Today's outline - January 13, 2022

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- Quantum postulates

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Reading Assignment: Chapter 3.2-3.3

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Homework Assignment #01:
Chapter 2:1,2,3,5,6,11
due Thursday, January 20, 2022

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After the measurement any photon that passed through the polarizer is now in the $|\uparrow\rangle$ state

More quantum principles

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Realizing an actual quantum computer requires a deep knowledge of quantum mechanics and experimental quantum systems

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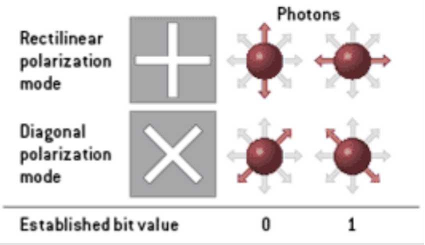
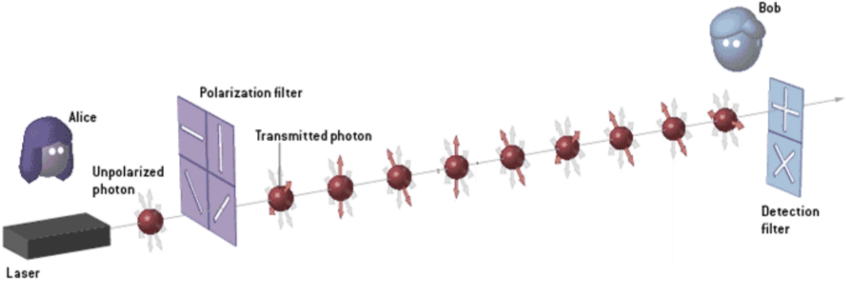
use photons polarized in two of three possible basis sets (rectilinear, diagonal, circular) and assign 0 and 1 bit values to each polarization direction possible



Quantum cryptography implementation



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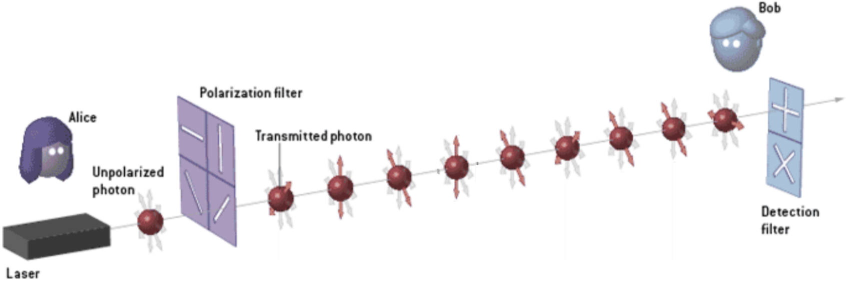

1. Alice chooses and records the filter type and the bit value for a series of photons sent

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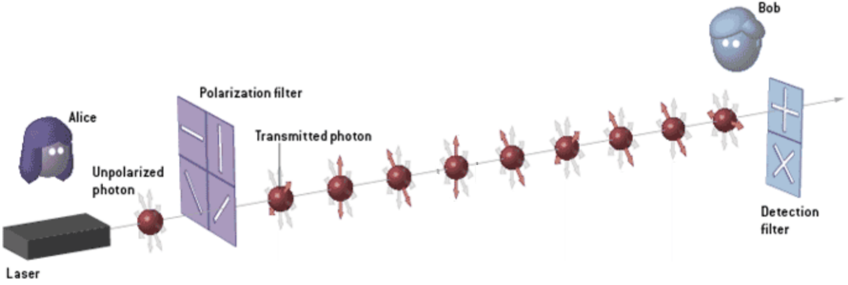

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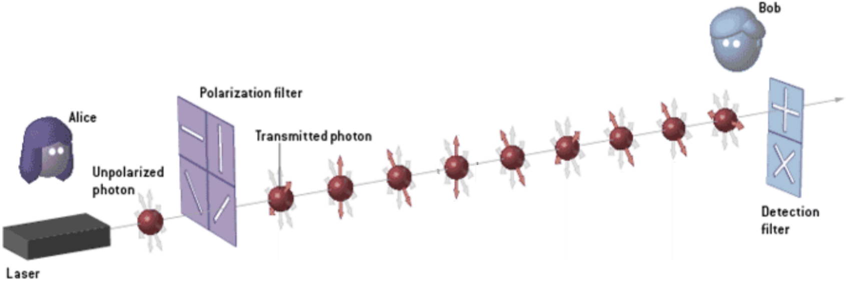

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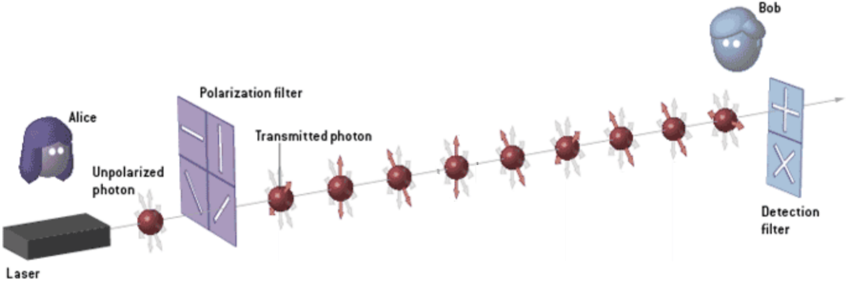

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2. Bob measures each incoming photon with a random choice of filter and records the choice and result
3. Bob tells Alice his filter choices on a public channel and Alice confirms which of his filters were correct
4. The remaining bits form the key that Bob and Alice can use

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Key distribution procedure

Alice's random bit	0	1	1	0	1	0	0	1

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Polarization sent	$\uparrow$	$\rightarrow$	$\searrow$	$\uparrow$	$\searrow$	$\nearrow$	$\nearrow$	$\rightarrow$

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Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$
Polarization measured	$\uparrow$	$\nearrow$	$\searrow$	$\nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$

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Polarization sent	$\uparrow$	$\rightarrow$	$\searrow$	$\uparrow$	$\searrow$	$\nearrow$	$\nearrow$	$\rightarrow$
Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$
Polarization measured	$\uparrow$	$\nearrow$	$\searrow$	$\nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$
Public discussion	Y		Y			Y		Y

