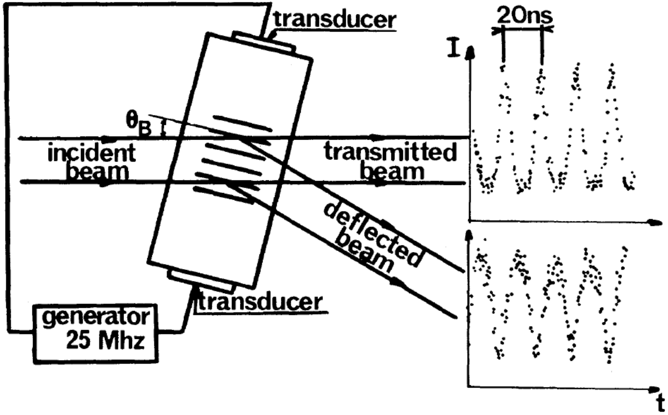

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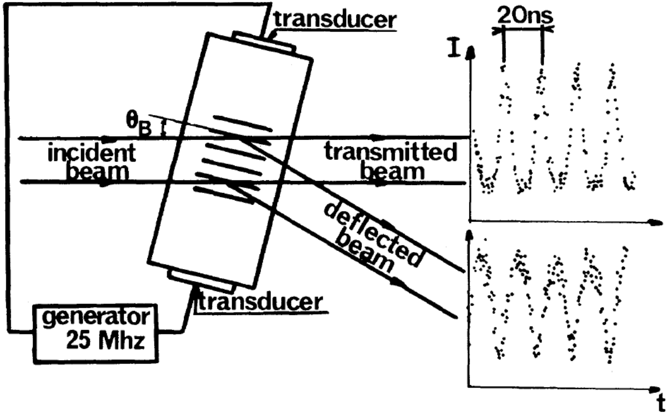

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Bragg diffraction occurs when the amplitude is maximum, twice each period

Bell's inequality confirmed

The results of the three different correlation experiments are used to compute the quantity S, which is corresponds to Bell's inequality

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for $(\hat{a},\hat{b}') = 67.5^\circ$ and all others 22.5° we have

$$S_{expt} = 0.101 \pm 0.020$$
$$S_{QM} = 0.112$$

Bell's inequality confirmed

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for $(\hat{a},\hat{b}') = 67.5^\circ$ and all others 22.5° we have

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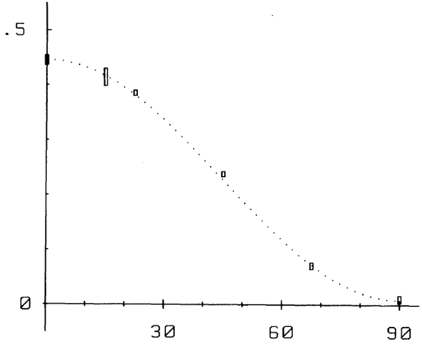

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Finally in 2015, three papers came out which closed both loopholes simultaneously