Today's outline - January 20, 2022

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- Outer products

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Reading Assignment: Chapter 4.3-4.4

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Reading Assignment: Chapter 4.3-4.4

Homework Assignment #02:
Chapter 3:1,4,8,10,14,15
due Thursday, January 27, 2022

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- Outer products
- Linear transformations
- Projection operators
- Qubit measurement revisited

Reading Assignment: Chapter 4.3-4.4

Homework Assignment #02:
Chapter 3:1,4,8,10,14,15
due Thursday, January 27, 2022

Homework Assignment #03:
Chapter 4:1,2,7,10,15,18
due Thursday, February 03, 2022

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$$a|0\rangle\langle 0| + b|0\rangle\langle 1| + c|1\rangle\langle 0| + d|1\rangle\langle 1| = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

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$$O|\psi\rangle = \left(\sum_i \sum_j a_{ij} |i\rangle\langle j| \right) \left(\sum_k b_k |k\rangle \right) = \sum_i \sum_j \sum_k a_{ij} b_k |i\rangle\langle j|k\rangle = \sum_i \sum_j a_{ij} b_j |i\rangle$$

the operator can be written in the same way for any basis $\{|\beta_i\rangle\}$ as $O = \sum_i \sum_j b_{ij} |\beta_i\rangle\langle\beta_j|$

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Note that we do not know the values of the qubits, just whether they are equal or not

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