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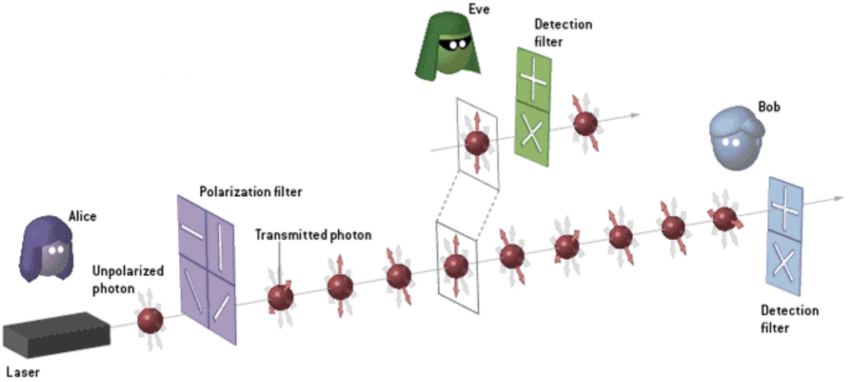
Polarization sent	$\uparrow$	$\rightarrow$	$\searrow$	$\uparrow$	$\searrow$	$\nearrow$	$\nearrow$	$\rightarrow$
Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$
Polarization measured	$\uparrow$	$\nearrow$	$\searrow$	$\nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$
Public discussion	Y		Y			Y		Y
Shared secret key	0		1			0		1

Eavesdropping scheme



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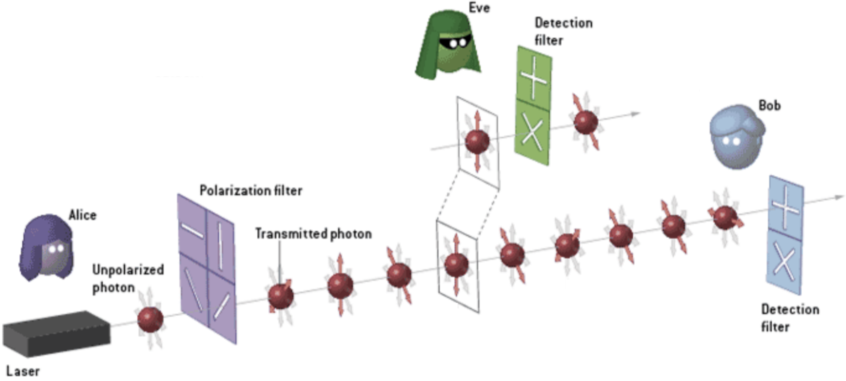

Suppose that Eve attempts to intercept a photon by measuring with a particular basis and then passing the resulting photon on to Bob

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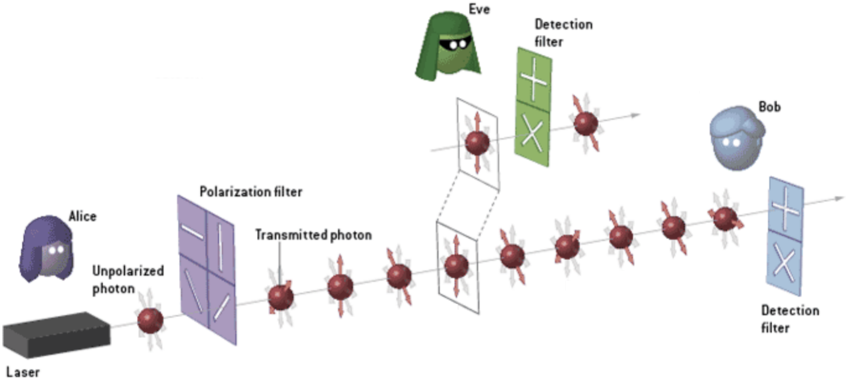

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An error may be created if Eve chooses the wrong filter

Key distribution with eavesdropper

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Alice's random basis	$+$	$+$	$\times$	$+$	$\times$	$\times$	$\times$	$+$
Polarization sent	$\uparrow$	$\rightarrow$	$\searrow$	$\uparrow$	$\searrow$	$\nearrow$	$\nearrow$	$\rightarrow$

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Polarization sent	$\uparrow$	$\rightarrow$	$\searrow$	$\uparrow$	$\searrow$	$\nearrow$	$\nearrow$	$\rightarrow$
Eve's random basis	$+$	$\times$	$+$	$+$	$\times$	$+$	$\times$	$+$

Key distribution with eavesdropper

Alice's random bit	0	1	1	0	1	0	0	1
Alice's random basis	+	+	×	+	×	×	×	+
Polarization sent	↑	→	↘	↑	↘	↗	↗	→
Eve's random basis	+	×	+	+	×	+	×	+
Eve's polarization	↑	↗	→	↑	↘	→	↗	→

Key distribution with eavesdropper

Alice's random bit	0	1	1	0	1	0	0	1
Alice's random basis	$+$	$+$	$\times$	$+$	$\times$	$\times$	$\times$	$+$
Polarization sent	$\uparrow$	$\rightarrow$	$\searrow$	$\uparrow$	$\searrow$	$\nearrow$	$\nearrow$	$\rightarrow$
Eve's random basis	$+$	$\times$	$+$	$+$	$\times$	$+$	$\times$	$+$
Eve's polarization	$\uparrow$	$\nearrow$	$\rightarrow$	$\uparrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\rightarrow$
Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$

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Eve's random basis	$+$	$\times$	$+$	$+$	$\times$	$+$	$\times$	$+$
Eve's polarization	$\uparrow$	$\nearrow$	$\rightarrow$	$\uparrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\rightarrow$
Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$
Polarization measured	$\uparrow$	$\nearrow$	$\nearrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\uparrow$	$\rightarrow$

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Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$
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Public discussion	Y		Y			Y		Y

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Eve's polarization	$\uparrow$	$\nearrow$	$\rightarrow$	$\uparrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\rightarrow$
Bob's random basis	$+$	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$
Polarization measured	$\uparrow$	$\nearrow$	$\nearrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\uparrow$	$\rightarrow$
Public discussion	Y		Y			Y		Y
Shared secret key	0		0			0		1

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To detect Eve with $P_d = 1 \times 10^{-9}$ requires $n = 72$

Experimental quantum cryptography

PHYSICAL REVIEW LETTERS

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Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

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An unknown quantum state $|\phi\rangle$ can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations. To do so the sender, "Alice," and the receiver, "Bob," must prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the unknown quantum system, and sends Bob the classical result of this measurement. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state $|\phi\rangle$ which Alice destroyed.

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The existence of long range correlations between Einstein-Podolsky-Rosen (EPR) [1] pairs of particles raises the question of their use for information transfer. Einstein himself used the word "telepathically" in this context [2]. It is known that *instantaneous* information transfer is definitely impossible [3]. Here, we show that EPR correlations can nevertheless assist in the "teleportation" of an intact quantum state from one place to another, by a sender who knows neither the state to be teleported nor the location of the intended receiver.

Suppose one observer, whom we shall call "Alice," has been given a quantum system such as a photon or spin-$\frac{1}{2}$ particle, prepared in a state $|\phi\rangle$ unknown to her, and she wishes to communicate to another observer, "Bob," sufficient information about the quantum system for him to make an accurate copy of it. Knowing the state vector $|\phi\rangle$ itself would be sufficient information, but in general there is no way to learn it. Only if Alice knows beforehand that $|\phi\rangle$ belongs to a given orthonormal set can she make a measurement whose result will allow her to make an accurate copy of $|\phi\rangle$. Conversely, if the possibilities for $|\phi\rangle$ include two or more nonorthogonal states, then no measurement will yield sufficient information to prepare a perfectly accurate copy.

A trivial way for Alice to provide Bob with all the information in $|\phi\rangle$ would be to send the particle itself. If she wants to avoid transferring the original particle, she can make it interact unitarily with another system, or "ancilla," initially in a known state $|a_0\rangle$, in such a way that after the interaction the original particle is left in a standard state $|\phi_0\rangle$ and the ancilla is in an unknown state $|a\rangle$ containing complete information about $|\phi\rangle$. If Alice now sends Bob the ancilla (perhaps technically easier than sending the original particle), Bob can reverse her actions to prepare a replica of her original state $|\phi\rangle$. This "spin-exchange measurement" [4] illustrates an essential feature of quantum information: it can be swapped from one system to another, but it cannot be duplicated or "cloned" [5]. In this regard it is quite unlike classical information, which can be duplicated at will. The most tangible manifestation of the nonclassicality of quantum information is the violation of Bell's inequalities [6] observed [7] in experiments on EPR states. Other manifestations include the possibility of quantum cryptography [8], quantum parallel computation [9], and the superiority of interactive measurements for extracting informa-

tion from a pair of identically prepared particles [10].

The spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission. Below, we show how Alice can divide the full information encoded in $|\phi\rangle$ into two parts, one purely classical and the other purely nonclassical, and send them to Bob through two different channels. Having received these two transmissions, Bob can construct an accurate replica of $|\phi\rangle$. Of course Alice's original $|\phi\rangle$ is destroyed in the process, as it must be to obey the no-cloning theorem. We call the process we are about to describe teleportation, a term from science fiction meaning to make a person or object disappear while an exact replica appears somewhere else. It must be emphasized that our teleportation, unlike some science fiction versions, defies no physical laws. In particular, it cannot take place instantaneously or over a spacelike interval, because it requires, among other things, sending a classical message from Alice to Bob. The net result of teleportation is completely prosaic: the removal of $|\phi\rangle$ from Alice's hands and its appearance in Bob's hands a suitable time later. The only remarkable feature is that, in the interim, the information in $|\phi\rangle$ has been cleanly separated into classical and nonclassical parts. First we shall show how to teleport the quantum state $|\phi\rangle$ of a spin-$\frac{1}{2}$ particle. Later we discuss teleportation of more complicated states.

The nonclassical part is transmitted first. To do so, two spin-$\frac{1}{2}$ particles are prepared in an EPR singlet state

$$|\Psi_{23}^{(-)}\rangle = \sqrt{\tfrac{1}{2}}\,(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle). \tag{1}$$

The subscripts 2 and 3 label the particles in this EPR pair. Alice's original particle, whose unknown state $|\phi\rangle$ she seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g., one or more may be photons, the polarization degree of freedom having the same algebra as a spin.

One EPR particle (particle 2) is given to Alice, while the other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair at this stage contains no information about $|\phi\rangle$. Indeed the entire system, comprising Alice's unknown particle 1 and the EPR pair, is in a pure product state, $|\phi_1\rangle\,|\Psi_{23}^{(-)}\rangle$, involving neither classical correlation nor quantum entanglement between the unknown particle and the EPR pair. Therefore no measurement on either member of the EPR pair, or both together, can yield any information about $|\phi\rangle$. An entanglement between these two subsystems is brought about in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle 1 and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of $|\Psi_{12}^{(-)}\rangle$ and

$$\begin{aligned}|\Psi_{12}^{(+)}\rangle &= \sqrt{\tfrac{1}{2}}\,(|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow_2\rangle),\\ |\Phi_{12}^{(\pm)}\rangle &= \sqrt{\tfrac{1}{2}}\,(|\uparrow_1\rangle|\uparrow_2\rangle \pm |\downarrow_1\rangle|\downarrow_2\rangle).\end{aligned} \tag{2}$$

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first particle as

$$|\phi_1\rangle = a\,|\uparrow_1\rangle + b\,|\downarrow_1\rangle, \tag{3}$$

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$\begin{aligned}|\Psi_{123}\rangle = {}&\frac{a}{\sqrt{2}}\,(|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle)\\ &+\frac{b}{\sqrt{2}}\,(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle).\end{aligned} \tag{4}$$

In this equation, each direct product $|\ _1\rangle|\ _2\rangle$ can be expressed in terms of the Bell operator basis vectors $|\Phi_{12}^{(\pm)}\rangle$ and $|\Psi_{12}^{(\pm)}\rangle$, and we obtain

$$|\Psi_{123}\rangle = \tfrac{1}{2}\,[|\Psi_{12}^{(-)}\rangle\,(-a|\uparrow_3\rangle - b|\downarrow_3\rangle) + |\Psi_{12}^{(+)}\rangle\,(-a|\uparrow_3\rangle + b|\downarrow_3\rangle) + |\Phi_{12}^{(-)}\rangle\,(a|\downarrow_3\rangle + b|\uparrow_3\rangle) + |\Phi_{12}^{(+)}\rangle\,(a|\downarrow_3\rangle - b|\uparrow_3\rangle)]. \tag{5}$$

It follows that, regardless of the unknown state $|\phi_1\rangle$, the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, respectively,

$$\begin{aligned}-|\phi_3\rangle \equiv -\begin{pmatrix} a \\ b \end{pmatrix},\quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}|\phi_3\rangle,\\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle,\quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle.\end{aligned} \tag{6}$$

Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state $|\phi\rangle$ which Alice sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do nothing further to produce a replica of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (6), corresponding, respectively, to 180° rotations around the z, x, and y axes, in order to convert his EPR particle into a replica of Alice's original state $|\phi\rangle$. (If $|\phi\rangle$ represents a photon polarization state, a suitable combination of half-

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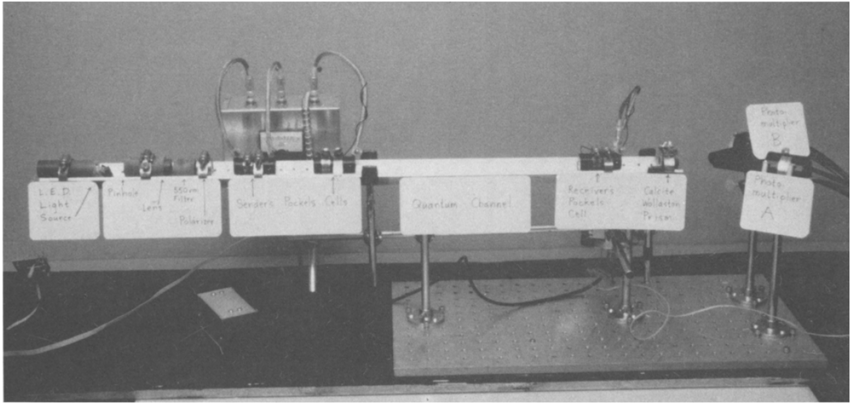
First experimental implementation (BB84 protocol)



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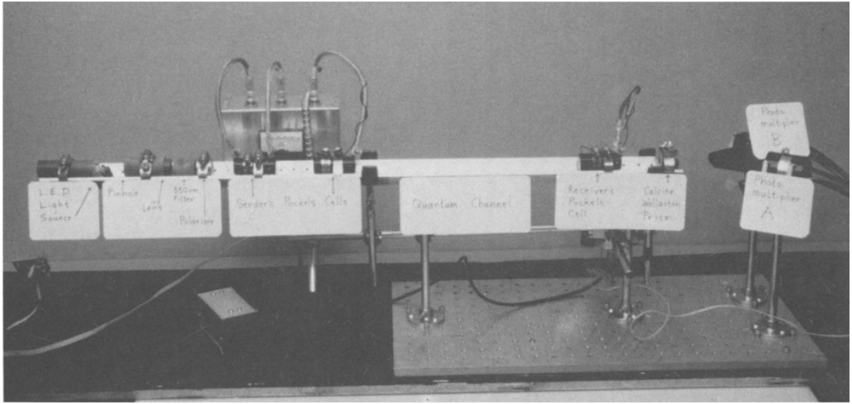

715,000 pulses $\rightarrow$ 2000 basis matches $\rightarrow$ 754 bit of shared key with eavesdropper having $< 10^{-6}$ bits of information

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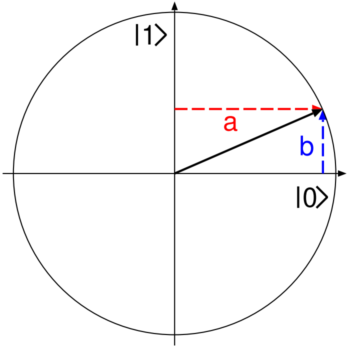

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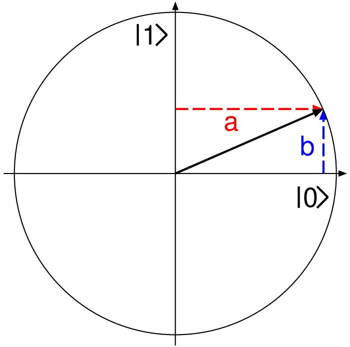

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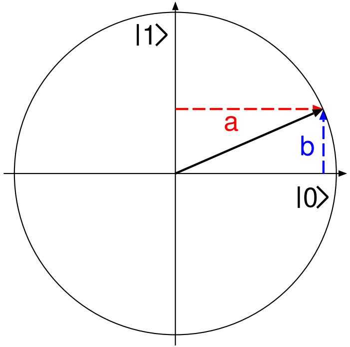

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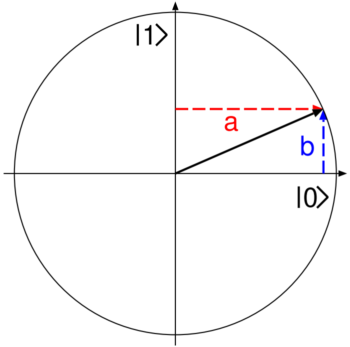

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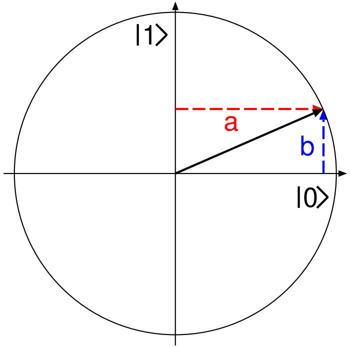

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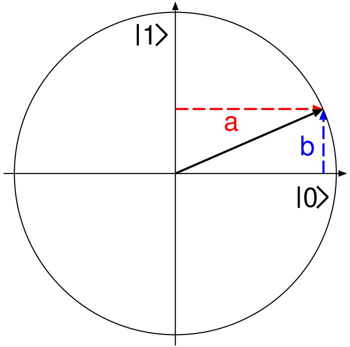

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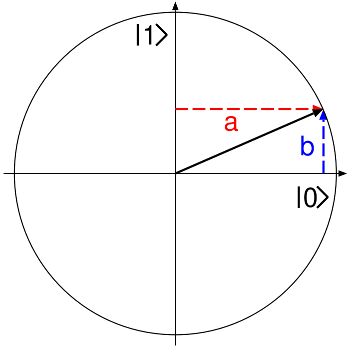

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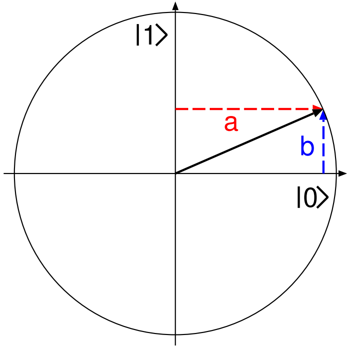

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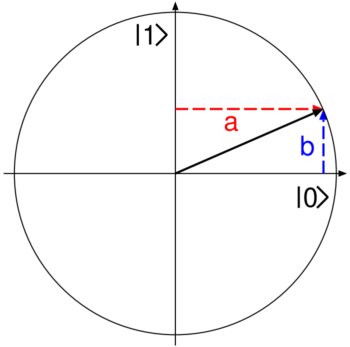

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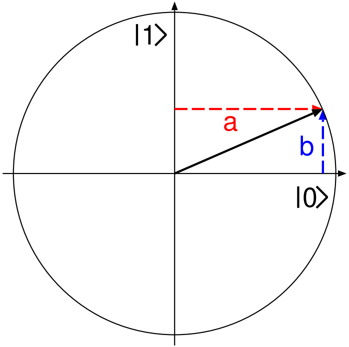

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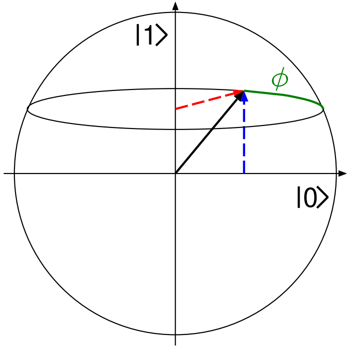

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the $\{|+\rangle, |-\rangle\}$ basis is also called the Hadamard basis and is sometimes represented by $\{|\nwarrow\rangle, |\nearrow\rangle\}$

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we define four additional special orthogonal single qubit states

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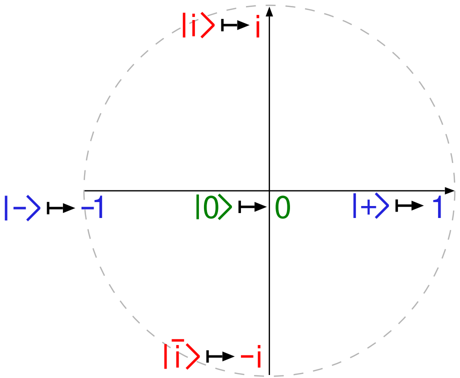
the problem with $|1\rangle$ can be solved by extending the complex plane

Extended complex plane

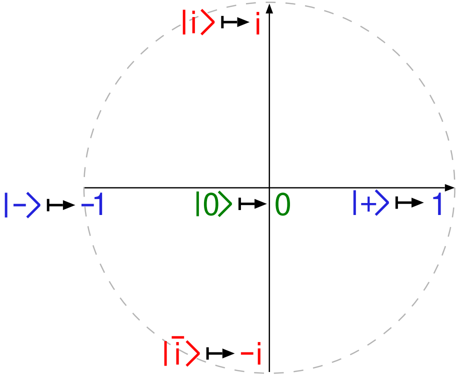
The qubit basis vectors which are properly mapped can be drawn on a unit circle in the complex plane



Extended complex plane

The qubit basis vectors which are properly mapped can be drawn on a unit circle in the complex plane

by adding an extra point called ∞ and defining the mapping: $|1\rangle \mapsto \infty$



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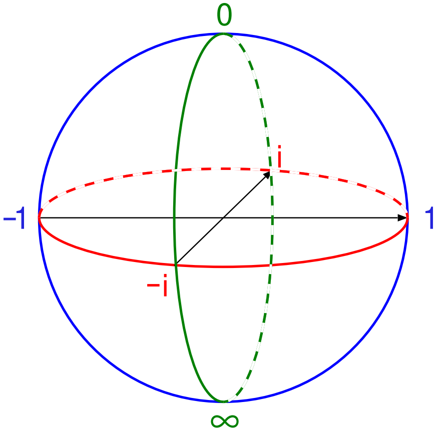
since each of the qubit basis vectors are normalized they have a magnitude of 1 the extended complex plane can be mapped to a sphere of radius 1



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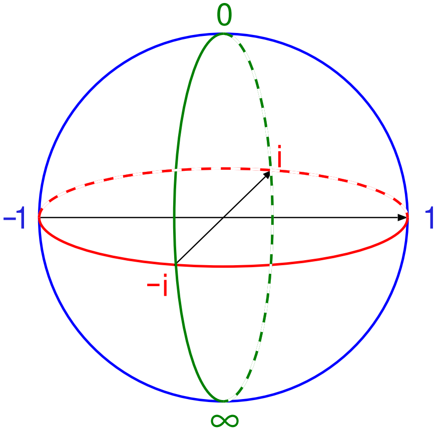

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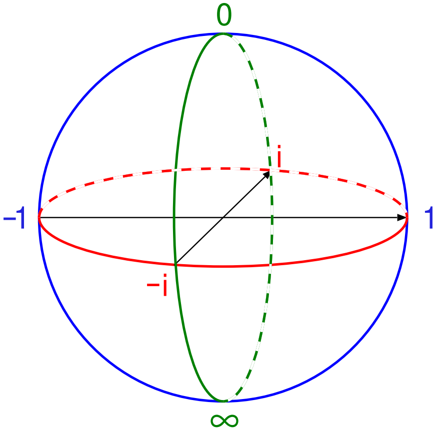

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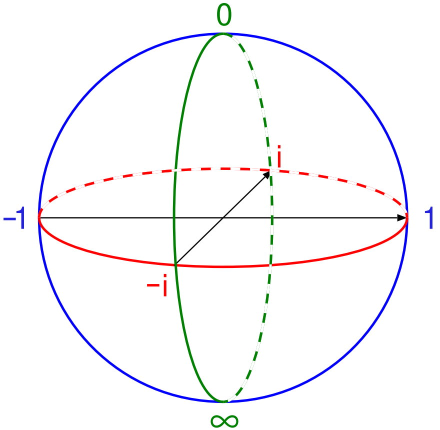

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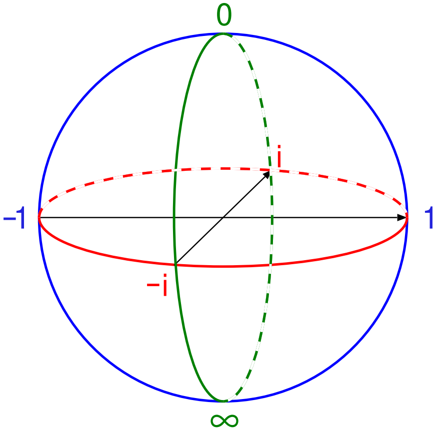
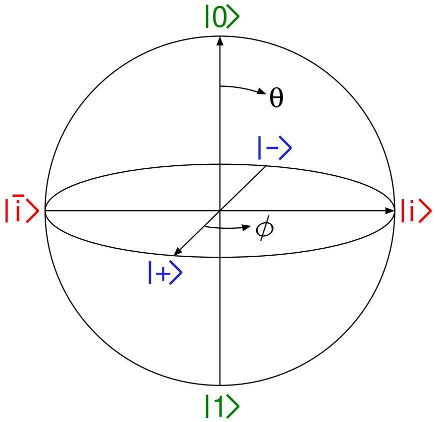


this maps an arbitrary single qubit state to a point on the surface of the Bloch sphere

Spherical coordinates & the Bloch sphere

Given the spherical representation of a general qubit, the three basis sets can easily be mapped onto the surface of the Bloch sphere

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right) e^{i\phi}|1\rangle$$



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Given the spherical representation of a general qubit, the three basis sets can easily be mapped onto the surface of the Bloch sphere

$|0\rangle = 1|0\rangle + 0|1\rangle \quad \longmapsto$

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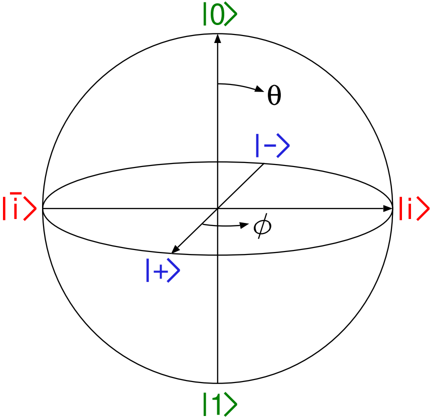

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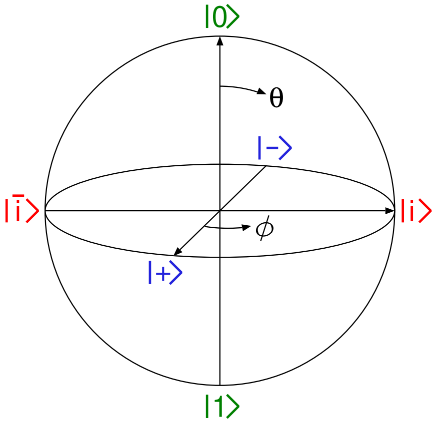

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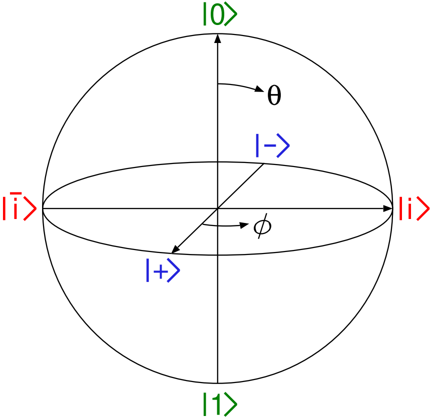

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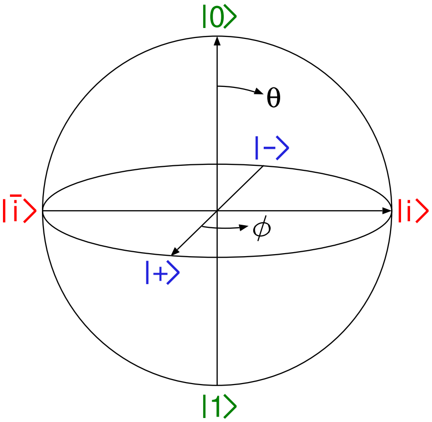
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle$$



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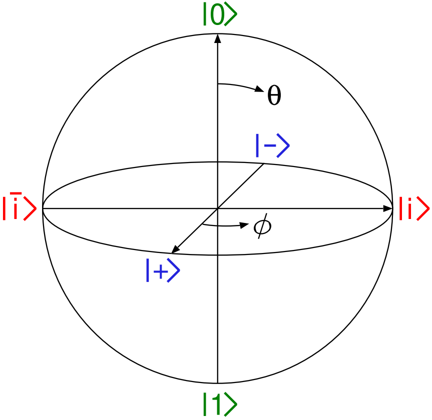

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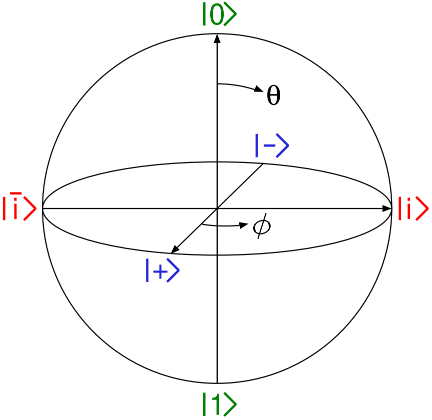

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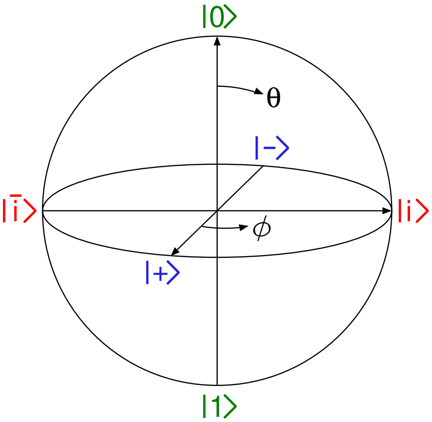

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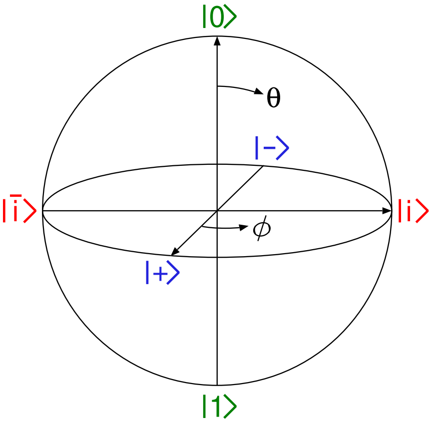

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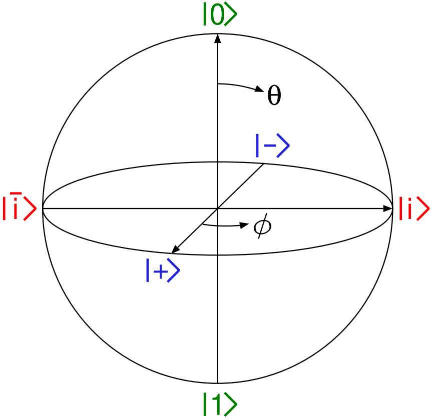

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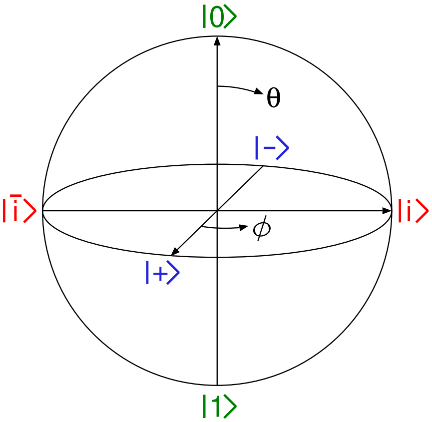

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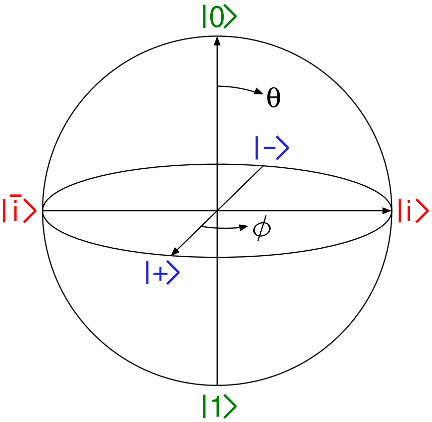

Spherical coordinates & the Bloch sphere

Given the spherical representation of a general qubit, the three basis sets can easily be mapped onto the surface of the Bloch sphere

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle$$

$$|0\rangle = 1|0\rangle + 0|1\rangle \quad \longmapsto \quad \theta = 0, \phi = 0$$

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$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad \longmapsto \quad \theta = \frac{\pi}{2}, \phi = 0$$

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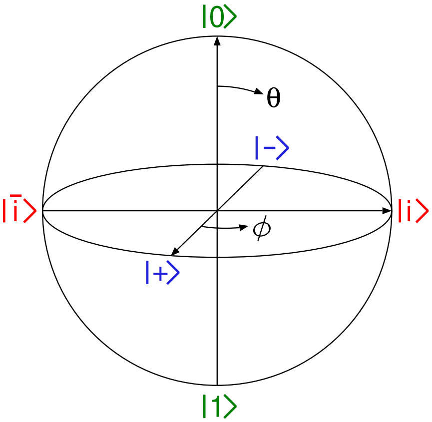

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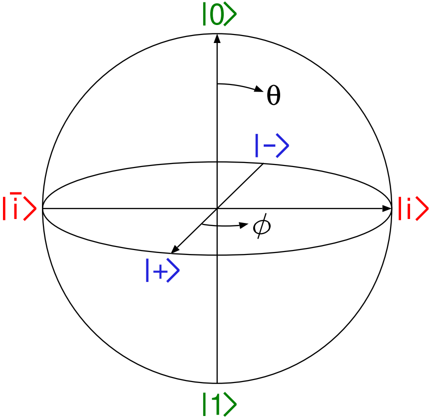
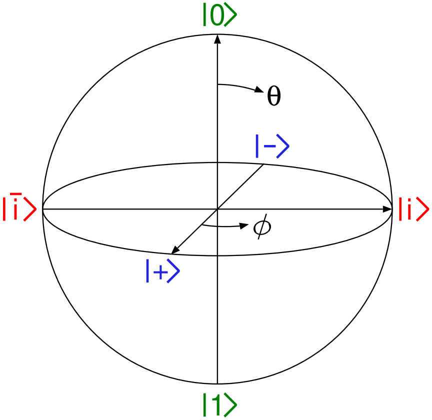


the points in the interior of the Bloch sphere have meaning for quantum information processing

Stereographic projection & the Bloch sphere

An alternative model is that of the stereographic projection which posits that $\alpha = s + it$ is complex



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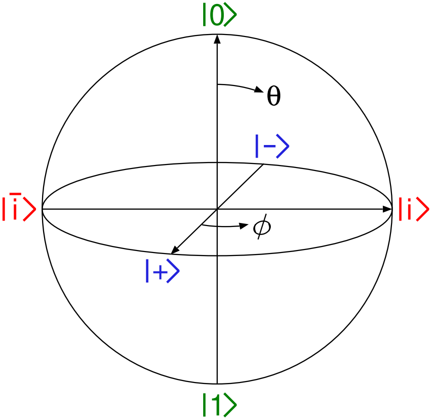

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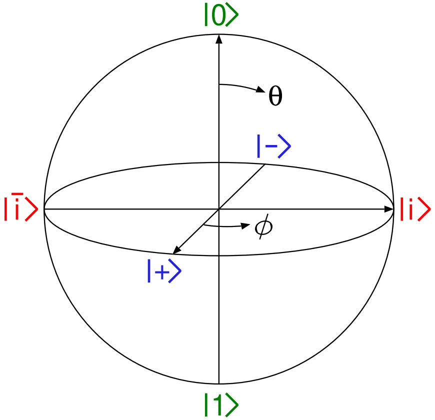

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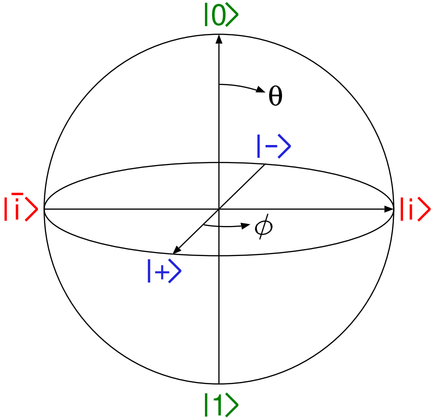

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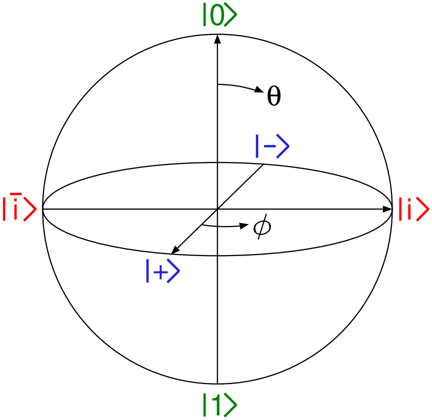

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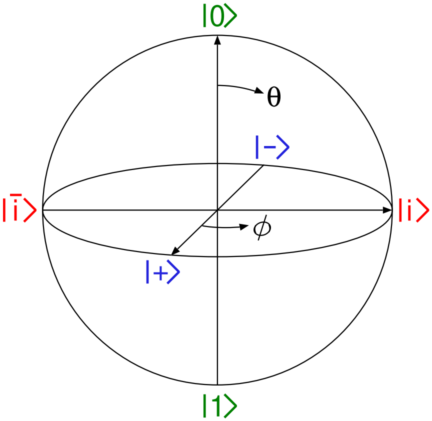

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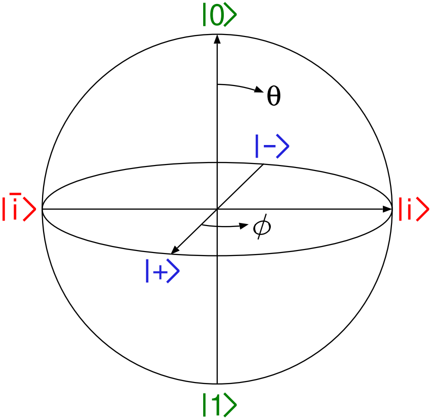

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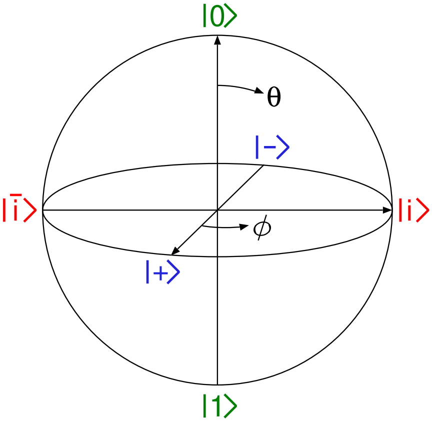

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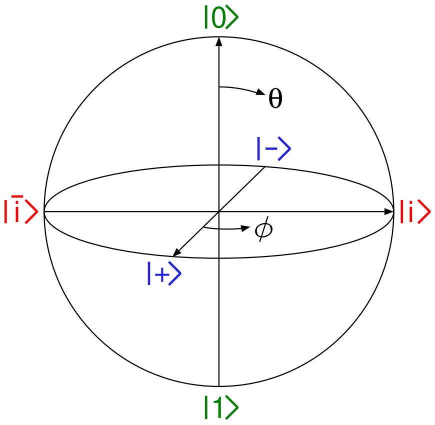

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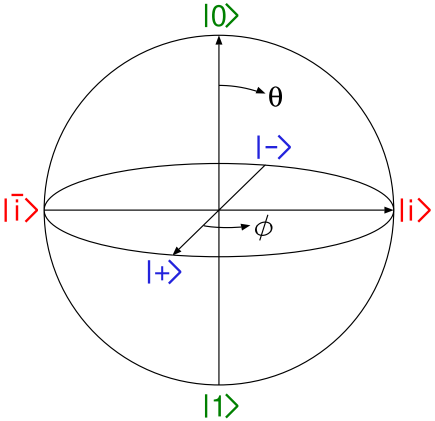

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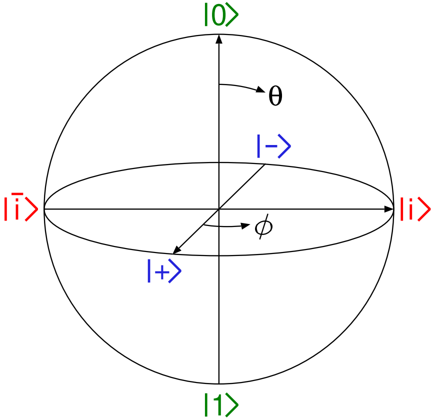

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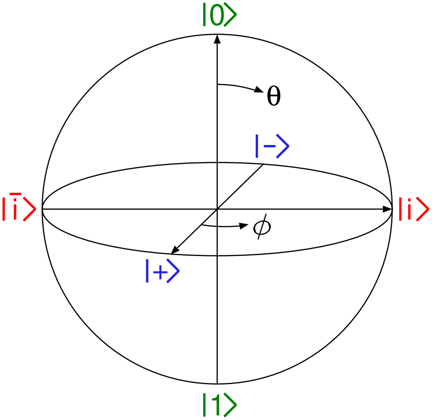

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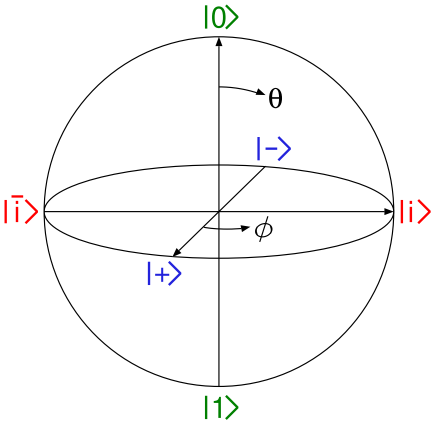

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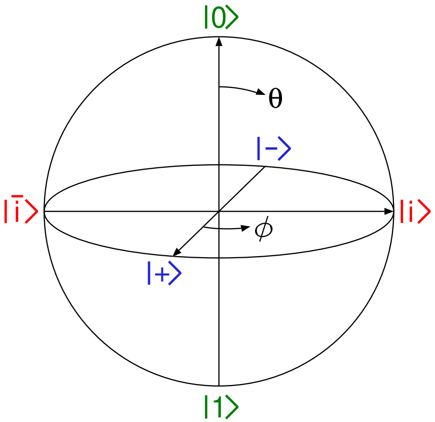

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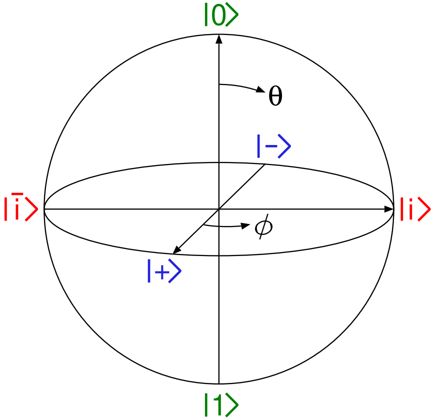

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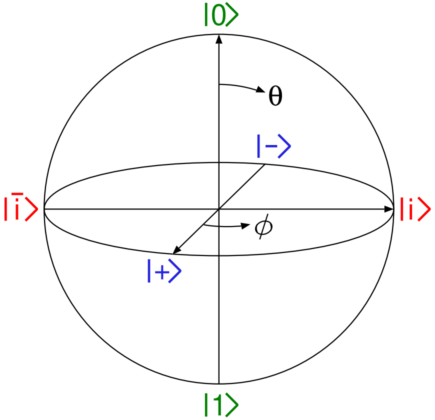

Direct sum of vector spaces

Consider two classical state spaces, V and W with bases

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thus, for a system of n two-state objects, the dimension of the state space of the system is $2n$, linear with the number of objects

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for $k = \min(n, m)$, all elements of $V \otimes W$ have the form

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|v\rangle \otimes (|w_1\rangle + |w_2\rangle) &= |v\rangle \otimes |w_1\rangle + |v\rangle \otimes |w_2\rangle \\
(a|v\rangle) \otimes |w\rangle &= |v\rangle \otimes (a|w\rangle) = a\,(|v\rangle \otimes |w\rangle)
\end{aligned}$$

for $k = \min(n, m)$, all elements of $V \otimes W$ have the form

$$|v_1\rangle \otimes |w_1\rangle + |v_2\rangle \otimes |w_2\rangle + \cdots + |v_k\rangle \otimes |w_k\rangle$$

Tensor product of vector spaces

Quantum systems, such as qubits combine as tensor products so for V and W with bases

$$A = \{|\alpha_1\rangle, |\alpha_2\rangle, \ldots, |\alpha_n\rangle\}, \qquad B = \{|\beta_1\rangle, |\beta_2\rangle, \ldots, |\beta_m\rangle\}$$

the tensor product $V \otimes W$ is an $n \times m$-dimensional space consisting of elements $|\alpha_i\rangle \otimes |\beta_j\rangle$

operations on such a vector space are now:

$$\begin{aligned}
(|v_1\rangle + |v_2\rangle) \otimes |w\rangle &= |v_1\rangle \otimes |w\rangle + |v_2\rangle \otimes |w\rangle \\
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$$|v_1\rangle \otimes |w_1\rangle + |v_2\rangle \otimes |w_2\rangle + \cdots + |v_k\rangle \otimes |w_k\rangle$$

the $\otimes$ symbol will often be dropped with the understanding that the tensor product is always implied: $|v\rangle \otimes |w\rangle \to |v\rangle|w\rangle$

More about tensor products

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the tensor product of two unit vectors is also a unit vector, and given orthonormal bases $\{|\alpha_i\rangle\}$ and $\{|\beta_j\rangle\}$ for V and W, the basis $\{|\alpha_i\rangle\} \otimes \{|\beta_j\rangle\}$ for $V \otimes W$ is also orthonormal

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for entangled states, it is meaningless to discuss the state of a single qubit that is part of the system